

Herbig Ae Stars with Algol-Like Minima: Modeling of the Spectral Energy Distribution and the Behavior of Colors at Minima

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Abstract—The properties of circumstellar dust shells around Herbig Ae stars with nonperiodic dimmings are discussed. A numerical model of radiative transfer in a dust medium is used to construct the infrared spectra and color–magnitude diagrams during minimum light. The method of modeling makes it possible to estimate the parameters of dust shells—the sizes, the radial distribution of matter, and the optical depth. Additional modeling of the circumstellar-extinction curves is needed to determine the properties of dust grains. The method is used to study the properties of the circumstellar shell around the Herbig Ae star WW Vul with Algol-like minima. The shell parameters are estimated from a comparison with observational data.

1. INTRODUCTION

Herbig Ae/Be stars are young moderate-mass stars surrounded by a considerable amount of gas and dust. A significant percentage (~25%) of the Herbig stars exhibit nonperiodic Algol-like light variations (Thé *et al.* 1994a). These stars can dim by as much as 2^m – 3^m in V ; for some of them, the color indices first increase during the minima and then decrease at a deep minimum. In order to explain the light variations in these stars, Wenzel (1969) proposed a model of variable circumstellar extinction in which dense clouds that revolve around the star occasionally obscure it from the observer. However, the most serious drawback of this model has long been the lack of answers to the following questions: (1) why do the color indices begin to increase by the end of deep eclipses? and (2) why are dimmings greater than 3^m not observed? Only recently has Grinin (1988) managed to solve these problems; he showed that the source of blue light is fine circumstellar dust that scatters the stellar emission. Grinin was also able to confirm this assumption by the observation of a considerable increase in the linear polarization (up to 10%) during the minimum light of several stars.

Varied observations of Herbig Ae stars, particularly intensive in recent years, have provided a large body of data on their dust shells:

– For a considerable number of stars, the energy distributions in the visible and in the near infrared (Cohen 1973; Glass and Penston 1974; Kilkenny *et al.* 1985; Hutchinson *et al.* 1994; Herbst *et al.* 1994), the far-infrared fluxes (Weintraub 1990; Weaver and Jones 1992), and, for some of them, submillimeter and millimeter data (Thé *et al.* 1994b) were obtained.

– Ultraviolet spectra were obtained for a number of stars from the IAU satellite, making it possible to construct the wavelength dependence of the circumstellar extinction [see a list of stars in Talavera and Verdugo (1994)].

– Photometric observations were performed for stars with Algol-like minima; for some of them (UX Ori, WW Vul, CQ Tau, etc.), linear-polarization observations were carried out during deep minima (Voshchinnikov *et al.* 1988; Berdyugin *et al.* 1990; Berdyugin *et al.* 1992).

– Images were obtained for several Herbig Ae stars at 50 and 100 μm (Natta *et al.* 1993b; Di Francesco *et al.* 1994). In addition, the profiles of the $\lambda 10 \mu\text{m}$ silicate band and other infrared bands are available for some of the Herbig Ae stars (Berrilli *et al.* 1992; Brooke *et al.* 1993; Wooden 1994); speckle interferometry and many other observations of the shells were performed [see Thé *et al.* (1994b) for details].

Despite the wealth of observational data on the dust shells around Herbig Ae stars with Algol-like minima, thus far nobody has succeeded in constructing a reliable theoretical model that would satisfactorily describe all of the available observations.

Voshchinnikov *et al.* (1988) and Voshchinnikov and Grinin (1991) used the model of variable circumstellar extinction to simultaneously model the light variations and variations in the polarization of dust-scattered light during eclipses for the stars UX Ori and WW Vul, respectively. This modeling made it possible to estimate some of the parameters of the shells and of an ensemble of dust grains. The calculations were performed under the assumption of a single scattering for shells with a uniform distribution of matter. However,

the subsequent scatterings can also be significant for optical depths $\tau_\nu \geq 0.2$ – 0.5 typical of the shells around Herbig stars (Voshchinnikov and Karyukin 1994; Voshchinnikov *et al.* 1995). In addition, modeling of the infrared spectra of Herbig Ae stars revealed a density gradient in their shells (Friedemann *et al.* 1992, 1993, 1994).

Friedemann *et al.* (1992, 1993, 1994) modeled the emission of dust in the range 1–100 μm for the stars WW Vul, SV Cep, and SZ Psc and considered the possible properties of dust grains in obscuring clouds. Based on a comparison with observations, these authors determined the distribution of matter and the shell sizes. However, they did not analyze the observational data on the behavior of the colors and polarization of the stars during eclipses.

The wavelength dependence of the circumstellar extinction for Herbig Ae stars was discussed by Voshchinnikov *et al.* (1995). Unfortunately, modeling the circumstellar-extinction curves gives rise to the problem of eliminating the interstellar component, which complicates the interpretation of the observations.

Note also that the infrared spectrum, including the profile of the $\lambda 10 \mu\text{m}$ silicate band, and the wavelength dependence of the circumstellar extinction were modeled by Sorrell (1990) for the Herbig Ae star AB Aur without minima. This joint study proves to be very useful in analyzing the properties of dust in the shells.

In this paper, we made an attempt to simultaneously analyze the behavior of the colors during minima and the spectral energy distribution in the infrared for stars with Algol-like minima. We modeled these observational data for the circumstellar dust shell around WW Vul, a typical Herbig Ae star with nonperiodic dimmings. We discuss the influence of model parameters on the results in detail.

The method of modeling and the basic assumptions and model parameters are outlined in Sec. 2. The observational data for the Herbig Ae star WW Vul are described in Sec. 3. The main results of our modeling and a discussion of the dependence of the results on model parameters are given in Sec. 4.

