The magnetic structure of sunspots and starspots

S.K. SOLANKI

Max-Planck-Institut für Aeronomie, Max-Planck-Str. 2, 37191 Katlenburg-Lindau, Germany

Received 2002 May 10; accepted 2002 July 3

Abstract. The single most important quantity determining the properties of sunspots and presumably starspots is their magnetic field. First an overview of the magnetic structure of sunspots is given, some of the progress made in recent years is described and some of the unsolved questions are pointed out. Both observational and theoretical aspects are dealt with. Finally, the magnetic structure of starspots is discussed. After presenting the evidence for (and against) their magnetic nature the signature of starspots in Zeeman Doppler images is described. It is pointed out that if the properties of sunspots are extrapolated to giant spots on rapidly rotating stars then the observed signature can be at least qualitatively explained.

Key words: Sunspots – solar magnetic fields – stars: spots – stars: magnetic fields

1. Introduction

The magnetic field is the central quantity determining the properties of sunspots. It permeates every part of a sunspot and by greatly reducing the convective transport of heat from below is finally responsible for sunspot darkness. Conversely, sunspots were the first astronomical objects recognized to harbour a magnetic field, by Hale (1908a, b). The magnetic field of sunspots is well studied and, at least in its basics, reasonably well understood. Starspots are thought to be magnetic phenomena similar to sunspots, although on a larger scale. In their case, however, the evidence of a magnetic nature is less direct, one major problem being that the usually employed magnetic diagnostics mainly sample the brighter gas outside the darkest parts of the starspots.

Here I first review the observed magnetic properties of sunspots, before touching on their theoretical description. This is followed by a brief discussion of arguments related to the magnetic nature of starspots. Finally, I point out how the strong azimuthal fields revealed by Zeeman Doppler Imaging on rapidly rotating, active stars may be explained by extrapolating the properties of sunspots to starspots. Not all aspects of sunspot magnetism are covered here. For example, little is said about magnetic fields in the upper atmosphere and I refer to the review by White (2002) for more on this topic.

Overviews of the observed magnetic structure of sunspots have been given by Martínez Pillet (1997), Skumanich et al. (1994), and may also be found in the volume edited by Thomas & Weiss (1992). A more comprehensive review of the magnetic (and other) properties of sunspots is in preparation (Solanki 2002).

Correspondence to: solanki@linmpi.mpg.de

2. Large-scale magnetic structure of sunspots

As an example of the magnetic structure of a regular sunspot a map of the magnetic field deduced from inversions of 1.56 μm lines is given in Fig. 1. Plotted are from top to bottom the vertical and radial component of the field, as well as its azimuthal direction.

2.1. The field strength

The magnetic field of sunspots has been measured in photospheric layers via the Zeeman splitting of absorption lines in the visible and the infrared. The magnetic field strength is largest near the geometrical centre of regular sunspots, i.e. sunspots with a single umbra that are reasonably circular. It drops monotonically outwards, reaching its smallest values at the outer penumbral edge. Almost all spectral lines show a smooth outward decrease of the field strength. This is in stark contrast to the brightness, which jumps at the boundary between the umbra and penumbra. Hence, the umbral boundary is not evident in the field strength. This simple picture of a relatively smooth magnetic distribution is valid for a spatial resolution of 2–3$''$ or lower. At higher resolution the fine-scale structure of the field becomes increasingly prominent (see Sect. 3).

Additional evidence that the field strength does not jump at the umbral boundary is provided by the relation between B and continuum intensity, I_c, or temperature, T (Kopp &

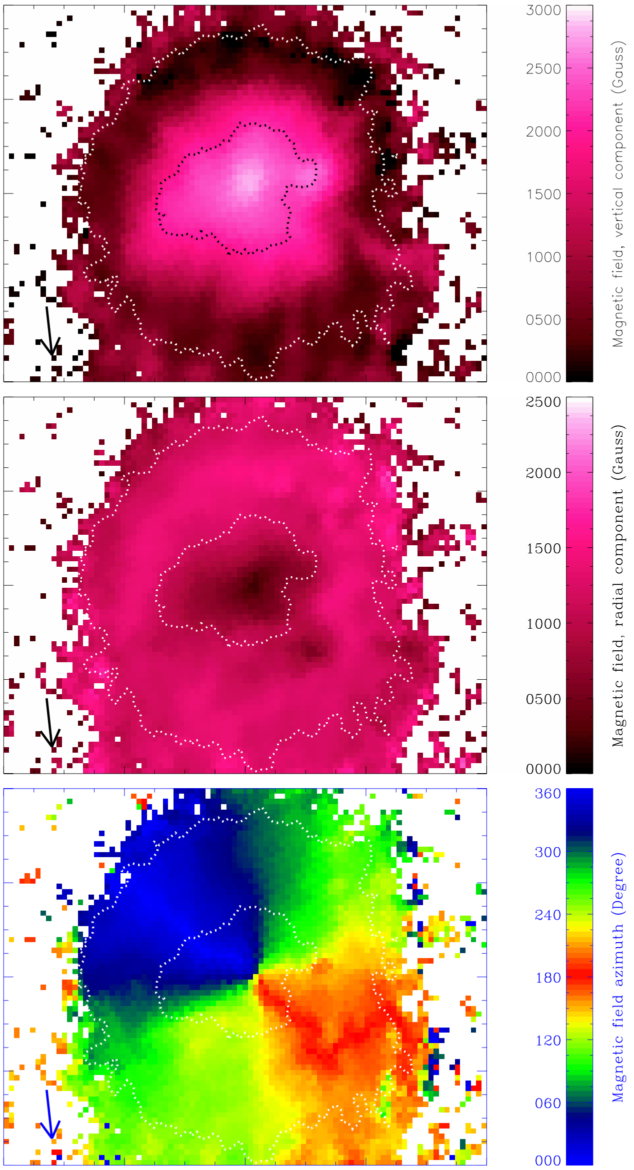

Fig. 1. Vertical and radial components of the magnetic vector, as well as the azimuth of the magnetic field in a regular sunspot (Figure kindly provided by S.K. Mathew).

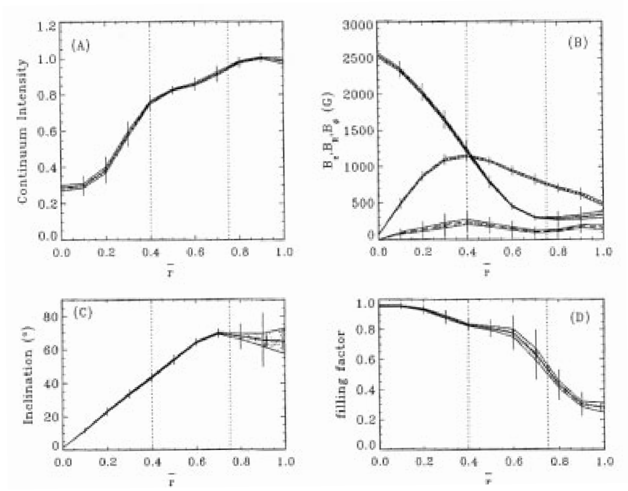

Fig. 2. Intensity and magnetic parameters vs. normalized radial distance, $\overline{r}$, from sunspot centre, as determined from 16 observations of sunspots. Vertical dotted lines indicate the umbra-penumbra (left) and the penumbra-canopy (right) boundaries. Plotted are the continuum intensity in Panel A, vertical (B_z, solid curve), radial (B_r, dotted curve) and azimuthal (B_ϕ, dashed curve) components of the magnetic field in Panel B, magnetic inclination in Panel C and magnetic filling factor in Panel D. $\overline{r}$ is normalized to the radius at which the canopy could not be seen anymore in the observations (figure from Keppens & Martínez Pillet 1996, by permission).

Rabin 1992, Martínez Pillet & Vázquez 1993, Solanki et al. 1993, Balthasar & Schmidt 1994, Stanchfield et al. 1997, Leka 1997, Mathew et al. 2002) which exhibits a discontinuous behaviour there. This demonstrates that the continuous distribution of the field strength at the boundary is not an artifact caused by smearing due to seeing or by strayligth, since the simultaneously measured intensity does indeed show a jump.

There is now a consensus on the general form of the normalized field-strength distribution, $B(r/r_p)/B_0$. Here r is the radial coordinate, r_p is the radius of the outer penumbral boundary and B_0 is the field strength at the centre of the sunspot (i.e. at $r = 0$). The radial dependence of the field strength (in an azimuthally averaged sense) has been measured and reported by many authors. The more recent such determinations include Lites & Skumanich (1990), Solanki et al. (1992), McPherson et al. (1992), Hewagama et al. (1993), Balthasar & Schmidt 1993, Keppens & Martínez Pillet (1996), Stanchfield et al. (1997) and Westendorp Plaza et al. (2001).

A set of recent measurements of $B(r/r_p)$ in regular, i.e. almost circular sunspots, are shown in Fig. 2. For comparison, the edge of the umbra, r_u/r_p lies at 0.4–0.5. Such regular, isolated sunspots do not appear to show global azimuthal twist of the field significantly above 20° (see Figs. 1 and 2).

These recent observations give for the field at the sunspot boundary $B(r_p) \approx$ 700–1000 G, which implies $B(r_p)/B_0 \approx$ 0.2–0.4. The fact that B is so large at the white-light boundary suggests that sunspots are bounded by a current sheet (Solanki & Schmidt 1993), although the raggedness of the sunspot boundary in white-light images means that the current sheet is not as smooth as one might picture on the basis of simple flux-tube models.

