Heliospheric Coordinate Systems

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Abstract

This article gives an overview and reference to the most common coordinate systems currently used in space science. While coordinate systems used in near-Earth space physics have been described in previous work we extend that description to systems used for physical observations of the Sun and the planets and to systems based on spacecraft location. For all systems we define the corresponding transformation in terms of Eulerian rotation matrices. We also give first order Keplerian elements for planetary orbits and determine their precision for the period 1950-2050 and describe methods to improve that precision. We also determine the Keplerian orbital elements for most major interplanetary missions and discuss their precision. We also give reference to a large set of web-sources relevant to the subject.
Discipline: b. Celestial Mechanics.

1 Introduction

Coordinate systems used in near-Earth space physics have been well covered by the works of Russell (1971) and Hapgood (1992). But there has been a lack of publicly available documentation on coordinate systems used in heliospheric space missions and in many cases the information does not seem comprehensive enough for reference purposes[1]. Specifically descriptions of systems based on the physical ephemeris of the Sun and planets and systems based on spacecraft position are currently not available in a form that makes the relation between both systems easy to understand. Experience shows that this deficiency leads to misunderstandings and errors in the production of spacecraft data sets. Another problem is the lack of information on the precision of transformations. This document tries to collect all information necessary for the calculation of coordinate transformations in space science and determines the precision of these transformations whenever possible.
We base all calculations on the current edition of the Astronomical Almanac (2000), hereafter cited as *A.*[2]and the Expl.Suppl. (1992), hereafter cited as *S.*. This means that the base system of astronomical constants used is the IAU(1976) system described in Astr.Alm.Suppl. (1984) implemented in the numerically integrated ephemeris DE200 (Standish, 1990). In general this paper does not describe methods applicable for spatial resolutions below the level of 1 arcsecond but the reader will be able to find the information necessary to achieve higher precision in the cited sources.
To achieve the highest precision in planetary positions one can either (1) implement the numerically integrated ephemeris DE200 or its more precise sequel DE405 (Standish, 1998a)[3], (2) implement a polynomial expansion of the ephemeris, for example the VSOP87 model(Bretagnon and Francou, 1988)[4], which is an expansion of DE200,

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[1]The American National Space Science Data Center(NSSDC) maintains a webpage at http://nssdc.gsfc.nasa.gov/space/helios/coor_des.html
[2]See the Nautical Almanac Offices webpage for details:
http://www.usno.navy.mil/and http://www.nao.rl.ac.uk/
[3]JPL Horizons System and DE200 at http://ssd.jpl.nasa.gov/horizons.html
[4]Data are available at the Institut de Mecanique Celeste at http://www.bdl.fr/

or (3) extend the formulae given in this paper to higher order in time using the values given by Simon et al. (1994) which are also based on VSOP87. Since the extraction code for DE200 is available in different computer languages, its implementation is easy (see e.g. Heafner (1999)) but the size of the corresponding data files may prevent its inclusion in distributed software. For the implementation of VSOP87 we recommend the book by Meeus (2000). In this paper we include first order mean orbital elements from Simon et al. (1994) and give the resulting precision with respect to DE200. The deviations are on the order of arcseconds while differences between DE200 and DE405 are only a few milliarcseconds.
We should point out that for purposes of spacecraft navigation or problems of planetary encounters it is recommended to install a tested software system whenever this is provided by the respective spacecraft navigation team. For most NASA missions such a system is available in the form of the JPL SPICE system [5]. The SPICE system is a software library which implements DE200 and other reference systems in the form of position and attitude data files ('SPICE kernel files') for solar system bodies and spacecraft. Unfortunately SPICE kernels do not cover all NASA missions and the precision of reconstructed trajectory data is usually not provided. Detailed documentation on SPICE is only available via software file headers, this paper may provide a useful introduction to the principles implemented in SPICE and similar software packages. Before considering implementing formulae given in this paper in your own software package, you might consider implementing the systems cited above, though these will not contain all the coordinate systems defined in our paper. Most data in this paper have been cross-checked by recopying them from the text into our software and comparing the results with tested data. To ease the software implementation of formulae given in this paper we are providing all data contained in the paper on our website [6], and will provide corrections and updates on that site as long as possible. The website also contains orbital plots used to determine the precision of data given in this paper.
We also cite the formulae and methods given by Hapgood (1992) for geocentric systems, which are based on the Astronomical Almanac for Computers (1988) which is no longer updated by the Nautical Almanac Offices. The formulae used by Hapgood (1992) are first order approximations of the third order formulae given in Expl.Suppl. (1961). We show later that they achieve a precision of about $34''$ for the timespan 1950-2050 if precession and nutation are included. For many practical purposes the first order approximation is sufficient, but a geocentric error of $34''$ corresponds to a distance of 230km at the L1 Lagrangian point which might be of importance for relative timings between spacecraft for geocentric systems. To keep the paper as compact as possible we will give formulae for planetary orbits to first order only but will point the reader to the sources for improving the precision. The formulae for nutation and precession are given to a precision of at least $2''$ for the period 1950-2050, which allows a higher accuracy transformation between inertial systems. Numerical values are either given in decimal degree (°) or arcseconds ($''$). Throughout this paper we use Eulerian matrix rotations to describe transformations denoted $E(\Omega,\theta,\phi)$ (see Appendix). A concise explanation of many terms and systems used in this paper may be found in section L of the Astronomical Almanac (2000).

2 Time

Table 1: Time-scales relevant in space science [see *A.* B4]

UT1	universal time, defined by the mean solar day
TAI	international atomic time, defined by SI seconds
UTC	coordinated universal time, TAI - leap seconds, broadcast standard
TT	terrestrial time, TT=TAI+$32^s.184$, basis for geocentric ephemeris
TDB	barycentric dynamical time, defined by the mean solar day at the solar system barycentre, basis for solar system ephemeris

The time-scales relevant for coordinate transformations are defined in Tab.1. Formulae in the J2000.0 reference system are in ephemeris time T_{eph} [*S.* 2.26, but see also Standish (1998b)], but for most purposes of space data analysis one may neglect the difference of less than 2 msec between T_{eph}, Barycentric Dynamical Time (TDB)

[5] JPL SPICE at http://naif.jpl.nasa.gov/naif.html
[6] http://www.space-plasma.qmul.ac.uk/heliocoords/

and Terrestrial Time (TT) [*A.* B5] and less than 0.1s between the two Universal Times (UTC, UT1). A difference between Atomic Time (TAI) and Coordinated Universal Time (UTC) is introduced by leap-seconds tabulated in Tab.2 [*A.* K9 for current table] [7].

Table 2: Leap seconds ΔA = TAI-UTC [see *A.* K9]

1972/1/1 +10s	1972/7/1 +11s	1973/1/1 +12s
1974/1/1 +13s	1975/1/1 +14s	1976/1/1 +15s
1977/1/1 +16s	1978/1/1 +17s	1979/1/1 +18s
1980/1/1 +19s	1981/7/1 +20s	1982/7/1 +21s
1983/7/1 +22s	1985/7/1 +23s	1988/1/1 +24s
1990/1/1 +25s	1991/1/1 +26s	1992/7/1 +27s
1993/7/1 +28s	1994/7/1 +29s	1996/1/1 +30s
1997/7/1 +31s	1999/1/1 +32s.	

Thus TDB or TT can be approximated from UTC by TDB=UTC+$32^s.184+\Delta A$ where ΔA is the number of elapsed leap seconds to date. For earlier dates Meeus (2000) gives different approximation formulae for UTC-TDB. Spacecraft data are usually given in UTC. Relative velocities of solar system objects are small enough ($<$ 100 km/s) to neglect the difference in time systems. Care must only be taken for problems of relative timing. If high precision timing ($<$ 0.1s) is requested the reader should refer to McCarthy (1996) and to the documentation of the SPICE system (see above). The reference points in time (*epochs*) for the ephemeris are given in Tab.3. Before 1984 the ephemeris referred to B1950.0 and many spacecraft trajectory data are still given in the older system (see Appendix). The actual position of solar system objects and spacecraft is usually given in an *epoch of date* system which means that coordinates refer to the orientation of the Earth equator or ecliptic at the time of measurement. We give formulae to convert from the reference epoch to the epoch of date in the following section 2.1.

Table 3: Epoch definitions [*S.* Table 15.3, *A.* B4]

J1900.0 =	1900 January 1, 12.00TDB =	JD 2415020.0
J1950.0 =	1950 January 1, 00.00TDB =	JD 2433282.5
J2000.0 =	2000 January 1, 12.00TDB =	JD 2451545.0
B1950.0 =		JD 2433282.42345905

The Julian Day Number (*JD*) starts at Greenwich mean noon 4713 Jan. 1, B.C. [*S.* 2.26]. The *epoch day number* is defined in this paper as the fractional number of days of 86400 seconds from the epoch:

$$d_0 = (JD - 2451545.0). \tag{1}$$

Formulae from *S.* and *A.* use Julian centuries (T_0) from J2000.0. One Julian century has 36525 days, one Julian year has 365.25 days, s.t. [*S.* T3.222.2]

$$T_0 = d_0/36525.0 \qquad \text{and} \qquad y_0 = d_0/365.25 \tag{2}$$

We use this notation throughout the paper. When the astronomical reference systems eventually switch to the next epoch (presumably J2050.0) formulae given in this paper have to be adapted.

2.1 Precession and Nutation

The two fundamental celestial reference systems used in heliospheric science are the ecliptic system defined by the mean orbit of the Earth at J2000.0 and the equatorial system defined by the mean orientation of the Earth equator at J2000.0 (see Fig.1).

