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Are sunspot penumbrae deep or shallow?

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Abstract. From the strength and inclination of the magnetic field measured across large symmetric sunspots we estimate the fraction of the total magnetic flux of the sunspot passing through the solar surface in the penumbra. It is found that on average approximately 1/2–2/3 of the total magnetic flux of the spot emerges in its penumbra. Sunspot penumbrae are therefore deep, i.e. the $\tau = 1$ level does not correspond to the lower magnetic boundary of the spot in its penumbra (except perhaps near its outer edge). Furthermore, the analysed data do not support the passage of any significant amount of magnetic flux through the solar surface (in either direction) at or beyond the edge of the sunspot. The observations support models of the sunspot magnetic field which are bounded by a relatively sharp current sheet. Evidence for a substantial deviation from a potential field in the penumbra is found in the analysed symmetric sunspots. Finally, at the height of line formation the field strength averaged over the whole umbra of all the analysed sunspots is approximately 2250 G, while the field strength averaged over the whole sunspot is roughly 1350 G. The latter value is similar to the field strength measured in small-scale magnetic features.

Key words: sunspots – Sun: magnetic fields – Sun: activity

1. Introduction

Sunspot models assume that penumbrae are either shallow (e.g. Schmidt et al. 1986), or deep (e.g. Jahn 1989). A penumbra is called shallow if the current sheet bounding the sunspot roughly corresponds to the $\tau = 1$ surface in the penumbra, i.e. when no (or only few) field lines cross the solar surface in the penumbra. In such a model the total magnetic flux of the sunspot emerging from the solar interiour passes through the umbra. On the other hand, a significant number of field lines do cross the solar surface within a deep penumbra. A rough (indirect) measure of the 'depth' of a penumbra is therefore the relative amount of magnetic flux emerging in it. It appears of basic importance to us to determine whether sunspot penumbrae are deep or shallow, since the energy transport mechanism in the two cases is expected to be quite different. Our analysis also allows us to determine other interesting global parameters of sunspots, such as the amount of return flux, the presence of a current sheet at the sunspot boundary and the field strength averaged over the umbra and the whole sunspot.

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2. Idea, data and procedure

The basic idea is to determine the total magnetic flux in the umbra, Φ_u, and to compare it with the magnetic flux in the penumbra, Φ_p. If $\Phi_p \ll \Phi_u$ then the penumbra is shallow, if $\Phi_p \gtrsim \Phi_u$, it is deep.

The determination of Φ_p and Φ_u is particularly simple in the case of large, symmetric sunspots. To determine the flux, we need the magnetic field strength, B, and inclination angle to the vertical, γ', measured as a function of radial distance r from sunspot centre. We have used $B(r)$ and $\gamma'(r)$ values published by Beckers & Schröter (1969), Wittman (1974), Kawakami (1983), Lites & Skumanich (1990) and Solanki et al. (1992b). These are, to our knowledge, the most thorough investigations of this type in the last two decades. The results of Adam (1990), although also of high quality, cannot be used, since she measures B only for $r/r_p \lesssim 0.7$, where r_p is the radius of the outer penumbral boundary. In this r/r_p range her B and γ' values are consistent with the values used by us. For simplicity we restrict ourselves to using azimuthally averaged curves of $B(r)$ and $\gamma'(r)$.

The $B(r/r_p)/B(r=0)$ curves used in the present investigation are plotted in Fig. 1. The corresponding $\gamma'(r/r_p)$ curves are shown in Fig. 2. Note that although Kawakami studied four sunspots he found differences between them only in the $B(r=0)$ and r_u/r_p values (r_u is the radius of the umbra); their $B(r/r_p)/B(r=0)$ and $\gamma'(r/r_p)$ are identical.

We first determine $\Phi(r)$, from which we can easily obtain

$$\Phi_u = \Phi(r_u), \qquad \Phi_p = \Phi(r_p) - \Phi(r_u). \tag{1}$$

$\Phi(r)$ is determined using two different methods.

