Limits on gravity-induced depolarization of light from the white dwarf Grw $+70°8247$

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(Received 27 May 1998; published 22 January 1999)

We use measurements of the polarization of light from a magnetic white dwarf to impose sharp constraints on the gravity-induced birefringence of space predicted by a broad class of nonmetric gravitation theories. Since gravity-induced birefringence violates the Einstein equivalence principle, our measurements test this foundation of general relativity and other metric gravitation theories in a new setting. [S0556-2821(99)01304-1]

PACS number(s): 04.80.Cc

Observing how propagation through a gravitational field affects light provides several classic tests of general relativity and other gravitation theories [1]. Measurements of the deflection of light and radio waves that graze the Sun's limb and of the closely related Shapiro delay are familiar examples. Until quite recently, however, a striking effect on light propagation predicted by some nonmetric alternatives to general relativity was overlooked. Only in the mid 1980s did Ni [2] note that nonmetric gravitational fields can single out linear polarization states of light that propagate with different speeds and use pulsar polarization data to impose rough constraints on this possibility.

Strong constraints on this kind of gravity-induced birefringence have been imposed since then by exploiting the way it can cause light's polarization to change as it propagates through a gravitational field. Gabriel *et al.* [3,4] used this approach to sharply constrain the strength of any birefringence induced by the Sun's gravitational field when they discovered that versions of Moffat's nonsymmetric gravitational theory (NGT) [5] predicted this phenomenon. If the birefringence were too pronounced, polarized light emitted from magnetically active regions near the Sun's limb would be depolarized as it propagates to an observer. Since one observes polarized radiation from such regions, any gravity-induced birefringence cannot be too strong. Solanki and Haugan [6] refined the resulting constraint, expressed as an upper limit on the Sun's NGT charge, to $l_\odot^2<(305\,\mathrm{km})^2$.

In this paper we show that observing polarized light from the white dwarf Grw $+70°8247$ imposes an analogous constraint on any birefringence induced by its strong gravitational field. Our constraint is a sharp one despite being conservative. It is expressed as an upper limit on the star's NGT charge, $l_*^2<(4.9\,\mathrm{km})^2$. It is conservative because it is based on worst-case assumptions regarding the size, shape and location on the stellar disk of the magnetically active region that emits the star's polarized light, assumptions that minimize depolarization caused by any gravity-induced birefringence.

We state our constraint in the form above to facilitate comparison with earlier ones. Note, however, that the significance of all these constraints goes far beyond testing versions of NGT. Predictions made by that theory merely provide concrete examples of gravity-induced birefringence, a phenomenon predicted by a broad class of the nonmetric theories encompassed by the χg formalism [4]. Haugan and Kauffmann [7] emphasize this generic quality of gravity-induced birefringence and show how to compute its effects using the χg representation of any gravitational field. They also emphasize that observations constraining the strength of gravity-induced birefringence complement more familiar tests of the Einstein equivalence principle, the Eötvös, gravitational redshift and Hughes-Drever experiments [1], for example.

For our purposes the gravitational field of a white dwarf is adequately approximated as static and spherically symmetric. This symmetry dictates that a light ray propagating through the star's gravitational field lies in a plane and that any birefringence induced by the field shows up as a difference between the phase velocity of light polarized with its magnetic field parallel to the ray's plane and that of light polarized with its magnetic field perpendicular to the ray's plane. This velocity difference varies as the ray's distance from the star and its orientation relative to the radial direction change. Any given nonmetric gravitation theory predicts a specific variation. The version of NGT considered by Gabriel *et al.* [3] predicts a fractional difference between the speed of light polarized with its magnetic field parallel to the ray's plane and propagating at an angle ϕ relative to the radial direction and the speed of light polarized with its magnetic field perpendicular to the ray's plane of

$$\frac{c_\phi - c_\perp}{c_\perp} = \frac{l_*^4}{2r^4}\sin^2\phi, \tag{1}$$

where l_* is the star's NGT charge and r is distance from the stellar core. To be definite, we use this expression in computations described below.

