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ASTRONOMY AND ASTROPHYSICS

Size distributions of dust in circumstellar debris discs

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Abstract. The size distribution of particles in a dust disc is determined by, and holds the key to, sources, sinks and dynamics of grains. Here we derive the size distribution in circumstellar debris discs, exemplified by the disc of β Pictoris, by modelling the dynamical evolution of the circumstellar dust, dominated by collisions. The whole disc is considered as consisting of two dust populations: larger grains moving in bound orbits (α-meteoroids) and smaller ones blown away by radiation pressure (β-meteoroids). Although β-meteoroids leave the disc in hyperbolic trajectories, they are continuously replenished by collisions, so that at any time the disc contains a substantial population of small particles. As a consequence, the fragmentation of α-meteoroids not only by mutual collisions, but also by impacts of β-meteoroids becomes significant. This flattens the distribution of α-meteoroids in the size regime adjacent to the blow-out limit and shifts the cross section-dominating sizes from a few micrometres to $\gtrsim 10\,\mu$m. The overall distribution shows essentially three different slopes: steeper ones for both β-meteoroids and large α-meteoroids and a gentler one for α-meteoroids with sizes just above the blow-out limit. This resembles the size distribution of interplanetary dust particles in the Solar system which, however, is shaped by different mechanisms. The basic features of the modelled size distribution (the presence of a substantial population of small hyperbolic particles in the disc, the dominance of grains $\sim 10\,\mu$m in size) well agree with the observational data available. Although particular calculations were made for the β Pic disc, our basic qualitative conclusions directly apply to the debris discs around other Vega-type stars with low gas contents and similar or somewhat lower optical depths.

Key words: stars: circumstellar matter – stars: individual: β Pic – stars: planetary systems

1. Introduction

Understanding the size or mass distribution of dust particles in dust environments – from cometary tails or dusty planetary rings to circumstellar debris discs – is a key to essential physical processes operating in a particular system. Indeed, the distribution of sizes fully reflects the dust production and loss mechanisms, as well as the forces and effects acting on the grains. The size distributions are not easy to retrieve, however. Derivation of them is usually only possible by combining various types of observational data with theoretical modelling that gives guidelines towards which type of size distribution to expect in one or another particular system.

An obvious example is the dust cloud of our Solar system. The size distribution of interplanetary dust is well established observationally at 1 AU from the Sun on the base of spacecraft in-situ measurements, microcratering of lunar samples and meteor data (Grün et al., 1985). At other heliocentric distances – from vicinities of the Sun to the Edgeworth-Kuiper belt (EKB) region – the observational data available to date are too scarce to permit determination of the size distribution. In these regions, only theoretical modelling provides estimates of the expected size distribution. Such a modelling was done for the Zodiacal cloud (Dohnanyi, 1972; Dohnanyi, 1973; Dohnanyi, 1978; Ishimoto, 1998; Ishimoto, 1999; Ishimoto & Mann, 1999; Ishimoto, 2000) and for the EKB dust (Stern, 1995; Stern, 1996).

Circumstellar debris discs represent another good example. Observations of main-sequence (Vega-type) and "old" pre-main-sequence (post-Herbig Ae/Be and post-T Tau) stars show that the distribution is broad and extends from a small fraction of micrometre to at least ~ 1 mm (e.g., Skinner et al. 1992; Sylvester et al. 1996; Sylvester & Skinner 1996; Li & Greenberg, 1998; Krivova et al. 2000a and references therein). Observational results alone, however, do not give direct clues to the shape of the distribution, and must be combined with theoretical assessments, which have not been undertaken so far.

In this paper, we attempt to access the size distribution in circumstellar discs, exemplified by the disc of β Pictoris, by modelling the collisional and dynamical evolution of the circumstellar dust. We compare and contrast the results with those for the Solar system dust cloud. Applications to other circumstellar discs are also discussed.