2. METHOD OF MODELING AND BASIC ASSUMPTIONS

2.1. A Model of the Dust Shell

We modeled the infrared spectra and the behavior of the colors during minima using a spherically symmetric model of the shell. The distribution of dust and gas in the shell was described by a power law: $n(r) \propto r^{-\alpha}$. We assumed that there was a cavity at the center of the shell around the star in which the temperature was higher than the dust evaporation temperature.

We also assumed that the Algol-like dimmings of the stars are a result of the obscuration of the star by dense dust clouds in the shell. Other effects from the presence of the clouds were assumed to be unimportant.

2.2. A Model of the Dust Mixture

In our calculations, we used the mixture of graphite and silicate particles proposed by Mathis *et al.* (1977) to explain the observed wavelength dependence of the interstellar extinction. The mixture is characterized by four parameters: the minimum and maximum grain radii (the grains are assumed to be spherical), the exponent q which describes the particle size distribution [$n(a) \propto a^{-q}$], and the ratio of the numbers of silicate and graphite particles in the mixture. We calculated the optical parameters of the dust particles using the Mie theory and the optical constants obtained by Draine (1985).

2.3. Method of Calculation

The total emission from a star surrounded by a dust shell I_λ is the sum of the emission from the star itself I_λ^* attenuated by the shell, the scattered emission I_λ^{sca} , and the dust thermal emission I_λ^{IR} ,

$$I_\lambda = I_\lambda^* e^{-\tau_\lambda^{(0)}} + I_\lambda^{\text{sca}} + I_\lambda^{\text{IR}}. \quad (1)$$

Here $\tau_\lambda^{(0)}$ is the optical depth of the shell at a wavelength λ .

The values of I_λ are given by

$$I_\lambda = I_\lambda^{\text{Kurucz}} \left[e^{-\tau_\lambda^{(0)}} + \frac{I_\lambda^{\text{sca}} + I_\lambda^{\text{IR}}}{I_\lambda^{\text{BB}}} \right], \quad (2)$$

where $I_\lambda^{\text{Kurucz}}$ is the intensity of the stellar radiation in accordance with the stellar atmosphere models computed by Kurucz (1979), and I_λ^{BB} is the intensity of the radiation from a blackbody with the temperature equal to the effective temperature of the star.

In order to calculate the second term in (2), we must simultaneously solve the equation of radiative transfer and the equation of thermal balance. For this purpose, we used the program of Chini *et al.* (1986), which makes it possible to compute radiative transfer in a spherically symmetric dust shell around a central source.

The change of the color indices during eclipses is calculated from

$$\begin{aligned} \Delta(X - Y) = & -2.5 \log \left(\frac{[I_X^* e^{-\tau_X^{(0)} - \tau_X^{(1)}} + (I_X^{\text{sca}} + I_X^{\text{IR}})]}{[I_Y^* e^{-\tau_Y^{(0)} - \tau_Y^{(1)}} + (I_Y^{\text{sca}} + I_Y^{\text{IR}})]} \right) \\ & + (I_Y^{\text{sca}} + I_Y^{\text{IR}}) [(I_X^* e^{-\tau_X^{(0)}} + (I_X^{\text{sca}} + I_X^{\text{IR}}))], \end{aligned} \quad (3)$$

where $\tau^{(1)}$ is the optical depth of an obscuring cloud, and X and Y are the wavelengths of the corresponding

Table 1. Model parameters

	Parameter	Designation	Value for WW Vul
Dust shell	Size of the inner cavity	R_i	3 AU
	Outer radius of the shell	R_o	10^4 AU
	Radial distribution of matter	α	1.5
	Optical depth of the shell in V	$\tau_V^{(0)}$	0.3
Dust mixture	Minimum grain size	a_{\min}	0.0075 μm
	Maximum grain size	a_{\max}	0.96 μm
	Particle size distribution	q	4.5
	Composition of the dust mixture	$n_C : n_{Si}$	1 : 1

bands. We chose such a wavelength dependence $\tau_\lambda^{(1)}$ that

$$\frac{\tau^{(1)}(\lambda_X)}{\tau^{(1)}(\lambda_Y)} \propto \left(\frac{\lambda_X}{\lambda_Y}\right)^{-\beta_{XY}}, \quad (4)$$

where the exponent β_{XY} is determined from the observed slope of the upper parts of the color tracks (Voshchinnikov 1989). This allows us not to consider the properties of dust in the clouds in detail.

Since the thermal emission of dust becomes important in the infrared ($\lambda \geq 0.7 \mu\text{m}$), in contrast to the previous papers (Voshchinnikov 1989; Voshchinnikov and Grinin 1991; Voshchinnikov *et al.* 1988), allowance for I_λ^{IR} makes it possible to analyze the behavior of the colors not only in the visible and in the ultraviolet, but also in the infrared. The behavior of the following color indices was modeled: $U-B$, $B-V$, $V-R$, $V-I$, $V-J$, $V-H$, $V-K$, and $V-L$. Unfortunately, we cannot make good use of these data thus far, because there are no regular observations of light variations in the J , H , K , and L bands.

For definiteness, we performed our calculations for a star with the same parameters as that of the typical Herbig Ae star with Algol-like dimmings WW Vul (Sp A0–A3e): the effective temperature is 9500 K, and the luminosity is $3 \times 10^{35} \text{ erg s}^{-1}$ (Friedemann *et al.* 1993).