The maximum field strength (i.e. B_0) increases almost linearly with sunspot diameter from $B_0 \approx$ 2000 G for the smallest to over 3500 G for the largest (e.g., Ringnes & Jensen 1961, Brants & Zwaan 1982, Kopp & Rabin 1992, Collados et al. 1994, Solanki 1997). Hence B_0 increases by a factor of roughly 2 as the amount of magnetic flux increases by a factor of 30. For the field strength averaged over the sunspot the variation is even smaller, being less than a factor of approximately 1.5. In addition, the average field strength, $\langle B \rangle \approx$ 1200–1700 G, is very similar to the field strength of small flux tubes (where, due to the finite spatial resolution of the observations the measured B is always an average over the flux tube cross-section). Therefore B averaged over flux tubes remains almost unchanged over 5–6 orders of magnitude of magnetic flux per feature, as pointed out by, e.g., Solanki et al. (1999).

A surprising result has been the discontinuous $B(r)$ relation indicated by infrared Ti I lines at 2.24μm. This is in contrast to all other diagnostics. These lines are simultaneously very sensitive to magnetic field and temperature, which makes them unique. They sample mainly the coolest components of the sunspot. So far only a single sunspot has been mapped in these lines. It exhibits a strong field ($B \approx 2700$ G), with comparatively small zenith angle in the umbra which jumps to an almost horizontal, weak field ($B < 1000$ G) in the penumbra. There is no sign of fields at intermediate strengths or inclinations in these lines (Rüedi et al. 1998a), although a raster in the almost equally Zeeman sensitive, but far less temperature sensitive Fe I line at 1.56μm displays the usual smooth decrease (Rüedi et al. 1999). These results suggest a connection between unresolved magnetic and thermal inhomogeneities.

2.2. Magnetic field orientation

The strongest magnetic field in a sunspot is in general also the most vertical. Inclination to the surface normal (i.e. zenith angle, ζ) increases steadily as the field strength decreases (see Figs. 1 and 2). Stanchfield et al. (1997) and Mathew et al. (2002) find an almost linear dependence of ζ on B (cf. Hale & Nicholson 1938, Beckers & Schröter 1969).

Recent investigations find an average inclination of 10–20° to the horizontal (i.e. $\zeta \approx$ 70–80°) at the boundary (Adam 1990, Lites & Skumanich 1990, Solanki et al. 1992, Lites et al. 1993, Title et al. 1993, Hewagama et al. 1993, Skumanich et al. 1994, Shinkawa & Makita 1996, Keppens & Martínez Pillet 1996, Westendorp Plaza et al. 2001).

Although the horizontal component of the field is mainly radial, a small residual twist of up to 10–15° still persists according to Lites & Skumanich (1990), Skumanich et al. (1994), Keppens & Martínez Pillet (1996) and Westendorp Plaza et al. (1997b; 2001).

A residual twist is not entirely surprising considering the fact that the sunspot superpenumbra seen in Hα (i.e. the sunspot canopy; see Sect. 2.4) is strongly twisted. This twist increases with increasing distance from the sunspot. Kawakami et al. (1989) show that the residual twist of the sunspot magnetic field is consistent with the twisted Hα fibrils seen in the superpenumbrae of at least some symmetric sunspots.

2.3. Subsurface magnetic structure

The magnetic structure of a sunspot below the solar surface is not directly observable, but can in principle be deduced from the change in the properties of p-modes in and around sunspots.

Observations by Braun et al. (1987, 1988, 1992), Bogdan et al. (1993), Braun (1995) and others of the change in amplitude and phase of p-mode waves passing through sunspots provide evidence of subsurface absorption and scattering of incoming waves by the magnetic and thermal inhomogeneity constituting the sunspot and carries information on the subsurface structure of the sunspot. The theory of the interaction of p-modes with complex magnetic structures (such as sunspots, if the fibril model of their subsurface field is correct, see Sect. 4) is not yet complete, but numerous simplified approaches have been taken (see Bogdan & Braun 1995, Bogdan 2000, 2002a, 2002b for reviews).

One attempt to distinguish between different models of the subsurface structure of sunspots has been made by Chen et al. (1997) on the basis of data from the Taiwan Oscillation Network (TON; Chou et al. 1995). They interpret the results of inversions in terms of the sunspot cross-section as a function of depth with the help of two simple models, one in which the flux tube underlying the sunspot is cylindrical, the other in which it is funnel shaped. The two models do give somewhat different signatures, but unfortunately both of these lie within the error bars. Chen et al. (1997) find nevertheless, that their results are consistent with Parker's fibril model of the subsurface field of sunspots (Sect. 4).

The subsurface flow crossing a sunspot deduced by Zhao et al. (2001) has also been argued to support the fibril model. Such a flow can weave its way between individual fibrils, but cannot cross a monolithic tube. More on the subsurface structure of sunspots deduced from local heliseismology is presented by Kosovichev (2002).

2.4. Magnetic canopy

In the solar atmosphere the magnetic field continues beyond the white-light boundary of sunspots. It forms an almost horizontal canopy with a base in the middle or upper photosphere, i.e. the field is limited to the upper part of the photosphere and higher atmospheric layers; it overlies field-free gas. The lower boundary of the magnetized layer in the superpenumbra is called the canopy base. The magnetic canopy is a natural result of the expansion with height of the magnetic flux tube underlying the sunspot. Recall that the visible sunspot is just a cross-section through this flux tube. In this picture the canopy base corresponds to the current sheet surrounding the sunspot.

Canopies are now regularly detected using different types of observations. The derived canopy base heights generally lie below 400 km (Giovanelli 1980, Jones & Giovanelli 1982, Giovanelli & Jones 1982, Solanki et al. 1992, 1994, 1999; Zhang 1994, Adams et al. 1993, Skumanich et al. 1994, Rüedi et al. 1995, Bruls et al. 1995, Keppens & Martínez Pillet 1996, Westendorp Plaza et al. (2001).

The intrinsic field strength in the canopy, $B(r/r_p > 1)$, is found to decrease continuously without any visible break at the white-light boundary of the sunspot (Solanki et al. 1992). The base height of the canopy increases relatively rapidly close to the sunspot. However, this rise soon slows, so that the canopy can be followed out to almost twice the sunspot radius using purely photospheric lines.

Recently Solanki et al. (1999) have demonstrated that the base height increases with r/r_p in a way consistent with the thin flux tube approximation, although sunspots definitely do not satisfy the conditions under which the thin-tube approximation is expected to be valid (namely that the width of the flux tube is smaller than the pressure scale height). This is illustrated in Fig. 3, where $R_{\rm FT}/R_{\rm FT}(\tau = 1)$ is plotted vs.

height. Here $R_{\rm FT}$ is the flux-tube radius. This normalized radius is seen to behave very similarly for the smallest and largest solar flux tubes. Hence, in some ways the largest and the smallest flux tubes behave in a surprisingly similar manner. Fig. 3 implies that $\tan\gamma(r_p) \sim r_p$, i.e. the larger the sunspot, the more horizontal the field at its boundary. If we extrapolate this relation to even larger starpots it suggests that these have nearly horizontal field at the penumbral edge.

Evidence is now emerging that the magnetic canopy is intimately connected with the so-called 'Moving Magnetic Features' (MMFs), originally studied in detail by Harvey & Harvey (1973). Recent studies of bipolar MMFs (Yurchyshyn et al. 2001, Zhang et al. 2002) have indicated that their orientation follows that of the magnetic canopy (as outlined by Hα fibrils), with the bipoles being directed such that the leading magnetic feature has the polarity of the parent sunspot. This has been interpreted by Zhang et al. (2002) to indicate a U-loop produced by a dip in the canopy. The simulations of Schlichenmaier (2002) indicate how such a structure may be produced.

2.5. Return flux and depth of the penumbra

Solanki & Schmidt (1993) find that approximately 1–1.5 times as much magnetic flux emerges in the penumbra as in the umbra. The penumbra is thus deep, in contrast to the model of a shallow penumbra in which the current sheet bounding the sunspot flux tube lies along the solar surface in the penumbra and no magnetic flux emerges there.

Observations have revealed the presence of return flux at some locations along the boundary of the sunspot (Westendorp Plaza et al. 1997a, 2001, Schlichenmaier & Schmidt 1999, Mathew et al. 2002). These locations are cospatial with downflows which are interpreted as a part of the Evershed flow draining down into the solar interior again. The 1-D inversions in conjunction with earlier investigations suggest that higher-lying field lines and the associated flowing gas pass on into the canopy, while at least some of the lower lying, almost horizontal field lines return to the solar interior at the sunspot boundary.

2.6. Vertical gradient of the magnetic field

Knowledge of the magnetic field at different heights allows the average vertical gradient of the field strength between these heights to be determined (and to be compared with the predictions of models).

The values of the obtained vertical gradient depend on both the horizontal position in the sunspot and the height range over which it is measured (i.e., the formation heights of the 2 diagnostics used).

The sum of all observations suggests that the field strength drops rapidly with height in the photosphere and more slowly at greater heights. The largest gradient is found in the central umbra, where $|\partial B/\partial z|$ values of up to 3 km s^{-1} have been found by Wittmann (1974), Balthasar & Schmidt (1993), Schmidt & Balthasar (1994), Pahlke & Wiehr (1990), Bruls et al. (1995), Rüedi et al. (1995), Penn & Kuhn (1995), Westendorp Plaza et al. (2001).