1. By adding together the vertical component of B for radial distances r' less than some specified r:

$$\Phi(r) = 2\pi \int_0^r B(r') \cos\gamma'(r') r' dr' \,. \tag{2}$$

This technique assumes that the Zeeman signal is produced in a perfectly horizontal layer. It does not take into account the Wilson depression with any associated inclination of the $\tau = 1$ surface. Due to this possible inclination, a small $B_z = B\cos\gamma'$ component may be present even in shallow penumbrae, so that the derived $\Phi(r)$ may be slightly too large for $r \gtrsim r_u$.

2. The flux emerging within a radial distance r of the sunspot centre must equal the flux passing through the specific spherical segment of radius $r/\sin\gamma'$, which intersects the solar surface at

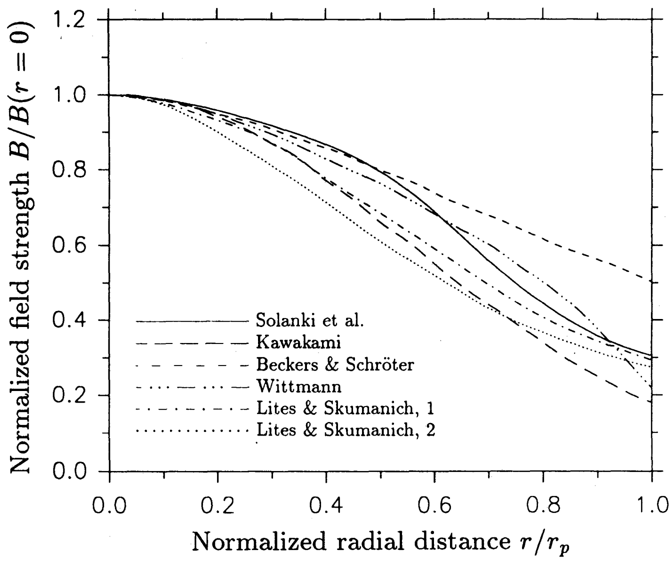

Fig. 1. Field strength normalized to its value at the centre of the sunspot, $B/B(r=0)$, vs. radial distance from sunspot centre, normalized to the radius of the outer penumbral boundary, r/r_p. Each curve represents the mean observed $B(r/r_p)/B(r=0)$ dependence for a given sunspot, or for a group of sunspots

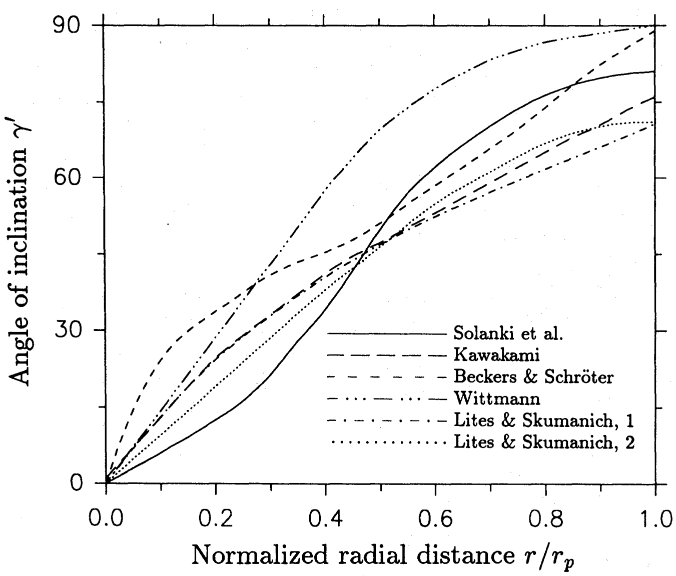

Fig. 2. Inclination angle of the magnetic vector to the vertical, γ', vs. r/r_p, based on the same observational data as Fig. 1

a distance r from the spot centre. If the field of the spot does not deviate too strongly from a potential field on this spherical segment then it (the field) will be approximately constant thereon and it will intersect the latter almost radially. The geometry is illustrated in Fig. 3. Thus,

$$\Phi(r) = B(r)A(r) = B(r) \cdot 2\pi \left(\frac{r}{\sin \gamma'(r)} \right)^2 (1 - \cos \gamma'(r)), \qquad (3)$$

where $A(r)$ is the area of the segment. This technique does not suffer from the same disadvantage as the first method (it should give $\Phi_p = 0$ for a shallow penumbra). On the other hand, it makes some assumptions about the structure of the field. A comparison with the sunspot model of Pizzo (1986), which is based on a numerical solution of the full magnetohydrostatic force balance, suggests that the assumptions underlying the second method are quite reasonable in the outer part of the sunspot, and are still acceptable in the umbra (see his Fig. 6). The two techniques are thus complementary to each other.