2.4. Model Parameters

All of the model parameters can be divided into two groups: the circumstellar-shell parameters and the dust-mixture parameters (Table 1). The first group includes the inner radius of the shell R_i (the radius of a dust-free cavity), the outer radius R_o (shell size), the radial distribution of matter (the exponent α , see Section 2.1), and the optical depth of the shell $\tau^{(0)}$. The second group of parameters includes the grain sizes a_{\min} and a_{\max} (the minimum and maximum radii, respectively), the particle size distribution (the exponent q ; see Sec. 2.2), and the composition of the mixture $n_C : n_{Si}$ (the proportion of graphite particles to silicate particles). Note that the parameters of the dust mixture determine the wave-

length dependence of the optical depth of the shell $\tau_\lambda^{(0)}$. In the subsequent analysis, we will fix only the optical depth in V . In Sec. 4, we will discuss the influence of each of these parameters on the results.

The last column in Table 1 gives the parameters for the model which is generally in good agreement with the observational data for WW Vul. In what follows, the parameters that are not given separately correspond to the values given in the table.

3. WW VUL: OBSERVATIONAL DATA

Let us briefly describe the observational data for WW Vul (similar data were also obtained for other Herbig Ae stars with Algol-like minima—UX Ori, CQ Tau, etc.). The photometric observations of WW Vul (HD 344361, BD+204136, IRAS 19238+2106) suggest that there are nonperiodic Algol-like dimmings by more than 2^m ; at deep minima, the $U-B$ and $B-V$ color indices first increase and then decrease (Zaitseva 1973, 1983; Timoshenko and Filip'ev 1983; Kardopolov and Filip'ev 1985). An analysis of the star's variability did not reveal any dominating period (Friedemann *et al.* 1993). The infrared observations revealed a considerable excess at these wavelengths compared to normal stars (Cohen 1973; Glass and Penston 1974; Weaver and Jones 1992).

The photometric observations of WW Vul at various wavelengths that we used to construct the spectral energy distribution in the visible and in the infrared and the color tracks during minima are summarized in Table 2. When modeling the spectral energy distribution in the visible and in the near infrared, we considered data for the brightest states of the star.

We used the standard reddening law (Savage and Mathis 1979) to correct the data for the interstellar extinction.

The photometric *uvby* observations of WW Vul performed by Friedemann *et al.* (1993) made it possible to determine the color excess for this star, $E_{B-V} = 0.345$. Assuming the normal reddening law, these authors found the distance to the object, $D = 550 \text{ pc}$, and the total absorption along the line of sight, $A_V^{\text{IS}} + A_V^{\text{CS}} \approx 1.7^m$

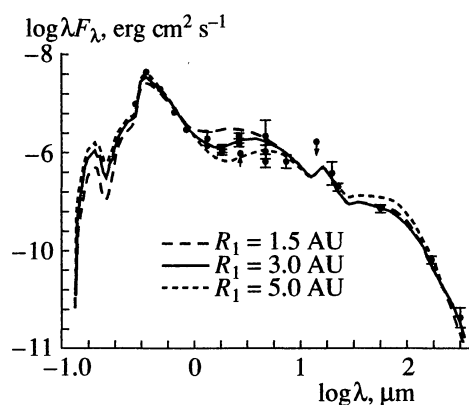


Fig. 1. Dependence of the results on the inner radius of the shell R_1 .

(here A_V^{IS} is the interstellar absorption, and A_V^{CS} is the absorption in the circumstellar shell). A comparison with the observations of other stars in the direction of WW Vul has indicated that $A_V^{CS} \approx 0^m.65$.

In what follows, the observations of WW Vul are shown in the figures that illustrate the influence of different parameters on the results of our modeling for comparison (they are marked by asterisks; the arrows indicate the upper and lower limits of the corresponding data).

4. RESULTS OF MODELING AND DISCUSSION

4.1. The Inner Radius of the Shell

In general, the size of the inner cavity is not a completely free parameter—it is determined by the fact that the dust temperature at the inner boundary of the shell cannot be greater than the particle evaporation temperature (~ 1500 K).

The dust emission at the inner boundary determines the shape of the spectrum in the wavelength range $\lambda\lambda \approx 1\text{--}5$ μm . Figure 1 shows the spectra for three shell mod-

els with different sizes of the inner cavity: 1.5, 3, and 5 AU. Given the sensitivity of the near-infrared flux to the dust temperature at the inner boundary, we can estimate the size of the inner cavity.

It should be noted that very fine ($a < 0.005$ μm) particles and polycyclic aromatic hydrocarbons (PAHs) can contribute to the shell emission in the near and mid-infrared. This issue was thoroughly studied by Siebenmorgen and Krügel (1992), Siebenmorgen *et al.* (1992), and Natta *et al.* (1993a). The latter modeled the spectral energy distribution for Herbig Ae/Be stars, taking into account the emission of fine particles and PAHs. An analysis of the results and their comparison with observations showed that the near-infrared emission is mainly determined by the parameters of the star itself and by the properties of coarse grains, while fine grains can give an appreciable contribution to the mid-infrared ($\lambda\lambda \approx 3\text{--}20$ μm) emission. The observations of Prusti *et al.* (1994) lend support to this inference: in three of the seven Herbig Ae/Be stars studied by these authors, the $\lambda\lambda \approx 3\text{--}20$ μm emission comes from extended regions.

Note, however, that the spectroscopic observations of WW Vul near $\lambda 3$ μm aimed at searching for PAH emission bands at $\lambda 3.29$ μm and $\lambda\lambda 3.4\text{--}3.6$ μm has produced negative results thus far (Brooke *et al.* 1993). In addition, the near-infrared (J, H, K) images obtained by Li *et al.* (1994) for sixteen Herbig Ae/Be stars, including WW Vul, suggest that almost all of the emission at these wavelengths comes from the inner layers of the shell. The authors believe that this does not confirm the assumption that fine particles and PAHs emit in this wavelength range, because in the latter case the emission must also come from other parts of the shell.

