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Research Note

Magnetic field strength in solar flux tubes: A model atmosphere independent determination

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Summary. The "line ratio method" (Stenflo, 1973) has been extensively used in the past to carry out measurements of the magnetic field strength in spatially unresolved magnetic flux concentrations. We present here a new variant of this technique, which is particularly simple as it does not depend on any radiative transfer calculations and thus the assumption of a model atmosphere is not required. General properties of the transfer equation lead us to a relationship between the circular polarization generated by two lines which are identical except for their Landé factors. This can be used to directly determine the field strength from the measured line profiles.

In order to test the method we have applied it to experimental data. A comparison with the traditional line ratio method is shown.

Key words: the Sun: faculae – magnetic fields

1. Introduction

In 1973, Stenflo proposed the "line ratio method" as a way of determining the magnetic field strength, B, of unresolved features in the solar atmosphere. It is based on the ratio between the circularly polarized profiles (Stokes V) of two spectral lines formed in the same atmospheric layers. This ratio has two important properties. On the one hand it does not depend on the surface fraction covered by the magnetic regions (filling factor), since the Stokes V signals of both lines come only from these zones. On the other hand, it is affected by the magnetic field strength due to the so-called Zeeman saturation effect (Stenflo, 1973). Through the use of two lines with identical thermodynamical properties, but different magnetic sensitivity, Stenflo made the procedure largely independent of the model atmosphere. He chose Fe I 5250 and Fe I 5247 as the best pair of lines with these characteristics. However, a value of the field strength can only be obtained from the line ratio if the transfer equation is explicitely solved.

We have derived an analytical expression relating the V profiles of two lines which are identical except for their Landé factors. With the help of this expression it is possible to determine the true field strength directly from the observed profiles, without having to carry out any radiative transfer model calculations. This makes the proposed method extremely efficient, allowing the field strength of a large number of regions on the Sun (or, alternatively, of one region as a function of time) to be rapidly measured. The availability of such a technique is particularly useful at the present moment, as we witness the advent of full-profile polarimeters coupled with two dimensional detectors. The method is applied to Fourier transform spectrometer (FTS) data (Stenflo et al., 1984), and the results are compared to those obtained with the classical line ratio technique.

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2. The method

It can be proved (Stenflo, 1971) that for a line with a triplet Zeeman pattern and under the assumption of a constant longitudinal magnetic field, $I+V$ and $I-V$ have the same transfer equation except for a constant shift in wavelength. I and V are the first and fourth Stokes parameters, intensity and circular polarization respectively. The solution of their transfer equations gives

$$I+V(\lambda-\lambda_0)=f(\lambda-\lambda_0+\Delta\lambda), \tag{1a}$$

$$I-V(\lambda-\lambda_0)=f(\lambda-\lambda_0-\Delta\lambda), \tag{1b}$$

where $\Delta\lambda=k\lambda_0^2 g B$, is the Zeeman shift with k a constant, λ_0 the wavelength of the transition giving rise to the line, g the Landé factor and B the field strength. The function $f(\lambda)$ is the profile of the Zeeman unsplit line which is related to the thermodynamical properties of the atmosphere, while all the information concerning the magnetic field is concentrated in $\Delta\lambda$.

These solutions can be used in a wide range of cases. They are based on the Unno equations (Unno, 1956), which do not include magneto-optical effects. However, the identities (1) remain valid even after taking these into account: the cross-talk terms which introduce the linear polarization into the circular vanish when the field is longitudinal (the dependence of magneto-optical terms on the inclination of the field can be found, for instance, in Wittmann, 1974).

Besides the assumptions mentioned above there are no conditions which a model atmosphere must fulfil. This means that a velocity field, which only changes the shape of the profiles, does not restrict the validity of the above noted solutions of the transfer equation.

The use of Eq. (1) allows us, as we will see below, to obtain a relation between the V profiles of two lines identical in their thermodynamical parameters, but with different g factors. This criterion is satisfied by Fe I 5250 and Fe I 5247 to a high degree.