2. Model

2.1. Kinetic approach

The spatial and/or size distribution of dust in a circumstellar disc can be modelled through the kinetic approach. Consider

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an axisymmetric, vertically uniform disc and denote by r the distance from the central star, by $\dot{\mathbf{r}}$ the velocity vector of a dust grain and by m its mass. Let $n(r, m)dm$ be the number density of the grains with masses $[m, m + dm]$ at a distance r. While a description of collisions is more convenient in terms of the *mass* distribution $n(r, m)$, discussion of astrophysical applications is easier in terms of the *size* distribution. In what follows we will often use the latter, expressed as $n(r, a) = n(r, m)4\pi\rho a^2$, where a is the radius of a grain and ρ is its bulk density.

The dust number density $n(r, m)$ satisfies the continuity equation

$$\frac{\partial n}{\partial t} + \mathrm{div}(\dot{\mathbf{r}}n) = \left(\frac{dn}{dt}\right)_{supply} - \left(\frac{dn}{dt}\right)_{loss} \qquad (1)$$

that describes the main transport mechanisms such as the Poynting-Robertson (P-R) effect through the divergence term, as well as the presumed sources and sinks of dust, including those caused by the grain-grain collisions. The latter play a dual role. On the one hand, they remove the material through catastrophic destruction and hence represent an important loss mechanism for larger grains. On the other hand, collisions generate small fragments and therefore act as "sources" of dust at smaller sizes. Thus in systems with sufficiently high dust densities the collisions are of primary importance for shaping the size and spatial distributions of dust and for controlling the overall dust budgets.

Solving the continuity equation (1) under the steady-state assumption $\partial n/\partial t = 0$ would yield both size and spatial distributions of dust, which are generally inseparable. However, solution of the whole problem is hampered by poor knowledge of the dust sources and by severe computational difficulties, so the solutions have been obtained only for certain particular cases. The simplest case is a collisionless system without any sources and sinks, but with the P-R force controlling the cloud. Provided the grain orbits are circular, the solution is $n \propto r^{-1}$, the size or mass distribution being arbitrary (e.g., Fessenkov, 1947; Briggs, 1962; Dohnanyi, 1978). Another simple case is a cloud of grains moving outward from the centre in hyperbolic trajectories (this applies to the so-called β-meteoroids in interplanetary space, see below), which has $n \propto r^{-2}$ (e.g., Lecavelier des Etangs et al., 1998; Ishimoto & Mann, 1999). A less trivial, yet simple, particular case is a system with P-R transport without sources but with mutual collisions, that are assumed to eliminate the grains without creating collisional fragments. For such a system, an analytic solution was found by Southworth & Sekanina (1973). Some other examples were discussed by Dohnanyi (1972; 1973; 1978), Rhee (1976), Leinert et al. (1983), Ishimoto (1998; 1999; 2000), Ishimoto & Mann (1999), among others. In all these cases, the radial transport of dust by the P-R force is of primary importance and accordingly, the continuity equation is often used to seek the spatial distribution of material rather than the size distribution. Such systems may be conventionally called *transport-dominated systems*.

The opposite particular case is *collision-dominated systems*, in which the collisional processes are so intensive that the transport mechanisms play an insignificant role. Then the divergence term in Eq. (1) can be omitted and, as long as the steady state is assumed ($\partial n/\partial t = 0$), the continuity equation reduces to the balance equation

$$\left(\frac{dn}{dt}\right)_{supply} - \left(\frac{dn}{dt}\right)_{loss} = 0. \qquad (2)$$

This equation was first solved analytically by Dohnanyi (1969) who considered a closed collisional system exemplified by the asteroidal belt and has shown that a power-law size distribution $n(a) \propto a^{-p}$ with the exponent $p = 3.5$ is settled in such a system, providing equilibrium between the gain and loss of collisional fragments at all sizes. Interestingly, the size distribution with a similar exponent is typical of a number of physical processes: so distributed are fragments of cratering (Fujiwara et al., 1977) and catastrophic (Davis & Ryan, 1990) collisions, dust produced by the activity of comets (Greenberg & Hage, 1990; Fulle et al., 1995), etc.

