Inversion of Stokes V Profiles:

The structure of solar magnetic fluxtubes and its dependence on the filling factor

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Summary: We present results from an inversion procedure that derives the temperature stratification, the turbulent velocity, and the magnetic field strength of the photospheric layers of magnetic fluxtubes from observed Stokes V spectra near disk center. In a first step the inversion has been applied to 10 Fe I and Fe II Stokes V profiles of a plage and a network region to obtain reliable models of the fluxtubes. In a second step the dependence of the fluxtube structure on the filling factor has been studied with spectra of 3 Fe I lines from 23 different regions based on the models derived in the first step. We find a temperature excess at equal geometrical height in the upper photosphere with respect to the quiet photosphere and a deficit at the level of continuum formation. There the magnetic field strength is on the order of 2000–3000 G and turbulent velocities are considerably larger in the fluxtubes than in the quiet photosphere. Fluxtubes are found to become cooler and their field strengths, at a given *optical* depth, to become larger as the filling factor increases. The field strength stratification along the *geometrical* height axis, on the other hand, is very similar for all the investigated regions. Some evidence is also found for a slight decrease of the non-stationary velocity amplitude with increasing filling factor.

1. Introduction

Most empirical fluxtube structures have been obtained by fitting synthetic Stokes I spectra from simple fluxtube models to observed facular spectra or by fitting the observed center-to-limb variations (CLV) of the continuum contrast of faculae (see the review by Solanki, 1990). However, these attempts have failed in finding a unique model that explains the observations and that is not heavily influenced by assumptions regarding the structure of the non-magnetic atmosphere between the fluxtubes (e.g. Walton, 1987). The few models derived from Stokes V profiles (Stenflo, 1975; Solanki, 1984, 1986) have the substantial advantage that the analysis can be performed independently of the spatial resolution. However, these efforts have been limited to just a few solar regions due to their reliance in the 1980s on FTS observations of a suitably large sample of carefully selected spectral lines. Therefore, comparatively little is known about the dependence of fluxtube structures on the filling factor α, which is defined as the fraction of the observed solar surface covered by strong magnetic fields. For practical reasons this type of investigation is best carried out with a small number of spectral lines. The dependence of some fluxtube properties on α has previously been studied by Stenflo and Harvey (1985). In Sect. 2 we present the inversion procedure, which is then applied to 10 lines of a plage and a network region near disk center (Sect. 3, see Keller et al., 1990, for more details) and in Sect. 4 to 23 spectra of different solar regions near disk center using only 3 lines (see Zayer et al., 1990, for more details).

2. Inversion procedure

The inversion of Stokes V profiles is based on the determination of a few model fluxtube parameters by a least-squares fitting algorithm. The fluxtube model used is the thin tube approximation (e.g. Roberts and Webb, 1978). The use of more elaborate models does not introduce any qualitative difference in the resulting fluxtube structures when using spectra from disk center (Keller et al., 1990). The free parameters of the models are the magnetic field strength at optical depth unity inside the fluxtube $B(\tau_i)$ (subscript i denotes the atmosphere within the fluxtubes), the temperature as a function of the geometrical height, and the macroturbulent velocity as a function of the line strength and the excitation potential. The microturbulent velocity is kept at a height independent value of 0.6 km s^{-1}. The inversion is started with prescribed initial values for the model parameters from which the corresponding fluxtube model is calculated. Then the synthetic Stokes V profiles resulting from the fluxtube model are compared with the observations. Based on this comparison improved model parameters are derived. We have tested the uniqueness of the derived fluxtube structures by starting the inversion with widely different initial values for the free model parameters. We always obtained the same final fluxtube structures within the accuracy of the program, which strongly supports the uniqueness of our solutions. Our experience with codes of different levels of sophistication lead to a rough estimate of the errors of the final code. They result in an uncertainty of 100 G in the magnetic field strength, of 200 K in the temperature stratification, and of 0.3 km s^{-1} in the macroturbulent velocity.