Gabriel *et al.* [4] established that this phase velocity difference implies that propagation from a point source on the star's surface to a distant observer introduces a phase shift between $\parallel$ and $\perp$ polarized light. Specifically,

$$\Delta\Phi(\mu)=\frac{\pi l_*^4}{\lambda R_*^3}\left\{\frac{3\pi}{16(1-\mu^2)^{3/2}}-\frac{\mu}{4}-\frac{3\mu}{8(1-\mu^2)}-\frac{3}{8(1-\mu^2)^{3/2}}\arcsin\mu\right\}, \tag{2}$$

where λ is the light's wavelength, R_* the star's radius and $\mu=\cos\theta$, with θ being the angle between the line of sight to the source point and the normal to the stellar surface at that point. This phase shift vanishes, as required by symmetry, for a point source at the center of the stellar disk ($\mu=1$) and increases monotonically as μ decreases to zero at the stellar limb.

The effect of this phase shift on light from a point source is to introduce cross-talk between circularly polarized light and linearly polarized light that has its magnetic field inclined at 45° relative to the plane in which the light ray propagates. An observer who defines Stokes parameters relative to a fixed direction in space rather than relative to the light ray's plane finds that the cross-talk is between Stokes V and a linear combination of Q and U. The linear combination depends on the location of the light source on the stellar disk because the plane in which the light propagates is perpendicular to the stellar limb at the point at which it is closest to the light source.

The effect of the phase shift (2) on polarized light from an unresolved, extended source is more complicated. In that situation, light reaching an observer from different parts of the source suffers different phase shifts and arrives with different polarizations. We must sum over these different components, using the additive property of the Stokes parameters, to determine the polarization the observer measures. The result is a reduction of the polarization observed relative to that of the light when it left the source. Specifically, $\sqrt{(V_{\rm obs})^2+(Q_{\rm obs})^2+(U_{\rm obs})^2}\leq\sqrt{(V_{\rm src})^2+(Q_{\rm src})^2+(U_{\rm src})^2}$, where the subscripts ''src'' and ''obs'' identify Stokes parameters defined using the flux of radiation propagating toward the observer in the neighborhoods of the source and of the observer, respectively. Equality of these polarizations implies the absence of gravity-induced birefringence, $l_*^2=0$.

While it is generally agreed that the polarization of white dwarfs is produced at the stellar surface as a result of the presence of Megagauss dipolar magnetic fields [8,9], we encounter a self-consistency problem when seeking evidence of depolarization in order to constrain gravity-induced birefringence. To determine whether this effect has depolarized light received from a white dwarf we must know the properties of the light the white dwarf emitted, but models of magnetic field distributions on white dwarfs and the data-fitting procedures used to determine source properties from such models ignore the possibility of gravity-induced birefringence. Consequently, the inferred source properties need not be valid if gravity-induced birefringence is significant. We break this vicious circle by assuming worst-case source properties that minimize depolarization caused by any gravity-induced birefringence. This allows us to use observational data to impose conservative, though still sharp, constraints on the strength of any such birefringence.

The polarization of a white dwarf can be accounted for by a large source region on its stellar disk emitting weakly polarized light or by a smaller region emitting more strongly polarized light. Other things being equal, gravity-induced birefringence causes less depolarization of light from smaller sources than from larger ones because the phase shift (2) varies less across a smaller source [4]. The least possible depolarization occurs for a circular source of completely polarized light centered on the stellar disk because such a source is as small as possible and because the phase shift (2) varies most slowly near its minimum at the stellar disk's center, $\mu=1$. For simplicity, we assume complete circular symmetry of our worst-case source. This implies that light emitted by the source is completely circularly polarized. Note that our conclusions are unaffected by the fact that linear white dwarf polarizations are observed. Gravity-induced birefringence causes cross-talk between Stokes V and one component of any linearly polarized light and depolarizes that component just as it does circularly polarized light. The fact that the other linearly polarized component is unaffected by gravity-induced birefringence is irrelevant in the present context because we focus on measurements of white dwarf circular polarization.