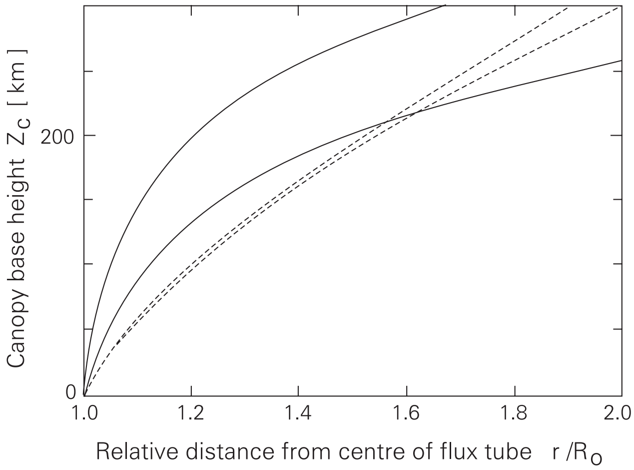

Fig. 3. Canopy base height, z_c, vs. distance from the centre of the flux tube normalized to its radius. The solid curves encompass the range of canopy base heights of sunspots, the dashed curves for slender flux tubes (from Solanki et al. 1999).

The measured umbral gradients are in reasonable agreement with simple theoretical predictions of $|\partial B/\partial z| = 0.5$–$1$ G km^{-1} at the flux-tube axis (Yun 1972). Theory also predicts both a decrease of the magnetic gradient with height and with increasing distance from the centre of a symmetric sunspot whose magnetic configuration is not too far from that of a potential field bounded by a current sheet.

3. Fine-scale structure of the magnetic field

The fine-scale structure visible in white-light images of sunspots is predominantly due to umbral dots and light bridges and to penumbral or superpenumbral filaments. As far as the magnetic field is concerned the main known inhomogeneities are concentrated in the penumbra.

3.1. Umbral dots

Various investigators have searched for the signature of umbral dots in the magnetic field. Whereas older observations showed a huge scatter in deduced field strength values, there has been convergence towards a rough consensus in recent years. Thus, Adjabshirzadeh & Koutchmy (1983), Pahlke & Wiehr (1990), Balthasar & Schmidt (1994), Lites & Scharmer (1989), Lites et al. (1991) and Tritschler & Schmidt (1997) all find no evidence of a significantly weaker field in umbral dots than in the surrounding umbra.

The fact that no significant decrease in B is seen above umbral dots does not necessarily mean that the Parker (1979c) and Choudhuri (1986) model of their production (Sect. 4.4) is wrong since, as Degenhardt & Lites (1993b) pointed out, only lines formed very deep in the atmosphere are expected to see the decrease in B in the dots predicted by these models. High resolution observations at 1.56μm are needed to resolve this question.

3.2. Fluted magnetic field in the penumbra

The azimuthal inhomogeneity of the penumbral magnetic field on small scales, in particular of its zenith angle, is now well established (e.g. Degenhardt & Wiehr 1991, Schmidt et al. 1992, Title et al. 1992, 1993). These authors found that the magnetic field is fluted or "uncombed" on small scales, in the sense that when travelling on a circle around the centre of the umbra of a regular sunspot the zenith angle of the field fluctuates by 10–40° on an arc s and sub-arc s scale. This basic result is confirmed by Lites et al. (1993), Hofmann et al. (1993, 1994), Rimmele (1995), Stanchfield et al. (1997), Westendorp Plaza et al. (1997b) and Wiehr (2000), while evidence supporting it has been provided by Solanki & Montavon (1993), and Martínez Pillet (2000).

Title et al. (1993) find a significant correlation of ζ with the velocity. The Evershed flow appears to be concentrated in the horizontal magnetic filaments. The correlation of magnetic vector inclination with brightness is far less clear, with different authors obtaining mutually opposite results. Similarly, there is no consensus on small-scale variations of field strength and its correlation with other quantities.

An alternative diagnostic of unresolved fine structure of the magnetic field is provided by broad-band circular polarization (BBCP) in sunspot penumbrae. BBCP was observed by Illing et al. (1974a,b, 1975), Kemp & Henson (1983), Henson & Kemp (1984), Makita & Ohki (1986), etc., and has been shown to be due to the blue-red asymmetry of Stokes V profiles of atomic spectral lines by Makita (1986) and Sánchez Almeida & Lites (1992).

The blue-red Stokes V asymmetry can be reproduced most easily by co-spatial line-of-sight gradients of the magnetic vector and the line-of-sight velocity (Illing et al. 1975, Auer & Heasley 1978, Sánchez Almeida & Lites 1992). However, as Solanki & Montavon (1993) point out, global gradients of the magnitude required would lead to magnetic curvature forces strong enough to destroy the sunspot. They demonstrated that the observations could be reproduced just as well by (more or less) horizontal flux tubes embedded in an inclined field, as sketched in Fig. 4. (cf. Martínez Pillet 2000). This picture has been taken up and developed by Schlichenmaier & Collados (2002), who propose a mixture of cool horizontal and hot inclined flux tubes (with flowing gas) embedded in an inclined background field. An inversion of 1.56 μm Stokes profiles including a possible horizontal flux tube by Borrero et al. (in preparation) has returned flux tubes that are approximately 200 km thick (cf. Sütterlin 2001) and form an arch from the inner penumbra to the outer boundary.

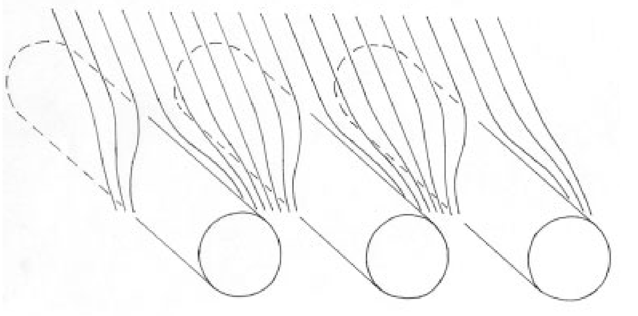

Fig. 4. Sketch of the local fine-scale structure of the magnetic field in sunspot penumbrae. The field is composed of two components, a flux-tube component, represented by the horizontal cylinders, and a more inclined magnetic field, indicated by the field lines threading their way between the flux tubes.

4. Models of the sunspot magnetic field

4.1. Introduction to the theoretical description of sunspots

Models of sunspots are of very diverse types and aim to either reproduce observed properties of sunspots, or to understand the physical processes occurring in them.

Unfortunately, the large size of sunspots and the small-scale structure dominating many of the dynamic processes within them conspire to make full fledged simulations with a modicum of physical realism beyond current reach. The large spectrum of timescales relevant to the problem make it even more intractible.

Hence, most models deal with some aspect of sunspots in detail, while neglecting or simplifying other aspects. Most numerous among the physical models are those describing the magnetic structure of a sunspot on the basis of an axially symmetric vertical flux tube in (approximate) magnetohydrostatic equilibrium.

Further details and references about models of the magnetic structure of sunspots are to be found in the following reviews: Moreno Insertis (1986), Thomas & Weiss (1992), Jahn (1992, 1997), Deinzer (1994) and Bogdan (2000), cf. Schüssler (1986).

Sunspots are thought to be the cross-sections at the solar surface of large, nearly vertically oriented magnetic flux tubes (Cowling 1934). Hence the large-scale magnetic structure of sunspots is generally represented by axially symmetric flux tubes. Most models of the magnetic structure are calculated in the magnetohydrostatic approximation, i.e. they neglect evolutionary aspects, convective motions, the Evershed effect, and the influence of waves and oscillations. For the overall structure of the magnetic field this is a satisfactory approximation since large mature sunspots evolve on time scales far longer than the time taken by disturbances travelling at the Alfvén or sound speed to cross them (the Alfvén transit time is on the order of an hour). Further arguments in favour of a static description of sunspots are given by Jahn (1997). Dynamic phenomena are important, however, for shaping the small-scale magnetic structure, which is particularly prominent in penumbrae (penumbral fibrils), but is also seen in umbrae (umbral dots). Most models of the whole sunspot neglect the fine-scale structure in the interest of tractability.

The sunspot magnetic field is confined horizontally by a combination of the excess gas pressure in the field-free surroundings of the sunspot and magnetic curvature forces. In contrast to small magnetic flux tubes the latter cannot be neglected, making the modelling of the sunspot magnetic field far more challenging. The magnetohydrostatic equilibrium is described by the force balance equation and one of Maxwell's equations:

$$\frac{1}{4\pi}\,\mathrm{curl}\,\mathbf{B}\times\mathbf{B} = \nabla p - \rho\mathbf{g}\,,$$

$$\mathrm{div}\;\mathbf{B} = 0\,,$$

where **B** is the magnetic vector, p is the gas pressure, ρ denotes the density and **g** gravitational acceleration. Hydrostatic equilibrium along field lines is already implicit in the force balance equation. Usually, significant additional assumptions are made, since the computation of the magnetic configuration without further assumptions requires the simultaneous and consistent solution for the magnetic and thermodynamic structures, which in turn makes it necessary to solve an energy equation in addition to the above equations (e.g. Alfvén 1943, Cowling 1957, Dicke 1970, Maltby 1977). Most such "comprehensive" solutions that have been attempted are not general since the magnetic structure is often partially prescribed and the thermodynamics are greatly simplified, although recently significant progress has been made on both accounts.

Various assumptions and simplifications have been used in the past to facilitate the description of the magnetic structure of sunspots. This includes the assumption of self-similarily (Schlüter & Temesvary 1958), return-flux models (Osherovich 1982), a constant-α force-free field description (Schatzmann 1965) and a MHS solution without a current sheet (Pizzo 1986). These approaches are not discussed further here. I refer to Solanki (2002) for a review of these techniques of calculating sunspot magnetic structure.