As a first step, we can derive from (1) that

$$V(\lambda-\lambda_0)=1/2\,[f(\lambda-\lambda_0+\Delta\lambda)-f(\lambda-\lambda_0-\Delta\lambda)]\,. \quad (2)$$

The same expression is valid for both lines except for different $\Delta\lambda$ and λ_0. A Taylor expansion of f then leads to the alternative form (cf. Eq. (2.1) of Solanki and Stenflo, 1984)

$$V(\lambda-\lambda_0)=\sum_{n=0}^{\infty} D^{2n+1}\,f(\lambda-\lambda_0)\,\Delta\lambda^{2n+1}/(2n+1)!\,, \quad (3)$$

where $D^n f(\lambda)$ signifies the n-th derivative of $f(\lambda)$ according to λ. By rearranging terms in (3) one obtains the following equation for the first derivative of $f(\lambda-\lambda_0)$

$$\Delta\lambda\,Df(\lambda-\lambda_0)=V(\lambda-\lambda_0)-\sum_{n=1}^{\infty} D^{2n+1}\,f(\lambda-\lambda_0)\,\Delta\lambda^{2n+1}/(2n+1)! \quad (4)$$

Consecutive differentiations of (4) allow us to express all the derivatives of f as derivatives of V and higher order derivatives of f. This means that the V profile of one of the lines (V_1) can be expressed as a function of the V profile of the other (V_2) and its derivatives, by inserting (4) and its derivatives for line 2 into (3) for line 1:

$$\begin{aligned}V_1(\lambda)&=\Delta\lambda_1/\Delta\lambda_2\,[V_2(\lambda)+D^2\,V_2(\lambda)\,(\Delta\lambda_1^2-\Delta\lambda_2^2)/6\\&\quad+D^4\,V_2(\lambda)\,(\Delta\lambda_1^2-\Delta\lambda_2^2)\,(\Delta\lambda_1^2-7/3\,\Delta\lambda_2^2)/120+\ldots]\\&=a_0\,V_2(\lambda)+a_2\,D^2\,V_2(\lambda)+a_4\,D^4\,V_2(\lambda)+\ldots\,,\end{aligned} \quad (5)$$

where $\Delta\lambda_1$ and $\Delta\lambda_2$ are the Zeeman splittings of the two lines respectively. In Eq. (5) it has been implicitly assumed that both lines have the same centre wavelengths, and these are zero. This is why, as a first step, the lines must be centered on each other before applying (5).

If we take Fe I 5250 as line 1 and Fe I 5247 as line 2 ($\Delta\lambda_1/\Delta\lambda_2\simeq 3/2$) the a_4 term is Eq. (5) becomes very small. A rough estimate gives $D^4\,V_2/D^2\,V_2\simeq 1/(\Delta\lambda_2^D)^2$, where $\Delta\lambda_2^D$ is the Doppler width of the Stokes I profile of line 2. For Fe I 5247 in solar magnetic elements $\Delta\lambda_2<\Delta\lambda_2^D$, because the magnetic field is not expected to be terribly strong. Then we obtain an upper limit for the importance of the fourth order term if we take $D^4\,V_2/D^2\,V_2\simeq 1/(\Delta\lambda_2)^2$. Thus $a_4\,D^4\,V_2/a_2\,D^2\,V_2\simeq 0.4\%$, and only the first two terms in (5) need be retained.

From the last equation one immediately obtains an expression for the field strength:

$$B^2=6a_2\,g_2\,\lambda_2^2/[k^2\,g_1\,\lambda_1^2\,(\lambda_1^4\,g_1^2-\lambda_2^4\,g_2^2)]\,. \quad (6)$$

The value of the coefficient a_2 can be derived easily in a realistic case. With the help of the observed V_1 and V_2, and after the numerically calculated second derivative of V_2, a least-squares fit to Eq. (5) gives a_2. This procedure can also be used to estimate the accuracy in the derived field strength by means of the law of propagation of errors:

$$\sigma_B=B\sigma_{a2}/(2a_2)\,. \quad (7)$$

It must be noted that this estimate only takes into account one source of errors: the accuracy of a_2. Other errors, due to non-fulfilment of the required conditions for the method, have to be added.