4.2. The Outer Radius of the Shell

The outer radius of the shell is not a completely free parameter either. The dust concentration in the outer parts of the shell cannot be lower than the mean concentration of matter in the diffuse interstellar medium.

Table 2. Observational data for WW Vul

Source	Observations
Berdyugin <i>et al.</i> (1992)	UBVRI light variations
Zaitseva (1983)	UBV light variations
Kardopolov and Filip'ev (1985)	UBVR light variations
Kolotilov <i>et al.</i> (1977)	UBVL magnitudes, JK light variations
Timoshenko and Filip'ev (1983)	UBVR light variations
Herbst <i>et al.</i> (1994)	UBVR light variations
Cohen (1973)	Magnitudes at $\lambda\lambda 3.5, 4.9, 8.4,$ and 11 μm
Glass and Penston (1974)	HKL magnitudes
Li <i>et al.</i> (1994)	JHK magnitudes
Weaver and Jones (1992)	Magnitudes at $\lambda\lambda 12, 25, 60,$ and 100 μm

Figure 2 shows the spectra of four model shells with different sizes: 10^3 , 3×10^3 , 10^4 , and 5×10^4 AU. Since the dust in the outer layers has a low ($T \approx 20\text{--}30$ K) temperature, the outer radius influences only the far-infrared ($\lambda \geq 30 \mu\text{m}$) flux.

It should be noted that the optical properties of the circumstellar dust in this spectral region are unreliable (Preibisch *et al.* 1993; Stognienko *et al.* 1995). Using a particular model of the dust mixture can noticeably affect the results of modeling. This introduces additional uncertainty into the estimate of the shell sizes.

4.3. The Radial Distribution of Matter

Figure 3 shows the spectral energy distribution (a) and the behavior of the color indices (b) during the obscuration of the star by a cloud for three model shells with the exponent α in the law of the radial distribution of matter, varying from 1.2 to 1.8. Recall that the other parameters, including the optical depth of the shell, are fixed.

The faster the dust density falls off outward, the later occur the turns of the color indices. However, as can be seen from Fig. 3b, the difference in all cases is relatively

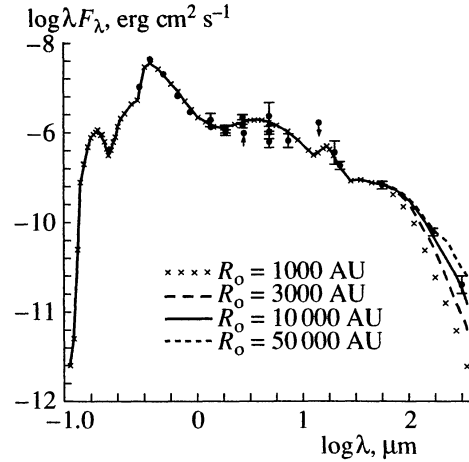


Fig. 2. Dependence of the results on the outer radius of the shell R_0 .

small. In contrast, the spectral slope in the range $\lambda \approx 10\text{--}50 \mu\text{m}$ is very sensitive to the parameter α .

Note that the result does not depend on the selected model of the dust mixture. Figure 4 shows the spectra and color-magnitude diagrams of the three model

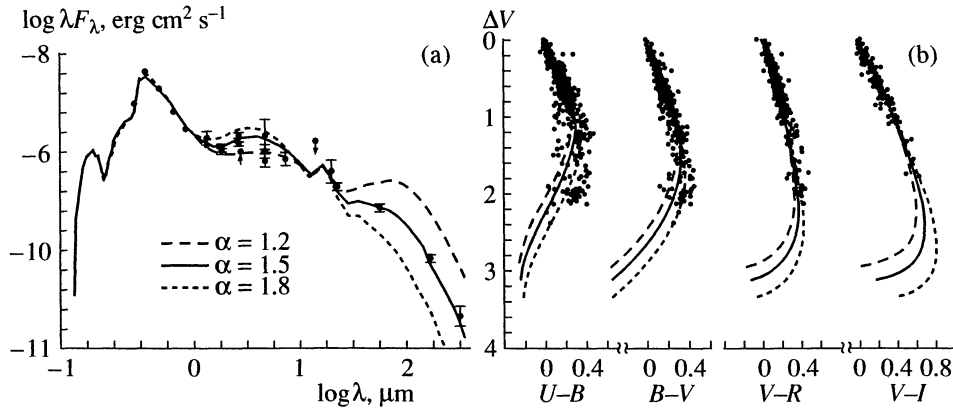


Fig. 3. Dependence of the results on the radial distribution of matter.

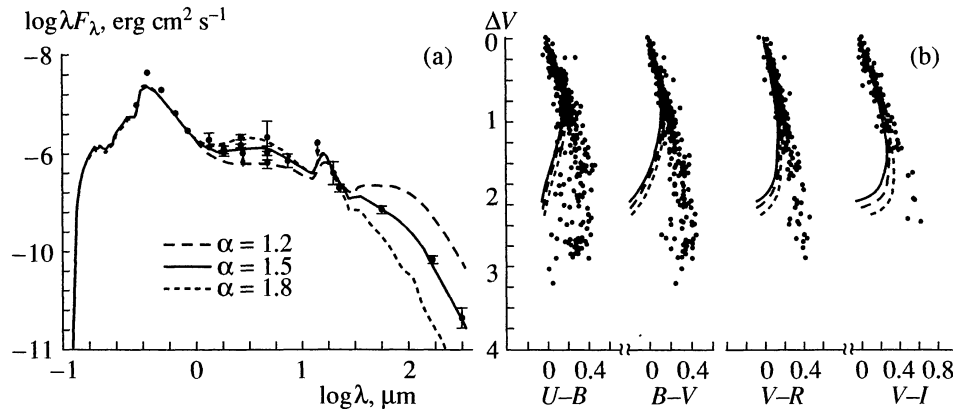


Fig. 4. Same as Fig. 3 for $q = 3.5$ and $\tau_V = 0.6$.