4.2. Monolithic vs. cluster model

One basic assumption underlying all attempts to quantitatively model the global magnetic structure of sunspots is the assumption that the sunspot is monolithic below the solar surface (e.g., Cowling), i.e., that it can be represented by a single flux tube. Since these layers are not directly accessible to observations, this assumption cannot be rigorously tested, although the techniques of local helioseismology can in principle set some constraints on the subsurface field.

Parker (1979a,b,c) proposed that just below the surface the magnetic field of a sunspot breaks up into many small flux tubes due to the fluting or interchange instability (Parker 1975, Piddington 1975). In this instability, the magnetic energy of the system is lowered by the fragmentation of a large flux tube (with strong magnetic curvature terms) into many small ones (with small curvature terms). In this picture, a sunspot can be described by a monolithic tube above the surface, but only by a crowd of small flux tubes (spaghetti) below the surface, as illustrated in Fig. 5. This model is often referred to as the spaghetti or jellyfish model.

Magnetic buoyancy can save sunspots ($\phi > 10^{20}$ Mx) from going unstable to fluting in the layers close to the solar surface (Meyer et al. 1977, Bünte et al. 1993). In deeper layers the interchange instability may still act. The depth at which the instability occurs and subsurface 'spaghetti' are

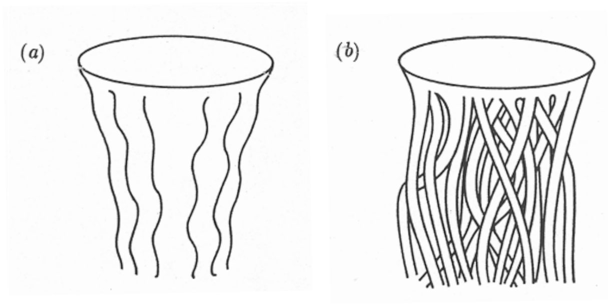

Fig. 5. Sketch of the monolithic (a) and cluster (b) models of the subsurface structure of sunspot magnetic fields (from Thomas & Weiss 1992, by permission).

produced depends on the total magnetic flux emerging in the sunspot and on details of the magnetic structure. In spite of this uncertainty a cluster model of sunspots has the advantage that it can explain the relatively high thermal flux seen in the umbra, as well as umbral dots in a natural manner (the latter as field-free intrusions into the sunspot, Parker 1979c, Choudhuri 1986). The complex magnetic structure in the penumbra, in which fibrils of field pointing in different directions are interlaced, suggests that the magnetic field is indeed concentrated into many small flux tubes. Further arguments for the cluster model have been presented by Choudhuri (1992). The patchy distribution of power of oscillations of the Zeeman signal in sunspots also suggests an inhomogeneous magnetic field in the subphotospheric layers (e.g. Rüedi et al. 1998b, Staude 1999, Balthasar 1999). Finally, as discussed in Sect. 2.3, local helioseismic investigations favour a cluster model of sunspots (Chen et al. 1997, Zhao et al. 2001).

4.3. Current-sheet models

The simplest consistent current sheet model is composed of a flux tube in whose interior the magnetic field is potential, i.e. current free, so that all the current is concentrated in a sheet at the boundary of the flux tube, termed the magnetopause. The main difficulty facing the modeler is the determination of the horizontal position of the magnetopause as a function of height in the presence of arbitrary stratifications in the flux tube and in its surroundings. Approximate solutions have been found and applied to sunspots and pores by Simon & Weiss 1970) and Simon et al. (1983). Wegmann (1981) proposed the first general solution to the free boundary problem. Schmidt & Wegmann (1983) were the first to apply this technique to sunspots.

There are, however, indications that a potential-field model bounded by a current-sheet at the magnetopause is too simple to describe sunspots. In particular, it is inadequate to describe the presence of the penumbra. Hence Jahn (1989) extended the Schmidt & Wegmann (1983) model to include body currents in addition to a current sheet at the boundary. The body currents were restricted to the outer part of the sunspot (corresponding approximately to the penumbra) and chosen such that the surface field matches the observations of Beckers & Schröter (1969). It turns out that the field deviates somewhat from potentiality, but this deviation is not very large anywhere (except at the boundary, of course). The deviation is nevertheless important for reproducing the observations. Although only a single boundary current sheet fails to reproduce the observations satisfactorily (Jahn 1989), combined sheet and body current models provide relatively good fits to the observations of the global magnetic structure of sunspots.

Jahn & Schmidt (1994), cf. Jahn (1992), considered a model very similar to that of Jahn (1989), but they replaced the body currents in Jahn's (1989) model by a current sheet located between the umbra and penumbra (in addition to the current sheet at the magnetopause). This structure allows for sharp thermal boundaries between umbra, penumbra and quiet Sun by specifying different mixing length parameters in each of these three domains. Between the current sheets the field is potential. This simplification of the magnetic structure relative to the model of Jahn (1989) is dictated by the aim of Jahn & Schmidt (1994) of obtaining a realistic thermal structure of the sunspot with distinctly different umbral and penumbral thermal transport mechanisms. These models are marred somewhat by the jump in the field strength at the boundary between the umbra and the penumbra, which is not present in the earlier models of Jahn (1989).

Finally, 2-D simulations of flux tubes with different amounts of magnetic flux, concentrated by the influence of convection in their surroundings (Hurlburt & Rucklidge 2000) indicate that a current sheet is automatically produced, with the current sheet becoming narrower as the magnetic Reynolds number R_m increases. The resulting field strength averaged over the flux tube cross-section is roughly independent of the total flux in the tube, in good agreement with the observational results of Solanki & Schmidt (1993) and Solanki et al. (1999). Unlike the observations, however, the maximum of the field strength is not reached at the flux tube axis, but rather near its boundary (where also the minimum plasma β is achieved in the models). This suggests that additional mechanisms besides concentration by convective cells are responsible for the formation of at least the larger solar flux tubes.

4.4. Uncombed fields and umbral dots

On a small azimuthal scale horizontal and inclined field lines are observed to alternate in the penumbra. Two ideas have been proposed to explain the origin of these so-called uncombed fields. Basically these ideas consider the small-scale magnetic structure to be dynamic and its complexity to result from an instability. One proposal, due to Wentzel (1992), has not been worked out quantitatively and consequently is not discussed further here.

Spruit (1981b) and Jahn (1992) proposed that the complex magnetic fine structure of the penumbra is due to the convective exchange of flux tubes. One possible scenario for interchange convection is the following: A flux tube near the magnetopause below the (deep) penumbra is heated by the field-free convective gas with which it comes into contact. The heated tube is buoyant and rises. At the surface it radiates away its excess energy, loses its buoyancy, becomes more horizontal and sinks again.

The first part of this scenario has been confirmed and quantified by numerical simulations (Schlichenmaier 1997, Schlichenmaier et al. 1998a,b). An illustration is given in Fig. 6. A thin flux tube lying at the magnetopause (i.e. the outer boundary of the penumbra) heats up, becomes buoyant and begins to rise (the background penumbral field and superadiabatically stratified gas was taken from the model of Jahn & Schmidt 1994). The part of the flux tube near the outer boundary of the penumbra reaches the surface first. Below the surface the tube rises almost adiabatically, but above the surface radiative losses make it denser and reduce the buoyancy. Also, the background stratification above the surface is no longer superadiabatic. Consequently, the parts of the flux tube above the surface come to rest, staying horizontal, while the surrounding field remains strongly inclined with respect to the surface. With time parts of the flux tube closer to the umbra emerge into the solar atmosphere and lengthen the horizontal portion of the flux tube. The horizontal flux tube remains in equilibrium, since, e.g., the negative buoyancy is balanced by the upward acceleration due to the expansion with height of the background field.

An observational signature of the formation of a horizontal flux tube is the movement of a bright point towards the umbra. Such moving bright points (called penumbral grains) have indeed been observed (Muller 1992, Sobotka et al. 1999). This model also predicts an outward gas flow along the horizontal flux tube, which is similar to that giving rise to the Evershed effect. Finally, Schlichenmaier (2002) has presented new simulations which suggest that outward streaming features in the outer penumbra, seen by some observers, may also be produced by this model, as well as MMF-like features beyond the white-light boundary of the sunspot.

The loss of buoyancy and return to its original more vertical state of the flux tube is, however, not produced by the simulations.

