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Letter to the Editor

Why rapid rotators have polar spots

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Abstract. Starspots on magnetically active, cool stars preferentially appear near the poles. We suggest that this preference of high latitudes is due to the rapid rotation of these stars which leads to a dominance of the Coriolis force over the buoyancy force in the dynamics of magnetic flux tubes. As a consequence, flux tubes erupting from the deep parts of the stellar convection zone follow a path nearly parallel to the axis of rotation and thus necessarily surface at high latitudes, unless their initial field strength exceeds a critical value for which buoyancy becomes dominant again. It is shown that for stars with rotation periods below about 10 days flux tubes with such large field strength (of the order of 10^6 G) cannot be formed and stored since they are unstable with respect to non-axisymmetric disturbances. Consequently, magnetically active stars with rapid rotation exhibit magnetic flux eruption at high latitudes and polar starspots.

Key words: stars: activity of – stars: magnetic field – stars: rotation of – magnetic flux tubes

1. Introduction

A major success of the last decade of cool-star research has been the surface imaging of rapidly rotating, active stars using the Doppler imaging technique (e.g. Vogt & Penrod 1983, Strassmeier 1990, Piskunov et at. 1990, Vogt & Hatzes 1991, Strassmeier et al. 1991, Dempsey et al. 1992, Kürster et al. 1992). The main feature of the Doppler images are large spots (they can easily cover 10% of the visible hemisphere) at high stellar latitudes. On most of the imaged stars the dominant spot actually straddles the stellar pole, in complete contrast to the sun, whose spots rarely appear at latitudes higher than 30°. The spot latitudes on rapid rotators still beg a physically founded explanation.

All available evidence suggests that, like sunspots, stellar spots are magnetic in nature. For example, a correlation between spots and magnetic fields has been inferred from Zeeman Doppler images (Donati et al. 1992, Saar et al. 1992). Therefore, we need to understand why the surface magnetic field on active stars is concentrated at high latitudes. In the present letter we point out that for sufficiently rapidly rotating stars the emergence of significant amounts of flux at their poles is unavoidable.

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2. Coriolis force and polar spots

In the case of the Sun observational evidence (e.g. Zwaan 1992) supports the view that magnetically active regions and sunspots form by the eruption of buoyant strands of magnetic flux from the deep layers of the convection zone (Parker 1955). We conjecture that the same mechanism is responsible for the formation of large spots on active cool stars with convective envelopes: a dynamo operating at the lower boundary of the convection zone (or in an overshoot layer below) creates a system of toroidal magnetic flux which develops buoyancy-induced instabilities (Spruit & van Ballegooijen 1982, Moreno-Insertis 1986), erupts towards the surface, and forms large magnetic starspots.

In a rotating system, we have to include the effect of angular momentum conservation which tends to suppress motion perpendicular to the axis of rotation via action of the Coriolis force. As first pointed out by Choudhuri & Gilman (1987), the ratio of Coriolis force ($\mathbf{F}_{\rm C}$) to buoyancy force ($\mathbf{F}_{\rm B}$) determines whether an erupting magnetic flux tube follows a radial path to the surface ($\mathbf{F}_{\rm B}$ dominates) or moves more or less parallel to the axis of rotation and surfaces at high stellar latitudes ($\mathbf{F}_{\rm C}$ dominates).[1] For a toroidal flux tube of internal density ϱ_i, which rises with velocity v the ratio of forces is of the order

$$\frac{|\mathbf{F}_{\rm C}|}{|\mathbf{F}_{\rm B}|} \simeq \frac{2\varrho_i v\Omega}{|\Delta\varrho|\, g}, \qquad (1)$$

where g is the gravitational acceleration, Ω is the angular velocity of the star, and $\Delta\varrho = \varrho_e - \varrho_i$ is the density difference between the surroundings of the tube and its interior. If the flux tube has the same temperature as the ambient gas and if the magnetic pressure is much smaller than the gas pressure, i.e. $\beta \equiv 8\pi p/B^2 \gg 1$ (which is a realistic assumption for the deep parts of the convection zones of cool stars), we may write

$$\frac{\Delta\varrho}{\varrho_i} \simeq \frac{B^2}{8\pi p_e}, \qquad (2)$$

where B is the magnetic field strength and p_e is the external gas pressure. Introducing the equipartition field strength with respect to the convective velocity, v_c, by $B_{\rm eq} = v_c\sqrt{4\pi\rho_e}$