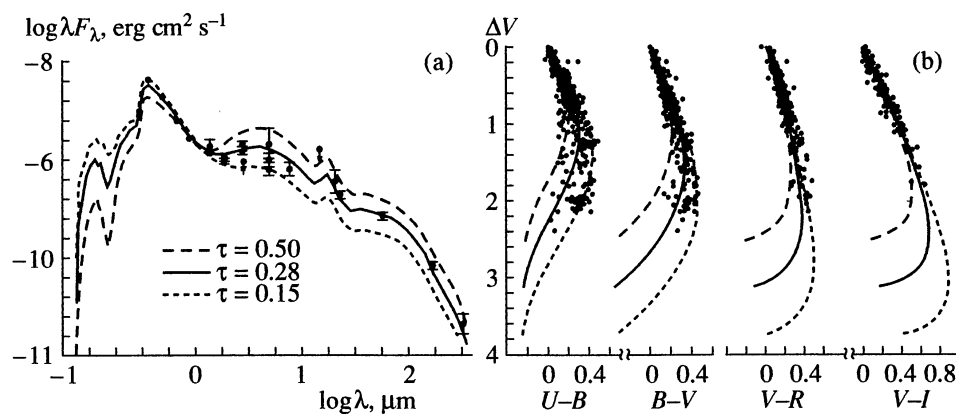


Fig. 5. Dependence of the results on the optical depth of the shell τ_V

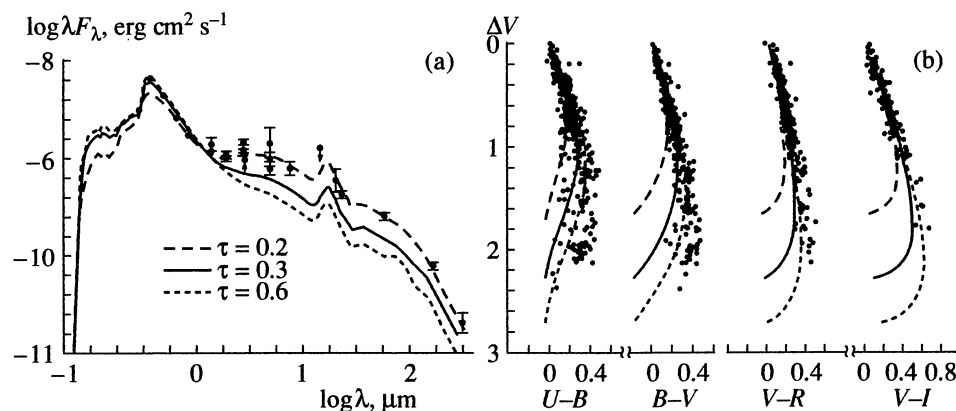


Fig. 6. Same as Fig. 5 for $q = 3.5$.

shells for a different dust mixture with a less steep size distribution ($q = 3.5$, as for the interstellar dust).

A comparison of the theoretical spectral energy distribution for different values of α with the observed energy distribution indicates that the exponent α can be determined from the observed spectral slope rather reliably.

4.4. The Optical Depth of the Shell

Figure 5 shows the spectra (a) and color tracks (b) during minima for three shell models with optical depths in V equal to 0.15, 0.28, and 0.50 and a particle size distribution with $q = 4.5$.

As can be seen from Figs. 5 and 6, the optical depth determines both the behavior of the color indices and the energy distribution in the infrared. Thus, based on a comparison with observations of the color indices and the infrared spectra, we can obtain independent estimates and constraints on the optical depth.

It should be recalled here that we ignore the shell ellipticity in our calculations. When modeling the lin-

ear polarization in a dust shell, its ellipticity plays a major role. The calculations performed by Voshchinnikov and Grinin (1991) for WW Vul show that the flattening of its shell is $\epsilon = A/B \approx 3$ (where A and B are the semimajor and semiminor axes of the ellipsoidal shell, respectively).

In an optically thin ($\tau < 1$) shell, the total volume of the emitting region is important, while the ellipticity effect turns out to be insignificant. Consequently, an ellipsoidal shell with a flattening ϵ can be replaced by a sphere of radius $R = B\epsilon^{1/3}$. Let us call the optical depth that corresponds to this radius "effective," τ_{eff} . The optical depth along the semimajor axis of the ellipsoid will then be

$$\tau_A = \tau_{\text{eff}} \epsilon^{-1/3}. \quad (5)$$

The fraction of light scattered in the shell also depends more likely on τ_{eff} than on the flattening. Figure 3 in Voshchinnikov and Karyukin (1994) shows a plot of the fraction of scattered light I_{sca}/I_* against the flattening calculated by these authors for various opti-

