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Noise Reduction in Astronomical Spectra: A New Wavelet-Based Method

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Abstract:

We present a wavelet technique to de-noise astronomical spectra based on non-orthogonal wavelet packets and compare it to Fourier-smoothing and other wavelet-based de-noising algorithms. It is found to give better results than any other tested de-noising algorithm and is particularly successful in recovering weak signals that are practically drowned by the noise.

1. Introduction

The wavelet transform decomposes a spectrum into waves of finite length. Compared to the Fourier transform the main advantage of a wavelet transform lies in the additional spatial resolution of the transformed spectrum. The wavelet transform represents the frequency content of the spectrum at finer and coarser resolutions (Mallat 1989). We present a new approach to the problem of noise reduction using non-orthogonal wavelet packets.

2. Technique

The wavelet packets analysis corresponds to a recursive application of the wavelet transform (Chui 1992; Wickerhauser 1991). For the base functions we make use of a set of non-orthogonal quadratic spline wavelets proposed by Mallat & Zhong (1992) which turn out to give excellent results for the cases considered here. Of importance is mainly the non-orthogonality of the wavelets.

Fig. 1 shows a two-level decomposition of a Stokes V (i.e. net circular polarization) spectrum with a noise level of $\sigma_{\rm noise} = 0.0056$. It clearly reveals some of the uncorrupted signal at scales 13 to 15 which is hidden by the noise at scale 1. Hence truncating scales 11 to 15, instead of scale 1, can recover features of the uncorrupted spectrum which are lost if scale 1 is truncated.

3. Results

We test the quality of our technique by applying it to originally low-noise solar Stokes I and V spectra which have been artificially contaminated by noise with a standard deviation of $\sigma_{\rm c} = 0.0014$ in the continuum of the Stokes V spectrum and $\sigma_{\rm c} = 0.014$ for the Stokes I spectrum.