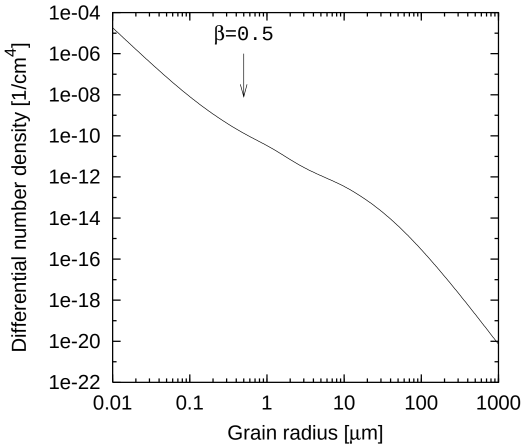

Fig. 1. Size distribution of dust in the Solar system at 1 AU from the Sun (Grün et al., 1985). The arrow marks the boundary between the particles in bound and unbound orbits.

We discuss now the applications of the kinetic approach to real dust complexes. For the distributions of interplanetary dust in the inner Solar system, such an investigation has been done by Ishimoto (1998), Ishimoto & Mann (1999) and Ishimoto (2000). Assuming the dust production from cometary and asteroidal sources and considering the P-R transport of dust and their collisional evolution, they wrote and solved numerically Eq. (1), reproducing the main features of the observed radial and size distributions of the Zodiacal dust. These studies particularly explain the typical size distribution of interplanetary dust derived from various observational datasets by Grün et al. (1985). This distribution (Fig. 1) can be closely fitted with a combination of three power laws with different slopes: ≈ 3.5 for $a \lesssim 0.5\,\mu$m (hyperbolic particles or β-meteoroids), ≈ 2 for $0.5\,\mu\mathrm{m} \lesssim a \lesssim$ 10–100 μm; ≈ 5 for $a \gtrsim 100\,\mu$m.

Both the Zodiacal cloud and the EKB of our Solar system exemplify dust discs with very low optical depths, $\tau \lesssim 10^{-6}$. For such systems, the P-R transport is the main evolutionary mechanism for dust grains over a broad range of sizes. According to estimates of Grün et al. (1985), the grains of $\sim 100\,\mu$m in size have collisional lifetimes comparable to the P-R times ($\sim 10^5$ years). Therefore, only very large grains are mainly lost to collisions, whereas most of the smaller particles are likely to drift all the way to the inner Solar system under the P-R force. This makes the problem especially complex: there is no collisional balance of the grains at any distance from the Sun. The size distribution results from an interplay between the sources, sinks, transport and collisional evolution of dust.

Paradoxically, the things are eased in denser systems with optical depths $\tau \gtrsim 10^{-4}$, but still much less than unity. This is the case for a number of circumstellar discs of young main-sequence, post-Herbig Ae/Be and post-T Tau stars currently observed – see, e.g., Backman & Paresce (1993), Sylvester et al. (1996), Fajardo-Acosta et al. (1998), Song et al. (2000). While still optically thin, such discs have much shorter collisional lifetimes. Catastrophic collisions between the grains represent the main loss mechanism of dust, which makes the P-R effect largely irrelevant to such systems (see, e.g., Artymowicz, 1997). Provided that the grains are moving in low-eccentricity orbits, each reasonably narrow radial zone of a disc can be treated as a closed system in dynamical equilibrium: material supplied by the sources at a certain distance stays at nearly the same distance, evolving through a collisional cascade to dust-sized grains which are eventually lost from the zone in the form of tiny debris, placed by the stellar radiation pressure in unbound orbits. This scenario suggests that the balance equation (2) and an approach similar to Dohnanyi's (1969) can be used to describe circumstellar systems, instead of the more general transport equation (1). It should be noted that the above-said implies gas-poor discs, in which the gaseous component is not able to appreciably damp the relative velocities, drastically reducing the role of catastrophic collisions. A number of discs around main-sequence and old pre-main sequence stars satisfy this condition as well (Zuckerman et al., 1995; Holweger et al., 1999; Liseau, 1999; Greaves et al., 2000).

Such collision-dominated discs are the subject of this paper. Particular calculations will be made for the disc of β Pic, with its maximum normal optical depth of $\sim 10^{-2}$ (Pantin et al., 1997). A wealth of the observational data, as well as a relatively good understanding of its physics, make β Pic a good object to apply our modelling effort.

