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The influence of vertical magnetic field gradients on the measured field strength and filling factor in late-type stars

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Abstract. The influence of a vertical gradient of the magnetic field in late type stars on the measurement of magnetic field strengths and filling factors is studied. Line profiles and contribution functions of spectral lines with large Landé factors are calculated in model stellar atmospheres in the presence of a magnetic field with a vertical gradient. It is found that the four lines, which have often been used to measure solar and steller magnetic fields in the past, are formed at heights sufficiently different to account for differences in apparent field strength of up to 1000 G if we assume the fields to be similarly structured as in the sun. Thus we conclude that the vertical gradient of the field may contribute to a good part of the discrepancy between different published measurements of field strength and filling factor in the K2 dwarf ϵ-Eri.

Key words: stars: magnetic fields – radiative transfer – Stokes profiles

1. Introduction

The measurement of the magnetic field strength and the magnetic flux is basic to the understanding of magnetic activity on late type stars. Beginning with Robinson et al. (1980) a number of stellar magnetic field measurements have been published (e.g. Marcy, 1984; Gondoin et al., 1985; Saar and Linsky, 1985, Saar et al., 1986a, Basri and Marcy, 1988; Mathys and Solanki, 1989; Marcy and Basri, 1989), all of which are based on comparing the profiles of Zeeman sensitive to Zeeman insensitive spectral lines. Unfortunately, the effect of the field on the line profile is small and the many simplifications made by the investigators render any comparison of their results difficult. As an example consider ϵ-Eri, the star with the best studied magnetic field. Marcy and Basri (1989) and Saar (1990) have compared the published magnetic field measurements of this star and find considerable discrepancies between the values of field strength and filling factor (1 kG to 3 kG and 10 % to 67 %). Such discrepancies have so far been attributed to intrinsic changes in the stellar magnetic field (e.g. Marcy, 1984; Saar et al., 1986b) and to differences between the techniques used to measure the fields (Marcy and Basri, 1989).

In the present contribution we investigate another source of uncertainty, namely the vertical gradient of the magnetic field. We assume the magnetic fields in the atmosphere of late type stars to occur in the form of small flux concentrations corresponding to those found in solar faculae. Since the magnetic field in such flux tubes is confined by the pressure of the ambient gas the field strength decreases with height h while the cross-sectional area of the magnetic elements and the magnetic filling factor, i.e. the fractional area covered by the field at a given height, increase with h. Because the various investigators have use different spectral lines, they must have obtained field strength and filling factor at different heights in the atmosphere which may explain, if only partly, the difference between their results.

2. Atmospheric models and spectral lines

For our analysis we used four Fe I lines whose properties are shown in Table 1.

Table 1. Fe I lines used in the present analysis

λ (Å)	$g_{\rm eff}$	E (eV)	$\log gf$
6173.3	2.5	2.22	−2.88
8468.4	2.5	2.22	−2.072
5250.2	3.0	0.12	−4.938
15648.5	3.0	5.43	−0.54

Listed are the wavelength, λ, the effective Landé factor, $g_{\rm eff}$, the excitation potential, E, and the logarithm of the statistically weighted oscillation strength, $\log gf$. The first line has been used extensively by Saar and co-workers (Saar et al., 1986a, 1986b; Saar, 1988) for the measurment of magnetic fields in late type stars, the second by Basri and Marcy (1988) and Marcy and Basri (1989) for the same purpose. The third and the fourth line have been employed a great deal for the investigation of solar magnetic fields (cf. e.g. Stenflo, 1989). The fourth line has also been used for stellar magnetic field measurements (Giampapa et al,.1983).

To determine the difference in the heights of formation of these lines we numerically solve the full Unno-Rachkovsky

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Send offprint requests to: S.K. Solanki equations in LTE and compute the Stokes parameters and their contribution functions with the technique described by Grossmann-Doerth et al. (1988). The choice of an appropriate model atmosphere is not straightforward. The temperature in the magnetic features of ϵ-Eri is virtually unknown, although there is some observational evidence that they are hotter than the non-magnetic atmosphere of the star (Solanki and Mathys, 1987; Mathys and Solanki, 1989). On the other hand, if we accept the conclusions of Basri et al. (1990), then the magnetic features forming the plages on late type stars like ϵ-Eri are not much hotter than their surroundings.