[1] In some cases it is more adequate to consider the ratio of total rise time of the erupting tube to rotational period. However, apart from numerical factors of order unity, the estimates obtained with such an approach agree with those given below.

and assuming hydrostatic stratification with scale height H, i.e. $p_e/H = \rho_e g$, we obtain from Eqs. (1) and (2)

$$\frac{|\mathbf{F}_C|}{|\mathbf{F}_B|} \simeq \left(\frac{B_{eq}}{B}\right)^2 \left(\frac{v}{v_c}\right) \left(\frac{2}{\mathrm{Ro}}\right), \quad (3)$$

where Ro is the *Rossby number* defined by

$$\mathrm{Ro} \equiv \frac{v_c}{2H\Omega}. \quad (4)$$

The Rossby number essentially is the ratio between the period of rotation and the convective turnover time and thus measures the degree of rotational influence on the convective motions. Noyes et al. (1984) suggest that stellar activity is correlated with the Rossby number[2] which would be in agreement with predictions from dynamo theory (e.g. Durney & Latour 1978). Since a rough estimate yields that the velocity of buoyant rise is of the order of the Alfvén velocity $v_A = B/\sqrt{4\pi\varrho_e}$ (Parker 1975) we may write $v/v_c = B/B_{eq}$ and finally obtain

$$\frac{|\mathbf{F}_C|}{|\mathbf{F}_B|} \simeq \left(\frac{B_{eq}}{B}\right) \left(\frac{2}{\mathrm{Ro}}\right) = \frac{2}{\mathrm{Ro_m}}, \quad (5)$$

where

$$\mathrm{Ro_m} \equiv \frac{v_A}{2H\Omega} \quad (6)$$

defines a 'magnetic Rossby number'. At the bottom of the solar convection zone we have Ro ≈ 0.2, so that for equipartition fields of the order of 10^4 G the r.h.s. of Eq. (5) has a value of 10. Consequently, flux tubes with equipartition field strength have to move parallel to the rotation axis and would erupt at high latitudes, in contradiction to the observed characteristics of solar activity. In fact, the simulations of Choudhuri and Gilman (1987, see also Choudhuri 1989) demonstrate the dominance of Coriolis forces for large equipartition flux tubes in the solar convection zone. The effects of turbulence and/or Kelvin-Helmholtz instability are unlikely to modify this conclusion since they lead to suppression of the polar escape of equipartition fields only for very small tubes below a few hundred km radius, much smaller than the sizes of sunspots let alone starspots (D'Silva & Choudhuri, 1991).

Eq. (5) shows that the simplest way to avoid a polar eruption in the case of the Sun is to assume that the actual field strength at the bottom of the convection zone is about ten times larger than B_{eq}, of the order of 10^5 G. As shown by Durney et al. (1990) such a large field strength is not incompatible with $\alpha\Omega$-dynamo models if a sufficiently large energy input for differential rotation is available. For dynamo models based on the instability of toroidal fields (e.g. Schmitt, 1987) large field strength actually is a necessary ingredient. The field strength which can be reached is in fact limited by the development of non-axisymmetric instabilities (see further below).

Active and spotted stars generally rotate much more rapidly than the Sun and thus the relevant Rossby numbers are much smaller, favouring a polar eruption of magnetic flux. Fig. 1 illustrates the dependence of the path taken by a rising flux tube on the rotation rate: the trajectory of an individual mass element of an axisymmetric toroidal flux tube (i.e., a flux ring) of $2 \cdot 10^5$ G starting at 20° latitude at the lower boundary of the solar convection zone is given for three values of the rotation rate, namely $\Omega = \Omega_\odot = 2.7 \cdot 10^{-6}$ s^{-1} (the actual solar value, dashed curve), $3\,\Omega_\odot$ (dotted curve), and $10\,\Omega_\odot$ (full curve). The trajectories which have been calculated by numerical integration of the equation of motion (see Moreno-Insertis et al. 1992) are depicted as curves in a meridional plane containing the (vertical) rotation axis. The flux tubes pierce this plane perpendicularly and stay toroidal during the whole evolution.