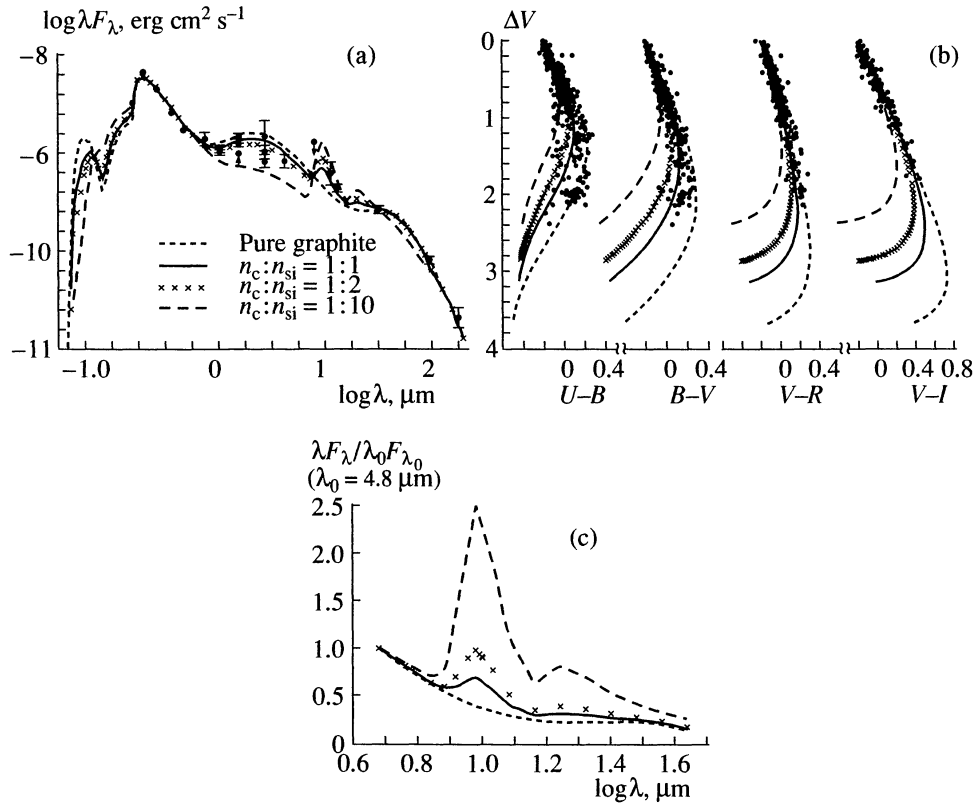


Fig. 7. Dependence of the results on the composition of the mixture n_C/n_{Si} .

cal depths. It can be seen from the figure that, to a good approximation,

$$I_{sca}/I_*(\tau_0, \epsilon_0 = 1) \approx I_{sca}/I_*(\tau = \tau_0 \epsilon^{1/3}). \quad (6)$$

Thus, the optical depth under discussion should be considered effective, and the total optical depth of the shell along the semimajor axis is related to it by Eq. (5).

4.5. Composition of the Dust Mixture

In order to study the influence of the chemical composition of the dust mixture on the results, we considered models with various relative abundances of silicate and graphite (Fig. 7).

At ultraviolet and visible wavelengths, the albedo of silicate particles is greater than that of graphite particles. Therefore, the larger is the number of silicate grains in the mixture, the earlier the turns of the color tracks occur. Figure 7 shows the results for four relative abundances of graphite and silicate in the mixture: $n_C/n_{Si} = 1/1, 1/2, 1/10$, and a purely graphite mixture. We see that the models of the behavior of the colors and the infrared spectra differ markedly only when the number of silicate particles is much greater than that of graphite particles. In this case, however, the models are inconsistent with the observations—the fraction of

dust-scattered light turns out to be too large. Therefore, it may well be inferred that $n_{Si}/n_C \leq 2$.

It appears that more information on the composition of the mixture could be obtained by analyzing the circumstellar-extinction curves. These curves must differ markedly for different compositions of the mixture. Graphite particles give an absorption peak around $\lambda 2200 \text{ \AA}$, while silicate particles produce an increase in the absorption at shorter wavelengths.

The presence of silicate grains in the shells around Herbig stars is also confirmed by spectroscopic observations. The spectra of some stars exhibit a fairly strong silicate $\lambda 10 \text{ \mu m}$ emission band (Sitko *et al.* 1981; Sorrell 1990; Berrilli *et al.* 1992; Wooden 1994). For example, the spectrophotometric observations of twelve Herbig Ae/Be stars carried out by Wooden (1994) in the range 8–13 μm revealed the above emission band in the spectra of stars of spectral type B7 or later. As can be seen from Fig. 7c, for this band to appear, the number of silicate particles in the mixture must be no smaller than that of graphite particles.

4.6. The Sizes of Dust Grains

Let us now consider the influence of grain sizes on the results of modeling.

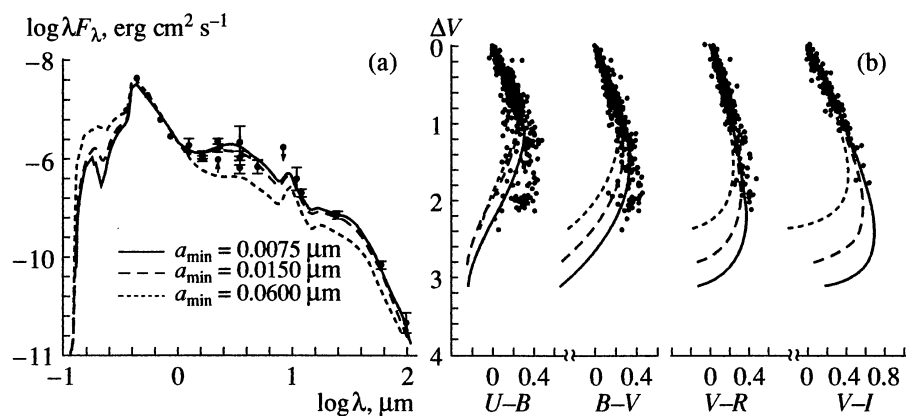


Fig. 8. Influence of the minimum grain size a_{\min} on the results.

