

# How magnetic is the solar chromosphere?

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**Abstract.** The lower solar chromosphere is thought to have a very inhomogeneous temperature structure, with hot magnetic flux tubes surrounded by cool ( $T < 4000$  K) non-magnetic gas (Ayres et al., 1986). The effect of such a thermally bifurcated atmosphere on the structure of the magnetic field in the chromosphere is considered. It is shown that magnetic flux tubes expand much more rapidly if the atmosphere is thermally bifurcated than if it is homogeneous. They merge and form a magnetic canopy with an almost horizontal base which does not exceed approximately 800–1000 km above  $\tau = 1$ , irrespective of the magnetic filling factor. Hence the middle and upper chromosphere is filled with a magnetic field almost everywhere on the Sun. The consequences of this result both for the Sun and for other late type stars are discussed.

**Key words:** the Sun: magnetic fields – the Sun: chromosphere – stars: magnetic fields – stars: chromospheres

## 1. Introduction

In many respects the magnetic field is to the solar surface what hedgerows are to the English countryside. It breaks the monotony and provides structure, as well as harbouring a whole zoo of interesting wildlife (overstable oscillations, a wide variety of MHD waves, spicules, H $\alpha$  fibrils, etc.). The nature of the magnetic field and its structure are well known in the solar photosphere (Spruit et al., 1989; Schüssler, 1989; Solanki, 1989b; Stenflo, 1989). There the field is concentrated mainly into flux tubes with kG field strengths (Stenflo, 1973; Frazier and Stenflo, 1972) which are distributed in a network along the edges of supergranular cells or are concentrated in active regions. Hence the total magnetic field covers only a small fraction of the solar surface, of the order of a percent, although the exact value remains uncertain. In the chromosphere, on the other hand, the situation is considerably less clear. Most theoreticians (Gabriel, 1976; Anzer and Galloway, 1983; Fiedler and Cally, 1990) expect the flux tubes to continue through the chromosphere, spreading continuously with increasing height until they cover almost the whole solar surface at the base of the corona, i.e. at a height of 1500–2500 km above the  $\tau_{5000} = 1$  level in the quiet photosphere (which serves as  $z = 0$ , i.e. as the origin of the height axis, for the rest of the present

paper). This picture seems to be supported by the indirect magnetic field indicators formed in the chromosphere (e.g. Spicules, H $\alpha$  fibrils, Ca II H2 and K2 intensity, etc.) which show the presence of considerable fine structure. In particular, the thin elongated H $\alpha$  fibrils suggest that the flux tubes continue basically unchanged into the chromosphere. However, the few magnetograms made in chromospheric lines (mainly in Ca II 8542 Å) appear to show the field spreading rapidly at a relatively low height (i.e. in the lower chromosphere or even the upper photosphere in some parts of active regions), thus forming a canopy overlying a mainly field free atmosphere (Giovannelli, 1980; Giovannelli and Jones, 1982; Jones and Giovannelli, 1983; Jones, 1985).

We propose that the key to the understanding of the magnetic structure of the chromosphere lies in the thermal inhomogeneity of the lower chromosphere deduced by Ayres and Testerman (1981) and Ayres et al. (1986) from spectra of the CO molecule and from their comparison with Ca II line spectra. According to these observations the lower chromosphere is divided into regions with a high temperature, higher than the temperature of “standard” average models (like the VAL models of Vernazza et al., 1981), most probably corresponding to magnetic features, and regions of low temperature (minimum temperatures below 4000 K), corresponding to the non-magnetic atmosphere (cf. Ayres et al., 1986; Ayres, 1989). This picture has been underpinned by numerous theoretical studies (e.g. Ayres, 1981; Kneer, 1983; Muchmore and Ulmschneider, 1985; Muchmore et al., 1988; Steffen and Muchmore, 1988; Anderson, 1989). The carbon monoxide molecule also plays a major role in the theoretical modelling, this time as the major cooling agent in those parts of the lower chromosphere where the temperature is sufficiently low for CO to form in significant quantity. Since CO cools the gas, which in turn causes more CO to form, an instability occurs. A rather cool end state is reached when all the atomic carbon is bound up in CO. Inhomogeneities in the chromospheric temperature cannot be explained by granulation according to Steffen and Muchmore (1988). In contrast, Massaglia et al. (1988) find that hot flux tubes can exist in a CO cooled atmosphere, even in the absence of non-radiative heating, due mainly to the lower gas density. The probable presence of large amplitude MHD waves with a sizeable energy flux in flux tubes (Solanki, 1989a; Solanki and Roberts, 1989) enhances the probability of flux tubes remaining hot in an otherwise cool chromosphere. In the following we adopt these findings, presuming the hot chromospheric component to be associated with magnetic fields and the cool component

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with the surrounding field free plasma. This picture is supported by the well established correlation between Ca II K2 brightness and the amount of magnetic flux in the form of magnetic elements (Skumanich et al., 1975; Schrijver et al., 1989).

Note that in the above and throughout the rest of this paper we refer to the chromosphere as the layer above the classical temperature minimum (as present in e.g. the VAL models, Vernazza et al., 1981) and below the transition zone, irrespective of its temperature. We also refer to the height of the temperature minimum in the sense of the VAL models (500–600 km above  $z=0$ ).

One aspect of CO cooling of the non-magnetic chromosphere which has so far not been considered is its influence on the structure of the magnetic field. Although Hasan and Kneer (1986) and Massaglia et al. (1988) have studied the influence of CO cooling on the thermal structure of flux tubes and their surroundings, they have assumed the field itself to be rigid and unaffected by the results. In the present paper we take the opposite approach. We use empirical or radiative equilibrium models for the two components of the chromosphere and consider how the thermal bifurcation affects the structure of the magnetic field, for example its expansion with height. Due to the uncertainties surrounding the mechanism(s) of chromospheric heating this approach is at present to be preferred over a completely self-consistent one.

## 2. Idea, method and models

### 2.1. Idea

Consider a magnetic flux tube in a unipolar magnetic region. In the photosphere, the concentrated field within the flux tube is confined mainly by a deficit in gas pressure within the flux tube, created by a convective instability (e.g. Parker, 1978; Webb and Roberts, 1978; Spruit and Zweibel, 1979; Nordlund, 1983; Venkatakrishnan, 1985; Hasan, 1985; cf. Schüssler, 1989, for a review). If the temperature is similar in the magnetic feature and in its surroundings, the pressure scale heights are also similar, so that the field strength decreases with a scale height corresponding to approximately half the pressure scale height. The area covered by a flux tube increases proportionally until it touches its neighbours and the fields merge. This height is generally referred to as the merging height, denoted here by  $h_m$ . The merging height increases steadily with decreasing filling factor at  $z=0$  (denoted by  $f$ ). Exact estimates for the average merging height on the Sun depend on the estimates of the global filling factor of the field near  $\tau_{5000}=1$ . For reasonable values of the global solar filling factor ( $0.002 \leq f \leq 0.02$ ) the merging height always lies in the upper chromosphere or in the transition region. The inclusion of magnetic tension does not change the above picture (Steiner et al., 1986; Steiner and Pizzo, 1989).