3. Fluxtube structure

We have applied the inversion procedure to the Fourier Transform Spectrometer observations of Stenflo et al. (1984) The selected data consist of the Stokes V profiles of 8 Fe I and 2 Fe II lines in the range from 5000 to 5500 Å observe in a plage and a network region near disk center. The true continuum contrast on the axis of fluxtubes is no well determined by observations, but has been chosen to be 1.3 for the plage and 1.4 for the network region. Th temperature stratification is parameterized at five grid points along the geometrical height scale.

The speed of convergence and the uniqueness of the solution of the iterative fit algorithm are determined b the way in which the comparison between observed and synthetic Stokes V profiles is performed. In the ideal case set of 'orthogonal' observables is extracted from the observed profiles such that each of the corresponding syntheti observables depends on only one fluxtube model parameter. By orthogonal Stokes V parameters we mean tha no two Stokes V parameters depend on one and the same fluxtube model parameter. Due to the extremely nor linear response of spectral line profiles to changes in the model parameters, it is not possible to find an ideal set observables. However, it is possible to find certain observables that are much closer to being 'orthogonal' to eac other than just the Stokes V profile values at various wavelengths. We anti-symmetrize all observed Stokes V profil around their zero-crossing, thus avoiding complicated models which can explain the observed Stokes V asymmet (Solanki, 1989).

The first observables that have been selected are the logarithmic ratios of the areas of the Stokes V wings the Fe I lines to the Stokes V wing areas of the Fe II 5197.6 Å line. Since strong Fe I lines are formed higher in t photosphere than weak Fe I lines, the temperature structure can be determined over different height ranges, by usi Fe I to Fe II ratios with Fe I lines of different strength. In the present work we form the ratios of the Stokes V areas Fe I 5048.4 Å, Fe I 5083.3 Å, Fe I 5127.7 Å, Fe I 5247.1 Å, Fe I 5250.6 Å, Fe I 5294.0 Å, Fe I 5383.4 Å, and Fe II 5414.1 to that of Fe II 5197.6 Å. Another observable is the well-known magnetic line ratio, i.e. the ratio of the Fe I 5247.1 Å the Fe I 5250.2 Å Stokes V amplitudes divided by the ratio of the corresponding Landé factors (Stenflo, 1973). Becau these two lines have nearly the same atomic parameters except for the Landé factor, their ratio is insensitive to model fluxtube parameters except for the magnetic field strength at their height of formation, the macroturbule velocity, and the angle of inclination. Stenflo et al. (1987) used the Fe I 5247.1 Å and Fe I 5250.6 Å lines to fo a thermal line ratio. These two lines have similar and not too large Landé factors (2.0 and 1.5, respectively) a similar line strengths in the quiet sun, so that they are affected by the magnetic field in much the same way. T difference in the excitation potential of their lower levels (0.09 eV as compared with 2.20 eV) gives rise to t substantial dependence of this line ratio on the thermodynamic properties of magnetic elements. To determine t macroturbulent velocity in fluxtubes we use the full width at half maximum (FWHM) of the Stokes V wings. T temperature in the higher photospheric layers of fluxtubes cannot be deduced from Fe I to Fe II ratios, because the observables are insensitive to temperature for strong Fe I lines formed in those higher layers. Since Stokes I li widths of strong lines are sensitive to the temperature we expect the wavelength separation of the Stokes V extre of strong lines to depend on the temperature as well (recall that $V \sim \partial I/\partial\lambda$).