Since fine dust grains are known to emit considerably better in the infrared than do coarse grains (Fig. 8a), the flux from a shell in which there are no fine ($a \leq 0.06 \mu\text{m}$) particles turns out to be lower. In this case, the turns of the color indices occur earlier (Fig. 8b). Thus, in the absence of particles smaller than $0.06 \mu\text{m}$, the amount of dust in the shell must be increased to explain the observed infrared fluxes from WW Vul. On the other hand, the amount of dust must be decreased in this case to explain the observed color-magnitude diagrams.

Similar results are also obtained for the standard particle size distribution with $q = 3.5$, even for $a_{\min} = 0.0075 \mu\text{m}$ (Fig. 6). The optical depth of the shell derived from the infrared data and from the color tracks is $\tau_V \approx 0.6$ and $\tau_V \approx 0.2$, respectively.

The maximum size of the particles in the shell affects the results of modeling to a lesser degree. The presence of coarse ($a > 0.2 \mu\text{m}$) particles has almost no effect on the energy distribution in the infrared. Since these particles scatter the visible and red light somewhat better, the turns of the $V-R$ and $V-I$ colors occur earlier in the presence of these particles (Fig. 9).

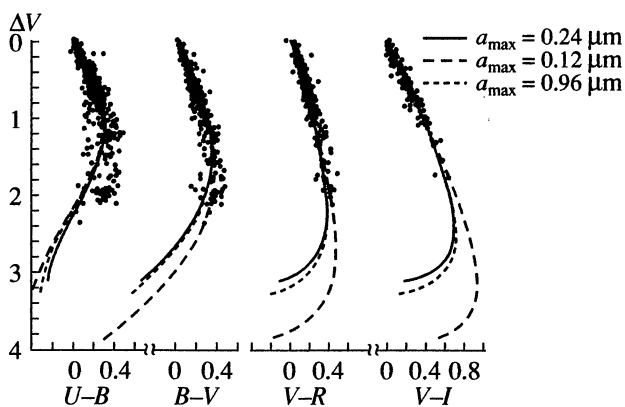


Fig. 9. Influence of the maximum grain size a_{\max} on the results.

Note that the observational data for WW Vul can still be explained for $q = 3.5$, but only if we assume that particles larger than $0.12 \mu\text{m}$ are completely absent and that the number of graphite particles is a factor of 4–5 greater than the number of silicate particles (Fig. 10b). The optical depths derived from the infrared data and from the behavior of the colors then turn out to be identical. It is obvious, however, that this case is not much different from the case with a steeper size distribution. In this case, we actually “artificially” increase the contribution of fine particles, while the results are less sensitive to the presence of coarse grains, as was already noted above.

4.7. A Model of the Shell around WW Vul

Let us summarize the results of our modeling for WW Vul. Figure 10 shows the infrared spectra and color-magnitude diagrams for two models that satisfactorily reproduce the observational data. The models mainly differ by the properties of dust particles. For both models, the parameters of the circumstellar shell—the inner and outer radii, the radial distribution of matter, and the optical depth—turn out to be essentially the same: $R_i \approx 3 \text{ AU}$, $R_0 \approx 10^4 \text{ AU}$, $\alpha \approx 1.5$, and $\tau_V \approx 0.3$ (see Table 1). The dust density at the inner boundary of the shell is $1.4 \times 10^{-17} \text{ g cm}^{-3}$, and the shell mass is $\sim 10^{-5} M_\odot$.

The derived inner radius of the shell and the exponent in the law of the radial distribution of matter are consistent with the models of Friedemann *et al.* (1993): they obtained $R_i = 2.7 \text{ AU}$ and $\alpha = 1.45$. This value of R_i is more than a factor of 2 smaller than the value derived by Voshchinnikov and Grinin (1991). The reason for the difference is that the latter performed no numerical modeling of the infrared data and obtained their estimate from simple analytical considerations. Numerical modeling showed that the dust temperature at the inner boundary of the shell for $R_i = 7 \text{ AU}$ is only $T_i \approx 900 \text{ K}$, while for our model this temperature is $T_i \approx 1300 \text{ K}$.