2.1. Flux of the penumbra

We have applied both methods to the data set of $B(r)$ and $\gamma'(r)$ curves mentioned in Sect. 2, covering a total of nine sunspots. The $\Phi(r)$ values derived from the 2nd technique, normalized such that $\Phi_{\max} = 1$, are plotted vs. r/r_p in Fig. 4. The $\Phi(r)$ curves derived from the first technique look similar. For a shallow penumbra we expect the curves to be flat for $r/r_p \geq r_u/r_p \approx 0.4 - 0.5$. This is obviously not the case. Only the observations of Kawakami (1983) show a flat portion in the outermost part of the penumbra ($r/r_p \gtrsim 0.8$). The $\Phi(r)$ curve derived from the data of Wittmann (1974) suggests the presence of some return flux in the outer part of the penumbra. The first method shows neither a pronounced flattening of $\Phi(r)$ (Kawakami's data), nor any sign of return flux (Wittmann's data). Indeed, it requires a change in polarity, which is not observed, for the first method to signal a return flux.

The difference between the $\Phi(r)$ curves derived from the various sources may, in principle, be due to intrinsic differences between the sunspots. However, the techniques used by the various investigators to determine $B(r)$ and $\gamma'(r)$ are rather heterogeneous and we suspect that differences in analysis are responsible for a considerable fraction of the scatter in $\Phi(r)$. The differences between the (unnormalized) $\Phi(r/r_p)$ curves are most pronounced for $r/r_p \gtrsim 0.8$, where the differences between the $B(r/r_p)/B(r=0)$ curves are also largest, probably due to the difficulty in measuring $B \lesssim 1000$ G using lines in the visible. Observations in the infrared are much better suited (e.g. Deming et al. 1991; Solanki et al. 1992a). The 12 μm (Deming et al. 1988) and 1.5 μm observations (Solanki et al. 1992b; McPherson et al. 1992) suggest that the $B(r_p)$ values of Wittmann (1974) and Kawakami (1983) are too low, while the $B(r_p)$ value of Beckers & Schröter (1968) is too high.

Leaving aside the details of the $\Phi(r)$ dependence, consider now the Φ_u and Φ_p values, derived using both methods. Table 1 lists the following sunspot parameters for each of the analysed sunspots: r_u/r_p, $B(r=0)$, Φ_u, Φ_p, $\Phi_p/\Phi_{\rm tot}$, $\langle B_{\rm umb} \rangle$ and $\langle B_{\rm spot} \rangle$. The Φ_u, Φ_p and $\Phi_p/\Phi_{\rm tot}$ determined by each of the two methods are listed in separate columns. The listed Φ_u and Φ_p values correspond to a sunspot with $r_p = 1$. $\langle B_{\rm umb} \rangle$ and $\langle B_{\rm spot} \rangle$ are the field strength averaged over the umbra and over the whole sunspot, respectively. They are discussed further in Sect. 3.6.

It is evident from the $\Phi_p/\Phi_{\rm tot}$ values listed in Table 1 that a sizable fraction of the magnetic flux of the sunspot passes through the penumbra, on the average roughly 1/2–2/3 of $\Phi_{\rm tot}$. Consequently, sunspot penumbrae are deep.