Denote the worst-case source's projected radius by $R_*\sqrt{1-\mu_p^2}$. Light emitted from the rest of the stellar disk is unpolarized. It follows that the net flux of polarized light emitted toward the observer at wavelength λ from the star's surface is

$$V_{\lambda,\rm src}=2\pi\int_{\mu_p}^{1}I_\lambda(\mu)\mu\,d\mu, \tag{3}$$

where $I_\lambda(\mu)$ is the intensity at wavelength λ emitted toward the observer from the projected radius $R_*\sqrt{1-\mu^2}$. To define a degree of circular polarization we divide this by the total stellar flux emitted toward the observer at wavelength λ:

$$F_\lambda=2\pi\int_{0}^{1}I_\lambda(\mu)\mu\,d\mu. \tag{4}$$

The function $I_\lambda(\mu)$ describes limb darkening. We will see that our constraints on gravity-induced birefringence are insensitive to differences between the forms this function is predicted to have by reasonable models of limb darkening, and so, for the moment, simply suppose that it has one of those forms. It is then easy to compute the flux of circularly polarized light an observer receives from a white dwarf given the size of its source of polarized light, μ_p, and the strength of birefringence induced by its gravitational field, l_*^2. To do so, let $V_{\lambda,\rm obs}(\mu)$ denote the flux of circularly polarized light reaching the observer from the ring on the stellar disk at $\mu(\leq\mu_p)$. This is determined by $I_\lambda(\mu)$ and $\Delta\Phi(\mu)$ [4]. Note that circular symmetry implies that light received from the ring has no net linear polarization. Summing contributions from the rings covering the polarized light source we find that the net flux of circularly polarized light of wavelength λ reaching the observer is

$$V_{\lambda,\mathrm{obs}}=2\pi\int_{\mu_p}^{1}V_{\lambda,\mathrm{obs}}(\mu)\mu\,d\mu. \tag{5}$$

To make contact with observations we divide this by F_λ to obtain an observed degree of circular polarization.

A measured value of a white dwarf's degree of circular polarization at wavelength λ implies, via Eq. (5), a relationship between its source size, μ_p, and the strength of any birefringence induced by its gravitational field, l_*^2. Since μ_p is not known *a priori*, we cannot simply use this relationship to determine l_*^2. However, we can infer a constraint on l_*^2 by recognizing that there is a largest value of this parameter that is consistent with the observed circular polarization of the white dwarf. To see this, imagine trying to use the relationship between μ_p and l_*^2 to determine the size of the star's polarized source for different values of l_*^2. This can certainly be done for small values since a unique source size can be found to account for the white dwarf's polarization in the absence of gravity-induced depolarization, $l_*^2=0$. Note that as the value of l_*^2 is increased from zero, the value of μ_p must decrease since a larger source causing more strongly polarized light to leave the star's surface is necessary to compensate for gravity-induced depolarization and account for the observed stellar polarization. There is, however, a limit to what can be achieved by decreasing μ_p since larger source size implies a greater degree of gravity-induced depolarization. Clearly, there is a largest value of l_*^2 consistent with any measured degree of circular polarization. In practice, we search for this value by evaluating Eq. (5) numerically for increasing values of l_*^2. The value we use to impose a limit on the strength of gravity-induced birefringence is the largest one for which a $1\geqslant\mu_p\geqslant0$ exists that predicts values of $V_{\lambda,\mathrm{obs}}/F_\lambda$ larger than or equal to that observed. The numerical evaluation of Eq. (1) reveals that the μ_p at which the largest $V_{\lambda,\mathrm{obs}}/F_\lambda$ is predicted depends on l_*^2.

The white dwarf we use to impose a constraint on gravity-induced birefringence is Grw $+70°8247$. This is a well-studied high-field magnetic white dwarf with a parallax of 0.076″ [10]. It was the first white dwarf found to have a magnetic field [11] on the basis of polarized observations [12]. Its most likely effective temperature T_{eff} of 14000 K [10,13] implies a radius of 0.0076 $R_\odot$, where $R_\odot$ is the solar radius, and a mass of about 1.0 $M_\odot$, where $M_\odot$ is the solar mass [14].