As pointed out by Pneuman et al. (1986) and Steiner and Pizzo (1989), if the temperature within the flux tube is much higher than in the surroundings, then the merging height can be lowered considerably. This is due mainly to the larger pressure scale height within the tube than in its surroundings. It causes the gas pressure within the tube to approach the gas pressure outside more rapidly with height than in the isothermal case, forcing the tube to expand anomalously. Finally, at some height the internal gas pressure becomes larger than the external pressure. Above this critical point, the gas pressure difference cannot confine even

the weakest magnetic field which expands to fill all of space above the critical height, irrespective of the filling factor at  $z=0$ .

Pneuman et al. (1986) and Steiner and Pizzo (1989) assumed that a model like the HSRA (Gingerich et al., 1971) or the VAL describes the non-magnetic atmosphere. They could then lower  $h_m$  by increasing the temperature in the flux tube in a simply parameterised manner. However, this leads to unrealistically high temperatures. They were, therefore, unable to produce a low lying magnetic canopy except for relatively large filling factors. The main shortcoming of their work was not to include the influence of CO cooling. Although flux tubes and the non-magnetic atmosphere have relatively similar temperatures in the photosphere (e.g. Solanki, 1986; Keller et al., 1989), in the chromosphere large temperature differences are produced by the presence of cool non-magnetic gas around the flux tubes (CO clouds). This causes the tubes to expand more rapidly (as first pointed out by Ayres, 1981) and may lead to small values of the merging height of magnetic elements even in the case of small filling factors.

### 2.2. Method

Although the basic effect we are interested in is due to gas pressure, the exact influence of tension forces can only be judged by explicit numerical calculation. We therefore use a code, described by Steiner et al. (1986), which solves the full magneto-hydrostatic (MHS) equations including all tension terms for a vertical flux tube. In the present analysis we prescribe the atmosphere at the axis of the flux tube and in its surroundings. The temperature is assumed to be horizontally constant within the flux tube and in the surroundings, with a discontinuous jump at the flux tube boundary. The magnetic field at the lower boundary of our computational domain (which lies at  $z=0$ ) is assumed to be horizontally constant within the flux tube, also with a sharp boundary; i.e. the tube is surrounded by a thin current sheet. Higher up the magnetic field distribution follows from the solution of the MHS equations. The stratification of the temperature, pressure and density in the surrounding atmosphere are taken from empirical or radiative equilibrium models. The pressure within the tube has been calculated assuming the atmosphere to be in hydrostatic equilibrium with the temperature and pressure scale height being taken from a model atmosphere described in the literature (cf. Sect. 2.3). This procedure amounts to a simple reduction of the gas pressure of the original model atmosphere by a constant factor which depends on the base magnetic field strength. Far above the merging height,  $h_m$ , the magnetic field is supposed to become vertical as a consequence of the presence of neighbouring flux tubes of equal magnetic polarity (upper boundary condition).

After prescribing the temperature throughout the atmosphere, three free parameters remain, namely the filling factor at  $z=0$ ,  $f$ , the radius of the model flux tube at  $z=0$ ,  $R^*$ , and the field strength at that level,  $B^*$ .<sup>1</sup> The most important of these is expected to be the filling factor and we have carried out all calculations for a range of filling factors, limited at the lower end to  $f=0.005$  by numerical problems arising due to the presence of

<sup>1</sup> In the following the superscript \* denotes any physical variable corresponding to the base of the atmosphere,  $z=0$ , which is at  $\tau_{5000}=1$  of the surrounding atmosphere.

almost horizontal field lines in the integration box. Although Fig. 2 gives a rough idea of how different field strengths affect  $h_m$ , we shall consider the influence of both  $B^*$  and  $R^*$  in detail in Sect. 3.2.

In view of the fact that the azimuthal component of the field has a negligible effect on  $h_m$  (Steiner et al., 1986) we have not considered twisted flux tubes in the present investigation. If anything, a twist will enhance the effect of the pressure and will lower the merging height even more.

### 2.3. Models

The average quiet Sun (i.e. 1-component) model tabulated by Maltby et al. (1986) (their Table 11, denoted as  $C'$  in the rest of the paper) and the flux tube model FLUXT of Ayres et al. (1986) are used to describe the atmosphere inside the tube. We regard  $C'$  as a very conservative lower limit to the flux tube temperature. Empirical models of the *average* temperature in plages derived from chromospheric lines (e.g. Shine and Linsky, 1974; Lemaire et al., 1981; or VALP of Ayres et al., 1986) lie between the two models we have chosen. This is to be expected, since average plage models are based on data which have contributions from both the magnetic and the non-magnetic components in the plage.

For the temperature outside the flux tube we have also chosen two atmospheres. One is the NLTE line blanketed radiative equilibrium model of Anderson (1989). It includes the effects of CO cooling and is abbreviated as RE in the following. The other is the empirical COOLC model of Ayres et al. (1986) based on observations of the centre to limb variation of CO line profiles above the temperature minimum. We consider that the RE model describes a lower limit to the temperature in the non-magnetic atmosphere, since it does not include any mechanical heating.

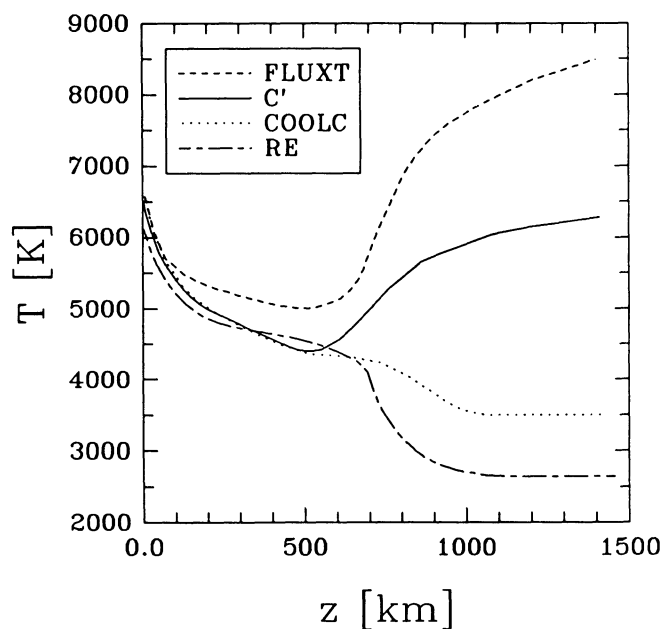


Fig. 1. Temperature profile of the model atmospheres of Maltby et al. (1986), Table 11, abbreviated as  $C'$  (—), Anderson (1989), abbreviated as RE (— · —), Ayres et al. (1986) (Fig. 12) FLUXT-model (---), and Ayres et al. (1986) COOLC-model (.....).

The influence of mechanical heating on CO cooled atmospheres has been studied by Muchmore and Ulmschneider (1985), Muchmore et al. (1988), Cuntz and Muchmore (1989) and Anderson and Athay (1989). The temperatures of the four models have been plotted as a function of height in Fig. 1. In those cases where the model atmosphere was not tabulated up to a height of 2000 km (FLUXT, COOLC, RE) the model has been extended upwards by appending an isothermal atmosphere in hydrostatic equilibrium, having a temperature equal to the last tabulated value. This extension, however, has no influence on the following results and was only necessary to keep the upper boundary of the flux tube model sufficiently above the merging height. Note that both the cool component atmospheres only dip below the  $C'$  curve above the height of the temperature minimum in that atmosphere, so that the criticism of the effectiveness of CO cooling at or below the temperature minimum made by Mauas et al. (1990) does not apply to them.