Theoretical concepts underlying umbral dots, in general consider them to be associated with some form of magnetoconvection. In models of a monolithic umbra the bright umbral dots are related to hot upflows, although it is not a priori clear why umbral dots should possess their characteristically small size. In the spaghetti model they are thought to be the protrusion of field-free material from below the surface into the penumbra (Parker 1979c, Choudhuri 1986, 1992). In this model, intrusions of field-free, convectively unstable gas are present between the numerous thin flux tubes (the 'spaghetti') below the umbra. If sufficient pressure builds up in this gas it rises, pushing the field lines aside. In some cases the gas can burst through the solar surface, becoming visible as an umbral dot. According to Choudhuri (1986) the system acts like a magnetic valve. Once sufficient gas has moved above the surface through the open valve the pressure from above is thought to increase again so that the valve should close. The main consequence for the magnetic field of these models, in particular that of Choudhuri (1986), is that at the continuum-forming layers there is a localized region of no field. However, only 100-200 km above that level the field is practically homogeneous again according to Degenhardt & Lites (1993a,b). The magnetic filling factor in the umbra reaches

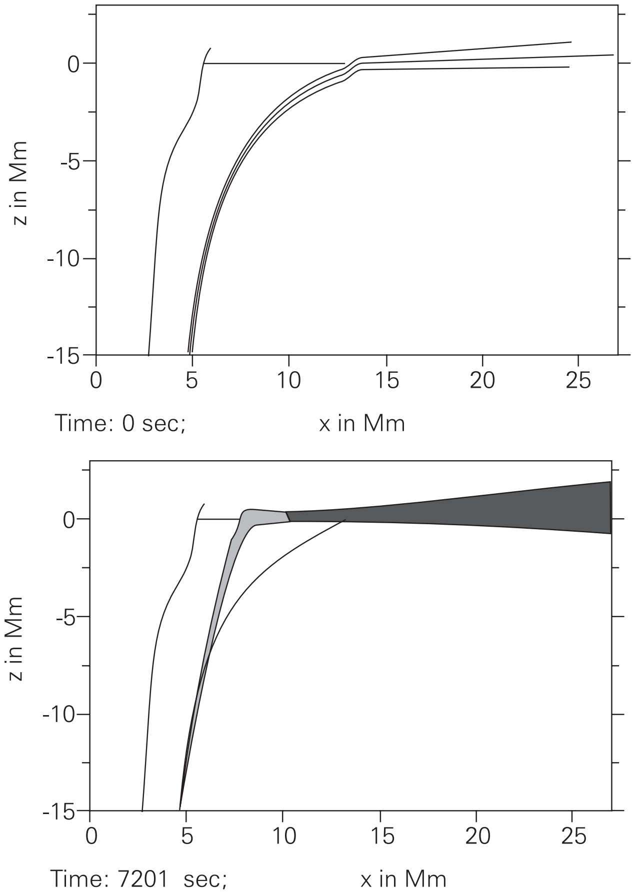

Fig. 6. Vertical cut through a model penumbra. Vertical axis: height z, horizontal axis: radial distance x from the axis of the sunspot (lying to the left). Indicated are the solar surface (horizontal lines near $z = 0$), the magnetopause between penumbra and quiet sun and the current sheet between umbra and penumbra. Shown is (a) a flux tube at its initial location at the magnetopause and its final position when it lies partially horizontally at the solar surface (adapted from Schlichenmaier 1997, by permission).

values above 95% in the mid- and upper photosphere, which is compatible with this model.

5. Starspots

5.1. Are starspots magnetic?

Compared to sunspots our knowledge of starspots is poor. Most of what we do know about them is based on their brightness contrast. Of their magnetic properties very little is known and it is even worthwhile to consider the question whether they really are magnetic features similar to sunspots. This is the assumption that is generally made, although the large polar spots on some rapidly rotating stars have been likened to solar polar coronal holes in some of their properties (Donati & Collier Cameron 1997). The evidence supporting the magnetic nature of starspots is on the whole indirect. Below I list the main evidence and arguments for and against this hypothesis, as well as making some other relevant comments.

1. Magnetic suppression of convective energy transport is the most efficient means of producing significant localized darkenings on the surface of a cool star. In particular, the evolution of the size, shape and number of starspots on the stellar surface is best understood in terms of magnetic features.
2. The detected starspots are much larger than even the largest sunspots. This unusual size can be explained with the larger amount of magnetic flux on these stars. However, due to the limited spatial resolution achievable with the employed detection techniques (e.g. Doppler Imaging) it is in general not possible to resolve sunspot-sized features. Hence, what appears as a single starspot may or may not be composed of multiple sunspot-sized spots (see Solanki 1999).
3. Another major difference between starspots, and sunspots, namely the high latitudes of the starspots compared with the near-equatorial location of sunspots has been explained in two different ways. Firstly, magnetic field generated at the base of the convection zone is susceptible to the enhanced influence of the Coriolis force on the rapidly rotating stars exhibiting high-latitude spots (Schüssler & Solanki 1992, Schüssler 2002). Secondly, meridional criculation causes the magnetic flux to concentrate increasingly near the stellar pole as the total amount of flux increases (Schrijver 2002). A combination of both effects may well be acting, but this needs to be studied.
4. The direct measurement of the magnetic field in individual starspots is difficult due to their low continuum intensity, in particular of the umbra. A possible magnetic signal from there is easily swamped by the higher signal from the bright faculae. For this reason it is likely that very little of the magnetic signal detected on active cool stars is umbral in origin. There may be an exception for very active stars for which the faculae to spot area ratio becomes small, as suggested by the work of Radick et al. (1989). Even for these stars, however the umbral, contribution to the disk-integrated spectral line profiles is difficult to measure. The penumbra, having a brightness close to that of the quiet photosphere, produces a far larger Zeeman signal even if the field strength there is lower. The factor of 3-4 larger penumbral area relative to umbral area (seen for sunspots) also enhances the penumbral contribution.
5. The argument that the measured field strength B on some stars considerably exceeds B_{eq}, just like the maximum B in sunspots does (see Fig. 7), and must therefore be due to starspots, is basically flawed. Here $B_{eq} = \sqrt{8\pi p_{\rm exp}}$, where $p_{\rm exp}$ is the gas pressure outside the sunspot or starspot. $B > B_{eq}$ is only true for the peak field strength in umbrae. Averaged over the whole sunspot $\langle B \rangle \approx B_{eq}$ is found, pretty much as in faculae. The similarity between flux-tube-averaged field strength over 6 orders of magnitude of surface area, is clearly visible in Fig. 8.

Zeeman Doppler Imaging (Semel 1989) and Magnetic Doppler Imaging (Saar et al. 1992) are the most advanced techniques for determining the distribution of the magnetic

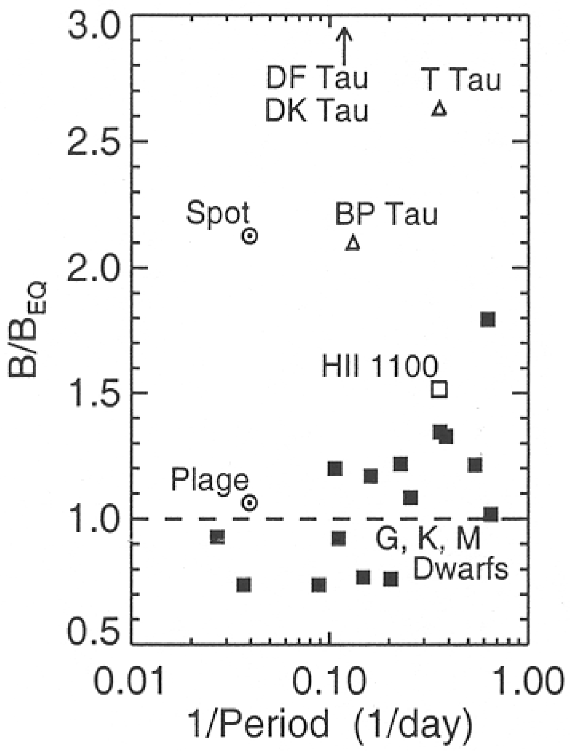

Fig. 7. Measured field strength, B, relative to the thermal equipartion value, B_{eq}, plotted vs. rotation rate. Each symbol represents a star. For comparison values for solar plage and the maximum value for the sunspot umbral field are also given (from Valenti and Johns-Krull 2001).

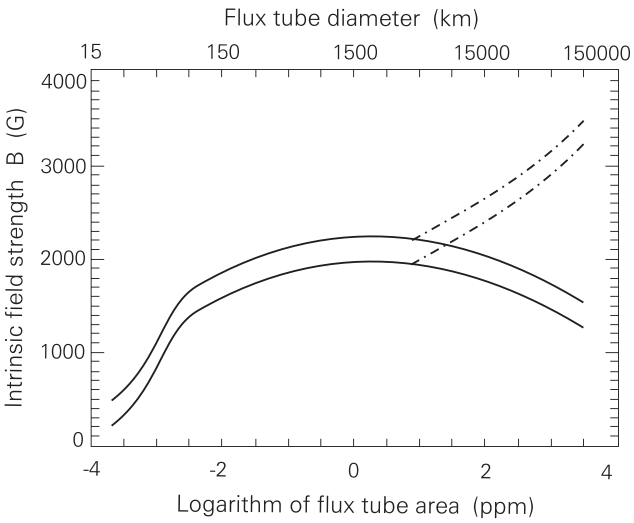

Fig. 8. Intrinsic field strength B of magnetic features vs. the (logarithmic) area of their cross section (lower axis, see Fig. 1) and their diameter (upper axis). The solid lines roughly enclose the observed range of values of the field strength averaged over the whole flux tube, including over both umbra and penumbra for sunspots. The dot-dashed lines represent the maximum field strength in the umbra.

field on the stellar surface. Of these, more effort has been put into developing Zeeman Doppler Imaging (ZDI) which has also been applied far more extensively. Typical for ZDI is that the magnetic signal is small for the darkest parts of a Doppler image (e.g. Donati et al. 1992; Donati & Collier Cameron 1997), i.e. to the parts corresponding to sunspot umbrae. E.g., for the young rapidly rotating star AB Dor the magnetic field is largest for regions of intermediate brightness. These may be similar to sunspot penumbrae.

In order to detect umbral fields it is necessary to employ spectral lines that are very weak outside the starspot. Zeeman sensitive molecular lines (see Berdyugina, 2002) are the lines of choice for this purpose. Note however, that they are most useful on stars that are not too cool, so that any molecular absorption must come from the sunspot.

