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# HOW ACCURATE ARE STELLAR MAGNETIC MEASUREMENTS? II. ANALYSIS OF DISK-INTEGRATED FLUX TUBE MODELS

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ABSTRACT We analyze disk-integrated line profiles generated in atmospheres embedded with fluxtubes to test the accuracy of simple radiative transfer (RT) analyses in determining stellar magnetic parameters.

## INTRODUCTION, MODELS AND ANALYSIS

A height independent, spatially uniform, two-component model of the photosphere is typically assumed in stellar magnetic analyses. This ignores, however, the intermittent flux tube topology and the decrease of the field strength $B$ with height. We test these assumptions by applying the simple analyses to realistic, disk-integrated theoretical line profiles, thereby extending our previous study of disk-center lines (Saar & Solanki 1992; 1993, hereafter SS93) and the work of Basri et al. (1990). As spots contribute little to the magnetic signal, even in the IR (SS93), we focus on bright plage/network regions for G2 ($T_{\rm eff}$ = 5750 K) and K4 ($T_{\rm eff}$ = 4500 K) dwarfs with magnetic region temperatures of $\Delta T = T_{\rm m} - T_{\rm eff}$ = 0 (the "N" models) and +250 K ("H" models).

The magnetic structure we adopt for our model consists of an array of vertical thin flux tubes (Roberts & Webb 1978), which fan out with height under pressure equilibrium ($p_e(z) = p_i(z) + B(z)^2/8\pi$) until they merge to form a uniform vertical field. We chose $\beta_0 = 8\pi p_i(0)/B^2(0) = 0.3$, (thereby prescribing the relative shift $\Delta z$ of the $\tau_c$ = 1-levels), $R_0$ = 100 km and three tube filling factors: $f$ = 0.0 (non-magnetic), 0.1 and 0.3. We compute line profiles along a bundle of 19 rays piercing at an angle $\theta$ to the vertical in a stellar model (Kurucz 1992; priv. comm.) embedded with an array of thin flux tubes (see Bünte et al. 1993 for details). The results of the RT calculations are then averaged (1.5-D radiative transfer). This procedure is repeated at 5 values of $\mu = \cos\theta$ and the resulting Stokes $I(\mu)$ are disk-integrated with 60 azimuthal divisions to give the final theoretical line profile. Throughout, we have used a microturbulence of $\xi = 1$ km s$^{-1}$ and a radial-tangential macroturbulence of $v_{\rm mac} = 2$ km s$^{-1}$. Three low/high Landé $g$ line pairs of similar $\lambda$, strength, and excitation potentials ($\chi_e$) were computed: 6240/6173Å, 6842/6843Å, and 15652/15648Å.

The analysis technique is an improved version of Saar's (1988), similar to the analysis in SS93. We solve the magnetic RT equations in a Milne-Eddington atmosphere, including Faraday rotation, and disk-integrate (as above) to produce the flux $F$. We assume the star is uniformly covered by a filling factor $f$ of magnetic regions emitting a flux $F_m$. Thus, model lines are fit with $F = (1-f)F_q + fF_m(B)$, where $F_q$ is the quiet region flux and $F_q = F_m(B=0)$. We used a conjugate gradient algorithm to search for the $\chi^2$ space minimum.

We use the "non-magnetic comparison star" approach to fix several free parameters. In this method, fits to lines in a presumed low $fB$ star of identical spectral type (here, the $f = 0$ models) are used to estimate some non-magnetic parameters (e.g., $\xi$, damping parameter $\alpha$, and $\beta = dS_\nu/d\tau$). We next separately fit the low $g_{\rm eff}$ line (to estimate line strength $\eta_0$, $v \sin i$, $v_{\rm mac}$) and high $g_{\rm eff}$ line (to estimate $\eta_0$, $f$, and $B$) in succession. A further iteration refines these, and a final fit is then made to both lines simultaneously.

## RESULTS AND DISCUSSION

A sample of the preliminary results (the iteration scheme can still be improved) is given in Table I (subscript F denotes the fit). In the table, we give $\mu = 1$