The pressure stratification for the RE model is plotted in Fig. 2, together with the pressure stratifications within magnetic flux tubes of three different field strengths and the same temperature profile as the  $C'$  model. Note the critical points at which the gas pressure within the flux tube equals the value of the external pressure. They all lie in the lower or middle chromosphere.

## 3. Results

### 3.1. The influence of temperature and filling factor on the merging height

Cross-sections through two flux tubes are plotted in Fig. 3. They both have the  $C'$  temperature stratification inside them,  $f=1\%$ ,  $R^*=100$  km and  $B^*=1500$  G. The two tubes differ, however, in the atmospheres of their surroundings. If the  $C'$  model is also

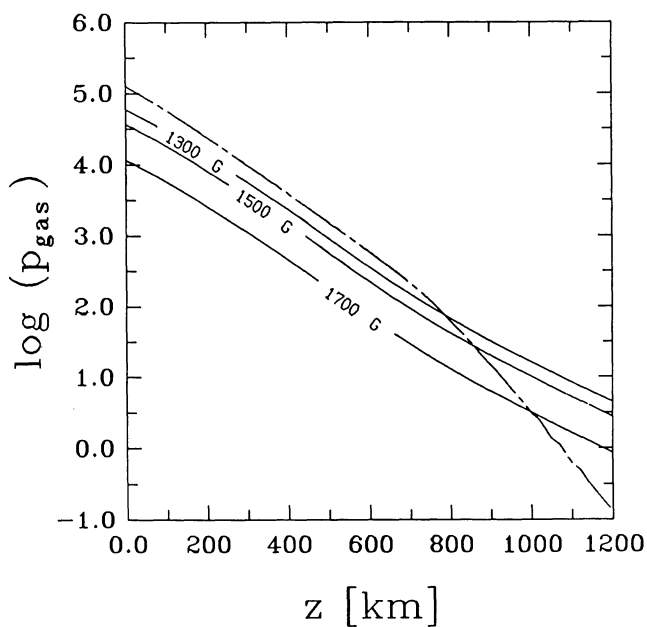


Fig. 2. Logarithm of gas pressure vs. height for the RE atmosphere (dot-dashed line) and flux tube atmospheres with various field strengths, as marked in the figure. The flux tube atmospheres have the same temperature profile as the  $C'$  model

used to describe the atmosphere outside the flux tubes, then the dashed field lines result, while the use of the RE model gives rise to the solid field lines. Above the height at which the  $C'$  and RE temperature stratifications diverge from each other the field lines of the two tubes also behave very differently. A comparison with Fig. 2 shows that the merging height for the  $C'/RE$  model combination is similar to the height of the critical point (for  $B^* = 1500$  G). The  $C'/RE$  tube also shows excess broadening (compared to the  $C'/C'$  one) lower in the atmosphere. As a matter of fact, it shows excess broadening even below 600 km, due to the slightly smaller pressure scale height of the RE model in comparison to  $C'$  below  $\approx 300$  km. This may be a result of the neglect of convection in the RE model.

In the following we concentrate on the merging height,  $h_m$ , of the flux tubes.<sup>2</sup> In Fig. 4  $h_m$  is plotted as a function of the filling factor  $f$  for various combinations of models for the magnetic and non-magnetic atmospheres. The dotted curve is produced when both the internal and external atmospheres have the same temperature. The  $h_m$  of this  $C'/C'$  combination increases unabated with decreasing  $f$  over the entire plotted range. It even steepens appreciably as  $f$  drops below 0.05, due mainly to the chromospheric temperature rise and the associated increase in the pressure scale height. Recall that the horizontal pressure difference is simply proportional to the pressure in this case.

The curves for the other cases behave rather differently. For  $f \geq 0.05$  they are almost indistinguishable from the  $C'/C'$  curve. The slight difference between the FLUXT/RE and FLUXT/COOLC combinations, on the one hand, and the rest in this high- $f$  regime is due to the higher temperature of FLUXT throughout the photosphere. The minuteness of this difference suggests that any uncertainty in the temperature of flux tubes or their surroundings in the photosphere should not significantly affect the merging height. The uncertainty is not expected to be larger than a few hundred degrees anyway (cf. Solanki, 1989b). As  $f$  drops below 0.05 the merging height begins to approach the critical height at which the internal and external gas pressures are equal and the  $h_m$  curves of the models with non-magnetic atmospheres described by COOLC or RE start to saturate.

As expected, the combination  $C'/COOLC$  produces the highest limit to  $h_m$ , lying above 1200 km, while FLUXT/RE gives the lowest, at around 750 km. The other two possible combinations  $C'/RE$  and FLUXT/COOLC give canopy bases lying in between, at around 900–1000 km. However, the main point is not the exact value of  $h_m$ , which depends considerably on the chosen models, but that given the presence of cool non-magnetic chromospheric gas, a relatively low-lying magnetic canopy is bound to form at or below a height given by the critical height of pressure crossing, *irrespective of the filling factor!* For empirical and theoretical models of the temperature stratification in the solar atmosphere this height lies in the bottom third of the chromosphere.

<sup>2</sup>  $h_m$  is defined here as the height where the flux tube radius has expanded to  $0.9 \times W$ , where  $W$  is the maximum radius to which the flux tube expands for a given filling factor.  $(1.8 \times W)^2$  equals about the circular cross section of the flux tube at the upper boundary,  $\pi W^2$  and corresponds to the area of a square which the flux tube would have to fill to merge with its neighbours if the flux tubes were distributed evenly over the solar surface at the corners of squares.

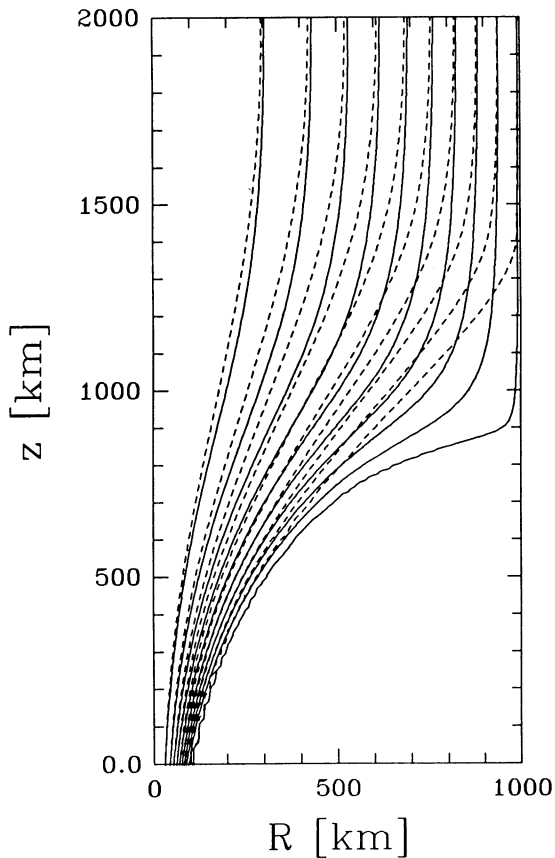
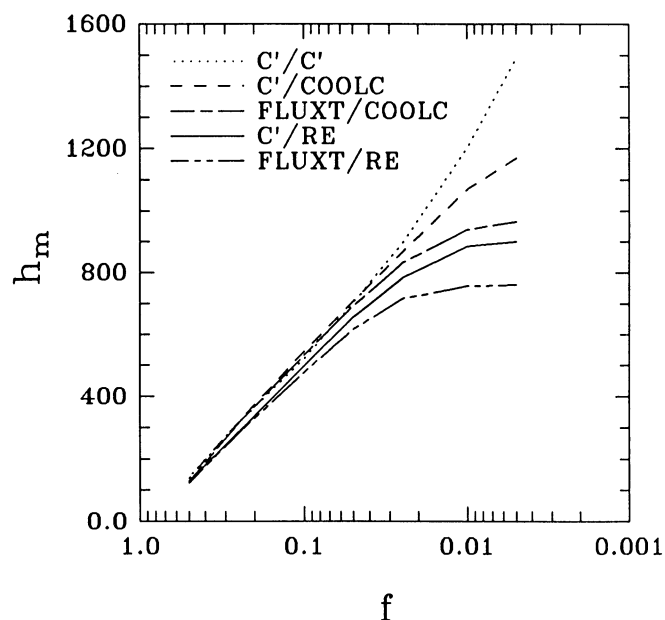