Can the fields in or near penumbra-like features in Zeeman Doppler Images be understood in terms of starspot magnetic structure? Basically 2 types of magnetic structure are seen that may be related to starspots. The first is a prominent ring of azimuthally oriented field (toridal field) usually forming a collar around a large polar spot, (e.g. on HR1099, (Donati et al. 1992). The Zeeman Doppler Image of this star is plotted in Fig. 9 (from Donati et al. 1992). Such a toroidal component at the stellar surface is, at least at first sight, unexpected and has been used to argue that the dynamo producing large-scale magnetic signals on these stars is not restricted to the bottom of the convection zone (e.g. Donati and Collier Cameron 1997). In Sect. 5.2 I discuss an alternative interpretation, based on the extrapolation of sunspot properties to the parameters typical of starspots and their parent stars.

The second type of feature are smaller, possibly not completely resolved structures exhibiting vertical fields on ZDI maps. These features are not well correlated with strong darkenings, but some of them do lie on top of mild darkenings (Donati & Collier Cameron 1997).

5.2. ZDI results: are they compatible with starspots having sunspot-like magnetic fields?

As the basis for this section I assume that the basic results from ZDI, as outlined in the last section, are correct. This is not a trivial statement since Stokes V-based ZDI follows the very ambitious aim of reconstructing a 6-dimensional quantity (surface maps of the full magnetic vector) from a two-dimensional data set (Stokes V as a function of wavelength and time). I refer to Piskunov (2002) and references therein for a critical look at the capabilities and limitations of ZDI.

Another basic premise underlying this section is that the properties of starspots can be roughly estimated by extrapolating from sunspots to larger features if the difference between the general properties of the active star and the Sun are also taken into account. I also assume that ZDI maps only show the field outside the umbra. The final assumption that I make is that in particular the polar spots seen in Doppler images are single spots and not conglomerates of smaller spots. This assumption, however, may be relaxed, as discussed later.

Let me begin by considering which component of a sunspot's magnetic vector is most likely to be visible in a

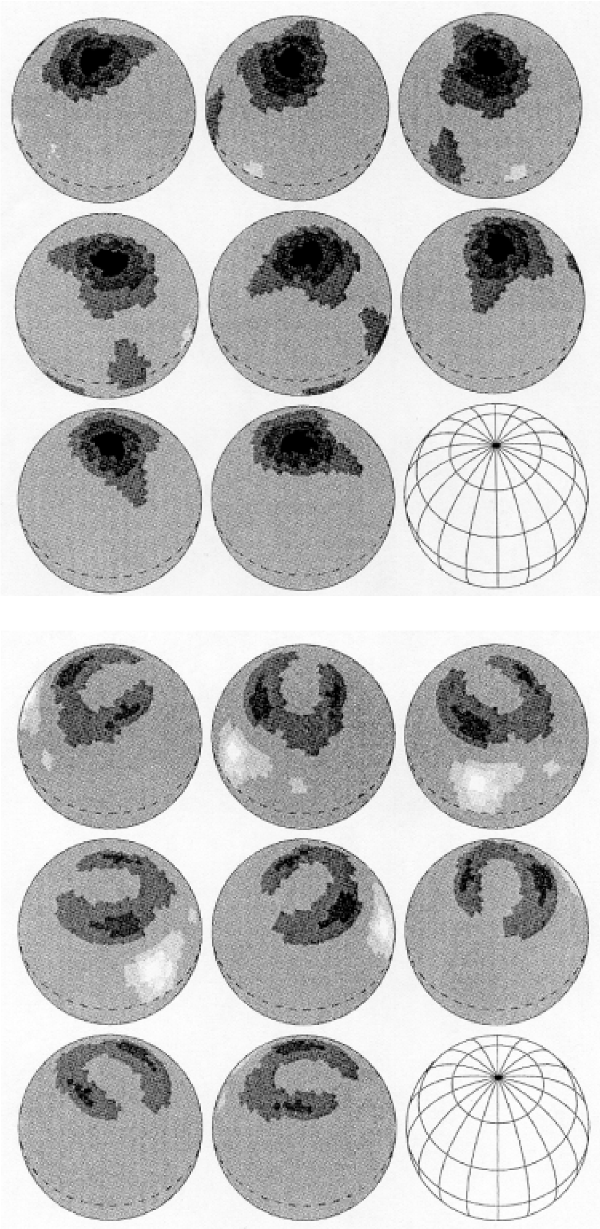

Fig. 9. Temperature (top) and toroidal field (bottom) maps of HR 1099 for the year 1990. The temperature steps correspond to 250 K, the magnetic field steps of 200 G. Dark grey corresponds to clockwise, light grey to counterclockwise directed field (from Donati et al. 1992, by permission).

Zeeman Doppler Image. In keeping with the ZDI literature the 3 components are described by the vertical, poloidal and toroidal components defined relative to the stellar surface and rotation axis.

First: is the observed field more likely to be vertical or horizontal?

Following Fig. 2 and assuming a linear dependence of the inclination angle ζ of the field to the vertical on radial distance r from the spot centre:

$$\zeta = \zeta_{\max} r / R_{\rm spot},$$

one obtains $\langle\zeta\rangle = 60°$ averaged over the whole spot if $\zeta_{\max} = 90°$ (which is reasonable for a big starspot according to Fig. 3). $\langle\zeta\rangle = 70°$ if B_z values of the penumbra alone are averaged. Since the umbra does not contribute significantly to the Stokes V signal in atomic lines, the detectable field is rather horizontal.

Now we need to distinguish between a starspots that are spatially resolved by ZDI and starspots that are not. For an unresolved starspot that is reasonably symmetric Stokes V signals from the horizontal components of the magnetic field will mutually cancel, so that mainly the vertical component will produce a net signal. The fact that the penumbral field is to a large part horizontal hence only leads to a reduction of the starspot's Zeeman signal. For very asymmetric starspots other field components may also be detectable, but probably to a smaller extent. For a resolved starspot this implies that the measured Zeeman signal is mainly horizontal. If the superpenumbral field is counted (see below), then the starspot field is even more horizontal.

Consider now a large polar spot (as a single starspot rather than a cluster of smaller spots). A direct comparison with the magnetic structure of sunspots illustrated in Figs. 1 and 2 would lead one to expect a poloidal field. However, before carrying out such a comparison we need to consider how sunspot properties would change when transported to a large spot at the pole of a rapidly rotating star. According to the sunspot analogy the penumbrae of starspots should also harbour an Evershed flow, i.e. a nearly horizontal outflow of matter. In a regular sunspot this outflow is nearly radial (twists of sunspot fields are typically less than 20°, see Fig. 2). At greater distances from sunspot centre (in the magnetic canopy), however, the field is strongly twisted (Sect. 2.4). Such a twist can be produced by the influence of the Coriolis force on the Evershed flow (Peter 1996). The following is an estimate of the length scale over which a significant twist is produced (due to M. Schüssler):

A packet of gas taking part in the Evershed outflow feels the Coriolis force

$$\frac{d\mathbf{v}}{dt} = \mathbf{v} \times \mathbf{\Omega}$$

where $\mathbf{\Omega} = (0, 0, \Omega)$ is the stellar rotation vector and $\mathbf{v} = (v_r, v_\rho, 0)$ is the Evershed flow (for simplicity the starspot has been assumed to be small compared to the stellar radius, so that the curvature of the stellar surface has been neglected). The solution of Eq. (1) is an oscillation of $\mathbf{v}$ between v_r and v_ρ (so-called inertial oscillations) with an amplitude:

$$\Delta r = \frac{v_o}{2\Omega} = \frac{1\text{km s}^{-1}}{2.3 \times 10^{-6}\text{s}^{-1}} = 1.8 \times 10^5\text{km}$$

for the solar case. Note that the numerical value of Δr corresponds to the superpenumbral radius. The physical interpretation of Δr is that it is the length scale over which the flow and consequently the associated magnetic field are deflected from the original direction to an azimuthal direction. For a star rotating at 10 times the solar rate ($\Omega = 10\Omega_\odot$) Δr is

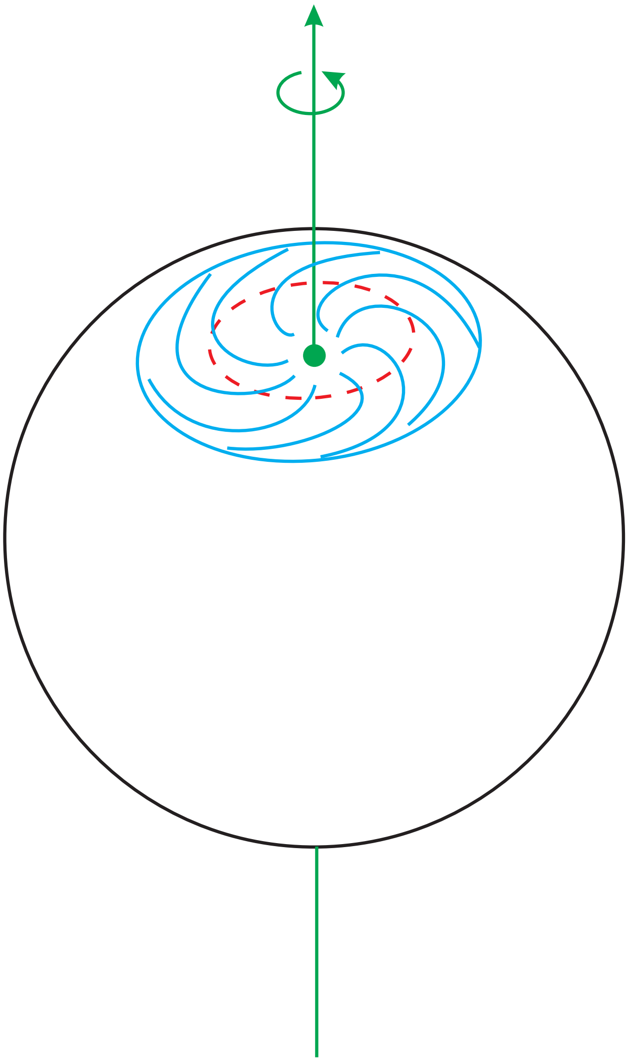

Fig. 10. Sketch of the expected magnetic structure of a large polar spot on a rapidly rotating star (such as HR 1099). The twisted magnetic structure is indicated, as are the umbral (dashed) and penumbral boundaries.

correspondingly shorter, while the starspot size is larger (typically R(sparspot) $\approx$ 10 R(sunspot)). Hence we expect the twist of the field to be very significant in the penumbrae of these spots. This would be visible as a strong azimuthal component of the field, producing a ring of nearly toroidal field in the penumbra of a polar spot. A sketch of the expected situation is given in Fig. 10.