[7] See also the webpage of the International Earth Rotation Service (IERS) at http://www.iers.org/

The intersection of the Earth equatorial plane and the Earth orbital plane (*ecliptic*) defines the line of the equinoxes (Fig.1). The ascending node of the geocentric ecliptic defines the vernal equinox (*first point of Aries*). The obliquity of the ecliptic at epoch J2000.0 with respect to the mean equator at epoch J2000.0 is given by [*A.* K6]

$$\varepsilon_{0J2000} = 23°26'21''.448 \approx 23°.439291111 \tag{3}$$

The orientation of both planes changes over time by solar, lunar and planetary gravitational forces on the Earth axis and orbit. The continuous change is called 'general precession', the periodic change 'nutation'. *Mean* quantities include precessional corrections, *true* quantities both precessional and nutational corrections.
The mean obliquity of the ecliptic of date with respect to the mean equator of date is given by [*S.* 3.222-1,*A.* B18]

$$\begin{aligned} \varepsilon_{0D} &= \varepsilon_{0J2000} - 46''.8150T_0 - 0''.00059T_0^2 + 0''.001813T_0^3 \\ &\approx 23°.439291111 - 0°.013004167T_0 - 0°.000000164T_0^2 + 0°.000000504T_0^3. \end{aligned} \tag{4}$$

The true obliquity of date $\varepsilon_D = \varepsilon_{0D} + \Delta\varepsilon$ includes the effects of nutation which are given to a precision of $2''$ for the period 1950-2050 by [*S.* 3.225-4]:

$$\Delta\varepsilon = 0°.0026\cos(125°.0 - 0°.05295d_0) + 0°.0002\cos(200°.9 + 1°.97129d_0). \tag{5}$$

For the calculation of true equatorial positions one also needs the *longitudinal nutation* which is given to first order by [*S.* 3.225-4]:

$$\Delta\psi = -0°.0048\sin(125°.0 - 0°.05295d_0) - 0°.0004\sin(200°.9 + 1°.97129d_0). \tag{6}$$

The corresponding rotation matrix from the mean equator of date to the true equator of date is then given by [S. 3.222.3]:

$$N(GEI_D, GEI_T) = E(0°, -\varepsilon_D, 0°) * E(-\Delta\psi, 0°, 0°) * E(0°, \varepsilon_{0D}, 0°). \tag{7}$$

To achieve higher precision one has to add further terms for the series expansion for nutation from [S. Tables .3.222.1-3.224.2][8].
The orientation of the ecliptic plane of date (ε_D) with respect to the the ecliptic plane at another date (ε_F) is defined by the inclination π_A, the ascending node longitude Π_A of the plane of date ε_D relative to the plane of date F, and the difference in the angular distances p_A of the vernal equinoxes from the ascending node. Values for J2000.0 are given in [*S.* Table 3.211.1]:

$$\begin{aligned} \pi_A &= (47''.0029 - 0''.06603T_0 + 0''.000598T_0^2)t + (-0''.03302 + 0''.000598T_0)t^2 + 0''.000060t^3 \\ \Pi_A &= 174°52'34''.982 + 3289''.4789T_0 + 0''.60622T_0^2 + (-869''.8089 - 0''.50491T_0)t + 0''.03536t^2 \\ p_A &= (5029''.0966 + 2''.22226T_0 - 0''.000042T_0^2)t + (1''.11113 - 0''.000042T_0)t^2 - 0''.000006t^3 \end{aligned} \tag{8}$$

where $T_0 = \varepsilon_F - \varepsilon_{J2000}$ and $t = \varepsilon_D - \varepsilon_F$ are the distances in Julian centuries between the fixed epoch ε_F and J2000.0 and between ε_D and ε_F respectively. The corresponding Eulerian rotation matrix is

$$P(HAE_{J2000}, HAE_D) = E(\Pi_A, \pi_A, -p_A - \Pi_A). \tag{9}$$

Coordinates defined on the equator of epoch are transformed to the equator of date by the Eulerian precession matrix

$$P(\varepsilon_F, \varepsilon_D) = E(90° - \zeta_A, \theta_A, -z_A - 90°) \tag{10}$$

The Eulerian angles are defined in [*S.* Table 3.211.1]:

$$\begin{aligned} \theta_A &= (2004''.3109 - 0''.85330T_0 - 0''.000217T_0^2)t + (-0''.42665 - 0''.000217T_0)t^2 - 0''.041833t^3 \\ \zeta_A &= (2306''.2181 + 1''.39656T_0 - 0''.000139T_0^2)t + (0''.30188 - 0''.000344T_0)t^2 + 0''.017998t^3 \\ z_A &= (2306''.2181 + 1''.39656T_0 - 0''.000139T_0^2)t + (1''.09468 + 0''.000066T_0)t^2 + 0''.018203t^3 \end{aligned} \tag{11}$$

where t and T_0 are defined as above. These formulae define the precession to the precision used for the Astronomical Almanac but may be easily reduced to lower order.

[8] Note that there is a typographic error in the mean lunar ascending longitude in [S. Tab.3.222.2], the first argument should read $\Omega = 125°02'40''.280$.

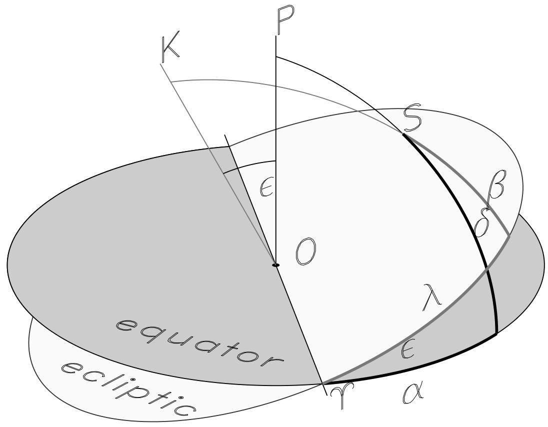

Figure 1: Ecliptic and Equatorial Systems: the ecliptic plane is inclined by the obliquity ε towards the Earth equatorial plane. The vernal equinox Υ defines the common +X-axis, the +Z-axes are defined by the Northern poles P and K. The position of an object S is defined by Right Ascension α and Declination δ in the equatorial system, by ecliptic longitude λ and latitude β in the ecliptic system.

Hapgood (1997) gives only the first order transformation between epoch of J2000.0 and epoch of date which is a reduction of the above formulae and also given to higher precision in [*A.* B18]:

$$\begin{aligned}
\theta_A &= 0^\circ.55675 T_0 - 0^\circ.00012 T_0^2 \\
\zeta_A &= 0^\circ.64062 T_0 + 0^\circ.0008 T_0^2 \\
z_A &= 0^\circ.64062 T_0 + 0^\circ.00030 T_0^2
\end{aligned} \tag{12}$$

For the heliocentric position of the Earth a complete neglect of precession results in an error of $1^\circ.0$ for the period 1950-2050, a neglect of nutation results in an error of $20''$. Using first order nutation and precession reduces the error to $2''.0$.

3 Description of Coordinate Systems

Each coordinate system we describe in the following is defined by the orientation of its three right handed cartesian axes in euclidean space and the position of its origin, relative to some other system. The +Z-axis always defines the polar axis of the respective spherical coordinates: latitudes are counted from the XY-plane (polar axis 90°), *co-latitudes* from the polar axis, longitudes are counted from the +X-axis (*prime meridian*) clockwise (left handed, +Y-axis −90°) or counter-clockwise (right handed, +Y-axis 90°) as specified.

3.1 Celestial Systems

- Geocentric Earth Equatorial GEI_{J2000} (Hapgood, 1995)
  This system is realized through the International Celestial Reference Frame (ICRF), which is the base system

for star catalogues and reference values of planetary positions (see the IERS webpage cited above).
XY-plane: Earth mean equator at J2000.0
+X-axis: First Point of Aries, i.e. vector(Earth-Sun) of vernal equinox at epoch J2000.0
Angles: Declination δ and Right Ascension α right handed.

- Mean Geocentric Earth Equatorial GEI_D (Hapgood, 1995)
  XY-plane: Earth mean equator of date.
  +X-axis: First point of Aries, i.e. vector(Earth-Sun) of vernal equinox of date.
  Transform: $T(GEI_{J2000}, GEI_D) = P(\varepsilon_D, \varepsilon_0)$ as defined in eqn.10.

- True Geocentric Earth Equatorial GEI_T (Hapgood, 1995)
  Base system for actual position of objects.
  XY-plane: Earth true equator of date.
  +X-axis: First point of Aries, i.e. vector(Earth-Sun) of vernal equinox of date.
  Transform: $T(GEI_D, GEI_T)$ as defined in eqn.7.

- Heliocentric Aries Ecliptic HAE_{J2000} (Fig.1)
  XY-plane: Earth mean ecliptic at J2000.0
  +X-axis: First point of Aries, i.e. vector(Earth-Sun) of vernal equinox at epoch J2000.0
  Angles: Celestial latitude β and longitude λ right handed.
  Transform: $T(GEI_{J2000}, HAE_{J2000}) = <\varepsilon_0, X> = E(0, \varepsilon_0, 0)$
  and subtraction of solar position vector if necessary.

- Heliocentric Aries Ecliptic HAE_D
  XY-plane: Earth mean ecliptic of date
  +X-axis: First point of Aries, i.e. vector(Earth-Sun) of vernal equinox of date
  Transform: $T(HAE_{J2000}, HAE_D) = E(\Pi_A, \pi_A, -p_A - \Pi_A)$ as defined in eqn.9 and
  $T(GEI_D, HAE_D) = E(0, \varepsilon_D, 0)$ where ε_D is defined by eqn.5.

3.2 Heliographic Systems

3.2.1 Solar Pole and Prime Meridian

Heliographic coordinate systems use the position of the solar rotation axis which is defined by its declination $\delta_\odot$ and the right ascension $\alpha_\odot$ with respect to the celestial pole ($GEI_{J2000} + Z$). Values for J2000.0 are [*S.* Table 15.7]:

$$\delta_\odot = 63^\circ.87 \qquad \alpha_\odot = 286^\circ.13 \tag{13}$$

The traditional definition refers to the ecliptic of date with the values for the inclination $i_\odot$ of the solar equator and longitude of the ascending node $\Omega_\odot$ [*S.* 7.2, note the typo]:

$$i_\odot = 7^\circ.25 \qquad \Omega_\odot = 75^\circ.76 + 1^\circ.397 T_0 \tag{14}$$

The ecliptic values for the polar axis have been in use since their first determination by Carrington. Newer measurements show that the axis direction is less well defined (Balthasar et al., 1987) but for the purpose of coordinate transformations one sticks with the original values. The same is true for the Solar rotation period for which the adopted values are [*A.* C3] :

$$r_{sid} = 25.38 days \qquad \text{and} \qquad r_{syn} = 27.2753 days, \tag{15}$$

where the sidereal period r_{sid} is relative to the celestial sphere, and the synodic relative to the rotating Earth (see also Rosa et al. (1995)). The time dependence in $\Omega_\odot$ takes approximate account of the ecliptic precession such that no further precessional transformation should be applied but there is of course a small difference between the ecliptic and the equatorial definition. In transformation of datasets always the equatorial values should be used.
Physical observations of the Sun refer to the apparent center of the visible disk from Earth (*subterrestrial point*) whose heliocentric ecliptic longitude is the apparent longitude of the Earth $\lambda_\odot = \lambda_{geo} - a$ defined in eqn.36 corrected for light aberration ($a \approx 20''$, see Appendix A.3).

3.2.2 Systems

As pointed out in section 4.3 heliographic systems should refer to a solar reference ellipsoid, but since the oblateness of the Sun is difficult to measure (Stix, 1989), for the following definitions the Sun is assumed to be spherical.