2.2. Two dust populations: α- and β-meteoroids

We consider the dust disc as consisting of two dust populations which, borrowing the terminology from the Solar system studies, may be called α- and β-meteoroids (Zook & Berg, 1975). The former are larger grains that move round the star in bound orbits, whereas the latter are smaller particles blown away from the star by the stellar radiation pressure. The boundary between them depends on the luminosity-to-mass ratio of the central star and the properties of dust grains. For β Pic and plausible compositions of grains in its disc, the boundary lies at a grain mass $m_0 \sim 10^{-10}$ g, or radius of $a_0 \approx 2$ to $3\,\mu$m (e.g., Artymowicz, 1997), provided the particles are compact.

2.3. Sources and collisional balance of α-meteoroids

As explained above, a reasonably narrow radial zone of a collision-dominated disc can be considered as a closed system which we, moreover, assume to be in a steady state. This means that $n_\alpha(r,m)dm$, the number density of dust grains with masses $[m, m+dm]$ ($m \ge m_0$) at a distance r, does not depend on time for all values of m. Therefore, this function obeys the balance equation (2):

$$\left(\frac{dn_\alpha}{dt}\right)_{sources} + \left(\frac{dn_\alpha}{dt}\right)_{\alpha-\alpha\ gain} - \left(\frac{dn_\alpha}{dt}\right)_{\alpha-\alpha\ loss} - \left(\frac{dn_\alpha}{dt}\right)_{\alpha-\beta\ loss} = 0, \quad (3)$$

where the terms mean respectively direct dust production rate by the parent bodies, production rate of fragments by mutual collisions of α-meteoroids, loss rate of α-meteoroids due to collisions with other α-meteoroids, loss rate of α-meteoroids due to collisions with β-meteoroids.

The main sources of the dust material believed to act in the discs are the activity of comets and collisions between planetesimals (Weissman, 1984; Beust et al., 1989; Lagrange-Henri et al., 1989; Lecavelier des Etangs et al., 1996b). Both mechanisms are known to produce initially most of the mass in the form of large fragments. This is expected for both collisional fragmentation of planetesimals (as laboratory experiments on disruptive impacts suggest – see, e.g., Davis & Ryan, 1990) and activity of comets (e.g., Sykes et al., 1986). The material produced is then collisionally reprocessed and after a collisional cascade eventually becomes "dust". Now we leave aside the range of larger masses, to which the material is supplied by parent bodies directly, and confine ourselves to smaller α-meteoroids with masses $m_0 \le m \le m_{max}$. In our calculations, we adopt $m_{max} \sim 10^{-2}$ g, which corresponds to the radius ~ 1 mm. Note that the exact value of m_{max} is not particularly important, because it does not affect the modelling results and only determines the upper applicability limit of the size distribution derived from the calculations. In the mass regime $m_0 \le m \le m_{max}$, the first term in Eq. (3) can be discarded:

$$\left(\frac{dn_\alpha}{dt}\right)_{\alpha-\alpha\ gain} - \left(\frac{dn_\alpha}{dt}\right)_{\alpha-\alpha\ loss} - \left(\frac{dn_\alpha}{dt}\right)_{\alpha-\beta\ loss} = 0. \quad (4)$$

This equation will be used to find the number densities of α-meteoroids at various distances from the star.

2.4. Collisional production of β-meteoroids

Now we assume that α-meteoroids create β-meteoroids through their mutual collisional fragmentation. Here we confine our analysis to destructive collisions and neglect the erosive, i.e. cratering events which are less important for the results (Dohnanyi, 1969). The number of β-meteoroids with masses $[m, m+dm]$ produced per unit time in a unit volume inside the disc at a distance r is (cf. Dohnanyi, 1969)

$$\frac{dn^+}{dt}(r,m)\ dm = dm \iint_{m_t,m_p \in D(r)} K(r,m_p,m_t)\ G(m_p,m_t) m^{-\eta} \times n_\alpha(r,m_p) n_\alpha(r,m_t) \left[m_p^{1/3}+m_t^{1/3}\right]^2 dm_t dm_p. \quad (5)$$