TABLE I Model parameters and sample results for G2 and K4 stars

| model | $\lambda$ | $B$ | $f$ | $B$ | $f$ | $\Phi$ | $B_F$ | $f_F$ | $\Phi_F$ | $\Delta\Phi_F$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | [$\tau_{\rm ext}$ = 1] | | [$\tau_{\rm int}$ = 0.01] | | | | | | |
| atms.[1] | [nm] | [G] | [%] | [G] | [%] | [G] | [G] | [%] | [G] | [%] |
| G2H0 | 620 | 1610 | 29 | 1182 | 38 | 462 | 1521 | 29.3 | 445 | −4 |
| G2H0 | 680 | 1599 | 29 | 1167 | 38 | 462 | 1791 | 19.5 | 349 | −24 |
| G2H0 | 1560 | 1610 | 29 | 1182 | 38 | 462 | 1711 | 20.3 | 349 | −24 |
| G2N0 | 620 | 1609 | 10 | 1193 | 13 | 160 | 1477 | 14.4 | 213 | 33 |
| G2N5 | | | | | | | 1777 | 6.2 | 110 | −31 |
| G2N0 | 680 | 1600 | 10 | 1178 | 14 | 160 | 2097 | 5.6 | 118 | −26 |
| G2N5 | | | | | | | .... | ... | ... | ... |
| G2N0 | 1560 | 1609 | 10 | 1193 | 13 | 160 | 1691 | 8.6 | 146 | −9 |
| G2N5 | | | | | | | 1799 | 9.1 | 163 | 2 |
| G2N0 | 620 | 1609 | 29 | 1193 | 38 | 462 | 1516 | 31.9 | 483 | 5 |
| G2N5 | | | | | | | 1323 | 31.7 | 420 | −9 |
| G2N0 | 680 | 1600 | 29 | 1178 | 38 | 462 | 1712 | 23.9 | 409 | −11 |
| G2N5 | | | | | | | 1123 | 33.5 | 377 | −18 |
| G2N0 | 1560 | 1609 | 29 | 1193 | 38 | 462 | 1682 | 23.6 | 396 | −14 |
| G2N5 | | | | | | | 1765 | 23.9 | 422 | −9 |
| K4H0 | 620 | 2021 | 29 | 1339 | 41 | 581 | 1903 | 21.0 | 399 | −31 |
| K4H0 | 680 | 1999 | 29 | 1319 | 42 | 580 | 2072 | 35.1 | 727 | 25 |
| K4H0 | 1560 | 2021 | 29 | 1339 | 41 | 581 | 2498 | 21.3 | 532 | −8 |
| K4N0 | 620 | 2022 | 29 | 1501 | 38 | 581 | 1827 | 23.7 | 433 | −25 |
| K4N0 | 680 | 2000 | 29 | 1481 | 38 | 580 | 2212 | 27.8 | 615 | 6 |
| K4N0 | 1560 | 2022 | 29 | 1501 | 38 | 581 | 2530 | 21.3 | 538 | −7 |

[1] e.g., G2H5 = G2 quiet atm., Hot ($\Delta T$ = 250K) mag. atm., $v \sin i$ = 5 km/s.

values of $B$ and $f$ at continuum $\tau_{\rm ext} = 1.0$ and $\tau_{\rm int} = 0.01$ (roughly the region of line formation). Note though, that the level of line formation varies as a complex function of not only $\eta_0$, $\lambda$, $\chi_e$ and $T_{\rm eff}$, but also $\mu$, $B$, and $f$. Thus, definitive statements about errors can only be made concerning the magnetic flux $\Phi = fB$ in the models. Grossmann-Doerth & Solanki (1990) give an indication of the possible errors in $f$ and $B$ due to wrong estimates of formation heights.

We find that the simple analysis determines the flux ($\Phi = fB$) to reasonable accuracy (±30%) in G2 stars, and is even better (± 10–20%) when the results of several line pairs with differing $T$ sensitivities and formation levels are averaged. In K4 stars, results are more strongly dependent on the lines used, especially for models with "hot" magnetic regions (where errors can reach ± 50% at $f = 0.1$), but again, line averaging can reduce errors to ±30%. Stronger, more saturated lines generally yield poorer results. $B$ tends to be overestimated and $f$ and $v_{\rm mac}$ underestimated on average. The dominance of the continuum enhancement relative to line weakening on cooler stars, which was found in SS93 (cf. Solanki 1992) is confirmed. This is due to the "hot" wall effect, where the magnetic continuum is enhanced by seeing the warmer layers through the partly evacuated fluxtube walls. Trends in the fitting errors for disk-integrated lines are sometimes *opposite* of those found for disk-center profiles, partly due to (1) the increased importance of the absorption in the external, non-magnetic atmosphere, (2) the increased continuum emission due to the hot tube walls, and (3) increased contribution of low $B$, higher altitude fields near the limb. Thus the integration over the disk introduces new effects which affect the line profiles and results derived from their inversion in subtle but significant ways. Low $f$ and high $v \sin i$ degrade $\Phi$ determinations, but the errors also depend on the lines involved and $B$ itself. We conclude that, under ideal conditions (no noise, no blends, symmetric profiles, perfect knowledge of the non-magnetic line profile) and with proper care, simple RT analysis of stellar magnetic fields can produce reasonably accurate results in a wide variety of realistic physical conditions.

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## REFERENCES

Basri, G.S., Marcy, G.W., Valenti, J., 1990, ApJ, **360**, 650

Bünte, M., Solanki, S.K., Steiner, O.: 1993, A&A, **268**, 736

Grossmann-Doerth, U., Solanki, S.K., 1990, A&A, **238**, 279

Roberts, B., Webb A.R., 1978, Solar Phys., **56**, 5

Saar, S.H., 1988, ApJ, **324**, 441.

Saar, S.H., Solanki, S.K. 1993, A&A, submitted (SS93)

Saar, S.H., Solanki, S.K. 1992, in *Cool Stars, Stellar Systems and the Sun*, eds. M. Giampapa, J. Bookbinder, 259

Solanki, S.K. 1992, *ibid.*, 211