- Heliographic Coordinates *HGC* (Expl.Suppl., 1961; Stix, 1989)
  Physical features on the surface of the Sun are located in *Heliographic coordinates* (Expl.Suppl., 1961, 11.B). Heliographic latitude is measured from the solar equator positive towards North, Heliographic longitude is defined westward (i.e. in the direction of planetary motion) from the solar prime meridian which passed through the ascending node on the ecliptic of date on 1854 Jan 1, noon (JD 239 8220.0). Heliographic longitude is sometimes identified with *Carrington longitude*, but this usage should be avoided since there have been different definitions of the later term over time.

XY-plane:	Solar equator of date
+X-axis:	ascending node on 1854 Jan 1, noon (JD 239 8220.0)
Angles:	Heliographic latitude Ψ and longitude Φ right handed.
Transform:	$T(GEI_{J2000}, HGC_{J2000}) = E(\alpha_\odot + 90°, 90° - \delta_\odot, W_0)$
	with the values from eqn.13 and $W_0 = 84°.10 + 14°.1844 d_0$[*S.* Table 15.7].

  Alternatively (but less exact) one may use the transformation from ecliptic coordinates:
  Transform: $T(HAE_{J2000}, HGC_D) = E(\Omega_\odot, i_\odot, w_0)$ where $\Omega_\odot$ and $i_\odot$ are defined in eqn.14 and the prime meridian angle is given by

  $$w_0 = (d_0 + 2415020.0 - 2398220.0)/25.38 \times 360° \tag{16}$$

- Solar Rotations(Expl.Suppl., 1961)
  Rotations of the Sun are counted in *Carrington rotations R*; a rotation starts when the heliographic prime meridian crosses the subterrestrial point of the solar disc. The angular offset θ between this point and the ascending node can be calculated from (Hapgood, 1992):

  $$\theta_\odot = \arctan(\cos i_\odot \tan(\lambda_\odot - \Omega_\odot)) \tag{17}$$

  such that the quadrant of θ is opposite that of $\lambda_\odot - \Omega_\odot$. Note that θ is called $L_0 - M$ in Expl.Suppl. (1961). The first Carrington rotation started on 1853 Nov 9 (JD 2398167.329), later start points can be calculated using the synodic period $r_{syn} = 27.2753$ days. The term *Carrington Time* has been used for the pair of numbers (R, L_0), where L_0 is the heliographic longitude of the subterrestrial point. For geophysical effects *Bartels rotations* have been used which start at 1832 Feb 8.00 (JD 239 0190.50) with a period of 27.0 days (Bartels (1952)).

- Heliocentric Earth Ecliptic *HEE* (Hapgood, 1992)

XY-plane:	Earth mean ecliptic of date.
+X-axis:	vector (Sun-Earth).
Transform:	$T(HAE_D, HEE_D) = E(0°, 0°, \lambda_{geo})$

  where λ_{geo} is the geometric ecliptic longitude of the Earth which can be determined by one of the methods described in section 4.2.1 or directly from eqn.36 to a precision of $34''$.

- Heliocentric Earth Equatorial *HEEQ* (Hapgood, 1992)

XY-plane:	Solar equator of date.
+X-axis:	Intersection between solar equator and solar central meridian of date.
Angles:	Heliocentric latitude Ψ and central longitude θ (increasing eastward) right handed.
Transform:	$T(HAE_D, HEEQ) = E(\Omega_\odot, i_\odot, \theta_\odot)$, where $\theta_\odot$ is defined in eqn.17.

- Heliocentric Inertial *HCI* (Burlaga, 1984)
  Burlaga (1984) originally defined a system, called heliographic inertial (*HGI*), with reference to the orientation of the Solar equator in J1900.0. We propose to call the system heliocentric and base it on J2000.0 instead:

XY-plane:	Solar equator of J2000.0.
+X-axis:	Solar ascending node on ecliptic of J2000.0.
Transform:	$T(HAE_{J2000}, HCI) = E(\Omega_\odot(T_0 = 0), i_\odot, 0°)$

- Heliocentric of Date *HCD*
  XY-plane: Solar equator of date.
  +X-axis: Solar ascending node on ecliptic of date.
  Transform: $T(HAE_D, HCD) = E(\Omega_\odot, i_\odot, 0°)$

3.3 Geocentric Systems

Geocentric systems have been described by Russell (1971) and Hapgood (1992) with corrections given in Hapgood (1995) and Hapgood (1997)[9]. You will also find a comprehensive introduction in Appendix 3 of Kivelson and Russell (1995). The ESA SPENVIS system contains an extensive description of geocentric systems [10]. A software package by J.-C. Kosik is also maintained and documented at the Centre de Données de la Physique des Plasmas [11]. We do not describe systems relevant for observations from the Earth surface, see [*S.*, Ch.4] for a decription of these systems.

3.3.1 Greenwich mean sidereal time

The Greenwich mean sidereal time is defined by the hour angle between the meridian of Greenwich and mean equinox of date at 0^h UT1: [*A.* B6]:

$$\Theta_{GMST} = 24110^s.54841 + 8640184^s.812866 T_U + 0^s.093104 T_U^2 - 6^s.2 \cdot 10^{-6} T_U^3, \tag{18}$$

in seconds of a day of 86400s UT1, where T_U is the time difference in Julian centuries of Universal Time (UT1) from J2000.0.
From this the hour angle in degree θ_{GMST} at any instant of time d_0 (Julian days from J2000.0) can be calculated by

$$\theta_{GMST} = \Theta_{GMST}(T_U(0^h)) \times 360°/86400^s + 180° + 360° * d_0 \tag{19}$$

For the precision needed in this paper we may neglect the difference between T_U and T_0, such that (Meeus, 2000):

$$\theta_{GMST} \approx 280°.46061837 + 360°.98564736629 d_0 + 0°.0003875 T_0^2 - 2°.6 \cdot 10^{-8} T_0^3. \tag{20}$$

3.3.2 Earth magnetic pole

The geographic position of the Earth magnetic pole and the dipole moment M_E can be calculated from the first three coefficients of the International Geomagnetic Reference Field (IGRF) published 5-yearly by IAGA Working Group 8[12]. For full precision interpolate the values g_{10}, g_{11}, and h_{11} for the date requested and determine the geographic longitude λ_D, latitude Φ_D and moment M_E by (Hapgood, 1992, 1997; Kertz, 1969):

$$\lambda_D = \arctan(h_{11}/g_{11}) \qquad \Phi_D = 90° - \arctan\frac{g_{11}\cos\lambda_D + h_{11}\sin\lambda_D}{g_{10}} \qquad M_E = \sqrt{g_{10}^2 + g_{11}^2 + h_{11}^2} * R_E^3, \tag{21}$$

where R_E= 6378.14 km is the Earth equatorial radius and λ_D lies in the fourth quadrant.
For the period 1975-2000 we derive following linear approximations with a precision of 0°.05:

$$\lambda_D = 288°.44 - 0°.04236 y_0 \qquad \Phi_D = 79°.53 + 0°.03556 y_0 \qquad M_E = 3.01117 - 0.00226 y_0 [10^{-6}\mathrm{T} \cdot R_E^3], \tag{22}$$

where y_0 are Julian years from J2000.0.

3.3.3 Systems

The following systems are referred to the true Earth equator or ecliptic of date, that is corrections for nutation and precession should be applied in transformations. We also give the bracket notation $<,>$ used by Hapgood (1992) (see Appendix).

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[9] See also their webpage at http://sspg1.bnsc.rl.ac.uk/Share/Coordinates/ct_home.htm
[10] ESA SPENVIS webpage at http://www.spenvis.oma.be/spenvis/
[11] See under MAGLIB at http://cdpp.cesr.fr
[12] See their webpage at http://www.ngdc.noaa.gov/IAGA/wg8/

- Geographic Coordinates GEO (Hapgood, 1992)
  XY-plane: True Earth equator of date.
  +X-axis: Intersection of Greenwich meridian and Earth equator.
  Angles: Geographic latitude and longitude (increasing westward) right handed, in the sense of a planetographic system (see section 4.3).
  Transform: $T(GEI_T, GEO) = <\theta_{GMST}, Z> = E(0°, 0°, \theta_{GMST})$
  where the θ_{GMST} is given by eqn.20.

- Geocentric Solar Ecliptic GSE (Hapgood, 1992)
  XY-plane: Earth mean ecliptic of date.
  +X-axis: vector Earth-Sun of date.
  Transform: $T(HAE_D, GSE) = <\lambda_{geo} + 180°, Z> = E(0°, 0°, \lambda_{geo} + 180°)$ with λ_{geo} from eqn.36 and subtraction of solar position vector if necessary.
  Also $T(GEI_D, GSE_D) = T(GEI_D, HAE_D)^{-1} * T(HAE_D, GSE)^{-1}$.

- Geocentric Solar Magnetospheric GSM (Hapgood, 1992)
  +Z-axis: projection of northern dipole axis on GSE_D YZ plane.
  +X-axis: vector Earth-Sun of date.
  Transform: $T(GSE_D, GSM) = <-\psi, X> = E(0°, -\psi, 0°)$ where $\psi = \arctan(y_e/z_e)$ and $\mathbf{Q}_e = (x_e, y_e, z_e)$ is the Earth dipole vector in GSE-coordinates.
  This can be calculated from the geographic position $\mathbf{Q}_g$ given in eqn.22 by
  $\mathbf{Q}_e = T(GEI_D, GSE_D) * T(GEI_D, GEO)^{-1}\mathbf{Q}_g$.

- Boundary Normal Coordinates LMN (Russell and Elphic, 1978)[13]
  +Z-axis: Normal vector to Earth Magnetopause.
  +Y-axis: cross-product of +Z-axis and GSM-Z-axis.

  The normal vector may be determined by a model or by minimum-variance analysis of data.

- Solar Magnetic SM (Chapman and Bartels, 1962)
  +Z-axis: Northern Earth dipole axis of date.
  +Y-axis: cross-product of +Z-axis and Earth-Sun vector of date.
  Transform: $T(GSM, SM) = <-\mu, Y> = E(90°, -\mu, -90°)$
  where $\mu = \arctan \frac{x_e}{\sqrt{y_e^2 + z_e^2}}$ with $\mathbf{Q}_e$ given above.

  The longitude of this system is also called *magnetic local time (MLT)* increasing eastwards from the anti-solar (0^h) to the solar (12^h) direction.

- Geomagnetic MAG (Chapman and Bartels, 1962)
  +Z-axis: Northern Earth dipole axis of date.
  +Y-axis: cross-product of Geographic North Pole of date and +Z-axis.
  Transform: $T(GEO, MAG) = <\Phi_D - 90°, Y> * <\lambda_D, Z> = E(\lambda_D + 90°, 90° - \Phi_D, -90°)$
  where Φ_D and λ_D are given in eqn.22.

  Geomagnetic latitude β_m and longitude λ_m (increasing eastward) refer to this system.