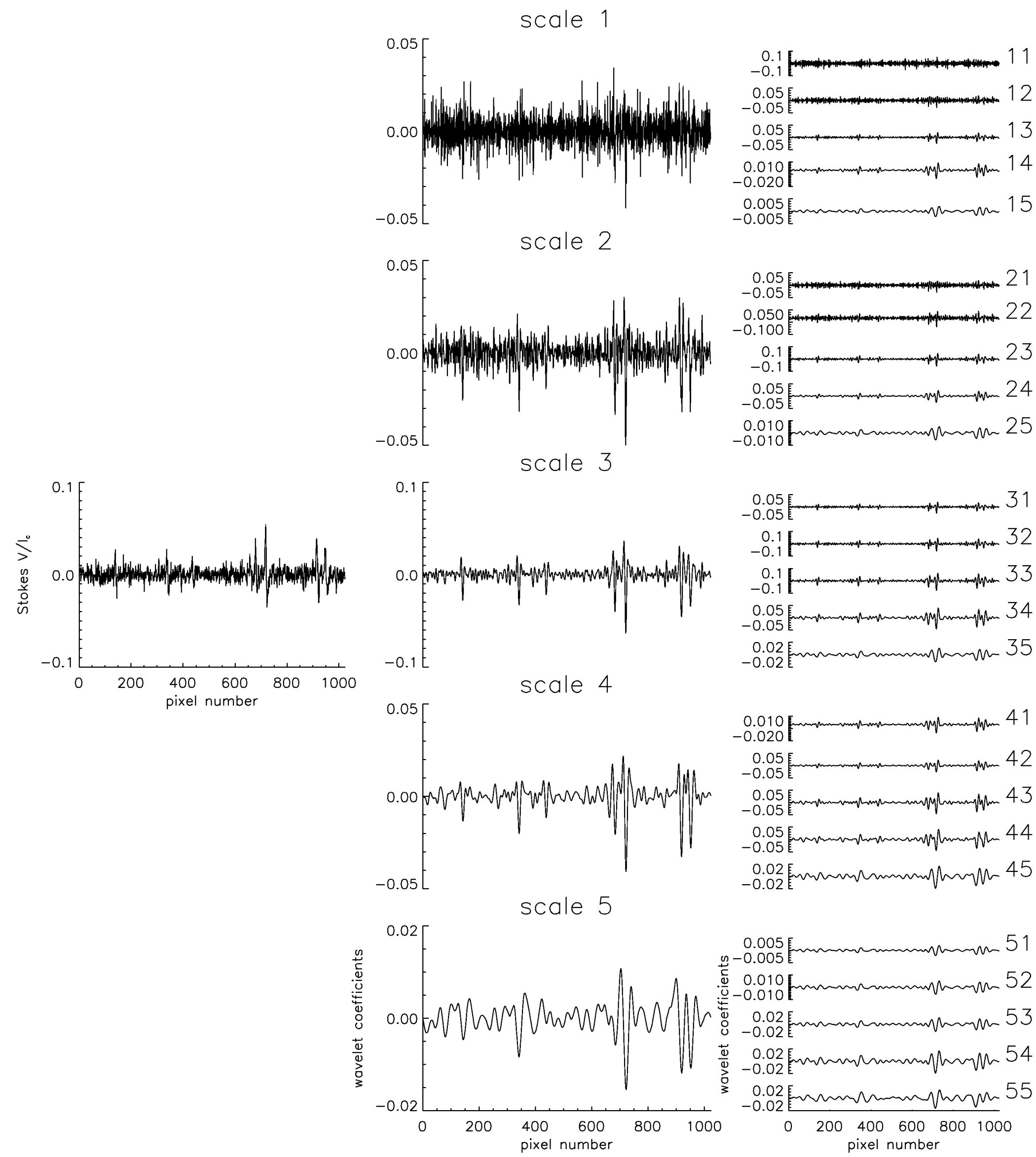

Figure 1. Two-level decomposition of a Stokes V spectrum. At the left the original signal, net circular polarization versus wavelength is plotted. At centre we show the non-orthogonal wavelet transform of this signal. Plotted are the wavelet coefficients as a function of wavelength. Scale 1 (top) shows coefficients of the highest frequency. The frequency decreases from top to bottom. At the right are plotted the wavelet transforms of each scale in central column. Thus the signals marked 11 to 15 represents the coefficients of the wavelet transform of scale 1, with 11 representing the highest frequencies and 15 the lowest.

While the Fourier smoothed spectrum shows the typical random oscillation within the low frequency parts of the spectrum, the wavelet packets are able to clean the continuum, so that a number of small features become recognizable (see Fig. 2).

4. Comparison

We compared the results of our technique to other wavelet-based de-noising methods, e.g., the ones by Starck & Bijaoui (1994; hierarchical thresholding), Bury et al. (1996) and Donoho & Johnstone (1994) and found it to be superior (see Table1).

The goodness of a de-noised spectrum is expressed by the variance σ^2 (Starck & Bijaoui 1994),

$$\sigma^2 = \frac{1}{N-1}\sum_{j=1}^{N}(x_j - \overline{x_j})^2, \tag{1}$$

where $\{x_j\}$, $j = 1...N$, represents the uncorrupted signal and $\{\overline{x_j}\}$, $j = 1...N$ stands for the de-noised version of the signal. N is the number of spectral points. Therefore the standard deviation of the remaining noise of a de-noised version of a spectrum is simply the square root of the variance.

Table 1. Numerical evaluation of a de-noised Stokes I spectrum (upper half of table) and Stokes V (lower half).

Stokes I	Fourier $\sigma^2[\cdot 10^{-5}]$	Hierarchical thresholding $\sigma^2[\cdot 10^{-5}]$	wavelet-packets 3-level decomposition $\sigma^2[\cdot 10^{-5}]$
Whole spectrum	7.4	4.9	4.4
Strong lines	11.4	9.1	7.0
Weak lines	12.2	8.9	6.9
"Continuum"	5.2	2.3	2.6
Stokes V	$\sigma^2[\cdot 10^{-7}]$	$\sigma^2[\cdot 10^{-7}]$	$\sigma^2[\cdot 10^{-7}]$
Whole spectrum	9.4	7.1	6.5
Strong lines	19.2	20.0	17.2
Weak lines	7.0	8.7	8.3
"Continuum"	6.4	1.8	2.0

5. Discussion

Of the tested methods our non-orthogonal wavelet packets technique attains the best results. The noise level of the continuum regions is reduced by a factor of 1.5 – 1.8 compared to the values obtained by Fourier smoothing. Averaged over the whole spectral range, this results in a gain of about 20% – 30% compared to the classical Fourier smoothing.