An exact solution of the force balance has been found by Peter (1996). It shows how the twist increases with increasing size of the sunspot. In particular, the exact solution shows that either direction of the twist is possible.

Note that a twisted field is also compatible with the magnetic structure produced at the pole by flux transport through meridional motion (see Schrijver 2002), in which a ring of one magnetic polarity surrounds a polar cap of the opposite polarity. Hence the presence of a toroidal field in association with polar spots does not imply that they must be single starspots.

Finally, another mechanism for producing a toroidal magnetic structure in a polar spot is differential rotation. The angular rotation rate is a function of latitude on active stars, just as it is on the Sun (Collier Cameron 2002). Hence the outer penumbra of a polar spot rotates a bit faster than the inner penumbra. This leads to a twist. The magnitude of the twist is enhanced by the large sizes of the polar spots and their long lifetimes, which allow the influence of the velocity shear to build up. The uncertain magnitude of the stellar differential rotation near the poles is the major uncertainty underlying this mechanism. Of course, both effects, the Coriolis force and differential rotation act in parallel and can either enhance or interfere with each other.

Finally allow me to consider the question whether only the penumbra contributes to the ZDI signal or if also the superpenumbra (the magnetic canopy described in Sect. 2.4) is visible. The usual (solar) definition of the superpenumbra is the sunspot's (starspot's) magnetic field outside the white-light boundary of the sunspot or starspot. On the Sun the boundary of the white-light sunspot is also the place where the magnetopause passes through the solar surface. Hence in the superpenumbra of a sunspot the magnetic canopy has its base in the solar atmosphere. On cooler stars this need not be the case.

On such stars the convection zone stops somewhat below the surface in the sense that Schwarzschild's convective instability criterion is no longer fulfilled in the layers immediately below the stellar surface. The cooling due to the magnetic field is only effective in the convection zone, however, since the magnetic field reduces the efficiency of convection, but hardly affects energy transport by radiation. Hence any subsurface expansion of the field in the radiatively dominated gas will not be visible as a darkening. In other words the size of the visible starspot is given by the cross-section of the magnetic flux tube when it intersects the upper boundary of the convection zone and *not* the stellar surface. The latter is thus relatively unimportant for determining the size of a dark starspot. Therefore, on a K star one can find a ring of magnetically permeated gas around a large starspot, which (the ring) does not appear dark.

How broad can such a ring be? This depends both on the depth at which the convection stops and on the angle of inclination of the field at the starspot boundary. If we take solar flux tubes as a guide we can roughly estimate this quantity. In particular, we extrapolate using the relation that $\tan\alpha \sim R$, which follows from Fig. 3. The convection always stops close to the stellar surface, at the most half a scale height below. For an almost horizontal field the expansion to the layer of line formation lying 50 km to 100 km above the continuum forming layer, can extend the starspot's magnetic boundary seen by a spectral line by 5%. This may partly explain why in ZDI the large polar spots (defined in brightness images) are often surrounded by an azimuthal (i.e. toroidal) field, partly lying outside the visible starspot.

6. Conclusion

The magnetic field of sunspots is now not only known to display a level of complexity not imagined even 15 years ago, it is beginning to serve as a prototype of the magnetic structure of starspots. Even relatively simple extrapolations from sunspots to starspots produce a qualitative agreement with Zeeman Doppler Images of polar starspots and their surroundings, *if* the difference in size and location of the spot and the properties of the parent star are taken into account. Nonetheless, the relative success of the simple estimates made in this paper point to the need for a more detailed and quantitative treatment.

Acknowledgements. I am grateful to J.-F. Donati, V. Martínez Pillet, R. Schlichenmaier and N.O. Weiss for allowing me to use figures from their publications and partly for providing the figures in electronic form.

References

Adam, M.G.: 1990, SoPh 125, 37
Adams, M.G., Solanki, S.K., Hagyard, M.J., Moore, R.L.: 1993, SoPh 148, 201
Adjabshirzadeh, H., Koutchmy, S.: 1983, A&A 122, 1
Akhmedov, Sh.B., Gelfreikh, G.B., Bogod, V.M., Korzhavin, A.N.: 1982, SoPh 79, 41
Auer, L.H., Heasley, J.N.: 1978, A&A 64, 67
Babcock, H.W.: 1951, ApJ 114, 1
Balthasar, H.: 1999, in: B. Schmider, A. Hofmann, J. Staude (eds.), *Magnetic Fields and Oscillations*, ASP Conf. Ser. 184, p. 141
Balthasar, H., Schmidt, W.: 1993, A&A 279, 243
Balthasar, H., Schmidt, W.: 1994, A&A 290, 649
Beckers, J.M., Schröter, E.H.: 1969, SoPh 10, 384
Berdyugina, S.: 2002, AN, in press
Bogdan, T.J.: 2000, SoPh 192, 373
Bogdan, T.J.: 2002a, AN 323, 196
Bogdan, T.J.: 2002b, AdSpR, in press
Bogdan, T.J., Brown, T.M., Lites, B.W., Thomas, J.H.: 1993, ApJ 406, 723
Bogdan, T.J., Braun, D.C.: 1995, in: V. Domingo, et al. (eds.), *Proc. 4th SOHO Workshop: Helioseismology*, ESA SP-376, p. 31
Brants, J.J., Zwaan, C.: 1982, SoPh 80, 251
Braun, D.C.: 1995, ApJ 451, 859
Braun, D.C., Duvall jr., T.J., LaBonte, B.J.: 1987, ApJ 319, L27
Braun, D.C., Duvall jr., T.L., LaBonte, B.J.: 1988, ApJ 335, 1015
Braun, D.C., Duvall jr., T.L., La Bonte, B.J., Jefferies, S.M., Harvey, J.W., Pomerantz, M.A.: 1992, ApJ 391, L113
Bruls, J.H.M.J., Solanki, S.K., Carlsson, M., Rutten, R.J.: 1995, A&A 293, 225
Chen, H.-R., Chou, D.-Y., and the TON Team: 1997, ApJ 490, 452
Chou, D.-Y., Sun, M.-T., Huang, T.-Y., et al.: 1995, SoP 160, 237
Choudhuri, A.R.: 1986, ApJ 302, 809
Choudhuri, A.R.: 1992, in: J.H. Thomas, N.O. Weiss (eds.), *Sunspots: Theory Observations*, Kluwer, Dordrecht, p. 243
Collados, M.x M., Martínez, Pillet, V., Ruiz Cobo, B., del Toro Iniesta, J.C., Vázquez, M.: 1994, A&A 291, 622
Collier Cameron, A.: 1996, MNRAS 278, 821
Degenhardt, D., Lites, B.W.: 1993a, ApJ 404, 383
Degenhardt, D., Lites, B.W.: 1993b, ApJ 416, 875
Degenhardt, D., Wiehr, E.: 1991, A&A 252, 821
Donati, J.-F., Brown, S.F., Semel, M., et al.: 1992 A&A 265, 682
Donati, J.-F., Collier Cameron, A.: 1997, MNRAS 291, 1
Giovanelli, R.G.: 1980, SoPh 68, 49
Giovanelli, R.G., Jones H.P.: 1982, SoPh 79, 267
Hale, G.E.: 1908a, ApJ 28, 100
Hale, G.E.: 1908b, ApJ 28, 315
Hale, G.E., Nicholson, S.B.: 1938, Papers of the Mt. Wilson Obs. V, Part 1, Carnegie Institution of Washington Publ. No. 489.
Harvey, K.L., Harvey, J.W.: 1973, SoPh 28, 61
Hewagama, T., Deming, D., Jennings, D.E., Osherovich, V., Wiedemann, G., Zipoy, D., Mickey, D.L., Garcia, H.: 1993, A&AS 86, 313
Hofmann, J., Deubner, F.-L., Fleck, B.: 1993, in: H. Zirin, G. Ai, H. Wang (eds.), *The Magnetic and Velocity Fields of Solar Active Regions*, IAU Coll. 141, ASP Conf. Ser. 46, 44
Hofmann, J., Deubner, F.-L., Fleck, B., Schmidt, W.: 1994, A&A 284, 269
Hurlburt, N.E., Rucklidge, A.M.: 2000, MNRAS 314, 793
Illing, R.M.E., Landman, D.A., Mickey, D.L.: 1974a, A&A 35, 327
Illing, R.M.E., Landman, D.A., Mickey, D.L.: 1974b, A&A 37, 97
Illing, R.M.E., Landman, D.A., Mickey, D.L.: 1975, A&A 41, 183
Jensen, E., Duvall jr., T.L., Jacobsen, B.H., Christensen-Dalsgaard, J.: 2001, ApJ 553, L193
Jones, H.P., Giovanelli, R.G.: 1982, SoPh 79, 247
Kawakami, S., Makita, M., Kurokawa, H.: 1989, PASJ 41, 175
Kemp, J.C., Henson, G.D.: 1983, ApJ 266, L69
Keppens, R., Martínez Pillet, V.: 1996, A&A 316, 229
Kopp, G, Rabin, D.: 1992, SoPh 141, 253
Kosovichev, A.: 2002, AN 323, 186
Leka, K.D.: 1997, ApJ 484, 900
Lites, B.W., Scharmer, G.B.: 1989, in: O. von der Lühe (ed.), *High Spatial Resolution Solar Observations*, National Solar Obs., Sunspot (NM), p. 286
Lites, B.W., Skumanich, A.: 1990, ApJ 348, 747
Lites, B.W., Bida, T.A., Johannesson, A., Scharmer, G.B.: 1991, ApJ 373, 683
Lites, B.W., Elmore, D.F., Seagraves, P., Skumanich, A.: 1993, ApJ 418, 928
Makita, M.: 1986, SoPh 106, 269
Makita, M., Ohki, Y.: 1986, Ann. Tokyo Astron. Obs. 21, 1
Martínez Pillet, V., Vázquez, M.: 1993, A&A 270, 494
Martínez Pillet, V.: 1997, in: B. Schmieder, J.C. del Toro Iniesta, M. Vázquez, (eds.), *Advances in the Physics of Sunspots*, ASP Conf. Ser. 118, p. 212
Martínez Pillet, V.: 2000, A&A 361, 734
Mathew, S.K., Solanki, S.K., Lagg, A.: 2002, in: *Poster Proc. 1st Potsdam Thinkshop on Sunspots and Starspots*, AIP, in press
McPherson, M.R., Lin, H., Kuhn, J.R.: 1992, SoPh 139, 255
Meyer, F., Schmidt, H.U., Weiss, N.O.: 1977, MNRAS 179, 741
Moreno Insertis, F.: 1986, A&A 166, 291
Muller, R.: 1992, in: J.H. Thomas, N.O. Weiss (eds.), *Sunspots: Theory and Observations*, Kluwer, Dordrecht, p. 175
Pahlke, K.-D., Wiehr, E.: 1990, A&A 228, 246
Parker, E.N.: 1975, SoPh 40, 291
Parker, E.N.: 1979a, ApJ 230, 905
Parker, E.N.: 1979b, ApJ 230, 914
Parker, E.N.: 1979c, ApJ 234, 333
Penn, M.J., Kuhn, J.R.: 1995, ApJ 441, L51
Peter, H.: 1996, MNRAS 278, 821
Piskunov, N.: 2002, AN, in press
Rimmele, T.R.: 1995, A&A 298, 260
Rimmele, T.R.: 1995, ApJ 445, 511
Ringnes, T.S., Jensen, E.: 1961, ApNr 7, 99
Royal Greenwich Observers: 1925, MNRAS 169, 35
Rüedi, I., Solanki, S.K., Livingston, W.: 1995, A&A 293, 252
Rüedi, I., Solanki, S.K., Keller, C.U., Frutiger, C.: 1998a, A&A 338, 1089
Rüedi, I., Solanki, S.K., Stenflo, J.O., Tarbell, T., Scherrer, P.H.: 1998b, A&A 335, L97