- Invariant magnetic shells (B_d, L_d)(McIlwain, 1966)
  These coordinates are used for functions of the magnetic field which are constant along the lines of force. For a position of radial distance R from the dipole center and magnetic latitude β_m in a dipolar field the magnetic field strength B_d and equatorial distance L_d of the line of forth are given by

$$B_d = \frac{M}{R^3}\sqrt{1 + 3\sin^2\beta_m} \qquad L_d = \frac{R}{\cos^2\beta_m}, \tag{23}$$

  where M is the magnetic moment of the dipole (see eqn.22 for Earth value). The offset between dipole center and gravity center ($\approx$ 500 km for Earth) has been neglected (Kertz, 1969).

[13] See also C.T. Russell's page at http://www-ssc.igpp.ucla.edu/ssc/tutorial/magnetopause.html

- Other magnetospheric coordinates [14][15][16]
  Many coordinate systems depend on a specific magnetic field model. For example Corrected Magnetic Coordinates (CGM)[17] are constructed by field line tracing. Magnetospheric Equatorial Coordinates (GME) use specific magnetotail models (Dunlop and Cargill, 1999). For a field model again (B, L) coordinates may be derived for which particle drift shells can be defined (McIlwain, 1966). See the references for details.

3.4 Position Dependent Systems

For the study of the local plasma environment of a spacecraft it is common to choose an axis system which depends on the position of the spacecraft. Widely used are Radial-Tangential-Normal systems defined by the radial vector from a central body to the spacecraft and the magnetic or rotational normal axis of that body. For highest precision one should use reference systems at the epoch of date.

- Heliocentric RTN System $HGRTN$ (Burlaga, 1984)
  This system was, for example, used by the Ulysses mission.
  +X-axis: vector (Sun-S/C).
  +Y-axis: cross-product of (heliographic polar axis) and +X-axis.
  Transform: $T(HCD, HGRTN) = E(\phi_{S/C} - 90°, \theta_{S/C}, 90°)$

  where $\phi_{S/C}$ and $\theta_{S/C}$ are the longitude and latitude of the spacecraft in the HCD system. Cartesian coordinates of this system are commonly called *Radial, Tangential, Normal* (RTN) coordinates.

- Dipole Meridian System DM (Kivelson and Russell, 1995)
  This system can be used in any dipolar field to separate radial and angular motions.
  +X-axis: vector (dipole Center-S/C).
  +Y-axis: cross-product of (dipole polar axis) and +X-axis.
  Transform: $T(MAG, DM) = E(\phi_{S/C} - 90°, \theta_{S/C}, 90°)$

  where $\phi_{S/C}$ and $\theta_{S/C}$ are the longitude and latitude of the spacecraft in the MAG system.

- Spacecraft solar ecliptic SSE [F.Neubauer (personal communication)]
  This system was for example used by the Helios mission.
  XY-plane: Earth mean ecliptic of date.
  +X-axis: projection of vector S/C-Sun on XY-plane.
  +Z-axis: ecliptic South pole.
  Transform: $T(HAE_D, SSE) = E(\phi_{S/C} - 90°, 180°, 90°)$

- Spin axis ecliptic SAE [NSSDC, Pioneer data pages]
  This spacecraft centered system was for example used by the Pioneer missions (under the acronym PE).
  +Z-axis: spacecraft spin axis $\mathbf{v}_A$ (towards Earth).
  +X-axis: cross-product of ecliptic polar axis of date and $\mathbf{v}_A$.
  Transform: $T(HAE_D, SAE) = E(\phi + 90°, \theta, 0.0)$

  where ϕ and θ are the ecliptic longitude and co-latitude of the spacecraft spin axis.

- Spin axis Sun pulse SAS
  This system is a fundamental reference system for most spinning spacecraft since the S/C - Sun meridian can easily be determined on board using a narrow-slit sun sensor. Thus a spacecraft-fixed instrumental system has only a longitudinal offset with respect to SAS linear in time.
  +Z-axis: spacecraft spin axis $\mathbf{v}_A$, right-handed orientation.
  +Y-axis: cross-product between +Z-axis and S/C - Sun vector $\mathbf{v}_S$.
  Transform: $T(HAE_D, SAS) = E(\phi + 90°, \theta, \Phi(\mathbf{v}_S))$

  where ϕ and θ are the ecliptic longitude and co-latitude of the spacecraft spin axis and $\Phi(\mathbf{v}_S)$ is the longitude of the vector $\mathbf{v}_{S0} = E(\phi + 90°, \theta, 0°) * \mathbf{v}_S$ and $\mathbf{v}_S$ is given in the ecliptic system.

[14] See also the APL Superdarn webpage http://superdarn.jhuapl.edu/aacgm/
[15] See also the University of Oulu spaceweb at http://spaceweb.oulu.fi/
[16] See also S. Haaland's page at http://gluon.fi.uib.no/ haaland/
[17] See also the NSSDC Modelweb at http://nssdc.gsfc.nasa.gov/space/cgm/

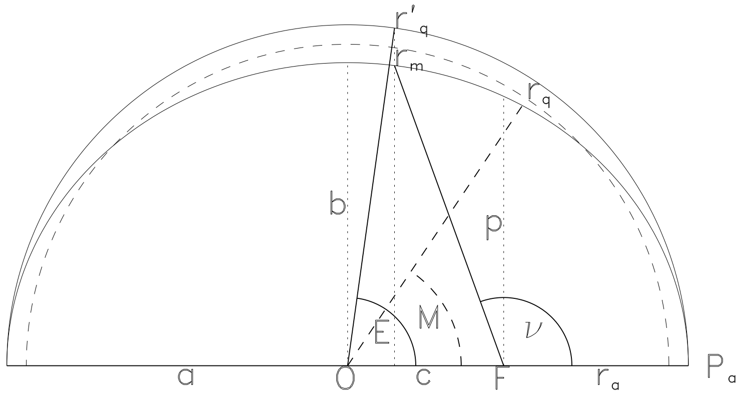

Figure 2: Keplerian orbital elements for the elliptical orbit of the point $\mathbf{r_m}$ around the focus F with true anomaly ν. Parameters of the ellipse are the axes a and b, the focal distance $c = ae$, the semi-latus rectum p and the point of periapsis P_a at distance r_a from F. Also shown are the concentric circles for the eccentric motion of the point $\mathbf{r'_q}$ with eccentric anomaly E, and mean motion of the point $\mathbf{r_q}$ with mean anomaly M (dashed).

4 Planetary Systems

4.1 Planetary Orbits

As pointed out in the introduction transformations based on classical Keplerian elements can only achieve a limited precision. But for many applications it is useful to have approximate positions available. For this reason we describe in the following the calculation of position and velocity of objects in Keplerian orbits. There are many textbooks on this subject – we recommend Murray and Dermott (2000) but e.g. Bate et al. (1971),Danby (1988) or Heafner (1999) are also very useful. There are also some good web sites devoted to the subject[18].

The gravitational motion of two bodies of mass M and m and position vectors $\mathbf{r_M}$ and $\mathbf{r_m}$ can be described in terms of the three invariants: *gravitational parameter* $\mu = \gamma_0(M+m)$, *specific mechanical energy* $\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}$, and *specific angular momentum* $h = |\mathbf{r} \times \mathbf{v}|$, where $\mathbf{r} = \mathbf{r_M} - \mathbf{r_m}$, $r = |\mathbf{r}|$ and $\mathbf{v} = \dot{\mathbf{r}}$. γ_0 is the constant of gravitation whose IAU1976 value is determined by [A. K6]:

$$\gamma_0 = k^2 \text{ with } k = 0.01720209895 \tag{24}$$

when masses are given in solar masses, distances in AU [1 AU = 149 597 870 km], and times in days.

The elements of the conical orbit (shown in Fig.2) are then given as *semi-major axis* $a = -\mu/\mathcal{E}$ and *semi-minor axis* $b = h/\sqrt{-\mathcal{E}}$, or alternatively as *semi-latus rectum* $p = b^2/a = h^2/\mu = a(1-e^2)$ and *eccentricity* $e = \sqrt{1-b^2/a^2} = \sqrt{1+\mathcal{E}h^2/\mu^2}$. Let the origin be at the focus $\mathbf{r_M}$, the vector $\mathbf{r}$ then describes the motion of the body $\mathbf{r_m}$. The *true anomaly* ν is the angle between $\mathbf{r}$ and the direction to the closest point of the orbit (*periapsis*) and can be determined from

$$r = \frac{p}{1+e\cos\nu}. \tag{25}$$

If there are two focal points (ellipse, hyperbola) their distance is given by $c = e \cdot a$, the distances of the periapsis and apoapsis are given by $r_p = a(1-e)$ and $r_a = a(1+e)$. An elliptical orbit has the *period* $P = 2\pi a\sqrt{a/\mu}$.

Mean elements of a body in an elliptical orbit ($e < 1$) are defined by the motion of a point $\mathbf{r_q}$ on a concentric circle with constant angular velocity $n = \sqrt{\mu/a^3}$ and radius $\sqrt{ab}$, such that the orbital period $P = 2\pi a\sqrt{a/\mu}$ is the same for $\mathbf{r_q}$ and $\mathbf{r_m}$. The *mean anomaly* $M = \sqrt{\mu/a^3}(t-T)$ is defined as the angle between periapsis and $\mathbf{r_q}$. Unfortunately there is no simple relation between M and the true anomaly ν. To construct a relation one introduces another auxiliary concentric circle with radius a and defines $\mathbf{r'_q}$ as the point on that circle which has the

[18]For example K.Burnett's site at http://www.btinternet.com/~kburnett/kepler/

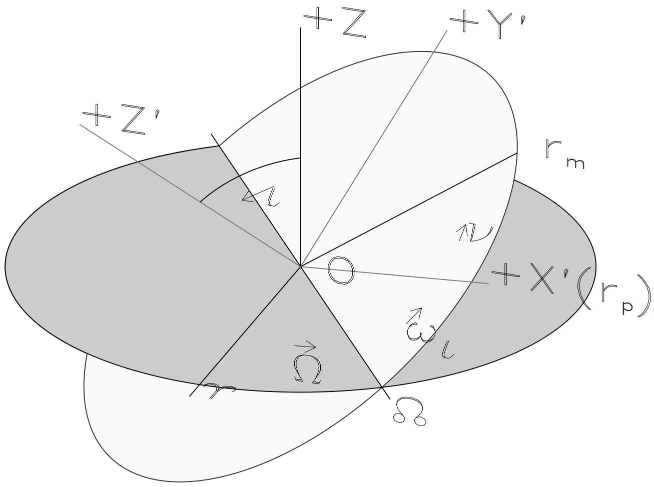

Figure 3: Orientation of a Keplerian orbit of the point $\mathbf{r_m}$ around the focus O with respect to the ecliptic plane. Symbols are given for the equinox ♈, the ascending node ☊ and its longitude Ω, the periapsis $\mathbf{r_p}$ and its argument ω, the inclination i, and the true anomaly ν. The perifocal system is denoted by (X',Y',Z').