2.2. Is the derived Φ_p significant?

Is a thin penumbra really incompatible with the data, i.e. do $B(r)$ and $\gamma'(r)$ produced by a simple model of a thin penumbra differ from the curves plotted in Figs. 1 and 2 by more than the scatter in the data points? We have tested this for the 1.5 μm observations of Solanki et al. (1992b). Figure 5 shows the $B(r/r_p)$ values derived

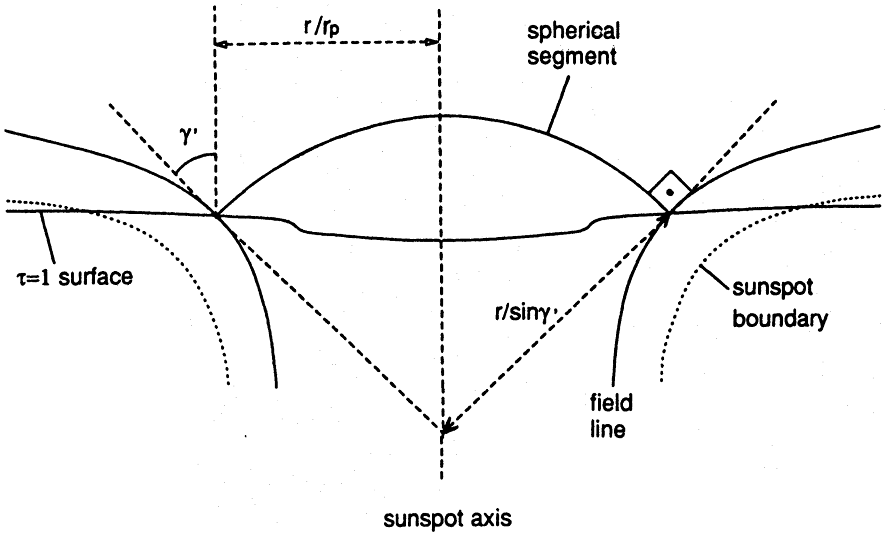

Fig. 3. Cutaway of a sunspot illustrating the parameters involved in the second method of flux determination

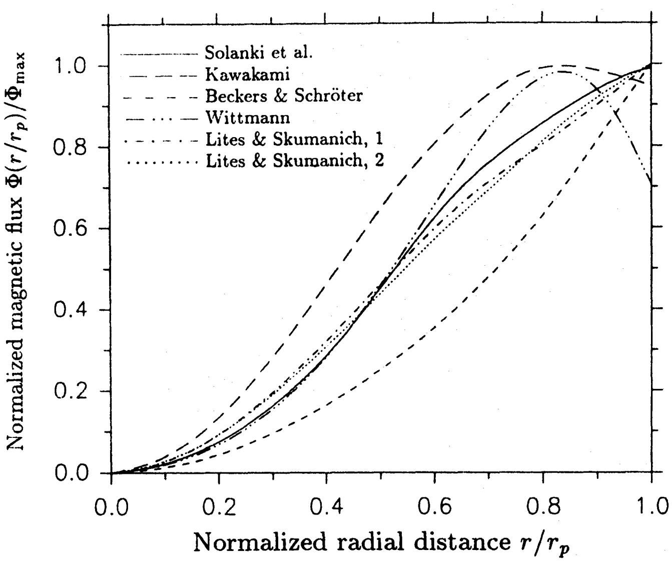

Fig. 4. Magnetic flux Φ emerging within radial distance r of the sunspot centre, normalized to a maximum value of unity, Φ/Φ_{max}, vs. r/r_p