Here,

$$K = \pi v_{imp} (4\pi\rho/3)^{-2/3}, \quad (6)$$

v_{imp} is the mean speed of a projectile grain with mass m_p with respect to a target grain of mass m_t. To compute the impact speed, a simple radiation pressure-driven kinematical model of Artymowicz & Clampin (1997) is used: $v_{imp} = v_{imp}(r,m_p,m_t) \approx \beta(m_p) v_k(r)$, where β is the radiation pressure to gravity ratio and $v_k(r)$ is the Keplerian speed at a distance r. Next, $G(m_p,m_t)m^{-\eta}$ is the mass distribution of collisional fragments of an impact between m_p and m_t. Plausible slopes are $\eta \sim 1.5$ to 2; see, e.g., Gault et al. (1963), Gault & Wedekind (1969), O'Keefe & Ahrens (1985) and discussion therein. The function G is taken to have the form

$$G(m_p,m_t) = (2-\eta)(m_p+m_t) m_x^{\eta-2}, \quad (7)$$

where m_x is the maximum mass of a collisional fragment (cf. Dohnanyi, 1969; Grün et al., 1985; Ishimoto, 2000). The integration domain D is $\{\max\{m_0, m_c(m)\} \le m_p \le m_t; m \le m_x\}$, where $m_c(m)$ is the minimum projectile mass that breaks up the target of mass m. The mass m_c also depends on the distance r, because so does the relative velocity of the projectiles with respect to the target. The quantities m_x and m_c are calculated with the formulae of Dohnanyi (1969) and Fujiwara et al. (1977) for basalt.

2.5. *Size and spatial distribution of β-meteoroids*

Next, we calculate the size and spatial distributions of the resulting β-meteoroids. To a reasonable accuracy we assume that β-meteoroids move radially outward from the star and, as stated earlier, that the disc is axisymmetric and vertically uniform, so that the number densities of both α- and β-meteoroids depend on the distance and not on the latitude and longitude. The expression for the number density of the β-meteoroids with a mass m at a distance r from the star then reads

$$n_\beta(r,m) = \frac{1}{r^2}\int_{r_{min}}^{r} r_0^2\ \frac{dn^+}{dt}(r_0,m)\ v^{-1}(r_0,r,m)\ dr_0. \quad (8)$$

Here, $v(r_0,r,m)$ is the velocity which a β-meteoroid born at a distance r_0 will develop at a distance r. Assuming that the initial speed of the β-meteoroid is the Keplerian speed of the disc rotation at the distance r_0, from the energy integral one obtains:

$$v(r_0,r,m) = \sqrt{GM_\star\left[\frac{2(1-\beta(m))}{r} + \frac{2\beta(m)-1}{r_0}\right]} \quad (\beta \ge 1/2), \quad (9)$$

$GM_\star$ being the gravitational parameter of the central star. The radiation pressure to gravity ratio $\beta(m)$ is taken as for the modelled "old cometary dust" of Wilck & Mann (1996), re-scaled from the Sun to β Pic with the aid of $\beta \propto L_\star/M_\star$, where $L_\star = 8.7 L_\odot$ and $M_\star = 1.75 M_\odot$ are the luminosity and mass of the star, respectively. The bulk density of this material is $\rho = 2.5\,\mathrm{g\,cm^{-3}}$.

2.6. *Lifetimes of α-meteoroids*

Important quantities are the lifetime of an α-meteoroid against destructive collisions with other α-meteoroids,

$$T_\alpha(r,m) = \left[\int_{\max\{m_0,m_c(m)\}}^{m_{max}} K(r,m_p,m)\ n(r,m_p)\left[m_p^{1/3}+m^{1/3}\right]^2 dm_p\right]^{-1}, \quad (10)$$

and the lifetime of an α-meteoroid against catastrophic impacts of β-meteoroids,

$$T_\beta(r,m) = \left[\int_{m_c(m)}^{m_0} K(r,m_p,m)\ n(r,m_p)\left[m_p^{1/3}+m^{1/3}\right]^2 dm_p\right]^{-1}. \quad (11)$$

In Eq. (10), K is given by Eq. (6). In Eq. (11), $K = \pi v_{\beta imp}(4\pi\rho/3)^{-2/3}$, where $v_{\beta imp}$ is the mean relative velocity between the α-meteoroids and the hyperbolic projectiles.