same perifocal x-coordinate as $\mathbf{r_m}$. The *eccentric anomaly* E is the angular distance between $\mathbf{r'_q}$ and the periapsis measured from the centre and is related to the mean and true anomalies by the set of equations:

$$M = E - e\sin E \qquad \textit{(Kepler equation)} \tag{26}$$

$$\cos\nu = \frac{e-\cos E}{e\cos E - 1} \qquad \cos E = \frac{e+\cos\nu}{1+e\cos\nu} = \frac{r}{p}(e+\cos\nu) \qquad r = a(1-e\cos E) \tag{27}$$

Thus, if the orbital position is given as an expansion in t_0 of the *mean longitude* $\lambda = \Omega+\omega+M$, the *true longitude* $\lambda_0 = \Omega+\omega+\nu$ can be found by an integration of the transcendental Kepler equation. In most cases a Newton-Raphson integration converges quickly (see Danby (1988) or Herrick (1971) for methods). For hyperbolic orbits ($e > 1$) one can as well define a mean anomaly $M_h = \sqrt{\mu/|a|^3}(t-T)$ but this quantity has no direct angular interpretation. The *hyperbolic eccentric anomaly* E_h is related to M_h and the true anomaly ν by

$$M_h = e\sinh E_h - E_h \qquad \cos\nu = \frac{e-\cosh E_h}{e\cosh E_h - 1} \qquad \cosh E_h = \frac{e+\cos\nu}{1+e\cos\nu} \qquad r = a(1-e\cosh E_h) \tag{28}$$

The orientation of an orbit with respect to a reference plane (e.g. ecliptic) with origin at the orbital focus is defined by the *inclination* i of the orbital plane, the *longitude of the ascending node* Ω, and the *argument of periapsis* ω which is the angle between ascending node and periapsis $\mathbf{r_p}$ (see Fig.3). The position of the body on the orbit can then be defined by its *time of periapsis passage* T, its true anomaly ν_0 at epoch t_0, or its true longitude $\lambda_0 = \Omega+\omega+\nu_0$ at epoch t_0. The *perifocal coordinate system* has its X-axis from the focus to the periapsis, and its Z-axis right-handed perpendicular to the orbital plane in the sense of orbital motion. In this system the position and velocity vector are given by

$$\mathbf{r} = r(\cos\nu, \sin\nu, 0) \qquad \mathbf{v} = \sqrt{\mu/p}(-\sin\nu, e+\cos\nu, 0). \tag{29}$$

Table 4: Heliocentric mean orbital elements of the planets in HAE_{J2000} to a precision of 10^{-7} or $1''$ for 1950-2050 after Simon et al. (1994). The elements are semi-major axis a [1 AU = 149 597 870 km], eccentricity e, mean longitude λ, longitude of periapsis ϖ, inclination i, and ascending node Ω. The time parameter T_0 is scaled in Julian centuries of 36525 days from J2000.0. EMB denotes the Earth-Moon barycentre. The first column μ gives the IAU1976 mass ratio Sun/planet.

	μ	a[AU]	$e[10^{-7}]$	$\lambda[°]$
Mercury	6023600	0.38709831	2056318+204T_0	252.2509055+149472.6746358T_0
Venus	408523.5	0.72332982	67719-478T_0	181.9798009+58517.8156760T_0
EMB	328900.5	1.0000010	167086-420T_0	100.4664568+35999.3728565T_0
Mars	3098710	1.5236793	934006+905T_0	355.4329996+19140.2993039T_0
Jupiter	1047.355	5.2026032	484979+1632T_0	34.3515187+3034.9056606T_0
Saturn	3498.5	9.5549092	555481-3466T_0	50.0774443+1222.1138488T_0
Uranus	22869	19.2184461	463812-273T_0	314.0550051+428.4669983T_0
Neptune	19314	30.1103869	94557+60T_0	304.3486655+218.4862002T_0

	$\varpi[°]$	$i[°]$	$\Omega[°]$
Mercury	77.4561190+0.1588643T_0	7.0049863-0.0059516T_0	48.3308930-0.1254227T_0
Venus	131.5637030+0.0048746T_0	3.3946619-0.0008568T_0	76.6799202-0.2780134T_0
EMB	102.9373481+0.3225654T_0	0.0 +0.0130548T_0	174.8731758-0.2410908T_0
Mars	336.0602340+0.4439016T_0	1.8497265-0.0081477T_0	49.5580932-0.2950250T_0
Jupiter	14.3312069+0.2155209T_0	1.3032670-0.0019877T_0	100.4644070+0.1767232T_0
Saturn	93.0572375+0.5665415T_0	2.4888788+0.0025514T_0	113.6655025-0.2566722T_0
Uranus	173.0052911+0.0893212T_0	0.7731969-0.0016869T_0	74.0059570+0.0741431T_0
Neptune	48.1202755+0.0291866T_0	1.7699526+0.0002256T_0	131.7840570-0.0061651T_0

These might directly be expressed by the eccentric anomaly E:

$$\mathbf{r} = a(\cos E - e, \sqrt{1-e^2}\sin E, 0) \qquad \mathbf{v} = \frac{\sqrt{\mu/a}}{r}(-\sin E, \sqrt{1-e^2}\cos E, 0) \tag{30}$$

In the hyperbolic case replace cos by cosh and sin by sinh. The transformation from the reference system to the perifocal system is given by the Eulerian rotation $E(\Omega, i, \omega)$ as defined in the Appendix. The ecliptic position of a planet is then given by $\mathbf{r_e} = E(\Omega, i, \omega)\mathbf{r}$.

4.2 Planetary Positions

Tab.4 gives the 6 orbital elements $a, e, \lambda, \varpi, i, \Omega$ and their time development for the 7 major planets and the Earth-Moon barycentre (EMB), where $\varpi = \Omega + \omega$ is the longitude of the periapsis. Values are reduced to a relative precision of 10^{-7} from Tab.5.8 in Simon et al. (1994). This precision is sufficient for the calculation of planetary positions to the highest precision possible with a single set of mean elements for the period 1950-2050. The resulting precisions in relation to the DE200 ecliptic position are given in Tab.5. The positions are calculated from the mean elements by determining the true anomaly from eqn.26 and applying eqns.30. The last three columns of Tab.5 are not corrected for disturbances by Jupiter and Saturn, while these disturbances are included in the first four columns using Tab.6 of Simon et al. (1994). As one can see from Tab.5 it is – at least for the outer planets – recommendable to apply these corrections. To save space we do not give the numerical values in this paper but refer the reader to Simon et al. (1994) or to our web-page (cited above). The last row of Tab.5 gives the loss in precision when using mean elements (not solving eqn.26) instead of true elements for the EMB: mean and true position differ by up to $2°.7$.

4.2.1 Position of Earth and Moon

Tab.5 gives also ecliptic positions of Earth and Moon. DE200 and VSOP87 give only the heliocentric position $\mathbf{r_{EMB}}$ of the EMB. DE200 gives in addition the geocentric position $\mathbf{r_{g_M}}$ of the Moon. If both values are given the

Table 5: Precision of planetary positions derived from orbital elements (Tab.4) for the period 1950-2060 compared to DE200 positions on the ecliptic of date. Maximal differences are given for heliocentric ecliptic latitude β, longitude λ, and distance r and orbital velocity v. Values 'With Disturbances' use the corrections given in Tab.6 of Simon et al. (1994).

	With Disturbances				Without Disturbances		
	$\delta\beta[''$]	$\delta\lambda[''$]	δr[1000km]	δv[m/s]	$\delta\beta[''$]	$\delta\lambda[''$]	δr[1000km]
Mercury	0.8	6.0	0.51	1.2	3.2	26	1.6
Venus	0.9	5.5	1.0	1.8	1.6	28	5.0
EMB	0.6	7.6	1.2	1.9	0.6	29	7.0
Mars	1.0	26	8.1	4.5	4.3	160	39
Jupiter	5.5	46	71	15	20	830	990
Saturn	14	81	170	37	62	2100	6700
Uranus	4.9	86	510	27	44	3600	8800
Neptune	1.7	10	170	22	69	2400	11000
Earth	1.0	7.9	1.8	3.6	1.1	29	7.2
Moon	51	64	62	180	51	84	67
Earth_{approx}	1.1	16	8.6	260	1.1	34	8.6
Earth-EMB	1.1	14	5.8	14	1.1	29	7.2
EMB_{mean}	1.1	6900	2500				

position of the Earth $\mathbf{r_E}$ can be calculated exactly (within the IAU1976 system) using the mass ratio Moon/Earth of $\mu_M = 0.01230002$[A. K6] (or its respective value used for the ephemeris) by:

$$\mathbf{r_E} = \mathbf{r_{EMB}} - \mathbf{r_{gM}} \frac{\mu_M}{1+\mu_M} \tag{31}$$

The velocity vector has the same transformation. For the VSOP87 system we apply following formula by J.L.Simon [personal communication] describing the rotation of the Earth around the EMB:

$$\lambda_E = \lambda_{EMB} + 6''.468 \sin D \qquad r_E = r_{EMB} + 4613 \cos D[\mathrm{km}], \tag{32}$$

where D is the *Delauney argument* from eqn.3.5 in Simon et al. (1994):

$$D = 297^\circ.8502 + P \cdot T_0, \tag{33}$$

where the rotation period is $P = 445267^\circ.11/\mathrm{century}$. The Earth velocity vector $\mathbf{v_E}$ can be calculated by

$$\mathbf{v_E} = \mathbf{v_{EMB}} + \Omega_E \times \mathbf{r_D}, \tag{34}$$

where $\mathbf{r_D} = \mathbf{r_E} - \mathbf{r_{EMB}}$ and the ecliptic angular velocity vector is given by $\Omega_E = 2\pi/P \cdot (0,0,1)$.
If only $\mathbf{r_{EMB}}$ and $\mathbf{r_E}$ are given the ecliptic position and velocity of the Moon can then be calculated by

$$\mathbf{r_M} = ((1+\mu_M)\mathbf{r_{EMB}} - \mathbf{r_E})/\mu_M. \tag{35}$$

But the precision of the resulting lunar velocity is rather low (190 m/s). Neglecting the difference between $\mathbf{r_{EMB}}$ and $\mathbf{r_E}$ increases the total error in the Earth position to $14''$ (Earth-EMB in Tab.5).
For slightly lower precision without solving the Kepler equation (26) the geometric ecliptic longitude of the Earth can be calculated by the approximation given for the Solar longitude in [A. C24]:

$$\lambda_{geo} = \lambda_{mean} + 1^\circ.915 \sin g + 0^\circ.020 \sin 2g \qquad r_{geo} = 1.00014 - 0.01671 \cos g - 0.00014 \cos 2g[\mathrm{AU}], \tag{36}$$

where λ_{mean} and the mean anomaly $g = \lambda_{mean} - \varpi$ for the EMB can be taken from Tab.4. This approximation has also been used by Hapgood (1992). The respective precision is $34''$ (Earth_{approx} in Tab.5).