In Fig. 1 the temperature (left panel) and magnetic field stratifications (right panel) of the plage (short dash

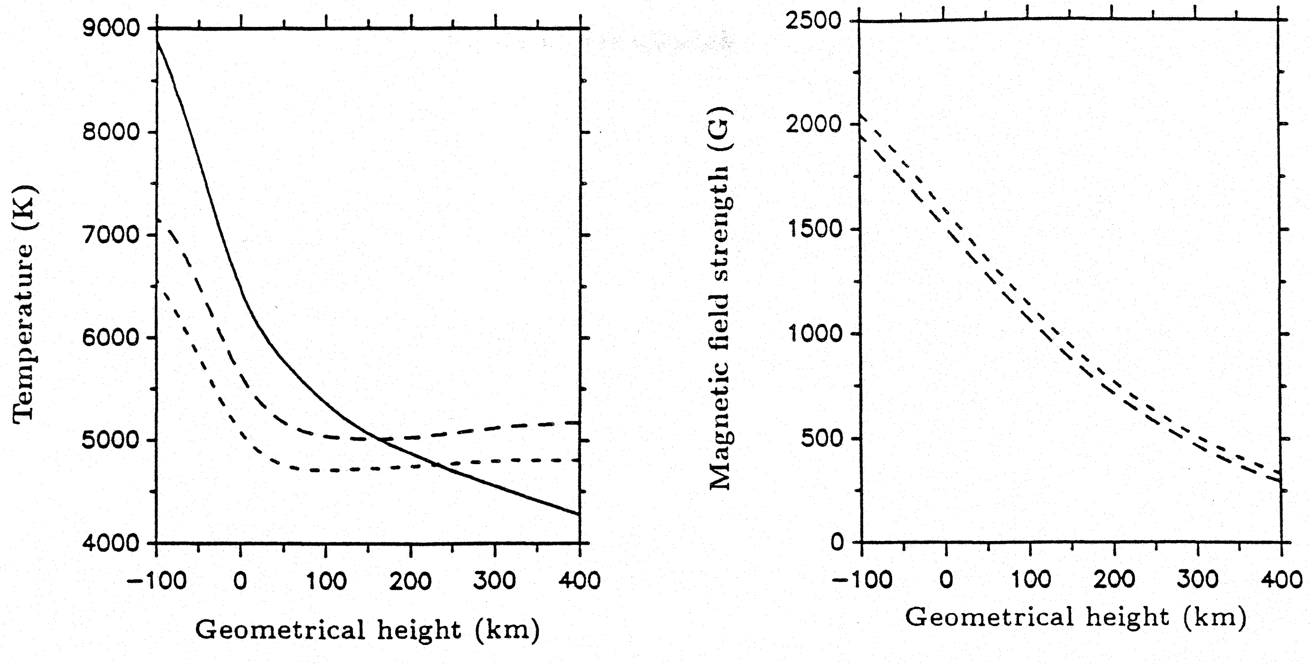

Fig. 1 Fluxtube models for the plage (short dashes) and the network region (long dashes) compared with the quiet photosphere (solid line) at equal geometrical height. The left panel shows the temperature, the right panel the magnetic field strength.

and the network (long dashes) regions versus the geometrical height are compared with the quiet photosphere (solid line). The behavior of $T(z)$ or $T(\tau_i)$ near $\tau_i = 1$ ($z = -165\,\mathrm{km}$) within the fluxtube should not be taken too literally, since there is only a single observational data point below $\log \tau_i = -1$ ($z = -65\,\mathrm{km}$) to constrain the temperature stratification, namely the continuum contrast. Bearing this and other constraints in mind, we feel that the present fluxtube models are reliable only within the range $-3 \lesssim \log \tau_i \lesssim -1$ ($-65\,\mathrm{km} \lesssim z \lesssim 200\,\mathrm{km}$). The macroturbulent velocities of weak lines are comparable with the values measured from Stokes I profiles in the quiet photosphere whereas for strong lines the velocities exceed the values found in the quiet photosphere by roughly $2\,\mathrm{km\,s^{-1}}$. It is rather difficult to estimate the influence of possible errors and idealizations on the derived fluxtube structures. The small set of grid points and lines, the assumption of horizontally homogenous temperature structures, LTE, and the continuum contrast values are probably the most severe limitations. We feel that the combined microturbulence and macroturbulence approach, the neglect of the angle between the magnetic field and the vertical, and the anti-symmetrization of the Stokes V profiles play only a minor role.