Rüedi, I. Solanki, S.K., Keller, C.U.: 1999, A&A 348, L37
Saar, S.H., Piskunov, N.E., Tuominen, I.: 1992, in: M.S. Giampapa, J.A. Bookünder (eds.), *Cool Stars, Stellar Systems and the Sun VII*, ASP Conf. Ser. 26, p. 255
Sánchez Almeida, J., Lites, B.W.: 1992, ApJ 398, 359
Schlichenmaier, R.: 1997, *Die Dynamik magnetische Flussröhren in Sonnenflecken*, PhD Thesis, Herbert Utz Verlag Wissenschaft, Munich
Schlichenmaier, R.: 2002, AN, in press
Schlichenmaier, R., Schmidt, W.: 1999, A&A 349, L37
Schlichenmaier, R., Jahn, K., Schmidt, H.U.: 1998a, ApJ 493, L121
Schlichenmaier, R., Jahn, K., Schmidt, H.U.: 1998b, A&A 337, 897
Schlichenmaier, R., Collados, M.: 2002, A&A 381, 668
Schmidt, W., Hofmann, A., Balthasar, H., Tarbell, T.D., Frank, Z.A.: 1992, A&A 264, L27
Schmidt, W., Balthasar, H.: 1994, A&A 283, 241
Schmidt, H.U., Wegmann, R.: 1983, in: B. Brosowski, E. Martensen (eds.), *Dynamical Problems in Mathematical Physics*, Verlag P. Lang, Frankfurt, p. 137
Schrijver, C.J.: 2002, AN 323, 157
Schüssler, M.: 1986, in: W. Deinzer, M. Knölker, H.H. Voigt (eds.), *Small Scale Magnetic Flux Concentrations in the Solar Photosphere*, Vandenhoeck & Ruprecht, Göttingen, p. 103
Schüssler, M.: 2002, AN, in press
Schüssler, M., Solanki, S.K.: 1992 A&A 264, L13
Shinkawa, T., Makita, M.: 1996, in: Y. Uchida, (ed.), *Magnetodynamic Phenomena in the Solar Atmosphere – Prototypes of Stellar Magnetic Activity*, Kluwer, Dordrecht, p. 615
Simon, G.W., Weiss, N.O.: 1970, SoPh 13, 85
Simon, G.W., Weiss, N.O., Nye, A.H.: 1983, SoPh 87, 65
Skumanich, A., Lites, B.W., Martínez Pillet, V.: 1994, in: R.J. Rutten, C.J. Schrijver (eds.), *Solar Surface Magnetism*, Kluwer, Dordrecht, p. 99
Sobotka, M., Brandt, P.N., Simon, G.W.: 1999, A&A 348, 621
Solanki, S.K.: 1997, in: B. Schmieder, J.C. del Toro Iniesta, M. Vázquez, (eds.), *Advances in the Physics of Sunspots*, ASP Conf. Ser. 118, p. 178
Solanki, S.K.: 1999, in: C.J. Butler, J.G. Doyle (eds.): *Solar and Stellar Activity: Similarities and Differences*, ASP Conf. Ser. 158, p. 109
Solanki, S.K.: 2002, A&AR, to be submitted
Solanki, S.K., Schmidt, H.U.: 1993, A&A 267, 287
Solanki, S.K., Rüedi, I., Livingston, W.: 1992, A&A 263, 339
Solanki, S.K., Montavon, C.A.P.: 1993, A&A 275, 283
Solanki, S.K., Walther, U., Livingston, W.: 1993, A&A 277, 639
Solanki, S.K., Montavon, C.A.P., Livingston, W.: 1994, A&A 283, 221
Solanki, S.K., Finsterle, W., Rüedi, I., Livingston, W.: 1999, A&A 347, L27
Spruit, H.C.: 1981, in: L.E. Cram, J.H. Thomas (eds.), *Physics of Sunspots*, NSO, Sunspot (NM), p. 359
Stanchfield II, D.C.H., Thomas, J.H., Lites, B.W.: 1997, ApJ 477, 485
Sütterlin, P.: 2001, A&A 374, L21
Thomas, J.H., Weiss, N.: 1992, in: J.H. Thomas, N. Weiss (eds.), *Sunspots: Theory and Observations*, Kluwer, Dordrecht, p. 3
Title, A.M., Frank, Z.A., Shine, R.A., Tarbell, T.D., Topka, K.P., Scharmer, G., Schmidt, W.: 1992, in: J.H. Thomas, N. Weiss, (eds.), *Sunspots: Theory and Observations*, Kluwer, Dordrecht, p. 195
Title, A.M., Frank, Z.A., Shine, R.A., Tarbell, T.D., Topka, K.P., Scharmer, G., Schmidt, W.: 1993, ApJ 403, 780
Tritschler, A., Schmidt, W.: 1997, A&A 321, 643
Valenti, J.A., Johns-Krull, C.: 2001, in: G. Mathys, et al. (eds.): *Magnetic Field Across the HR-diagram*, ASP Conf. Ser. 248, p. 179
Westendorp, Plaza C., del Toro Iniesta, J.C., Ruiz Cobo, B., Martínez Pillet, V., Lites, B.W., Skumanich, A.: 1997a, Nature 389, 47
Westendorp, Plaza C., del Toro Iniesta, J.C., Ruiz Cobo, B., Martínez Pillet, V., Lites, B.W., Skumanich, A.: 1997b, in: B. Schmieder, et al. (eds.): *Advances in the Physics of Sunspots*, ASP Conf. Ser. 118, p. 202
Westendorp, Plaza C., del Toro Iniesta, J.C., Ruiz Cobo, B., Martínez Pillet, V., Lites, B.W., Skumanich, A.: 2001, ApJ 547, 1130
White, S.: 2002, AN 323, 265
Wiehr, E.: 2000, SoPh 197, 227
Wittmann, A.D.: 1974, SoPh 36, 29
Yun, H.S.: 1972, SoPh 22, 137
Yurchyshyn, V.B., Wang, H., Goode, P.R., 2001: ApJ 550, 470
Zhang, H.: 1994, SoPh 154, 207
Zhang, J., Solanki, S.K., Wang, J.: 2002, A&A, submitted
Zhao, J., Kosvichev, A.G., Duvall jr., T.L.: 2001, ApJ 557, 384