Table 6: Physical Ephemeris of the Planets GEI_{J2000}[*S.* Table 15.7, *A.* E87]: (α_0, δ_0) is the position of the North pole in GEI_{J2000}, $(\dot{\alpha}, \dot{\delta})$ its change per Julian century T, W_0 is the position of the prime meridian at GEI_{J2000}, $\dot{W}$ its change per day

Name	$\alpha_0[°]$	$\dot{\alpha}[°/T]$	$\delta_0[°]$	$\dot{\delta}[°/T]$	$W_0[°]$	$\dot{W}[°/d]$
Sun	286.13		63.87		84.10	+14.1844000
Mercury	281.01	-0.003	61.45	-0.005	329.71	+6.1385025
Venus	272.72		67.15		160.26	-1.4813596
Earth	0.00	-0.641	90.00	-0.557	190.16	+360.9856235
Mars	317.681	-0.108	52.886	-0.061	176.868	+350.8919830
Jupiter III	268.05	-0.009	64.49	+0.003	284.95	+870.5360000
Saturn III	40.58	-0.036	83.54	-0.004	38.90	+810.7939024
Uranus III	257.43		-15.10		203.81	-501.1600928
Neptune	299.36	+0.70sin$N$	43.46	-0.51cos$N$	253.18	536.3128492-0.48sin$N$
(where N=	359.28	+54.308)				
Pluto	313.02		9.09		236.77	-56.3623195

4.3 Planetocentric Systems

For solar system bodies the IAU differentiates between *planetocentric* and *planetographic* body-fixed coordinates : Planetocentric latitude refers to the equatorial plane and the polar axis, planetographic latitude is defined as the angle between equatorial plane and a vector through the point of interest that is normal to the biaxial ellipsoid reference surface of the body. Both latitudes are identical for a spherical body. Planetocentric longitude is measured eastwards (i.e. positive in the sense of rotation) from the prime meridian. Planetographic longitude of the sub-observation point increases with time, i.e. to the west for prograde rotators and to the east for retrograde rotators. All systems defined in the following are planetocentric.

Table 6 gives the orientation of the planetary rotation systems for all major planets at epoch GEI_{J2000} and their change with time. These are defined by the equatorial attitude (α, δ) of the rotation axis and the *prime meridian angle* w_0. Data are taken from [*S.* Table 15.7] which is identical to the table given by Davies et al. (1996). The ascending node right ascensions are given by $\Omega = \alpha + 90°$. The respective transformation matrices are

$$\begin{aligned} T(GEI_{J2000}, PLA_{J2000}) &= E(\alpha_0 + 90°, 90° - \delta_0, W_0) \\ T(GEI_{J2000}, PLA_D) &= E(\alpha_0 + \dot{\alpha}T_0 + 90°, 90° - \delta_0 - \dot{\delta}T_0, W_0 + \dot{W}d_0) \end{aligned} \quad (37)$$

where d_0 and T_0 are defined in eqn.1 and 2.

4.3.1 Jovian Systems

Since we have used different Jovian coordinate systems in previous work (Krupp et al., 1993) we include a description of these systems. Most of these systems are discussed in Dessler (1983). The Jovian pole of rotation is defined by the values (α, δ) given for "Jupiter III" in Table.6. The transformation from GEI_{J2000} can be calculated from

$$T(GEI_{J2000}, JUP_X) = E(\alpha + 90°, 90° - \delta, w_0) \quad (38)$$

where the *prime meridian angle* w_0 is given in the following table. Note that longitudes are counted left-handed (clockwise) from the prime meridian in the following Jovian systems.

- System I JUP_I, mean atmospheric equatorial rotation [*S.* Table 15.7]
  +Z-axis: Pole of rotation. p-angle: $w_{0I} = 67°.1 + 877°.900 d_0$

- System II JUP_{II}, mean atmospheric polar rotation [*S.* Table 15.7]
  +Z-axis: Pole of rotation. p-angle: $w_{0II} = 43°.3 + 870°.270 d_0$

- System III JUP_{III}, magnetospheric rotation [*S.* Table 15.7]
  +Z-axis: Pole of rotation. p-angle: $w_{0III} = 284°.95 + 870°.536 d_0$
  Transform: $T(GEI_{J2000}, JUP_{III}) = E(\alpha + 90°, 90° - \delta, w_{0III})$

  This is the 1965 definition of System III, the Pioneer missions used the 1957 definition:

  $$w_{01957} = w_{0III} + 106°.31209 + 0.0083169 d_0$$

  which can be calculated from eqn.7c in Seidelmann and Divine (1977) and was originally defined by the magnetospheric rotation period.

- System III fix Sun Line
  +Z-axis: Pole of rotation. +Y-axis: cross-product of +Z-axis and vector (Jupiter-Sun)

- Magnetic Dipole System JUP_D (Dessler, 1983)
  +Z-axis: dipole axis defined by its System III latitude and longitude:

  $$lat_D = (90° - 9°.8) \qquad \lambda_D = 200°$$

  +X-axis: intersection of System III prime meridian and magnetic equator.
  Transform: $T(JUP_{III}, JUP_D) = E(\lambda_D + 90°, 9°.8, -\lambda_D - 90°)$ (approximately).

- Centrifugal System JUP_C(Dessler, 1983)
  +Z-axis: centrifugal axis defined by its System III latitude and longitude:
  $lat_C = (90° - 7°.0) \qquad \lambda_C = 200°$
  +X-axis: intersection of System III prime meridian and centrifugal equator.
  Transform: $T(JUP_{III}, JUP_C) = E(\lambda_C + 90°, 7°.0, -\lambda_C - 90°)$ (approximately).

- Magnetic Dipole System fix Sun line
  +Z-axis: dipole axis. +Y-axis: cross-product of +Z-axis and vector (Jupiter-Sun)

- Magnetic Dipole $r\theta\phi$ System
  +X-axis: vector (Jupiter-S/C) +Z-axis: (dipole axis)$\times$+X-axis
  This system depends on the S/C-position.

5 Spacecraft Elements

To determine approximate positions of spacecraft relative to each other or to planets without using positional data files it is useful to have orbital elements of spacecraft in Keplerian orbits. This excludes most near Earth missions since their orbits are not Keplerian. In Tab.7 we list orbital elements for most major interplanetary missions. We have fitted these elements to trajectory data provided by NSSDC[19]. Not much accuracy is claimed by NSSDC for the propagated trajectories of any heliospheric spacecraft. But random cross-comparison with published papers had revealed mismatches of $< 0°.1$ in angles or $< 1\%$ in radial distanceHEE (R.Parthasarathy, personal communication). We used the vector method given in ch.2 of Bate et al. (1971) to calculate initial values for the elements which we then fitted to achieve the smallest maximal deviation from the position data. The deviations are listed in the last three columns of Tab.7. The spatial resolution of the NSSDC position data is only 0.1° and the temporal resolution 1 day. This results in a poor precision of the orbital elements at perihelion specifically for the Helios mission where the spacecraft moves 8°/day. For this reason we re-calculated the Helios orbits by integration from cartesian state vectors provided by JPL and then fitted elements to the re-calculated orbits. See also the JPL Voyager home page[20]for more Voyager orbital elements, and the ESA Ulysses home page[21]for a discussion of Ulysses orbital elements.

[19]NSSDC at http://nssdc.gsfc.nasa.gov/space/helios/heli.html
[20]Voyager at http://vraptor.jpl.nasa.gov
[21]Ulysses at http://helio.estec.esa.nl/ulysses/

Mission	Period	a[AU]	e	λ[°]	ϖ[°]	i[°]	Ω[°]	Δr[AU]	$\Delta\phi$[°]	$\Delta\theta$[°]
Galileo	1990.4-1990.9	0.982	0.298	$195.36 + 366.670 y_0$	182.17	3.39	76.51	0.014	1.2	0.10
Galileo	1991.2-1992.8	1.572	0.439	$304.32 + 181.146 y_0$	-240.47	4.57	-103.37	0.036	0.7	0.09
Galileo	1993.8-1996.0	3.113	0.700	$180.16 + 64.938 y_0$	-277.61	1.68	-105.39	0.014	0.8	0.09
Helios1	1977.0-1986.0	0.6472	0.5216	$126.77 + 691.475 y_0$	-101.84	0.004	70.18	0.001	0.39	0.016
Helios2	1977.0-1981.0	0.6374	0.5436	$147.76 + 707.453 y_0$	294.58	0.024	121.85	0.002	0.89	0.004
Pioneer10	1972.4-1973.9	3.438	0.715	$291.99 + 56.479 y_0$	160.02	2.08	-17.06	0.019	0.18	0.07
Pioneer10	1974.3-2005.0	-6.942	1.727	$111.81 + 19.700 y_0$	-42.02	3.14	-28.57	0.019	0.02	0.007
Pioneer11	1973.5-1974.8	3.508	0.7166	$220.69 + 54.797 y_0$	195.46	3.05	16.64	0.013	0.10	0.06
Pioneer11	1975.0-1979.6	16.729	0.7767	$180.91 + 5.264 y_0$	55.05	15.29	-5.24	0.012	0.25	0.20
Pioneer11	1979.7-2000.0	-8.059	2.161	$127.99 + 15.668 y_0$	173.21	16.63	160.40	0.06	0.23	0.12
Ulysses	1991.1-1992.1	9.035	0.8905	$143.48 + 13.272 y_0$	21.13	1.99	13.57	0.014	0.07	0.06
Ulysses	1992.2-2005.0	3.375	0.6032	$256.31 + 58.073 y_0$	-22.93	79.15	-21.85	0.007	0.92	0.50
Voyager1	1978.0-1979.1	5.020	0.8009	$332.66 + 31.820 y_0$	-17.71	0.93	-11.4	0.010	0.44	0.12
Voyager1	1979.2-1980.8	-4.109	2.258	$302.05 + 43.088 y_0$	112.12	2.46	113.23	0.010	0.23	0.06
Voyager1	1980.9-2005.0	-3.203	3.742	$332.47 + 62.642 y_0$	157.35	35.71	178.95	0.034	0.10	0.11
Voyager2	1977.9-1979.4	3.624	0.7244	$65.98 + 52.225 y_0$	-20.65	0.84	-33.03	0.013	0.13	0.06
Voyager2	1979.6-1981.6	-17.345	1.2905	$216.12 + 5.000 y_0$	110.80	2.58	120.05	0.017	0.10	0.06
Voyager2	1981.7-1986.0	-3.913	3.4537	$324.52 + 46.379 y_0$	189.87	2.66	77.65	0.017	0.12	0.05
Voyager2	1986.1-1989.3	-2.902	6.0618	$7.18 + 72.400 y_0$	-144.23	2.81	-98.07	0.034	0.12	0.12
Voyager2	1990.7-2000.0	-4.021	6.2853	$256.56 + 44.661 y_0$	231.66	78.92	101.65	0.045	0.13	0.10

Table 7: Heliocentric mean orbital elements of major interplanetary spacecraft in HAE_{J2000} fitted to data provided by NSSDC. The elements are semi-major axis a, eccentricity e, mean longitude λ, longitude of periapsis ϖ, inclination i, and ascending node Ω. The time parameter y_0 is scaled in Julian years of 365.25 days from J2000.0, periods are given in decimal Julian years from J2000.0+2000.0. The last three columns contain the precision of positions determined from the elements relative to NSSDC position data: Maximal difference in HAE distance, longitude and latitude over the period given.