4. Dependence on the filling factor

To investigate the dependence of the fluxtube structures on the filling factor we use data from three different sources, namely the two FTS spectra used in Sect. 3, which contain the three chosen Fe I lines at 5247.1 Å, 5250.2 Å, and 5250.6 Å, the data obtained by Stenflo and Harvey (1985) with the vertical grating spectrometer of the McMath telescope, and data obtained with the Horizontal Telescope of the Arosa Astrophysical Observatory (HAT). The latter consist of simultaneous recordings of the Stokes I and V profiles of the three selected lines. In total 23 spectra obtained from regions of varying filling factors at $\mu \geq 0.9$ are analysed in detail.

We assume a geometrical height independent temperature difference, ΔT, with respect to the plage model from Sect. 3, which is used as a reference. Note that by choosing an appropriate ΔT we can approximately reproduce the network model from Sect. 3. Therefore the simple temperature parameterization introduced above can be regarded as a rough interpolation between the temperature structures of magnetic elements in regions of quite different filling factors. Since Fe I 5247.1 Å and Fe I 5250.2 Å are sufficiently similar only two free parameters for the macroturbulen velocity are required, ξ_{mac_1} for Fe I 5247.1 Å and Fe I 5250.2 Å, and ξ_{mac_2} for Fe I 5250.6 Å. The observables extracte from the three lines are the magnetic and the thermal line ratio as well as the FWHM of the Stokes V wings o the three lines. Note that the use of only one line ratio as a proper temperature diagnostic does not allow reliabl information on further temperature parameters (e.g. gradients) to be obtained in a straightforward manner.