Polarization measurements of Grw $+70°8247$ have been published by Landstreet and Angel [15], Angel *et al.* [16], and Allen and Jordan [17]. Its polarization is time independent. Since $\Delta\Phi$ is proportional to $1/\lambda$, we find the wavelength λ at which $V_{\lambda,\mathrm{obs}}/\lambda F_\lambda$ is greatest in order to impose the sharpest possible constraint on l_*^2. For observations in the visible spectral range, analyzed and discussed in detail by Angel *et al.* [16], this condition is satisfied by $V_{\lambda,\mathrm{obs}}/F_\lambda=-6\pm0.25\%$ at 449 nm. The larger this observed degree of polarization is, the stronger the resulting constraint on l_*^2, and so it is interesting to note that recent Hubble Space Telescope spectropolarimetry in the ultraviolet has revealed high levels of circular (12%) and linear (20%) polarization between 130 and 140 nm, with the absorption feature at 134.7 nm being particularly prominent [17]. To be conservative, we assume a large absolute error of 1.0% on these measurements, and use $V_{\lambda,\mathrm{obs}}/F_\lambda=11\%$ at 134.7 nm.

As noted above, our evaluation of Eq. (5) depends on the limb darkening of Grw $+70°8247$. Since the surface of this star cannot be resolved, this cannot be measured directly. However, we know that the broadband spectrum of Grw $+70°8247$ is well represented by blackbody radiation [10] and by radiative equilibrium models [18,13]. This suggests that its limb darkening should be well represented by a simple law like the one describing the directly observed solar limb darkening. We have chosen to use [19]

$$\frac{I_\lambda(\mu)}{I_\lambda(\mu=1)}=1+(\mu-1)g+(\mu^2-1)h, \tag{6}$$

with

$$0\leqslant g+h\leqslant1.$$

Requiring the sum of the free parameters g and h to be unity imposes the maximum possible limb darkening, i.e. $I_\lambda(\mu=0)=0$.

Our search for the maximum value of l_*^2 compatible with $V_{\lambda,\mathrm{obs}}/F_\lambda$ for $\lambda=449\,\mathrm{nm}$ yields the constraint $l_*^2\leqslant(7.8\,\mathrm{km})^2$ when using the limb darkening coefficients $(g,h)=(0,1)$. Neglecting limb darkening yields the constraint $l_*^2\leqslant(7.4\,\mathrm{km})^2$. For other values of the (g,h) pair the constraint on l_*^2 falls between these extremes. Clearly, our constraint is not sensitive to assumptions about limb darkening. The UV observations of Allen and Jordan [17] yield the tighter constraint $l_*^2\leqslant(4.9\,\mathrm{km})^2$.

There is considerable scope for using polarization measurements of white dwarfs to impose sharper constraints on gravity-induced birefringence. One obvious approach is to make a proper off-center dipole model of the magnetic field of the white dwarf and to determine its parameters via a fit to observations taking into account possible cross-talk caused by gravity-induced birefringence. Since many properties of the dipolar field can be derived from the total flux (see, e.g., the analysis by Wickramasinghe and Ferrario [13] for Grw $+70°8247$), which is unaffected by gravity-induced birefringence, this should work well. It would be particularly interesting to perform such an analysis on data from a white dwarf whose magnetic dipole axis, unlike that of Grw $+70°8247$, is not aligned with its rotation axis. The polarization of such stars is modulated by their rotation. This temporal modulation provides additional constraints on the geometry of the magnetic field on the stellar surface. As long as the condition $\delta\geqslant\pi/2-i$ is satisfied, where δ is the angle between the dipole and rotation axes and i is the angle between the line of sight and rotation axis, the source of the greatest circular (and linear) polarization periodically lies close to the stellar limb where cross-talk caused by gravitational birefringence is most pronounced. At those times the star's polarization should be extremely sensitive to l_*^2. Spectropolarimetry of such stars in the UV would be particularly useful.

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