Therefore we have chosen the scaled solar K2 dwarf atmosphere of Basri and Marcy (1988) which they employed for the analysis of their ϵ-Eri spectra. To test the influence of the temperature of the atmospheric model on our conclusions we have also used an extremely hot model, the solar network flux tube model of Solanki (1986). The field strength in both models is calculated using the thin tube approximation (e.g. Parker, 1979), with $B(\tau = 10^{-2}) \approx 2000$ G, the field strength typically measured on ϵ-Eri using lines in the visible (τ is the continuum optical depth at 5000Å within the magnetic feature). For the ambient non-magnetic atmosphere we have used the Basri and Marcy K2 dwarf model in both cases. For simplicity all the calculations are carried out at the centre of the stellar disk and only a single ray along the flux tube axis is considered.

3. Calculations and Results

In order to simulate the actual observations of stellar magnetic fields we have used two methods of deriving a value of the "effective" magnetic field strength from our computed Stokes parameter profiles.

In the first method we compute the field strength B from the wavelength difference, $2\Delta\lambda_m$, between the minima of the two σ-components of Stokes I (or between the two Stokes V maxima if Stokes I does not show sufficiently distinct σ-minima) by the formula:

$$B = \frac{\Delta\lambda_m}{4.67 \times 10^{-13} g \lambda^2}.$$

Here $\Delta\lambda_m$ and λ, the wavelength, are in Å and B is in G. The field strengths derived by this method are listed in Table 2 under the heading $B(\Delta\lambda_m)$.

Table 2. Magnetic field strengths derived from different lines

Line	K2 dwarf		Solar flux tube	
	$B(\Delta\lambda_m)$	$B(fit)$	$B(\Delta\lambda_m)$	$B(fit)$
5250.2	1290	1800	1600	1800
6173.3	2030	2200	1950	2200
8468.4	1190	1600	1500	1750
15648.5	2690	2500	2650	2600

The considerable difference between these values may be explained by the contribution functions of Stokes I at the wavelength of the σ- minima (Figures 1 and 2) and by the variation of the field strength with height. The combination of these two features implies that the field strength at the height of the maximum of the contribution function varies from line to line.

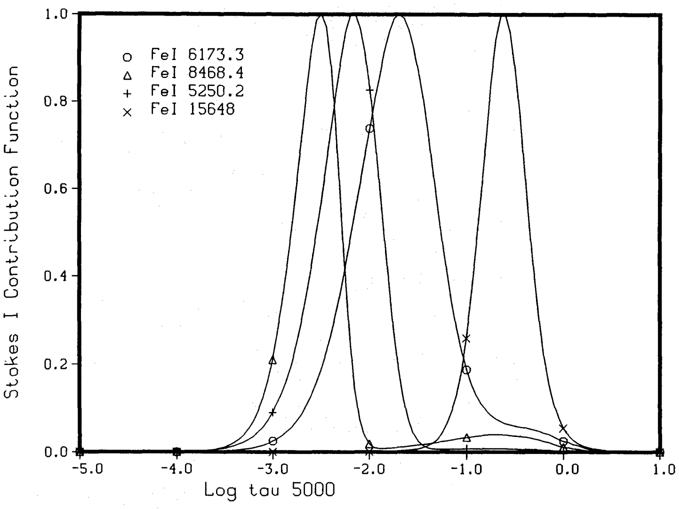

Fig. 1. The Stokes I contribution functions per unit of length in a K2 dwarf magnetic flux tube model

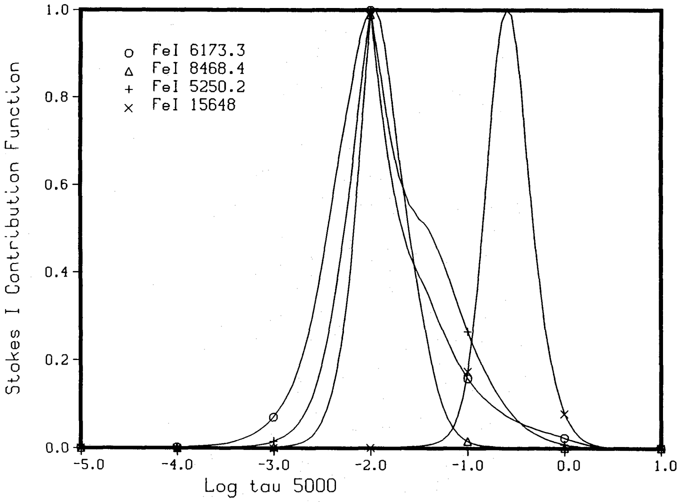

Fig. 2. The Stokes I contribution functions per unit of length in a solar network magnetic flux tube model embedded in a K2 dwarf atmosphere