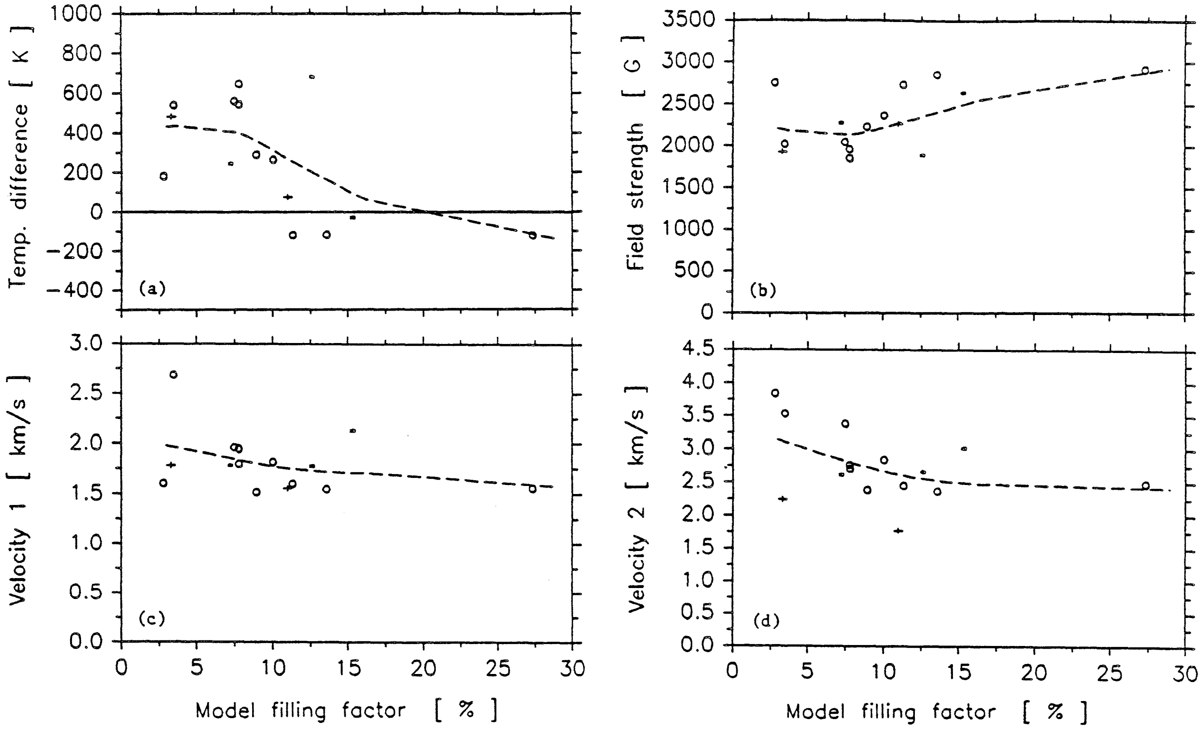

Fig. 2 Results of the Stokes V inversion using three lines only. **(a)** Temperature difference with respect the plage model of Sect. 3, **(b)** field strength at continuum optical depth unity, **(c)** macroturbulence velocity Fe I 52547.1 Å and Fe I 5250.2 Å, **(d)** macroturbulence velocity of Fe I 5250.6 Å. All quantities are plotted as a functi of the filling factor as determined from the comparison of the observed and the synthetic spectra (stars: HAT; circl McMath vertical grating spectrograph; crosses: FTS). The dashed curves are smoothed spline fits to the data.

The resulting values of the four free model parameters are shown in Fig. 2 as functions of the filling factor, whi has been obtained from the ratio between the synthetic and the observed Stokes V amplitudes. Only those regio are represented which yield sufficiently good fits. For the other spectra (generally with high noise levels) we do n consider the derived atmospheric parameters to be reliable. Note the substantial decrease in temperature and t simultaneous increase in the magnetic field strength with increasing filling factor. The small ΔT of the FTS pla data suggests that the present inversion procedure gives results consistent with those of the more elaborate one Sect. 3 and increases our confidence in the simpler version of the inversion used here. The deduced macroturbulen velocities decrease slightly with increasing filling factor, although a constant ξ_{mac_2} cannot be completely ruled ou

It is instructive to consider the behaviour of the temperature and the magnetic field of the best fit models alo the continuum optical depth axis, τ_{5000}, and also along the geometrical height axis. The temperature $T(\tau_i)$ a the field strength $B(\tau_i)$ are plotted in Fig. 3a and b, respectively, while $T(z)$ and $B(z)$ are plotted in Fig. 3c and respectively. The $T(z)$ curves simply reflect the temperature parameterization. The difference between the $T(z)$ a the $T(\tau_i)$ curves is a measure of how strongly the continuum opacity is affected by temperature. The curves of t magnetic field strength $B(z)$ all lie very close together. The relatively small scatter of the $B(z)$ curves derived for t various regions is of the order of magnitude expected from the noise in the data and uncertainties in the inversi