3. A test for the method

In order to test the reliability of the method, we have used FTS data of a strong plage and an enhanced network region. The data have been described in detail by Stenflo et al. (1984), and Solanki (1987). Note that Stenflo et al. (1984) called the enhanced network region a weak plage. These spectra have a very high S/N ratio and are suitable for our purposes. They were obtained close to disk center, so that it is expected that the observed field is nearly longitudinal because the buoyancy keeps the fluxtubes perpendicular to the surface (see e.g. Spruit, 1981; Schüssler, 1986). So the requirements of the method are met except for the constancy of the field. According to Spruit (1981), in the case of a thin fluxtube in hydrostatic equilibrium (no magnetic tension), the magnetic field varies with a scale height ($H_B^{-1}=d\,(\ln B)/dz$) which is twice the pressure scale height: $H_B\simeq 300$ km. Magnetic tension tends to increase the scale height (e.g. Pneuman et al., 1986). Since the chosen lines, Fe I 5250 and Fe I 5247, are strongly weakened inside the magnetic elements due to the high temperature, their contribution functions have a rather small extent, over which the field strength does not change too strongly. The assumption of constant field strength has been made in all applications of the line ratio method in the past, except by Solanki et al. (1987). Since the field varying with height requires a numerical solution of the transfer equations along many lines of sight, it is impractical for application to more than a few line profiles. For the present we neglect the observed area asymmetry in Stokes V, which cannot be produced by velocity gradients for a constant longitudinal magnetic field.

First of all, we centered the two spectral lines on each other by calculating the center of gravity of their intensity profiles and then, we shifted one line until the wavelength difference between them became zero. Next, the second derivative of the V profile of Fe I 5247 was evaluated. It was done in the Fourier domain multiplying by $(2\pi\,\mathrm{i}s)^2$, where s is the frequency (Bracewell, 1965).

Figures 1 and 2 show the results of the technique applied to data from a plage and a network region. One of them presents a V signal $\simeq 4.5$ times stronger than the other, an effect mainly due to the difference in the filling factor because, as we will see below, B is more or less the same for both. We have plotted all the terms of Eq. (5): $_1$, $a_0\,V_2$, $a_2\,D^2\,V_2$ and $a_0\,V_2+a_2\,D^2\,V_2$. The good fit to V_1 by $a_0\,V_2+a_2\,D^2\,V_2$ proves that the $D^4\,V_2$ term in Eq. (5) is indeed negligible compared to the two lower order terms. Table 1 summarizes the numerical results of this test. We wish to remark that, within the error bars, the first coefficients of these fits are both 3/2. This is, as expected, the ratio between the g factors of the two lines (3 for Fe I 5250 and 2 for Fe I 5247). The table also shows the derived values of the field strength together with those obtained with the traditional line ratio method (see Solanki et al., 1987). The agreement between them is to be expected. Both variants of the basic technique are based on the same effect, namely Zeeman saturation and it is not surprising that a similar magnetic field is obtained when they are applied to the same data.

4. Conclusions

The technique which we have described here to measure the field strength of the unresolved magnetic component of a plage, although based on assumptions similar to those used in the classical line ratio method, presents some simplifications. First, a radiative transfer calculation is not required. Only general pro-