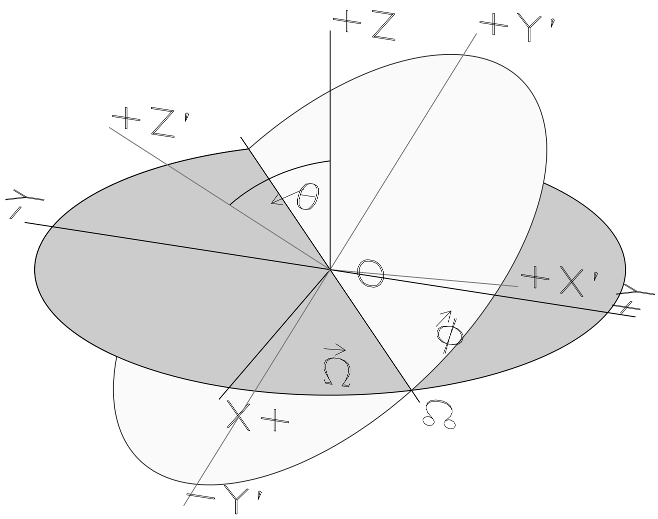

Figure 4: Eulerian Rotation $E(\Omega,\Theta,\Phi)$(after Madelung (1964)): the transformation between system $S(X,Y,Z)$ and system $S'(X',Y',Z')$ can be expressed by the three right-handed principal rotations: 1. $<\Omega,Z>$ around the Z-axis towards the ascending node ☊, 2. $<\vartheta,X>$ around the ascending node axis, 3. $<\phi,Z>$ around the Z'-axis towards the $+X'$-axis.

6 Summary

We have collected formulae relevant for the transformation between planetocentric and heliocentric coordinate systems and determined the precision of these transformations for the period 1950-2060, most relevant for space science. We give a very short but complete description of orbit determination from Keplerian orbital elements. With the simple set of formulae given in this paper (and adapted from Simon et al. (1994)) the positions of the inner planets can be determined to $160''$ though for the Earth this precision can be increased to $29''$. Adding disturbance terms from Simon et al. (1994) increases the precision to $8''$. This sets the limits for the precision to be achieved by a single set of Keplerian elements. For higher precision the installation of an integrated ephemeris is recommended.

We also determined Keplerian orbital elements for major interplanetary spacecraft within the precision limits given by the NSSDC data source. These allow quick approximate calculations of spacecraft positions and also allow to cross check existing position data sets. Formulae given in this paper can easily be implemented in software. Programs used in preparation of this paper have been written in the IDL language and are available from our website [22].

A Appendix

A.1 Eulerian Rotation

In this paper we describe transformations between cartesian coordinate systems in Euclidean space. Let system S be defined by the orthonormal right-handed basis vectors X,Y,Z and system S' by the orthonormal right-handed basis vectors X',Y',Z' with a common origin O. The position of system S' in system S is then defined by the angular coordinates of its pole ($Z' = (\theta, \Psi = \Omega - 90°)$) and the prime meridian angle ϕ (see Fig.4) which is the angular distance between *prime meridian* X' and *ascending node* ☊. The Eulerian transformation matrix from S to S' is then defined by (Madelung, 1964):

$$E(\Omega,\theta,\phi) = \begin{pmatrix} \cos\phi\cos\Omega - \sin\phi\sin\Omega\cos\theta & \cos\phi\sin\Omega + \sin\phi\cos\Omega\cos\theta & \sin\phi\sin\theta \\ -\sin\phi\cos\Omega - \cos\phi\sin\Omega\cos\theta & -\sin\phi\sin\Omega + \cos\phi\cos\Omega\cos\theta & \cos\phi\sin\theta \\ \sin\Omega\sin\theta & -\cos\Omega\sin\theta & \cos\theta \end{pmatrix} \tag{39}$$

Such that a vector $\mathbf{v}$ given in S has coordinates $\mathbf{v}' = E * \mathbf{v}$ in S'. This corresponds to three principal rotations:

$$E = R_3(\phi) * R_1(\theta) * R_3(\Omega) = <\phi, Z> * <\theta, X> * <\Omega, Z> \tag{40}$$

in the notation of Hapgood (1992) where '*' denotes matrix multiplication. The three principal rotations are on the other hand given by

$$R_1(\zeta) = <\zeta, X> = E(0,\zeta,0) \qquad R_2(\zeta) = <\zeta, Y> = E(90°,\zeta,-90°) \qquad R_3(\zeta) = <\zeta, Z> = E(0,0,\zeta). \tag{41}$$

Note that all rotation matrices are orthogonal, s.t. $E^{-1} = E^T$ and transformations between all systems defined in this paper can easily be calculated by a series of matrix multiplications.

A.2 Velocity Transformations

While position and magnetic field vectors are independent of the relative motion of the coordinate system this is not true for other vectors for example for the solar wind velocity vector. Usually this vector is originally given in a spacecraft reference frame. For solar wind studies it is advisable to subtract the effect of the spacecraft motion relative to a heliocentric inertial system. If the spacecraft velocity vector is not provided together with the positional data the velocity can be calculated from the temporal derivative of the position time series. The velocity vector in the transformed system is generally given by

$$\mathbf{v}' = \dot{E}\mathbf{r} + E\mathbf{v} - \mathbf{v_c}, \tag{42}$$

where $\mathbf{v_c}$ is the relative speed of the system origins and $\dot{E}$ is the temporal derivative of the rotation matrix:

$$\dot{E}(\Omega,\theta,\phi) = A * E\dot{\Omega} + B * E\dot{\theta} + E * A\dot{\phi} \tag{43}$$

$$A = \dot{R}_3 R_3^T = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad B = R_3(\phi)\dot{R}_1 R_1^T R_3^T(\phi) = \begin{pmatrix} \cos^2\phi & \cos\phi\sin\phi & \sin\phi \\ -\cos\phi\sin\phi & -\sin^2\phi & \cos\phi \\ \sin\phi & -\cos\phi & 0 \end{pmatrix}$$

For the transformation into planetocentric systems ϕ is the only angle changing rapidly such that $\dot{E}(\Omega,\theta,\phi) \approx E * A\dot{\phi}$.
One of the most common transformations is the transformation from a heliocentric inertial system like HAE_D to a geocentric rotating system like GSE_D. Since $\dot{\lambda}_E \approx 1°/day \approx 2 \cdot 10^{-7} rad/s$ the rotational part of the velocity transformation can be neglected for geocentric distances of less than $5 \cdot 10^6$ km to keep an accuracy of $\approx$ 1km/s. In that case the transformation reduces to the subtraction of the orbital velocity of the Earth which in the ecliptic system is given by

$$\mathbf{v_E}_{HAE} = v_0 * (\cos(\lambda_{geo} + 90°), \sin(\lambda_{geo} + 90°), 0°) \tag{44}$$

where $v_0 = 29.7859$ km/s is the mean orbital velocity of the Earth and λ_{geo} the Earth longitude defined in eqn.36.

[22] http://www.space-plasma.qmul.ac.uk/heliocoords/

Table 8: Numerical example for a geocentric S/C position vector in different coordinate systems . Positions are in geocentric cartesian coordinates in units of Earth equatorial radii [R_E = 6378.14 km] for the date Aug 28, 1996 16:46:00 TT.

System	X [R_E]	Y [R_e]	Z [R_E]
GEO_T	6.9027400	-1.6362400	1.9166900
GEI_T	-5.7864335	-4.1039357	1.9166900
GEI_D	-5.7864918	-4.1039136	1.9165612
HAE_D	-5.7864918	-3.0028771	3.3908764
HAE_{J2000}	-5.7840451	-3.0076174	3.3908496
GEI_{J2000}	-5.7840451	-4.1082375	1.9146822
HGC_{J2000}	-5.4328785	4.1138243	2.7493786
HEE_D	-4.0378470	-5.1182566	3.3908764
$HEEQ_D$	-4.4132668	-5.1924440	2.7496187
HCD	-4.3379628	5.2555187	2.7496187
GSE_D	4.0378470	5.1182566	3.3908764
GSM_D	4.0378470	6.0071917	1.2681645
SM_D	3.3601371	6.0071917	2.5733108
MAG_D	3.3344557	6.0215108	2.5732497
$HGRTN_E$	4.0360303	5.1931904	-3.2771992

A.3 Light Aberration

For physical effects which depend not on the geometric relative position of two objects B_1, B_2 but on the apparent position of B_1 relative to B_2 one has to take light travel into account. The relativistic deflection of light by the Sun is only larger than $1''$ for angular distances from the Sun of less then 0°5 (see [S. Table 3.26.1]) and may be neglected for our purposes. The change in position during the light travel time (for example $20''$ between Sun and Earth) can be calculated by iteration by determining the geometric position at time $t_1 = t_0 - R(t_0)/c$ where $R(t_0)$ is the distance between B_1 and B_2 at t_0 and c is the speed of light [S. 3.314-315]. The light aberration is caused by the relative speeds of the observer B_1 to the light coming from object B_2 and the aberrated position of B_2 moving with relative speed $\mathbf{v}$ can be calculated by $\mathbf{r_2} = \mathbf{r_{20}} + R\mathbf{v}/c$ [S. 3.317].

B Numerical Example

In the following we give a numerical example for the application of some formulas given in the paper for comparison with software implementations. As pointed out in the introduction all numerical values in this paper will be available through our website.

B.1 Position Transforms

Note that in the version of this paper published in Planetary&Space Science, 50, 217ff, the date taken for this example is erroneously given as Aug 28, 1996 16:46:00 UTC, not TT.