Fig. 3. Representative examples of the model magnetic flux tubes considered in this paper. The dashed curves are magnetic field lines delineating a flux tube with equal temperature within and outside the tube. The temperature profile has been taken from the  $C'$  model. The solid curves correspond to the field lines belonging to a flux tube surrounded by the cool RE atmosphere. The temperature within the flux tube is again given by the  $C'$  model. The field strength and the radius at the base ( $z=0$ ) are  $B^* = 1500$  G and  $R^* = 100$  km, respectively. The filling factor at  $z=0$  is  $f = 0.01$ .

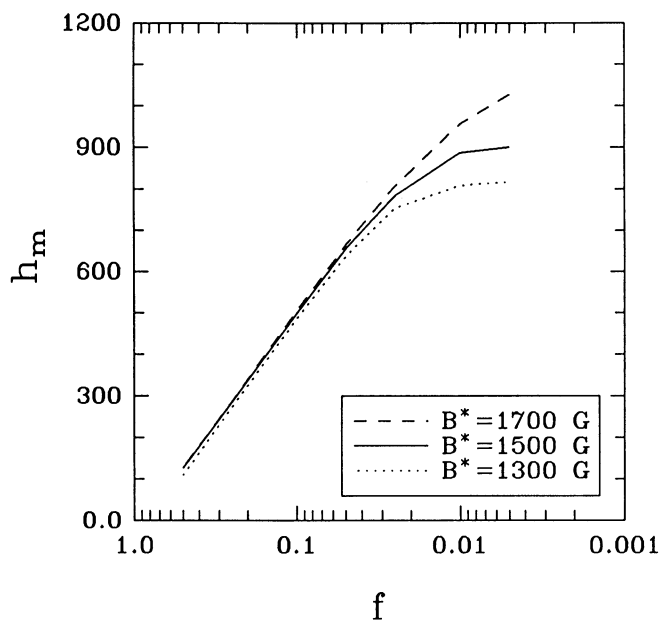
### 3.2. The influence of field strength and radius on the merging height

Let us now consider the influence on  $h_m$  of the remaining two free parameters in our model,  $B^*$  and  $R^*$ . The influence of  $B^*$  is illustrated in Fig. 5.  $h_m$  is plotted vs.  $f$  for the  $C'/RE$  atmosphere combination and three different values of  $B^*$ , namely 1300 G, 1500 G and 1700 G. The pressure stratifications for these cases are plotted in Fig. 2. The close correspondence between the asymptotic  $h_m$  value for small  $f$  and the critical height can be deduced by comparing Figs. 2 and 5. As expected, the larger the  $B^*$ , the higher the  $h_m$  limit.

Fortunately, the photospheric field strength is one of the more precisely measured parameters of small scale solar magnetic features (Solanki et al., 1987; Zayer et al., 1989, 1990; Keller et al., 1990), so that the dependence of  $h_m$  on  $B^*$  does not significantly add to the uncertainty in the height of the canopy base. In the most extensive series of precision field strength measurements to date Zayer et al. (1990) find  $B^* \approx 1450 - 1650$  G for all the studied regions, with a slight dependence on filling factor. Regions with smaller  $f$ , i.e. those which give rise to



**Fig. 4.** Merging height,  $h_m$ , as a function of filling factor,  $f$ , for various combinations of atmospheres within and surrounding the flux tubes. The temperature profile combinations are, respectively, FLUXT/RE (-----), C'/RE (—), FLUXT/COOLC (— · — · —), C'/COOLC (·····), and C'/C' (.....)



**Fig. 5.** Merging height,  $h_m$ , as a function of filling factor,  $f$ , for flux tubes with a temperature profile taken from the C' model, surrounded by the RE atmosphere. The three curves correspond to flux tubes with a base field strength of  $B^* = 1500$  G (—),  $B^* = 1700$  G (-----), and  $B^* = 1300$  G (.....).  $R^* = 100$  km in all three cases

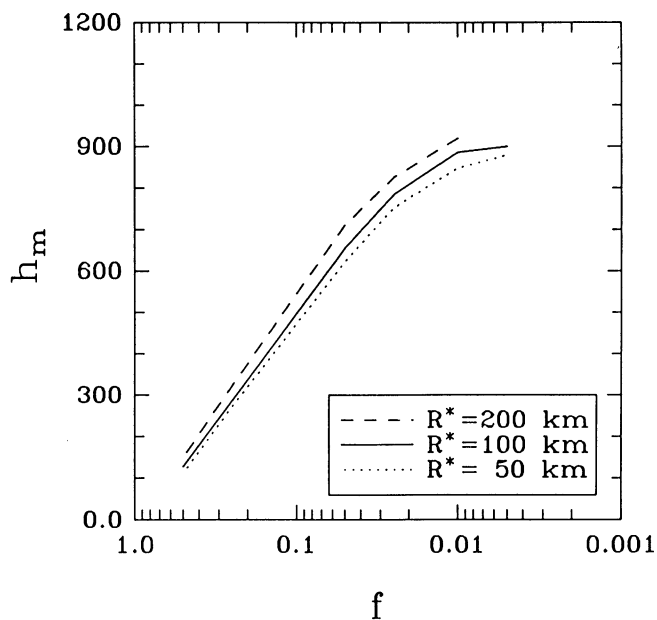
canopies near the critical height tend to have  $B^*$  values at the lower end of the range. The uncertainty in the canopy height introduced by the uncertainty in the measured field strength is estimated to be approximately 50 km.

Finally, we consider the influence of  $R^*$ . Increasing  $R^*$ , while keeping  $f$  fixed, increases the magnitude of the magnetic tension forces (cf. Pneuman et al., 1986; Steiner and Pizzo, 1989). Therefore, by varying  $R^*$  we can obtain an idea of the importance of magnetic tension for  $h_m$ . Figure 6 shows  $h_m$  vs.  $f$  for tubes with three different radii  $R^* = 50$  km (dotted), 100 km (solid) and 200 km (dashed). Although the curves diverge from each other even for large filling factors, nowhere is this divergence significant. The difference in  $h_m$  between the two extreme models is less than 100 km. Since a diameter of 400 km can be considered an upper limit to the size of magnetic elements (Ramsey et al., 1977; Von der Lüh, 1989; Zayer et al., 1989), we conclude that the exact size of magnetic elements is of little relevance to the height of the canopy base. Consequently, magnetic tension forces also play a minor role in determining this parameter.