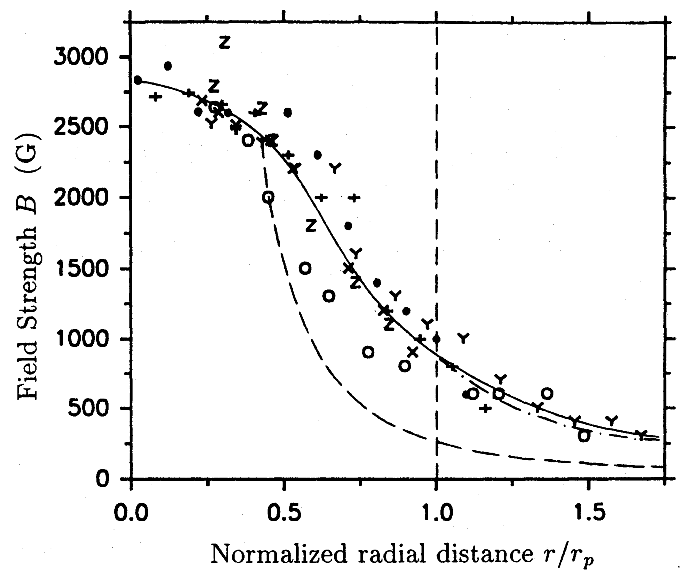

Fig. 5. Field strength B vs. r/r_p. Circles, dots, plusses, crosses, 'Y' and 'Z' symbols represent the observations of Solanki et al. (1992b) along different slices through a sunspot. Solid curve: Eyeball fit through the data points, dashed curve: $B(r)$ expected if no flux passes through the solar surface in the penumbra. Dot-dashed curve: $B(r)$ if no flux passes through the solar surface outside the penumbra

from the data (the various symbols refer to different parts of the sunspot, see Solanki et al. 1992b for details). The solid curve is an eyeball fit to the data points, while the dashed curve is the expected $B(r)$ for $\Phi_p = 0$, calculated according to the second method, assuming that the measured $\gamma'(r)$ is correct. The dashed curve lies well outside the scatter of the data points. The condition $B_z(r \gtrsim r_u) \approx 0$ (the signature of a thin penumbra, according to method 1) is equally inconsistent with the data. Similarly, the synthetic γ' curves produced by assuming that $\Phi_p = 0$ and that the measured $B(r)$ is correct are also incompatible with the data. We conclude that the observations deviate from the predictions of a thin penumbra by a significant amount.

2.3. Flux conservation beyond the outer penumbral boundary

Solanki et al. (1992b) measured $B(r)$ and $\gamma'(r)$ of the sunspot field beyond the visible boundary of the spot (out to $r/r_p \approx 1.7$). Thus, for this spot it is possible to determine how much flux passes through the solar surface at or just outside r_p, so that the return-flux model proposed by Osherovich (1982) and Flå et al. (1982) can be tested.

We find no sign of flux disappearance at $r = r_p$ (there is no obvious jump in either B or γ' there) or in the superpenumbra within the uncertainties of the measurements. The dot-dashed curve in Fig. 5 is the $B(r)$ expected from method 2 if the magnetic flux is conserved at $r/r_p > 1$. Thus our analysis suggests that no flux appears or disappears at and outside the visible outline of the sunspot. Solanki et al. (1992b) reached a similar conclusion using a simpler, less reliable technique.

Table 1. Magnetic flux and field strength in sunspots

Investigators	r_u/r_p	$B(r=0)$ (G)	Φ_u Method 1	Φ_p Method 1	Φ_u Method 2	Φ_p Method 2	Φ_p/Φ_{tot}(%) Method 1	Φ_p/Φ_{tot}(%) Method 2	$\langle B_{umb} \rangle$ (G)	$\langle B_{spot} \rangle$ (G)
Beckers & Schröter	0.35	2550	800	1600	1000	6900	67	87	2360	1700
Wittmann	0.4	2705	1000	700	1450	2200	41	60	2430	1550
Kawakami	0.45	2100	1000	1000	1150	850	50	43	1770	990
	0.47	2700	1400	1200	1550	950	46	38	2240	1270
	0.44	2600	1200	1300	1350	1050	52	44	2200	1220
	0.42	2500	1050	1350	1200	1100	56	48	2180	1200
Lites & Skumanich	0.4	2480	900	1400	1000	2200	61	69	2070	1210
	0.4	2630	1200	1700	1200	2450	59	67	2250	1350
Solanki et al.	0.42	2850	1350	1500	1500	3300	53	69	2620	1680
Average	0.42	2570	1100	1300	1250	2350	54	59	2240	1350

2.4. Are sunspots bounded by current sheets?

It is evident from Fig. 1 that $B(r_p) \neq 0$. In particular, infrared measurements suggest that $B(r) \approx$ 800–900 G (Deming et al. 1988; McPherson et al. 1992; Solanki et al. 1992b). Furthermore, there is no evidence for the presence of additional magnetic flux at $r \gtrsim r_p$. These two facts, taken together, support sunspot models with a sharp boundary, i.e. a current sheet at the interface to the non-magnetic atmosphere. Such sunspot models have been constructed by e.g. Jahn (1989) and Pizzo (1990), cf. Schmidt & Wegman (1983). Models without a sharp magnetic boundary (e.g. Deinzer 1965; Yun 1971; Pizzo 1986) predict either that additional magnetic flux must emerge at $r > r_p$, or that the field strength drops to zero at $r = r_p$.