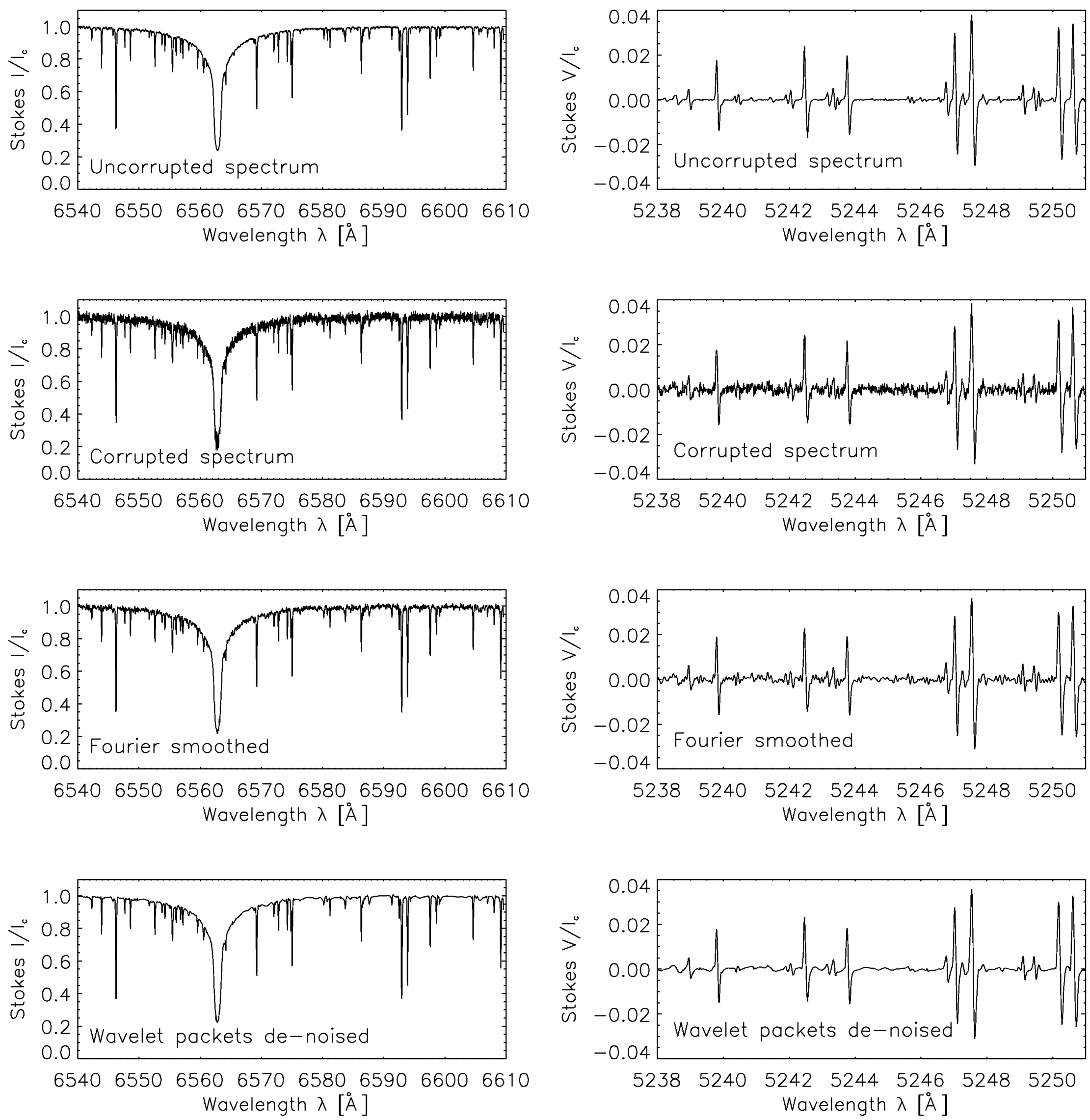

Figure 2. Direct comparison of the two de-noising methods. The Fourier smoothed spectrum shows clearly the typical random oscillations within the low frequency parts. The wavelet-packets de-noised version, using a three-level decomposition, shows a clean continuum and even a number of small features of the original spectrum are at least qualitatively recovered

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