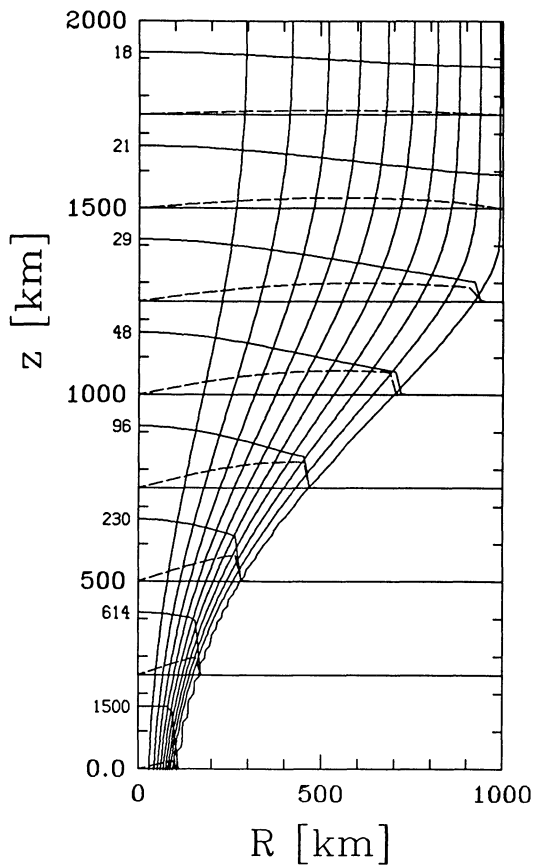
The investigations described in this section have shown that in the quiet Sun (i.e. for small filling factors) the only parameters not strictly fixed by observations which can significantly affect the height of the canopy base are the temperatures of the tubes and their surroundings. For empirically or theoretically derived temperatures the height of the canopy base lies below  $z \approx 750$ –1200 km. In active regions and within the confines of network elements, i.e. wherever  $f$  is large, the filling factor is the main parameter determining the merging height. In the quiet Sun, on the other hand, the merging height is fixed by the critical height and is independent of filling factor.

### 3.3. The internal structure of the magnetic features

It is instructive to compare the magnetic field distributions within flux tubes surrounded by a hot and by a cool chromospheric atmosphere, respectively. Figure 7 shows the field lines of a model with a C'/C' atmosphere combination (Fig. 7a) and of a



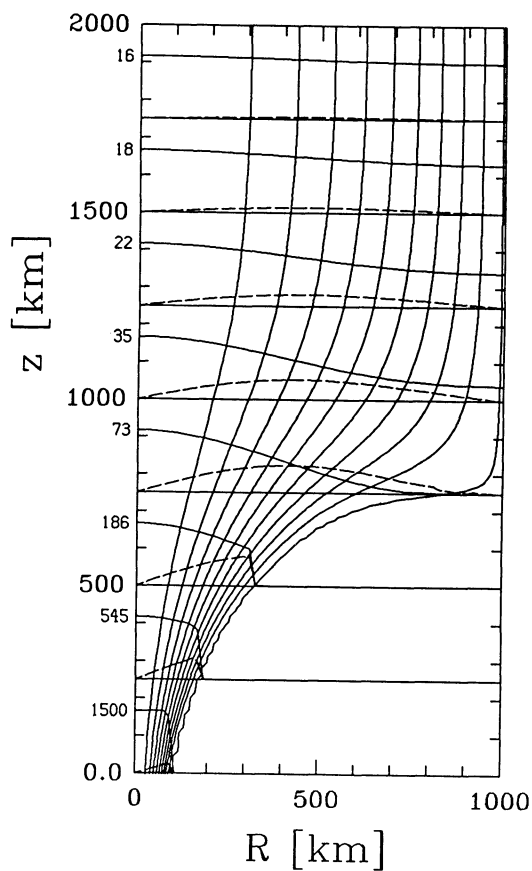
**Fig. 6.** Merging height,  $h_m$ , as a function of filling factor,  $f$ , for flux tubes with a temperature profile taken from the C' model, surrounded by the RE atmosphere. The three curves correspond to flux tubes with a radius at the base of  $R^* = 100$  km (—),  $R^* = 200$  km (-----), and  $R^* = 50$  km (.....).  $B^* = 1500$  G in all three cases



**Fig. 7a.** Model magnetic flux tube with equal temperature within and outside the tube. The temperature profile has been taken from the C' model. Superimposed on the field lines are horizontal curves showing the radial variation of the vertical,  $B_z$  (solid lines), and the radial,  $B_r$  (dashed lines), components of the field at eight different heights in the atmosphere. Both  $B_z(r)$  and  $B_r(r)$  are normalized to  $B_z$  at the axis, the value in Gauss of which is indicated to the left of each curve. The heights to which each  $B_z(r)$  and  $B_r(r)$  pair belongs is marked by a horizontal straight line.  $R^* = 100$  km,  $f = 0.01$ , and  $B^* = 1500$  G

model with a FLUXT/RE combination (Fig. 7b).  $R^* = 100$  km,  $B^* = 1500$  G and  $f = 1\%$  in both figures. Superimposed on the field lines are the  $B_z(r)/B_z(0)$  (solid) and  $B_r(r)/B_z(0)$  (dashed) curves at eight different heights in the atmosphere. The height to which each  $B_z(r)$  and  $B_r(r)$  pair refers is marked by a horizontal straight line.  $B_z$  and  $B_r$  are the vertical and radial components of the magnetic field, respectively, and  $r$  is the radial coordinate.

Not surprisingly the  $B_z(r)$  and  $B_r(r)$  curves are rather different for the two models, particularly in the vicinity of the merging height of the FLUXT/RE model combination. Due to the rapid expansion of the field near this height  $B_r$  becomes much larger than  $B_z$  near the outer boundary of the FLUXT/RE flux tube. In addition, the field strength near the boundary is much smaller than its value at the axis. The field strengths at the axes of the two flux tubes actually lie less than 23% apart at all heights in the atmosphere. Therefore, the rapid decrease of the external/internal gas pressure difference towards the critical point and the anomalous expansion it produces only affects the outer parts of the tube, leaving the 'core' or central cylinder almost unchanged. Consequently, although flux tubes themselves no longer exist above the merging height, for a few 100 km above it the magnetic field