2.5. Deviations from a potential field

Table 1 shows that although the Φ_u values derived from both methods are similar, for 5 out of 9 analysed sunspots the absolute value of Φ_p (method 1) is considerably smaller than Φ_p (method 2). An inclined $\tau = 1$ surface in the penumbra would lead us to expect the opposite. One possibility to explain this anomaly is that the measured B or γ' values are wrong. Particularly interesting are possible errors in γ', since by decreasing γ' in the penumbra Φ_p obtained with method 1 is increased while Φ_p obtained with method 2 is decreased. We find that γ' has to be reduced by at least 15–20° throughout the penumbra before Φ_p(method 1) > Φ_p(method 2) for all observed sunspots. An error of this magnitude is too large to be realistic (e.g. it would imply an inclination of only 50–60° at r_p), even considering that fields with different inclinations in the penumbra ("uncombed fields", Degenhardt and Wiehr 1991) could systematically affect γ' measurements. The most plausible remaining explanation is that the field at $r \gtrsim r_u$ deviates strongly from a potential configuration. The rapid flattening of the field lines with increasing r, found by Solanki et al. (1992c) near $r = r_u$, is completely consistent with this conclusion and may well provide an explanation for the deviations found between the two methods.

2.6. Average field strengths

The last two columns of Table 1 list the field strength averaged over the umbra $\langle B_{umb} \rangle$ and over the whole spot $\langle B_{spot} \rangle$. First the field strength $\langle B(r) \rangle$ averaged over the suface area within a radial distance r of the sunspot centre was determined.

$$\langle B(r) \rangle = \frac{2}{r^2} \int_0^r B(r') r' \, dr'. \tag{4}$$

Then it was straightforward to obtain $\langle B_{umb} \rangle = \langle B(r_u) \rangle$ and $\langle B_{spot} \rangle = \langle B(r_p) \rangle$. Although $\langle B_{umb} \rangle$ values lie in a range familiar for sunspots, the $\langle B_{spot} \rangle$ values are surprisingly small. They are of the same magnitude as the field strengths measured in small-scale magnetic features (e.g. Stenflo & Harvey 1985; Rabin 1992; Rüedi et al. 1992).

3. Conclusions

From an analysis of the radial dependence of the field strength and inclination angle of nine sunspots we conclude that a significant fraction (approximately 1/2–2/3) of the magnetic flux of a sunspot emerges in the penumbra; i.e. sunspot penumbrae are deep. This is in agreement with most magnetohydrostatic models of the sunspot magnetic field (e.g. Schlüter & Temesvary 1958; Deinzer 1965, Yun 1971; Pizzo 1986; Jahn 1989). Models of a shallow penumbra (e.g. Schmidt et al. 1986; cf. Nordlund & Stein 1989) and observations suggesting a penumbral canopy (e.g. Giovanelli 1982) are not compatible with our analysis. Spruit (1981) reached qualitatively the same conclusion, based on a comparison of the inclination angles measured by Beckers & Schröter (1969) with estimates of the Wilson depression.

There is no evidence for any flux directly connected to the sunspot passing through the solar surface outside the penumbra, close to the sunspot. Our analysis therefore rules out return-flux models (e.g. Osherovich 1982; Flå et al. 1982). It also does not support the disappearance of flux at the penumbral edge, which has been invoked to reconcile mass conservation and the Evershed effect. However, more observations are required to decide this point. The observations further suggest that only models bounded by a current sheet are realistic. The present results are consistent with the conclusion of Solanki et al. (1992c) that magnetic curvature forces are important, particularly near the umbral boundary. Finally, we find that the average field strength of a typical large sunspot is relatively similar to the typical field strengths measured in small flux tubes.

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