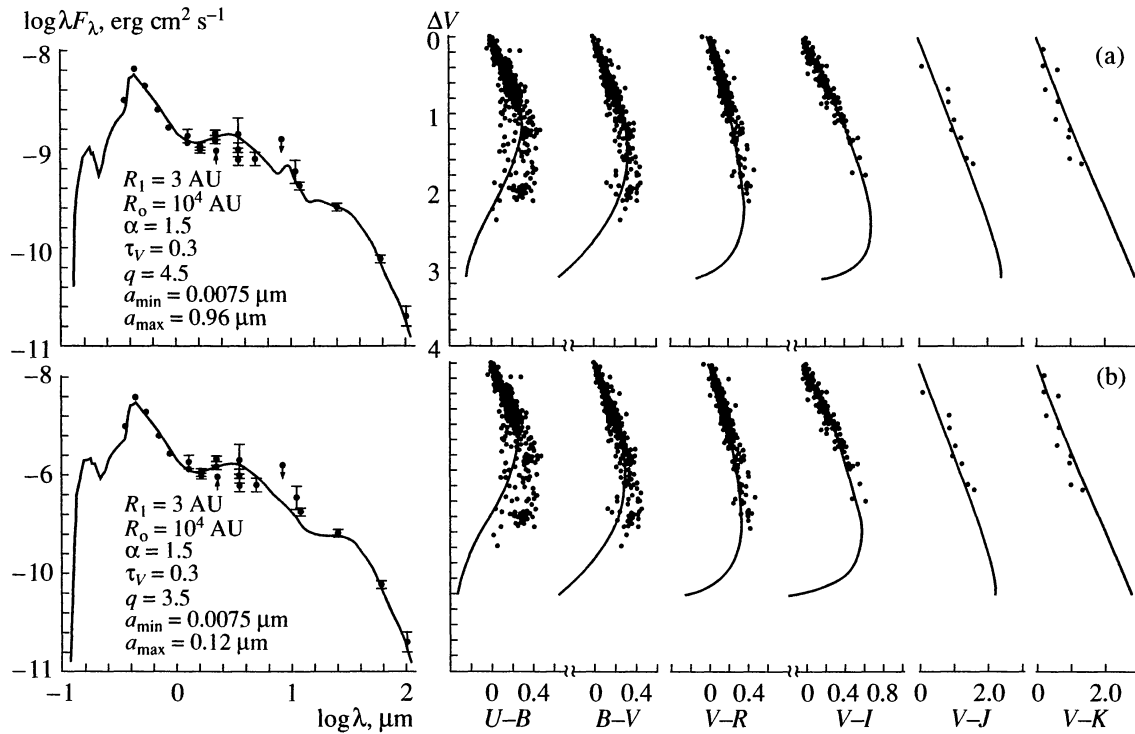


Fig. 10. Results of our modeling for WW Vul, a Herbig Ae star with nonperiodic light minima.

The derived outer radius of the shell is greater than the value obtained by Friedemann *et al.* (1993), $R_0 = 4.7 \times 10^3$ AU. This difference probably results from the fact that the data for the optical constants of dust in the far infrared are unreliable (see Sec. 4.2). In addition, the model constructed by these authors gives a flux at 100 μm that is lower than the observed flux, whereas allowance for the outermost layers of the shell leads to an increase in the flux precisely in the far infrared.

The difference between the estimates for the optical depth of the shell is more important: $\tau_V = 0.65$, as derived by Friedemann *et al.* (1993). However, these authors modeled the infrared spectra using the standard dust mixture with $q = 3.5$, $a_{\min} = 0.0075 \mu\text{m}$, and $a_{\max} = 0.24 \mu\text{m}$. This model indeed yields $\tau_V = 0.6$ (Fig. 8a), but, as was already noted above, it cannot account for the observed behavior of the colors at minimum light. In this case, the turns of the color indices (Fig. 8b) would occur even at shallow minima with $\Delta V < 1^m$, while at minima with $\Delta V \approx 1.5^m$ we would see the occurrence of the turns of the $V-R$ and $V-I$ color indices, which is at variance with the observations.

Voshchinnikov and Grinin (1991), who modeled the behavior of the colors and the degree of linear polarization at minima, obtained $\tau_V = 0.55$. This result is in satisfactory agreement with our value. Recall that we considered the effective optical depth, and for the flattening $\epsilon = 3$ derived by Voshchinnikov and Grinin from polar-

ization observations, the optical depth along the semi-major axis of the shell they analyzed is a factor of $\sim 3^{1/3} \approx 1.4$ greater than the effective optical depth. It should also be noted that they considered a different wavelength (in general, $\tau_U > \tau_V$).

As for the model of the dust mixture, it appears that no definitive conclusions can be reached about it in terms of the model under consideration. We showed that the models of shells in which there are no fine ($a < 0.06 \mu\text{m}$) particles cannot explain the observations. Note also that the fraction of fine particles must be fairly large, and that they must have a rather steep size distribution. The inference that the size distribution is steeper than that in the interstellar medium is consistent with the models of Voshchinnikov and Grinin (1991). However, the polarization data modeled by these authors can be better explained in the absence of grains smaller than $0.05 \mu\text{m}$.

5. CONCLUSION

Let us summarize the main results of our study.

We modeled the energy distribution in the infrared and the behavior of the color indices during minima for Herbig Ae stars with nonperiodic dimmings. We studied the influence of the parameters of dust shells and an ensemble of dust grains on the results. We showed that:

(1) The parameters of circumstellar shells—the inner and outer radii, the radial distribution of matter, and the optical depth—have a significant effect on the energy distribution in the infrared.

In this case, the inner radius and the distribution of matter can be determined rather accurately from the observed spectral energy distribution. The outer radius of the shell can be reliably determined only if the optical parameters of dust in the far-infrared are well known. The optical depth of the shell also strongly affects the behavior of the color indices during eclipses. This makes it possible to determine fairly reliably the optical depth by modeling both types of observations.

(2) The parameters of the dust mixture (composition and particle sizes) cannot be reliably determined from the observed infrared spectra and light variations at various wavelengths during minima.

We compared our models with the observational data for the Herbig Ae star WW Vul. For its circumstellar dust shell, we obtained the following parameters: the inner radius of the shell $\approx 2\text{--}3$ AU; the shell size $\sim 10^4$ AU; the law of the outward density fall-off $\propto r^{-1.5}$; the effective optical depth of the shell $\tau_V \approx 0.3$; and its mass $\sim 10^{-5}M_{\odot}$. We confirmed the conclusion of Voshchinnikov and Grinin (1991) that the particle size distribution is steeper than that in the interstellar medium: $q \approx 4.5\text{--}5.0$. However, their assumption that there are no small ($\leq 0.05 \mu\text{m}$) dust grains makes it impossible to reach agreement with all of the available observational data.

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