The second method of deriving a field strength consists of computing the Stokes parameter profiles in an atmosphere with a *uniform* magnetic field and varying this field strength until the best agreement is achieved with the profiles resulting from the models with a vertically varying magnetic field strength. This method is a rough simulation of the techniques used to derive field strengths from stellar data. The field strengths thus derived are listed in Table 2 under the heading $B(fit)$.

For the K2 dwarf model the difference between $B(fit)$ derived from Fe I 8468Å and Fe I 6173Å is approximately 600 G, between the B values derived from Fe I 8468Å and Fe I 15648Å it is 1000 G. 600 G is sufficient to explain a substantial part, but not all, of the discrepancy between the results of Basri and Marcy (1988) and Marcy and Basri (1989) based on Fe I 8468Å on the one hand, and the results of Saar et al. (1986a, b), and Mathys and Solanki (1989), based on Fe I 6173.3Å and on other lines in the visible, on the other hand.

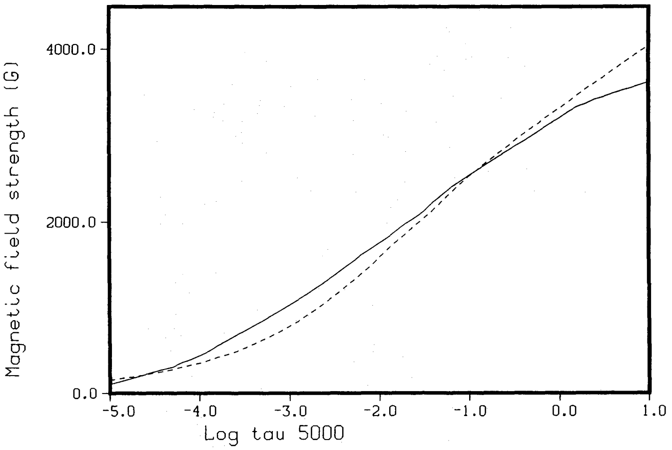

Fig. 3. The magnetic field strength, B, as a function of continuum optical depth at 5000Å. Solid curve: Flux tube with a K2 dwarf atmosphere. Dashed curve: Solar network flux tube. Both models were assumed to be embedded in an undisturbed K2 dwarf atmosphere

The difference in the heights of formation of *disk integrated* profiles of Fe I 6173Å and Fe I 8468Å may be larger than we calculate. The solar limb darkening decreases steadily between 5000Å and 9000Å (e.g. Pierce and Waddell, 1961), mainly due to the decreasing temperature sensitivity of the Planck function. Therefore the disk integrated profile of Fe I 8468Å should obtain a larger contribution from regions near the stellar limb than the corresponding profile of Fe I 6173Å. Since lines are formed higher in the atmosphere near the stellar limb than at disk centre, we expect the difference between the field strengths felt by the disk integrated profiles of the two lines to be even larger than suggested by our calculations. A rough estimate of this effect shows it to be of the order of 50–100 G.

Another consequence of the decrease of the field strength with height is the expansion of magnetic features with height. Thus the fraction of the stellar surface covered by magnetic field (i.e. the filling factor) increases with height. Therefore the filling factor derived from the diverse lines should depend on their heights of formation. For example, for the K2 dwarf model Fe I 8468Å should give a 40% larger filling factor than Fe I 6173Å.

For the solar flux tube model the differences in heights of formation are considerably smaller (Fig. 2) - in fact, three of the contribution functions almost coincide. These differences can only explain a small part of t discrepancy between the various measurements. However, this model is probably too hot for a proper representation of flux tubes on ϵ-Eri. Therefore, the differences between the field strengths felt by the various lines found for this model should be considered a conservative lower limit for ϵ-Eri.