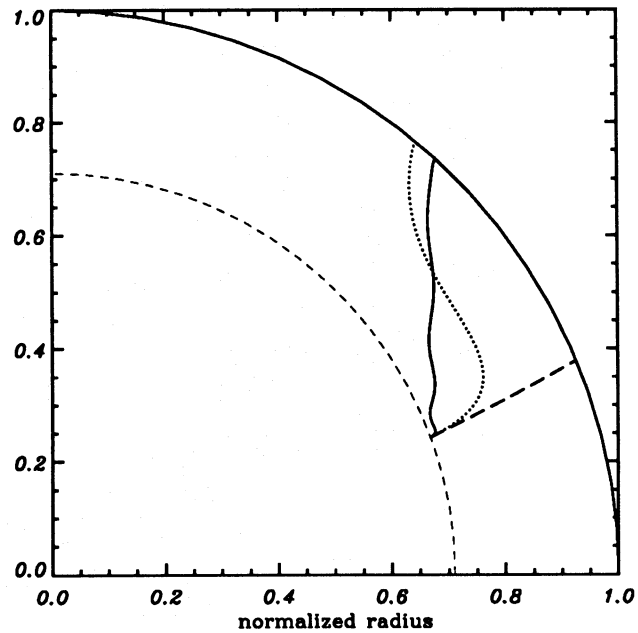

Fig. 1. Motion of a toroidal flux tube with (initial) field strength $B = 2 \cdot 10^5$ G starting at a latitude of 20° at the bottom of the solar convection zone for different values of the rotation rate. The curves show the trajectories of a single mass element of the axisymmetric flux ring in a meridional plane which contains the (vertical) axis of rotation. The dashed curve is for a rotation rate of $\Omega = \Omega_\odot = 2.7 \cdot 10^{-6}$ s^{-1} (the actual solar rotation rate, 27d period), the dotted curve for $\Omega = 3\,\Omega_\odot$ (9d period), and the full curve for $\Omega = 10\,\Omega_\odot$ (3d period). The tube rises almost radially for $\Omega = \Omega_\odot$, but the Coriolis force becomes important for the larger values of the rotation rate. For $\Omega = 10\,\Omega_\odot$, the tube practically moves parallel to the axis of rotation.

While the tube follows an almost radial trajectory for $\Omega = \Omega_\odot$, the Coriolis force constrains the motion more and more for larger values of Ω: the tube has to move parallel to the axis of rotation and erupts at high latitudes (about 50° in this case). The amplitude of the inertial oscillation which is superposed on the axial motion decreases with increasing rotation rate. For K and M stars which have deeper convection zones than the Sun, the latitude of eruption is correspondingly larger and the magnetic flux surfaces in the vicinity of the poles, even if the erupting tubes start near the equatorial plane.

Eq. (5) defines a minimum magnetic field strength, $B_{\min}$, which must be exceeded for radial eruption of magnetic flux (dominating buoyancy), viz.

$$B_{\min} = 4H\Omega\sqrt{4\pi\rho_e}. \quad (7)$$

If B is smaller than $B_{\min}$, the flux tubes tend to move parallel to the rotation axis and erupt in the polar regions. Using values appropriate for the bottom of the solar convection zone,

[2] See Schrijver (1992) for a deviating point of view.

i.e. $H_\odot = 5.7 \cdot 10^9$ cm and $\rho_\odot = 0.23$ g·cm^{-3} (Spruit, 1977), we may rewrite Eq. (7) as

$$B_{\rm min} \simeq 10^5 \left(\frac{\Omega}{\Omega_\odot}\right)\left(\frac{H}{H_\odot}\right)\left(\frac{\rho}{\rho_\odot}\right)^{1/2} {\rm G}. \tag{8}$$

A model of a K5 dwarf of 0.6 solar mass (Kippenhahn & Thomas 1964) yields $H = 4.4 \cdot 10^9$ cm and $\rho = 1.98$ g·cm^{-3} so that

$$B_{\rm min} \simeq 2.3 \cdot 10^5 \left(\frac{\Omega}{\Omega_\odot}\right) {\rm G}. \tag{9}$$