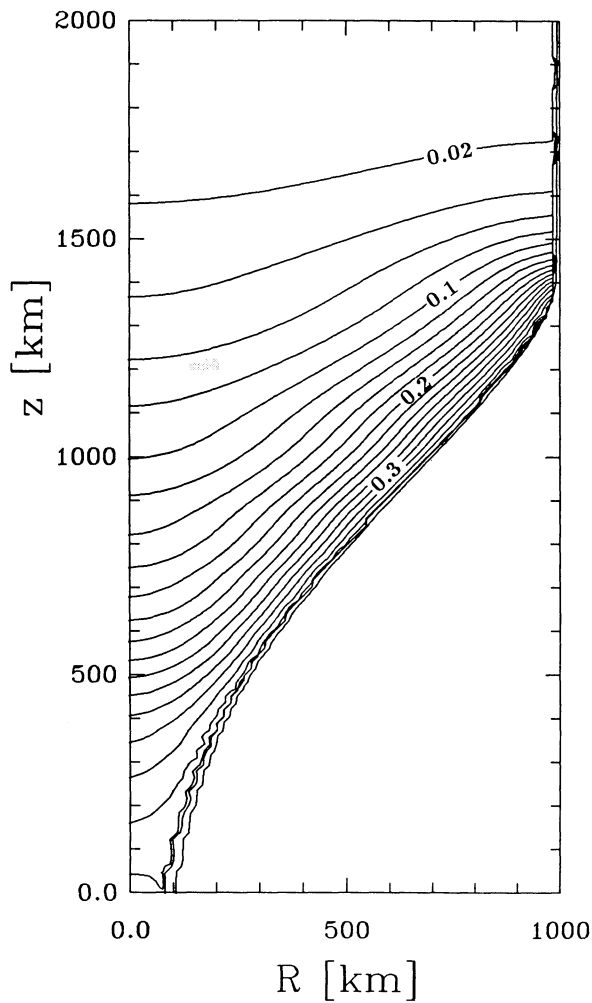


**Fig. 7b.** The same as Fig. 7a, but this time for the FLUXT/RE model combination, so that the temperature in the surroundings is different from that in the flux tube

remains highly structured, with the strongest fields occurring cospatially with the location of the flux tubes in the photosphere. The field becomes horizontally homogeneous at a height determined mainly by the filling factor and practically uninfluenced by the thermal structure of the atmosphere outside the flux tube. So, although the field is practically horizontally homogeneous within the flux tube in the lower photosphere according to observations (Zayer et al., 1989), it loses this property completely in the lower chromosphere. Note, however, that a horizontal distribution of temperature within the flux tube may well change the field distribution above the merging height (cf. Sect. 4).

Let us now consider the plasma  $\beta = 8\pi P_i/B_z^2$  which Fig. 7 suggests may be quite different for the two cases (cf. Steiner and Pizzo, 1989). Figures 8a and b depict the curves of equal  $\beta$  for the same models as in Fig. 7a and b, respectively. In Fig. 8a  $\beta$  decreases with increasing height, i.e. the magnetic field becomes energetically ever more dominant the higher one goes in the atmosphere. However, note that the curves are far from horizontal except above the merging height ( $z \geq 1500$  km). Below that they run more or less parallel to the boundary, except near the centre of the flux tube. The  $\beta$  curves must cut the axis of symmetry at right angles due to the boundary condition at  $r = 0$ :  $dB/dr = dP_i/dr = 0$ .

The situation is very different for the FLUXT/RE case, particularly near the merging height. Since the excess gas pressure



**Fig. 8a.** Map of the spatial variation of the plasma  $\beta$  for the flux tube of Fig. 7a. Both the flux tube and its surroundings have the same temperature profile (the C' model).  $R^* = 100$  km,  $f = 0.01$ ,  $B^* = 1500$  G and the base value of the plasma  $\beta$  is  $\beta^* = 0.38$

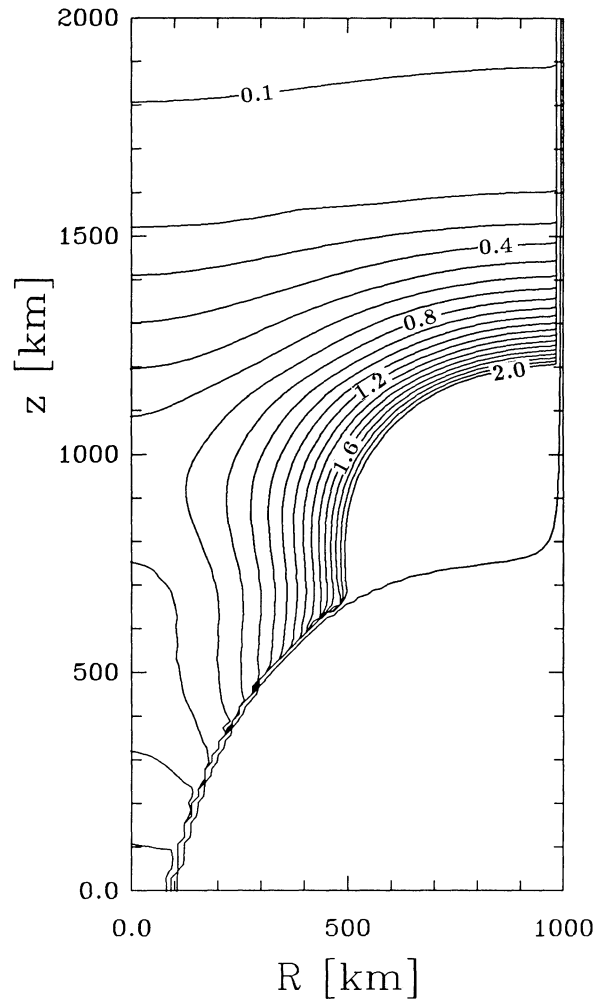
within the magnetic feature forces the magnetic field outwards, it becomes rather weak in the outer parts of the tube (cf. Fig. 7b). Accordingly, it is not surprising that  $\beta$  increases to very large values there. Thus, the gas dominates over the magnetic field over a large fraction of the canopy base, so that fluctuations in gas pressure can easily change the canopy height.

The behaviour of  $\beta$  in Fig. 8 can be understood by keeping in mind that the pressure must be exactly balanced across the boundary of the tube, i.e.

$$P_e(R) - P_i(R) = \frac{B_i^2}{8\pi}. \quad (1)$$

$R$  is the radius corresponding to the boundary of the tube. Using this relation it is possible to rewrite the expression for  $\beta$  at the boundary as

$$\beta(R) = \frac{P_i(R)}{P_e(R) - P_i(R)}. \quad (2)$$



**Fig. 8b.** Map of the spatial variation of the plasma  $\beta$  for the flux tube of Fig. 7b. The flux tube has a temperature profile equal to that of the FLUXT model and is surrounded by the RE atmosphere. Contours of  $\beta > 2.0$  at the sharp bend where the flux tube meets the computational boundary have been deleted for clarity.  $R^*$ ,  $f$ ,  $B^*$  and  $\beta^*$  have the same values as in Fig. 8a

Since  $P_i = cP_e$  if  $T_i = T_e$  at all heights<sup>3</sup> ( $c < 1$  is a constant of proportionality),  $\beta(R)$  is itself independent of  $z$ . This is exactly what is seen in Fig. 8a, where the iso- $\beta$  curves lie parallel to the flux tube boundary.

In the case represented in Fig. 8b, on the other hand, as  $P_e(R)$  approaches  $P_i(R)$  near the merging height  $\beta(R)$  tends to infinity according to Eq. (2), being limited only by the merging of the tubes.

To avoid negative values of  $\beta$  at the boundary (negative  $\beta$  implies either negative gas pressure, or an imaginary field strength), it follows from Eq. (2) that, the field of neighbouring tubes must merge at or below the critical point. This explains why the merging height is so insensitive to any flux tube parameter which does not affect the critical point, e.g., magnetic tension. Note, that due to numerical uncertainties (finite grid

<sup>3</sup> We here assume the mean molecular weight to be equal within and outside the flux tube.

spacings, etc.) the merging heights for small filling factors in Figs. 3–6 may sometimes slightly exceed the critical height.