2.7. *Iterative solution of the equations*

The calculations are accomplished in two steps. In the first approximation, we neglect the losses of α-meteoroids due to impacts of hyperbolic particles and therefore solve Eq. (4) without the last term, and then find the distribution of β-meteoroids resulting from the mutual collisions of α-meteoroids from Eqs. (5)–(9). In the second approximation, we include the catastrophic fragmentation of α-meteoroids by β-meteoroids and, solving Eq. (4) simultaneously with the Eqs. (5)–(9) of production and distribution of β-meteoroids, construct a self-consistent model for both populations of dust in the disc.

3. Solution to the equations in the first approximation

3.1. *Distribution of α-meteoroids*

In the first approximation, we neglect the catastrophic breakup of α-meteoroids by β-meteoroids, so that Eq. (4) reduces to

$$\left(\frac{dn_\alpha}{dt}\right)_{\alpha-\alpha\ gain} = \left(\frac{dn_\alpha}{dt}\right)_{\alpha-\alpha\ loss}. \quad (12)$$

This equation was solved analytically by Dohnanyi (1969), who considered a closed collisional system exemplified by asteroids in the main belt. The solution is a power law

$$n_\alpha(r,m) = A(r) m^{-p} \quad (13)$$

with the exponent $p = 11/6$. To find the factor A, we assume that the number density of large grains is uniform vertically within the disc with the half-opening angle $\epsilon = 7°$, and make use of the radial profile of normal optical depth in the form (Artymowicz & Clampin, 1997)

$$\tau(r) = 2\tau_m[(r/r_m)^{-p_{inner}} + (r/r_m)^{p_{outer}}]^{-1}, \tag{14}$$

where $\tau_m \equiv \tau(r_m)$ is the maximum normal optical depth reached at the distance r_m (60 AU for β Pic), while p_{inner} and p_{outer} describe the radial slopes in the inner ($r \ll r_m$) and outer ($r \gg r_m$) parts of the disc. Then, provided that $p \neq 5/3$,

$$A(r) = \frac{(4\pi\rho/3)^{2/3}|p - 5/3|}{2\pi Q_{ext}\sin\epsilon \left|m_0^{5/3-p} - m_{max}^{5/3-p}\right|} \, \frac{\tau(r)}{r}, \tag{15}$$

where $\tau(r)$ is given by Eq. (14), $Q_{ext} \approx 2$ is the extinction efficiency, and m_{max} is the maximum mass of α-meteoroids considered. For $p = 5/3$, a logarithmic dependence results.

For the distribution of α-meteoroids given by Eqs. (13)–(15), the density of β-meteoroids is computed with the aid of Eqs. (8), (5), and (9).

3.2. Results

The model contains a number of parameters, such as: the maximum normal optical depth of the disc, τ_m; the slopes of the optical depth in the inner and outer parts of the disc, p_{inner} and p_{outer}; the power-law index of the mass distribution of the fragments generated in a destructive collision, η; the exponent of the mass distribution of large grains assumed to be in collisional equilibrium, p. As a "standard" set of values for β Pic, we take:

$$\begin{aligned} &\tau_m = 10^{-2}, \\ &p_{inner} = 2.0, \quad p_{outer} = 2.0, \\ &\eta = 11/6, \quad p = 11/6. \end{aligned} \tag{16}$$

These values will be justified later, in Sect. 4. Relevant ranges for these parameters, as well as dependence of the modelling results on their values, will also be discussed there.

For the standard model (16), the computed size distribution at various distances from the star is depicted in Fig. 2. The basic result is that the size distribution of larger grains cannot be simply extrapolated to smaller sizes: the grains with sizes just below the blow-out limit are typically somewhat (but not strongly) depleted, and the slope of their size distribution may differ from that of the larger grains. The contribution of smaller particles grows with the distance from the star. This result is readily understandable: β-meteoroids are blown radially away from the star, thus the β-meteoroids produced at a certain distance from the star contribute to the number density at all larger distances.