A recent model of a 0.6 solar mass star provided by F. D'Antona (cf. D'Antona and Mazzitelli 1985) gives $H = 3.3 \cdot 10^9$ cm and $\rho = 3.7$ g·cm^{-3} which results in the same $B_{\rm min}$ as the Kippenhahn-Thomas model. The relations (8) and (9) are illustrated in Fig. 2 which gives $B_{\rm min}$ as a function of rotation period $P = 2\pi/\Omega$ for the Sun (lower curve) and for the K5 dwarf (upper curve). We see that rapidly rotating, active stars with periods of the order of a few days have their magnetic flux erupting near the poles unless the flux tubes have fields well in excess of 10^6 G at the bottom of their convection zones.

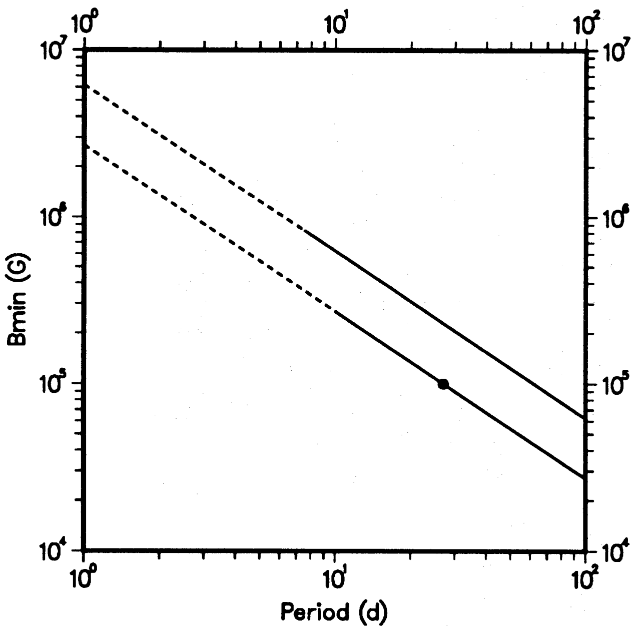

Fig. 2. Minimum field strength for non-polar eruption of flux tubes as function of stellar rotation period. For $B < B_{\rm min}$ the Coriolis force dominates and rising flux tubes have to move parallel to the axis of rotation and erupt in the polar regions. The lower curve is for a solar model (the point corresponding to its present rotation period of 27 days is indicated by the dot) while the upper curve applies to a K5 dwarf. The dashed parts of the curves indicate flux tube instability with respect to non-axisymmetric disturbances; in these cases, tubes with $B > B_{\rm min}$ cannot be formed and polar eruption becomes unavoidable.

It is not known whether differential rotation in rapidly rotating stars is strong enough to produce such large field strengths. Even if this would be possible, it requires very large values of the subadiabaticity in the overshoot region in order to store flux tubes with megagauss fields (Moreno-Insertis et al. 1992). In any case, an upper limit to the field strength is given by the development of non-axisymmetric instabilities (Spruit & van Ballegooijen 1982, van Ballegooijen 1983, Ferriz-Mas and Schüssler 1992): a flux tube which is stored in a stably stratified convective overshoot region and gets gradually amplified by differential rotation becomes unstable if the field strength exceeds a critical value, flux loops erupt towards the surface and form active regions.

The dashed parts of the curves in Fig. 2 indicate instability of flux tubes with $B \geq B_{\rm min}$ predicted by the criteria given by Ferriz-Mas & Schüssler (1992). For rapidly rotating stars, the critical field for the onset of instability is actually much smaller than $B_{\rm min}$. We have assumed here that the flux tubes are stored in a subadiabatic overshoot region with $\nabla - \nabla_{\rm ad} = -5 \cdot 10^{-6}$ which seems a reasonable value for both cases (cf. Skaley & Stix 1991; Zahn 1991). We conclude from Fig. 2 that for G and K dwarfs with periods below 8–10 days high-latitude spots are unavoidable: flux tubes become unstable and erupt before they reach the necessary field strengths $B > B_{\rm min}$ for buoyancy to dominate.

3. Discussion and conclusions

The simple physical considerations described in Sect. 2 imply that on cool stars with sufficiently large rotation rate (small magnetic Rossby number) the surface magnetic flux is concentrated at high latitudes.