#### 4. Discussion and conclusions

##### 4.1. Discussion of the results and of their consequences

The main result of the present investigation is that in the presence of a thermally inhomogeneous (bifurcated) chromosphere, as produced by CO cooling of the non-magnetic gas, magnetic fields expand and merge *below* a height of between approximately 700 and 1200 km. Therefore, a large and possibly a dominant part of the chromosphere is filled with magnetic fields. In addition, the merging height increases with decreasing magnetic filling factor only up to a critical height. If the filling factor is decreased further, the merging height remains practically unchanged. Due to the rapid expansion of the field near this critical height a canopy with an almost horizontal base is produced for sufficiently small filling factors. A low lying magnetic canopy, as deduced from observations by Giovanelli (1980) and, using the more accurate calibration curves of Jones and Giovanelli (1982), by Giovanelli and Jones (1982) and Jones and Giovanelli (1983), is a natural outcome of a thermally bifurcated atmosphere. This is the only mechanism known to us which produces, in a theoretically consistent manner, a magnetic structure that can be termed a canopy. With a hot non-magnetic chromospheric component the magnetic field never becomes as horizontal as the extensive observations of Giovanelli and Jones suggest.

In the calculations presented here the merging height produced by a cool non-magnetic chromosphere is approximately 500–800 km lower than that produced by a uniformly hot atmosphere for a global solar filling factor of approximately 0.005. Note, however, that on the real Sun the difference is expected to be considerably larger (for the same global filling factor) due to the very uneven distribution of the magnetic field on the solar surface. The majority of the flux tubes are clumped together in active regions and network elements at supergranular boundaries. The filling factor in such regions (averaged over say 5–10′) varies between 5% and 25% (outside sunspots). Between such concentrations of magnetic flux lie the supergranule cell interiors with a considerably smaller average flux density (in the form of “weak” intranetwork fields, Livingston and Harvey, 1975; Martin, 1989). If, say, the average field strength of the Sun is 15 G (corresponding to a global filling factor of 1% at  $z=0$ ) and regions of concentrated field (active regions, network) cover 9% of the surface with an average field strength of approximately 150 G ( $f \approx 10\%$ ), then the rest of the solar surface (over 90% of it!) would have an average filling factor of the order of only 0.1%. For a thermally homogeneous hot chromosphere and such an irregular concentration of the field, the merging height over large tracts of the solar surface would lie well above 2000 km, i.e. in the transition region, leaving the chromosphere largely field free. In contrast, in the cool non-magnetic chromosphere scenario now favoured by observations and theory, the merging height does not exceed approximately 800–1000 km anywhere on the Sun.

In addition to the just described direct results there are a number of additional consequences of our study.

Firstly, since the thermal bifurcation is a purely chromospheric phenomenon on the Sun (cf. Mauas et al., 1990, who show that CO cannot cool the temperature minimum region significantly) the proposed mechanism (involving a critical height)

cannot produce canopies at or below the traditional temperature minimum level. Photospheric canopies appear only to be possible for magnetic filling factors larger than approximately 10%. Therefore, although the present study supports observations of canopies in the lower chromosphere, it cannot explain canopies in the photosphere except in very active regions where the large filling factors force the tubes to merge at a height of a few hundred km. Very low lying canopies (100–300 km above  $z=0$ ) have only been observed in the vicinity of sunspots and have been explained by their superpenumbrae (Giovanelli and Jones, 1982; Giovanelli, 1982).

A somewhat more quantitative comparison with the data shows that our calculations explain quite naturally the observation of Jones and Giovanelli (1983) that the median canopy base height is almost independent of mean flux for  $\langle B \rangle \leq 30$  G (i.e.  $f \lesssim 2\%$ ) and gives the right general value for the canopy base height ( $\lesssim 1000$  km) in unipolar regions. All the same, we do not attempt to compare our canopy heights with those derived by Giovanelli and Jones point by point in detail, not only due to the uncertainties in the available models, but also because the canopy observations were interpreted without taking the thermal bifurcation into account.

Secondly, maps of the plasma  $\beta$  produce some interesting results.  $\beta$  is small everywhere except when the merging height approaches the critical height. Then in the outer parts of the flux tube from approximately the merging height to a few hundred km above it,  $\beta$  can reach values well above unity. This means that in the outer canopy the energy is concentrated in the gas and not in the magnetic field. The latter must follow fluctuations in, e.g., the gas pressure produced by an acoustic wave or a magnetoacoustic surface wave travelling along the flux tube boundary (cf. Edwin and Roberts, 1983, for a description of the wave modes in a thick tube).<sup>4</sup> Acoustic or magnetic waves or even the 5 minute oscillations will, therefore, move the outer canopy considerably, but cannot affect the main body of the flux tube. To picture the behaviour of flux tubes in the presence of a dynamic photosphere and chromosphere one can compare them to water lilies. These have a firm central stem unaffected by motions in the water. The stem fans out at the surface into a broad, supple leaf which rides water surface waves, flapping up and down along with them. Due to the propensity of acoustic and magnetoacoustic waves in the solar atmosphere, generally accompanied by pressure fluctuations, we expect the canopy base to be in a dynamic state of rise and fall.

Interestingly, acoustic waves in the non-magnetic atmosphere may play a dual role and may also help to keep the canopy in place. According to Muchmore et al. (1988, Fig. 2), acoustic waves manage to counteract the CO cooling most efficiently at just about the height of the canopy base. They may thus provide some of the energy to keep the atmosphere above the canopy base hot and thus help maintain the status quo.

Thirdly, the fact that magnetic tension does not influence the merging height implies that it can also be determined reasonably

<sup>4</sup> Whether the kinetic energy of the wave will be sufficient to affect the canopy height directly is uncertain. At  $z=900$  km the kinetic energy density of a wave with an amplitude of  $1 \text{ km s}^{-1}$  is  $E_k = \rho v^2/2 \approx 1 \text{ erg cm}^{-3}$ , which corresponds to the magnetic energy density of a field of 5 G, a value which is easily reached in the outer canopy for all but the smallest filling factors.



accurately from the thin tube model (i.e. pure horizontal pressure balance). It is also possible to obtain a rough idea of the merging height through a simple analytical formula obtained by interpolating between expressions for the merging height in the two extreme cases of very large and very small filling factors. For an isothermal atmosphere (i.e. in the absence of a critical height) Spruit (1983) derived the following expression for the merging height, which we use to represent the limit of very large  $f$ ,

$$h_m = -2h_p \ln f.$$

where  $h_p$  is the pressure scale height. The limit for very small filling factors simply reads:

$$h_m = h_c,$$

where  $h_c$  is the critical height. The interpolation formula found to give the most satisfactory representation of the numerical results for a wide range of  $h_p$  and  $h_c$  combinations reads:

$$h_m = \left( -\frac{f^{h_p/h_c}}{h_p \ln f} + \frac{(1-f)^{h_p/h_c}}{h_c} \right)^{-1}. \quad (3)$$

However, note that the correct internal distribution of the field is only obtained by including magnetic tension explicitly in the calculations.

Fourthly, the presence of a rapidly expanding field and a relatively low lying magnetic canopy limits the vertical extent of the CO clouds and may largely determine their geometry. One of the main assumptions underlying all theoretical and empirical models of CO cooled atmospheres has been a fixed, height independent magnetic coverage (this includes zero-coverage, i.e. a purely field free atmosphere). For example, Ayres et al. (1986) derive a fixed value of 7.5% for the magnetic coverage in the quiet Sun. However, from Fig. 7b we can see that for  $f=1\%$  the covering fraction of the magnetized plasma changes from about 10% at  $z=500$  km to 100% at the merging height of about 750 km. This example demonstrates that the expansion of the flux tubes must be taken into account in future modelling. The derived small  $f$  value by Ayres et al. (1986) may be due to an excessively hot flux tube component which is derived by assuming a fixed plage filling factor of 50%, a value which may be much larger above the temperature minimum. Anderson (1989) argues that this small filling factor may be due to an overly hot non-magnetic cool component.