In Fig. 3 we show the lifetimes of α-meteoroids against mutual collisions (Eq. 10) and against catastrophic impacts of β-meteoroids (Eq. 11). For the "typical" disc grains (several micrometres in radius), the former are on the order of ten orbital periods at 60 AU (where τ peaks) and increase closer to and farther out from the star, in agreement with Artymowicz (1997).

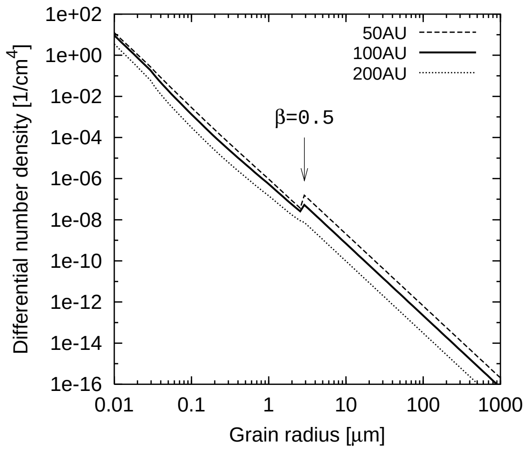

Fig. 2. Size distribution of dust in the β Pic disc at different distances from the star. Shown is the solution in the first approximation, in which the destruction of α-meteoroids by β-meteoroids is ignored.

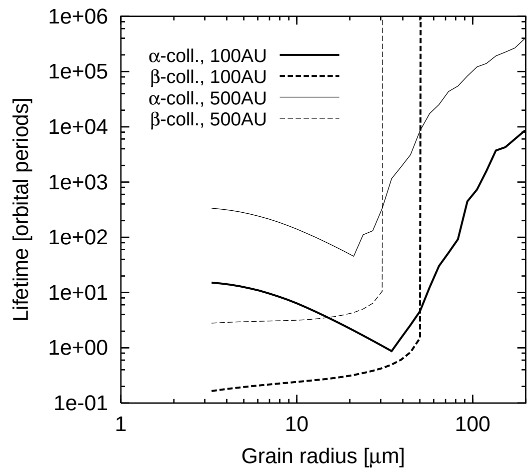

Fig. 3. Lifetimes of α-meteoroids against mutual collisions (bold lines) and catastrophic impacts of β-meteoroids (dashed lines). Shown is the solution in the first approximation.

Interestingly, the lifetimes reach the minimum, which may be as short as one orbital period, for grains several tens of micrometres in size. This can easily be explained by the size dependence of the number density and collisional cross section.

A quite important result is that the collisional lifetimes of α-meteoroids against catastrophic impacts of β-meteoroids turn out to be much shorter. Fig. 3 particularly shows that at 100 AU α-meteoroids with radii between 3 and $50\,\mu$m are broken up by impacts of β-meteoroids typically before they complete one revolution about the star. (Larger grains can only be eroded but not destroyed by hyperbolic grains, which explains why the lifetimes tend to infinity at $\approx 50\,\mu$m.) This should affect the size distribution of α-meteoroids in the zone adjacent to the blow-out limit. It necessitates constructing a self-consistent model for distributions of both populations of particles, which we accomplish in the next section. Yet before we make necessary calculations, we can argue that the effect described above would lead to a change of the slope p in (13) for the smallest α-meteoroids

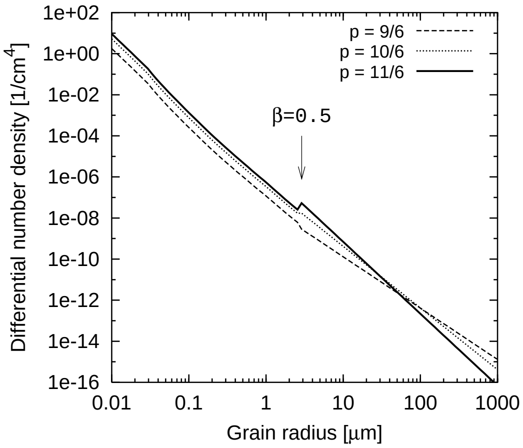

Fig. 4. Size distribution at 100 AU for three different exponents p. Shown is the solution in the first approximation.