Almost all stars with a rotation period $P_{\rm rot} \lesssim 5 - 10$ days and spot latitudes determined using Doppler imaging techniques clearly exhibit high-latitude or polar spots e.g. UX Ari (K0 IV, $P_{\rm rot} \approx 6\overset{d}{.}4$), EI Eri (G5 IV, $1\overset{d}{.}9$), V711 Tau (K1 IV, $2\overset{d}{.}8$), HD 199178 (G5 IV, $3\overset{d}{.}3$), HD 155555 ($1\overset{d}{.}7$) and possibly YY Men (K1 IIIp, $9\overset{d}{.}5$).[3] In addition, high latitude spots have been identified photometrically on a number of rapid rotators, e.g. BY Dra (M0 Ve, $3\overset{d}{.}8$), AU Mic (M2 Ve, $4\overset{d}{.}9$), II Peg (K2 IV-V, $6\overset{d}{.}7$), SV Cam (G3 V, $0\overset{d}{.}6$), RT And (F8 V, $0\overset{d}{.}6$) and BH Vir (F8 IV–V, $0\overset{d}{.}8$).[4] On the other hand, the evidence for polar spots is still viewed as controversial by some authors (e.g. Byrne 1992, Patterson et al. 1992).

Taken at face value, our analysis predicts polar spots for all these stars. Thus there is no need to invoke qualitative differences in the dynamo action between moderately active (relatively slowly rotating) and strongly active (rapidly rotating) stars to explain high-latitude spots. Nor is it neccessary to arbitrarily invoke a migration of stellar spots to the poles after their emergence near the equator. High-latitude magnetic concentrations, for which starspots serve as a proxy, are the natural product of the rapid rotation, or, more precisely, small magnetic Rossby numbers of these stars. For such stars, the main problem now is to explain the presence of the *low*-latitude spots seen in many Doppler images.

Strictly speaking, our results are only valid for single main-sequence stars and we can only guess at the critical $P_{\rm rot}$ for giants and pre-main-sequence stars. A more rigorous treatment of giants using relevant stellar structure models will be the subject of a subsequent paper. With the help of such data, we hope

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3 Conflicting evidence regarding polar spots on YY Men = HD 32918 is found by Piskunov et al. (1990, no polar spot) and Kürster et al. (1992, a large polar spot).

4 Photometrically determined spot latitudes are, in general, less reliable than those derived from Doppler imaging.

to decide whether the high-latitude spots on relatively slowly rotating giants, like σ Gem (K1 III, $P_{\rm rot} = 19.4$ d, Dempsey et al. 1992), are due to the effects described here.

Close binaries with synchronous rotation form an important subgroup of active stars (e.g. RS CVn systems) and many of the stars exhibiting polar spots actually belong to such systems. Since the Coriolis force is independent of the location of the axis of rotation, the dynamics of erupting flux tubes in rotationally synchronized binaries is determined basically by the same effects as described in Sec. 2 for single stars. However, the tidal forces felt by the stars are asymmetric with respect to their axes and may therefore lead to preferred ('active') longitudes for flux tube eruption (Caligari, Moreno-Insertis and Schüssler, in preparation). It remains to be seen whether these asymmetries may also open paths for low latitude eruption of flux tubes. Observational evidence exists for the presence of active longitudes on a number of rapidly rotating binaries. Four of the best studied systems, the eclipsing binaries SV Cam, RT And, BH Vir and WY Cnc (all with $P_{\rm rot} < 1$d), possess two active longitudes located at phases 0.25 and 0.75 (Zeilik et al. 1988, 1989, 1990a,b).

Since, for rapidly rotating single stars, the majority of the magnetic flux must surface near the pole, the polar spots are expected to be complex magnetic structures containing both magnetic polarities, in order to ensure magnetic flux conservation. Thus, our suggestions are consistent with the hypothesis that polar spots are in reality conglomerates of densely packed smaller spots.

We have illustrated the polar eruption of magnetic flux tubes in rapidly rotating stars with the help of crude estimates. A much more detailed study based on a large number of stellar models and incorporating a more complete description of the flux tube dynamics is neccessary in order to obtain reliable quantitative information and to investigate whether the result holds also for stars of other spectral types and luminosity classes.

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