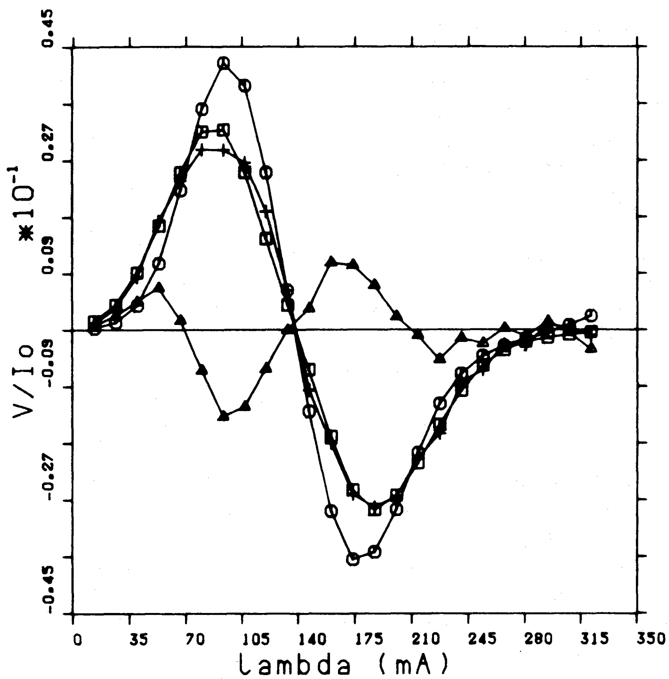

Fig. 1. Here we show the results of the fit of Eq. (5) using data of a strong active region plage: V_1 (□), $a_0 V_2$ (○), $a_2 D^2 V_2$ (△) and $a_0 V_2 + a_2 D^2 V_2$ (+). It is clearly visible how the a_2 term goes in the sense of correcting the differences between the a_0 term and V_1 (see text and Table 1)

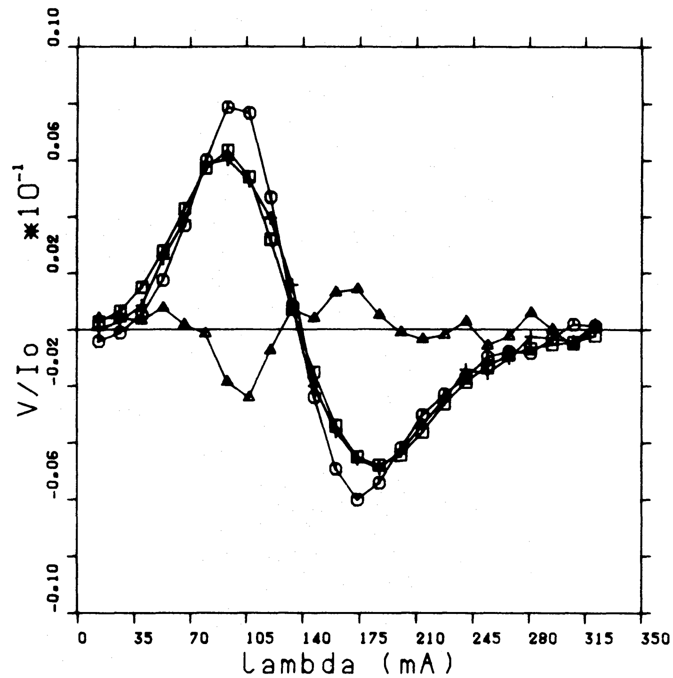

Fig. 2. Same as for Fig. 1 but for an enhanced network region

Table 1. Results of applying our variant of the line ratio method and the classical technique to FTS data. a_0 and a_2 (in units of pix^2 where 1 pix = 13.4 mÅ) are the coefficients of the fit (5) (see text). They are followed by estimates of their errors. B is the magnetic field (in Gauss) obtained by us. As a comparison the field strength derived with the line ratio method involving numerical radiative transfer is listed under B(LRM)

	a_0	a_2	B	B(LRM)
Enhanced network	1.445 ± 0.040	1.18 ± 0.16	$1010. \pm 70$	~ 1150
Strong plage	1.445 ± 0.040	1.71 ± 0.17	$1220. \pm 60$	$1150.-1200$

perties of the transfer equation are used to express, explicitly, the relationship existing between the V profiles of two lines which are identical except for their Landé factors. With this equation, a fit of the observed V profile of Fe I 5250 to the V of Fe I 5247 and its derivatives is enough to obtain the field strength. The above is also valid if a velocity field exists in the region where the lines are formed. As the method avoids numerical solution of the transfer equation, no model atmosphere has to be assumed. This distinguishes the present technique from the classical line ratio method. The use of both methods with the same data gives the same results to within 10%.

The variant proposed here also has an advantage over the one proposed by Stenflo et al. (1987), since the latter assumes the unsplit line profile shapes to be the same inside and outside the fluxtubes, while we make no such assumption. We conclude that the proposed technique is a simple, reliable and efficient means of determining the field strength in solar magnetic fluxtubes near disk center.

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