We assume that a spacecraft position is given in true geographic coordinates (GEO_T) on the date Aug 28, 1996 16:46:00 TT (JD 2450324.19861111). Numerical results are given in Tab.8 (We have chosen this date and position because software by M. Hapgood [personal communication] uses these values as a reference set.) The Julian century for this date is $T_0 = -0.0334237204350195$ (eqn.2). In the following we apply the formulas of section 3.3.3. To convert from GEO_T to GEI_T we calculate $\theta_{GMST} = 228°.68095$ by eqn.20. To convert from the true equator of date to the mean equator of date we have to apply the nutation matrix (eqn.7) with $\varepsilon_{0D} = 23°.439726, \Delta\psi = 0°.0011126098, \Delta\varepsilon = -0°.0024222837$. Then we apply $E(0, \varepsilon_{0D}, 0)$ to transform to the mean ecliptic of date (HAE_D), the precession matrix (eqn.9) to transform to the ecliptic of J2000 (HAE_{J2000}) and

Table 9: Heliocentric position and velocity vectors of the Earth and the Ulysses spacecraft on Jul 31, 1994 23:59:00 UTC.

Vector	Source	System	Units	X	Y	Z
$\mathbf{r}_{\mathbf{U}N}$	NSSDC	GEI_{B1950}	km	-135 927 895.1	126 880 660.0	-340 567 928.0
$\mathbf{v}_{\mathbf{U}N}$	NSSDC	GEI_{B1950}	km/s	18.54622396	-8.287477214	2.89468231
$\mathbf{r}_{\mathbf{U}P}$	Tab.7	HAE_{J2000}	km	-134 998 220	-208 472 970	-362 990 550
$\mathbf{v}_{\mathbf{U}P}$	Tab.7	HAE_{J2000}	km/s	18.624175	-6.2275813	5.9890246
$\mathbf{r}_{\mathbf{U}J}$	Tab.7	GEI_{J2000}	km	-134 998 220	125 262 330	-341 329 890
$\mathbf{v}_{\mathbf{U}J}$	Tab.7	GEI_{J2000}	km/s	18.624175	-8.0959913	3.0176330
$\mathbf{r}_{\mathbf{U}B}$	Tab.7	GEI_{B1950}	km	-135 246 410	126 772 910	-340 673 310
$\mathbf{v}_{\mathbf{U}B}$	Tab.7	GEI_{B1950}	km/s	18.546949	-8.3037658	2.9273224
$\mathbf{r}_{\mathbf{U}S}$	SPICE	GEI_{B1950}	km	-135 922 227	126 877 772	-340 564 861
$\mathbf{v}_{\mathbf{U}S}$	SPICE	GEI_{B1950}	km/s	18.546466	-8.287645	2.894987
$\mathbf{r}_{\mathbf{EMB}}$	Tab.4	HAE_{J2000}	km	94 752 993	-118 648 910	-1 355
$\mathbf{v}_{\mathbf{EMB}}$	Tab.4	HAE_{J2000}	km/s	22.792	18.478	0.00025
$\mathbf{r}_{\mathbf{E}}$	Tab.4	HAE_{J2000}	km	94 750 228	-118 652 770	-1 355
$\mathbf{v}_{\mathbf{E}}$	Tab.4	HAE_{J2000}	km/s	22.802	18.471	0.00025
$\mathbf{r}_{\mathbf{E}J}$	Tab.4	GEI_{J2000}	AU	0.633 3661	-0.727 6925	-0.315 5032
$\mathbf{r}_{\mathbf{E}AA}$	AstrAlm	GEI_{J2000}	AU	0.633 3616	-0.727 6944	-0.315 5035

$E(0, \varepsilon_{0J2000}, 0)$ to transform to the equator of J2000 (GEI_{J2000}). (The vector is still geocentric since we did not apply a translation. So it might better be called GAE_D etc. but we stick with the H to avoid more acronyms.)
We use $T(GEI_{J2000}, HGC_{J2000})$ and $T(HAE_D, HCD)$ of section 3.2.2 to transform to the heliographic systems. To transform to geocentric Earth Ecliptic (HEE_D) coordinates we use $T(HAE_D, HEE_D)$ from section 3.2.2, for $HEEQ_D$ we use $\theta_\odot = 259^\circ.89919$ (eqn.17). To transform to GSE_D with low precision we use the ecliptic longitude of the Earth $\lambda_{geo} = -24^\circ.302838$ (eqn.36). To transform to GSM_D we use the Earth dipole position $\lambda_D = 288^\circ.58158, \Phi_D = 79^\circ.411145$ (eqn.22) and angles $\psi_D = -21^\circ.604166, \mu_D = 20^\circ.010247$.
To proceed to position dependent systems we now determine the Earth position to a higher precision using the orbital elements of the EMB from Tab.4 corrected by Tab.6 of Simon et al. (1994) (values available on our website):

$$a = 1.0000025, \lambda = -22^\circ.769425, e = 0.016710039, \varpi = 102^\circ.92657, i = -0^\circ.00043635047, \Omega = 174^\circ.88123. \quad (45)$$

Using eqns.26 and 30 with $\mu_E = 1/332946$(Tab.4), we get the EMB position in HAE_{J2000}:

$$\lambda_{EMB} = -24^\circ.305587, \beta_{EMB} = -0^\circ.00014340633, r_{EMB} = 1.0099340[\mathrm{AU}]. \quad (46)$$

Using the Delauney argument $D = -184^\circ.63320$ we get the Earth position in HAE_{J2000} (eqn.32):

$$\lambda_E = -24^\circ.305442, r_E = 1.0099033[\mathrm{AU}]. \quad (47)$$

We apply the precession matrix (eqn.9) to get the Earth position vector in HAE_D (1AU = 149 597 870km, 1R_E =6378.14km):

$$X_E = 21579.585[R_E], Y_E = -9767.205[R_E], Z_E = 0.000016[R_E] \quad (48)$$

Adding this vector to the geocentric position (GAE_D) and transforming to HCD we get the HCD longitude and latitude of the spacecraft:

$$\Phi_{S/C} = -100^\circ.11050, \Theta_{S/C} = 7^\circ.1466473, \quad (49)$$

from which we calculate the S/C-centered position vector of the Earth $HGRTN_E$.

B.2 State Vectors

Note that in the version of this paper published in Planetary&Space Science, 50, 217ff, the values calculated from Tab.4&7 are calculated for Jul 31, 1994 23:59 TT, not UTC.

The position and velocity vectors (*state vector*, $\mathbf{r}_{\mathbf{U}N}, \mathbf{v}_{\mathbf{U}N}$ in Tab.9) of the Ulysses spacecraft which we used to determine the orbital elements in Tab.7 was provided by NSSDC for the Julian date $JD = 2449565.5000137$ (Jul 31, 1994 23:59:00 UTC) in heliocentric earth-equatorial coordinates for epoch ε_{B1950}.

In the following we describe how to derive the state vectors for Ulysses and Earth from the orbital elements for this date and compare the values with the respective data of the JPL SPICE system. The Julian century for this date is $T_0 = -0.054195755956093$ (eqn.2). From Tab.7 we take the values for the orbital elements for Ulysses in HAE_{J2000}:

$$a = 3.375d, \lambda = 256^\circ.31 + 58^\circ.073 \cdot T_0 \cdot 100, e = 0.6032, \varpi = -22^\circ.93, \Omega = 79^\circ.15, i = -21^\circ.84 \tag{50}$$

Using eqns.26 and 30 and 1 AU= 149597870km we calculate the HAE_{J2000} state vector ($\mathbf{r}_{\mathbf{U}P}, \mathbf{v}_{\mathbf{U}P}$).

This position is in agreement with the ecliptic position available from the spacecraft Situation Center for day 213, 1994: ($\lambda_{HAE_{J2000}} = 188^\circ.8, \beta_{HAE_{J2000}} = -69^\circ.4, r = 2.59$ AU). To compare this vector with the NSSDC value ($\mathbf{r}_{\mathbf{U}N}, \mathbf{v}_{\mathbf{U}N}$) we have first to transform from the ecliptic HAE_{J2000} system to the equatorial GEI_{J2000} system using $T(HAE_{J2000}, GEI_{J2000}) = E(0, -\varepsilon_0, 0)$. Since GEI_{B1950} refers to the orientation of the Earth equator at $\varepsilon_{B1950}(T_{0B1950} = -0.50000210)$ we have to calculate the precession matrix using eqn.10 :

$$P(0.0, \varepsilon_{B1950}) = \begin{pmatrix} 0.99992571 & 0.011178938 & 0.0048590038 \\ -0.011178938 & 0.99993751 & -2.7157926 \cdot 10^{-5} \\ -0.0048590038 & -2.7162595 \cdot 10^{-5} & 0.99998819 \end{pmatrix} \tag{51}$$

Finally we derive the Ulysses state vector in GEI_{B1950} ($\mathbf{r}_{\mathbf{U}B}, \mathbf{v}_{\mathbf{U}B}$).

The distance to the original NSSDC position ($\mathbf{r}_{\mathbf{U}N}, \mathbf{v}_{\mathbf{U}N}$) is 697950 km (0.0046 AU), the difference in velocity 36 m/s in agreement with the precision cited in Tab.7 for the orbital elements. The respective position provided by the JPL SPICE system is ($\mathbf{r}_{\mathbf{U}S}, \mathbf{v}_{\mathbf{U}S}$), which deviates by 7062 km and 0.42 m/s from the NSSDC state vector.

Now, we calculate the HEI_{J2000} state vector of the Earth at the same time. From Tab.4 we get the undisturbed orbital elements of the EMB:

$$a = 1.0000010, \lambda = -50^\circ.546769, e = 0.016710876, \varpi = 102^\circ.91987, \Omega = 174^\circ.88624, i = -0^\circ.00070751475 \tag{52}$$

To increase precision we apply the disturbance corrections by Tab.6 of Simon et al. (1994) (values available on our website) and get:

$$a = 0.99998900, \lambda = -50^\circ.549526, e = 0.016710912, \varpi = 102^\circ.91987, \Omega = 174^\circ.88624, i = -0^\circ.00070754223 \tag{53}$$

Using eqns.26 and 30 with $\mu_E = 1/332946$(Tab.4), we get the EMB state vector in HAE_{J2000} ($\mathbf{r}_{\mathbf{EMB}}, \mathbf{v}_{\mathbf{EMB}}$). Given the low precision of the Ulysses position this would already be good enough to get the geocentric Ulysses state vector but to compare with SPICE data or the Astronomical Almanac we now apply eqn.32 to get the Earth state vector in HAE_{J2000} ($\mathbf{r}_{\mathbf{E}}, \mathbf{v}_{\mathbf{E}}$), where we used the Delauney argument $D = -73^\circ.746062$. Finally we transform from HAE_{J2000} to GEI_{J2000} using $E(0, -\varepsilon_0, 0)$ as above to get $\mathbf{r}_{\mathbf{E}J}$, which can be compared with the value given in section C22 of the Astronomical Almanac for 1994 ($\mathbf{r}_{\mathbf{E}AA}$,which agrees with the value given by the SPICE system).

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