The reason for the different heights of formation is threefold. Firstly, the continuum opacity changes considerably with wavelength. For stars with an effective temperature lower than 6000K 8468Å lies close to the H^- b-f opacity maximum, 15648Å lies close to the opacity minimum and 6173Å and 5250Å lie at intermediate opacities (e.g. Unsöld, 1968, p. 181ff). From this effect alone we expect Fe I 8468Å to be formed highest and Fe I 15648Å to be formed lowest. Secondly, the height of formation increases with increasing line strength (e.g. Magain, 1986), which for equal excitation potential implies mainly increasing $\log gf$. Due to its larger $\log gf$ Fe I 8468Å is again expected to be formed higher than Fe I 6173Å. Thirdly, the height of formation generally decreases with increasing excitation potential. The second and third effects are expected to be considerably larger than the first one (Grossmann-Doerth et al., in preparation).

The difference between the heights of formation in the two models has partly to do with the particular $T(\tau)$ stratifications (temperature gradients affect heights of formation considerably), but mainly with the fact that in the network flux tube model all the lines are relatively weak due to the high temperature and weak Fe I lines are formed at relatively similar τ values.

4. Conclusions

We have shown that the observed strength of a magnetic field in a stellar atmosphere depends on the spectral lines used to measure it — provided the field is similarly structured as in the sun. This effect could be responsible for a part of the discrepancies between diverse observations and it should be taken into account in future analysis, in particular if better data become available. It is particularly important to keep in mind when comparing results based on different lines.

On the other hand, with our method it is, in principle, possible to measure the vertical gradient of the magnetic field strength on late type stars by comparing the field strengths determined by different lines. In this fashion the theory of magnetic field confinement by gas pressure on stars other than the sun could be tested.

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References

Basri, G.S., Marcy, G.W.: 1988, *Astrophys. J.* **330**, 274
Basri, G.S., Marcy, G.W., Valenti, J.A.: 1990, *Astrophys. J.* in press
Giampapa, M.S., Golub, L., Worden, S.P.: 1983, *Astrophys. J.* **268**, L121
Grossmann-Doerth, U., Larsson, B., Solanki, S.K.: 1988, *Astron. Astrophys.* **204**, 266
Gondoin, P., Giampapa, M.S., Bookbinder, J.A.: 1985, *Astrophys. J.* **297**, 710
Magain, P.: 1986, *Astron. Astrophys.* **163**, 135
Marcy, G.W.: 1984, *Astrophys. J.* **276**, 286
Marcy, G.W., Basri, G.: 1989, *Astrophys. J.* **345**, 480
Mathys, G., Solanki, S.K.: 1989, *Astron. Astrophys.* **208**, 189
Parker, E.N.: 1979, *Cosmical Magnetic Fields*, Clarendon Press, Oxford
Pierce, A.K., Waddell, J.: 1961, *Mem. Roy. Astron. Soc.* **68**, 89 W
Robinson, R.D., Worden, S.P., Harvey, J.W.: 1980, *Astrophys. J.* **236**, L155
Saar, S.H.: 1988, *Astrophys. J.* **324**, 441
Saar, S.H.: 1990, in *Solar Photosphere: Structure, Convection, Magnetic Fields*, J.O. Stenflo (Ed.), Kluwer, Dordrecht, *IAU Symp.* **138**, 427
Saar, S.H., Linsky, J.L.: 1985, *Astrophys. J.* **299**, L47
Saar, S.H., Linsky, J.L., Beckers, J.M.: 1986a, *Astrophys. J.* **302**, 777

Saar, S.H., Linsky, J.L., Duncan, D.K.: 1986b, in *Cool Stars Stellar Systems and the Sun*, M. Zeilik, D.M. Gibson (Eds.), Springer Verlag, New York, p. 275

Solanki, S.K.: 1986, *Astron. Astrophys.* **168**, 311

Solanki, S.K., Mathys, G.: 1987, in *Activity in Cool Star Envelopes*, O. Havnes et al. (Eds.), Reidel, Dordrecht, p. 39

Solanki, S.K., Steiner, O.: 1990, *Astron. Astrophys.* in press

Steiner, O., Pizzo, V.J.: 1989, *Astron. Astrophys.* **211**, 447

Stenflo, J.O.: 1989, *Astron. Astrophys. Review* **1**, 3

Unsöld, A.: 1968, *Physik der Sternatmosphären*, Springer Verlag, Berlin

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