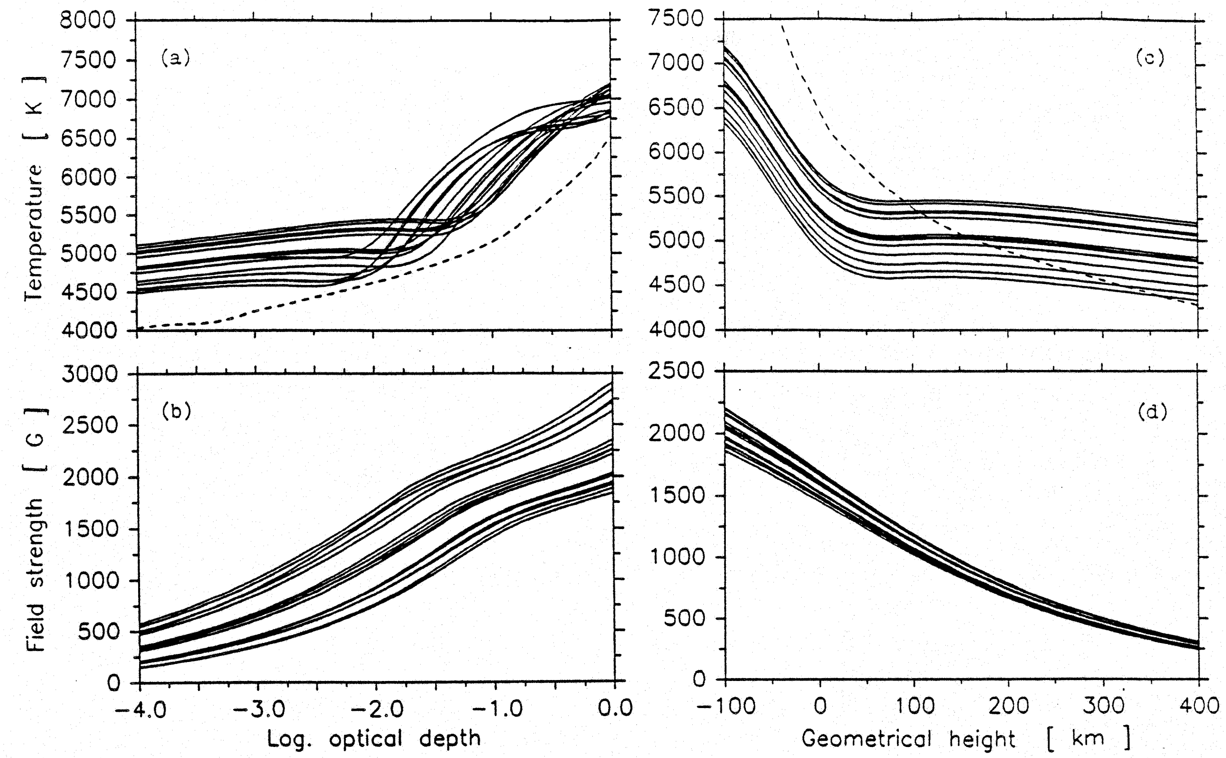

Fig. 3 Temperature (a) and field strength (b) as functions of $\log(\tau_{5000})$, and temperature (c) and field strength (d) as functions of geometrical height, z, resulting from the inversion of Stokes V data (solid curves). The dashed curves in a and c represent the temperature stratification of the quiet sun.

procedure. It therefore appears that magnetic elements have a unique or an almost unique field strength at a given height in the atmosphere. This is due to the strong evacuation of the fluxtubes in all the observed regions, so that the derived $B(z)$ curves all resemble the asymptotic case of a completely evacuated fluxtube for which $B(z)$ is exclusively determined by the pressure stratification of the surrounding non-magnetic atmosphere. The similarity between the various $B(z)$ curves is in striking contrast to the large variation of $B(\tau_i)$ in Fig. 3b. The latter is thus clearly induced by changes of the optical depth scale and the heights of formation of the spectral lines due to temperature variations alone.

Theoretical model calculations by Knölker and Schüssler (1988) suggest that there are two possible explanations for the dependence of the temperature within fluxtubes on the amount of magnetic flux. Their models show that fluxtubes grow cooler with increasing size. A decrease in temperature with α may, therefore, be due to a greater average size of fluxtubes in regions with more magnetic flux. They also argue that fluxtubes could be cooler in regions of larger magnetic flux due to the denser packing, even if their size remains unaltered. Unfortunately, the present data cannot easily distinguish between the two proposed mechanisms. $B(\tau_i = 1)$ varies approximately from 2000 to 3000 G as the filling factor increases from 3% to 30%. However the field strength as a function of geometrical height hardly varies at all with α. Since the temperature stratification changes with the filling factor, the variation of $B(\tau_i)$ with α is a simple consequence of the α independent $B(z)$ structure and the varying temperature stratification.

The amplitude of the non-stationary velocity within magnetic elements (simulated by macroturbulence) is found to decrease somewhat with increasing filling factor. Thus it seems that the excitation mechanism for disturbances is reduced in regions with larger α. One of the main proposed mechanisms for the generation of longitudinal waves is the compression of fluxtubes by granules (Musielak et al., 1989). Observations indeed suggest that granular velocities are reduced in active regions (Title et al., 1989), so that the efficiency of this mechanism is reduced as α increases.

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