Fifthly, the present analysis can also provide a rough idea of the temperature within flux tubes in the chromosphere. In the lower and middle photosphere the temperature within flux tubes is relatively well determined, both from observations (Solanki, 1986; Keller et al., 1989) and from theoretical modelling (Grossmann-Doerth et al., 1989; Steiner and Stenflo, 1989), but little or nothing is known about it higher in the atmosphere. In the low chromosphere below the merging height we expect the temperature in the flux tubes to be considerably higher than in an average quiet Sun or average plage model (Shine and Linsky, 1974; C'; Lemaire et al., 1983; VALP of Ayres et al., 1986). The Ayres et al. (1986) FLUXT model can serve as a rough guide for the flux tube temperature in the upper photosphere and lower chromosphere below the merging height. Note, however, that this model was derived from Stokes  $I$  data by assuming a fixed filling factor, and it must therefore be considered with some caution. Above the merging height the temperature within the magnetic region should approach that given by averaged models of plages or the quiet Sun (depending on  $f$ ), since the very cool CO

components are no longer present. However, note that the concept of a single temperature within flux tubes above the merging height becomes somewhat dubious (cf. Sect. 4.2).

Sixthly, although the calculations presented here are for a unipolar region on the Sun, their results can be applied directly to bipolar regions due to the almost negligible effect of magnetic tension on the height of the canopy base. To transfer our results to this case we must replace the term “merging height” by the “maximum height of the lower boundary” of the field and the inverse of the “filling factor” by the “separation between the footpoints” of the loop. If the footpoint separation of an isolated bipolar region is small then the maximum height of the lower boundary between the two poles will increase with increasing separation. In other directions the field will simply expand normally until it reaches the critical height, where it becomes almost horizontal. For larger separations between the foot points the canopy base height will everywhere be given by the critical height.

Seventhly, the observations and calculations of Wiedemann et al. (1987) suggest that the chromospheres of other late type stars are also thermally bifurcated. Numerous late type stars are also known to possess a magnetic field (Linsky, 1985; Saar, 1989) which, for the only star for which we have direct observational evidence ( $\epsilon$  Eri), is concentrated mainly into bright and hot flux tubes corresponding to solar plages or network elements (Mathys and Solanki, 1989). Therefore, we expect relatively low lying canopies to form on such stars as well. Since the critical height is determined mainly by the height at which CO begins to cool the atmosphere, the maximum height of the canopy base on an early G-type star should not be too different from the solar value, almost independently of the global magnetic filling factor of the star. The only exceptions may be very active stars (global stellar  $f > 10-20\%$ ), or stars with abnormally low C or O abundances (less efficient CO cooling).

For cooler stars CO is expected to be an effective cooling agent at increasingly lower heights (e.g. Massaglia et al., 1988), so that, assuming that the magnetic flux tubes are hot in such atmospheres too (recall that  $\epsilon$  Eri is a K2 dwarf) magnetic canopies should lie increasingly lower in the atmospheres. One can speculate that for sufficiently cool stars (M dwarfs), the canopy may actually lie at a height at which the spectral lines used to measure magnetic fields are formed. This may contribute to solving the problem of the anomalously large filling factors (up to 90%) measured on some M dwarfs (Saar and Linsky, 1985, 1987).

Note, however, that very low lying canopies *cannot* be the *only* explanation of the large  $f$  values measured on some M-dwarfs. Due to the very presence of different temperatures in the magnetic and non-magnetic features a two component model, with a good knowledge of the temperature of both components, is required to reliably derive the filling factor. Since the true temperatures are not known, Saar and Linsky assumed the same temperature in both components. Probably the derived filling factors have to be revised. Recall that even on the Sun  $f$  is uncertain by a factor of two (Schüssler and Solanki, 1988).

#### 4.2. Limitations

One of the major limitations of the present investigation is the assumption of horizontally constant temperature within the magnetic elements. This may be a fairly reasonable assumption in

the photosphere. However, in the chromosphere temperature inhomogeneities (within the magnetic elements) may arise due to inhomogeneities in the heating rate. For example, heating may be stronger near the boundaries of the tubes due to enhanced dissipation of waves there, or due to reconnection. This would lead to horizontal gas pressure and field strength gradients within the flux tube and could affect the merging height.

Indeed, temperature diagnostics like the UV continuum at 1600 Å, the Ca II K2 peak intensity and UV transition region lines invariably show the presence of small bright features. However, not all these diagnostics necessarily contradict a horizontally constant temperature within the flux tubes. The continuum at 1600 Å is formed near the classical temperature minimum, where flux tubes have only expanded by a factor of three in radius (compared to  $z=0$ , e.g. Fig. 3) and the spatial resolution (approximately 1") of the best filtergram observations of Foing and Bonnet (1984) is still insufficient to resolve the internal structure of flux tubes, in particular since within flux tubes the 1600 Å continuum may arise deeper in the atmosphere, as has been argued by Foing and Bonnet (1984).

In a standard quiet Sun model the source function of the temperature sensitive peak of Ca II K2 begins to decouple from the temperature only approximately 100 km above the classical temperature minimum (Uitenbroek, 1989), i.e. still below the canopy height, although this result may change for other temperature structures. Therefore, fine structure of 1–2" in Ca II filtergrams does not contradict relatively low lying canopies, and at present provides, at the most, only weak evidence for horizontal temperature variations inside magnetic flux tubes.

Fine structure seen in the C IV lines formed at approximately  $10^5$  K are a different matter entirely, since they are formed well above the merging height of the field. Dere et al. (1984) find bright C IV patches correlated with the magnetic network. They are smaller than 2" and cover 16% of the surface at that height. This suggests that the magnetic atmosphere above the merging height is not thermally homogeneous and consequently neither is the gas pressure (see also Fiedler and Cally, 1989). Since the field over much of the canopy has a rather large  $\beta$  for a few hundred km above the merging height, horizontally inhomogeneous gas pressure at these heights will lead to a horizontal redistribution of the field. The exact field distribution will depend on the thermal structure. If, as the observations suggest, the part of the canopy field above the flux tubes (i.e. over the photospheric magnetic network) is hotter in the middle and upper chromosphere, then we would expect the field just above the canopy base to be more homogeneous than Fig. 7b suggests.

If the temperature within the flux tubes is inhomogeneous at photospheric levels, then the merging height itself can be affected. Since, as discussed above, there is no concrete observational evidence for such a distribution we do not attempt such solutions.

We have also not considered the whole question of how strongly the height of the transition region varies from point to point on the surface (e.g. Feldman et al., 1979; Fiedler and Cally, 1989), as well as the well known (magnetic field related?) inhomogeneities seen in H $\alpha$  (fibrils, spicules).

The question of how the hot flux tube atmosphere interacts with its cool surroundings below the merging height can only be addressed with a multi-dimensional treatment of the radiative transfer. It is clear that there will be a transition layer of a certain thickness across the flux tube surface with a smoothly changing

temperature, which again may influence the merging height. An investigation of this problem is planned.

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