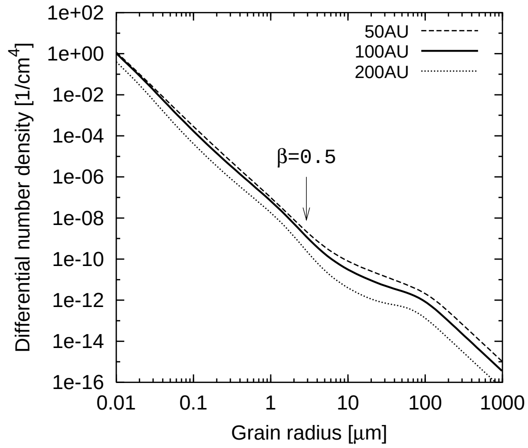

Fig. 5. The same as in Fig. 2, but in the second approximation. Instead of showing the "raw" outcome of the model calculations, in this and subsequent figures we plot curves after smoothening to remove notches, which are artefacts of the calculation technique. Notches like the ones seen in Fig. 2 or 4 arise near the boundary between the α- and β-meteoroids; others appear near the boundary between those α-meteoroids that can be destroyed by the largest β-meteoroids and those that can only be eroded by the hyperbolic projectiles.

with respect to its value for larger grains. This rises a question, whether or not such a change would lead, in turn, to a substantial change in the population of β-meteoroids. The answer is rather negative. We have made test calculations of the size distribution of β-meteoroids for several values of the slope p in Eq. (13) (Fig. 4). The results suggest a weak dependence of $n_\beta(r,m)$ on the exponent p. This helps converging the iterations described in the next section.

4. Solution to the equations in the second approximation

4.1. Collisional balance of α- and β-meteoroids

Now we should put back the last term in Eq. (4). Let us re-denote the number density of α-meteoroids (13), found in the first approximation, by $n_0(r,m)$. Recall that this function satisfies Eq. (12):

$$\left(\frac{dn_0}{dt}\right)_{\alpha-\alpha\ gain} = \left(\frac{dn_0}{dt}\right)_{\alpha-\alpha\ loss}. \tag{17}$$

On the other hand, the "actual" number density $n(r,m)$ that we seek should satisfy Eq. (4):

$$\left(\frac{dn_\alpha}{dt}\right)_{\alpha-\alpha\ gain} = \left(\frac{dn_\alpha}{dt}\right)_{\alpha-\alpha\ loss} + \left(\frac{dn_\alpha}{dt}\right)_{\alpha-\beta\ loss}. \tag{18}$$

Note that the left-hand side terms in Eqs. (17) and (18) are approximately equal, because the collisional gain of the grains in the considered size range results from the destruction of much larger α-meteoroids, which stay immune to impacts of small hyperbolic particles. Consequently, equating the left-hand sides of Eqs. (17) and (18) gives

$$\left(\frac{dn_0}{dt}\right)_{\alpha-\alpha\ loss} = \left(\frac{dn_\alpha}{dt}\right)_{\alpha-\alpha\ loss} + \left(\frac{dn_\alpha}{dt}\right)_{\alpha-\beta\ loss} \tag{19}$$

or, in the expanded form (cf. Dohnanyi, 1969),

$$\begin{aligned} & n_0(r,m) \int_{\max\{m_0, m_c(m)\}}^{m_{max}} K(r,m_p,m)\ n_0(r,m_p) \\ & \quad \times \left[m_p^{1/3} + m^{1/3}\right]^2 dm_p \\ & = n_\alpha(r,m) \int_{\max\{m_0, m_c(m)\}}^{m_{max}} K(r,m_p,m)\ n_\alpha(r,m_p) \\ & \quad \times \left[m_p^{1/3} + m^{1/3}\right]^2 dm_p \\ & + n_\alpha(r,m) \int_{m_c(m)}^{m_0} K(r,m_p,m)\ n_\beta(r,m_p) \\ & \quad \times \left[m_p^{1/3} + m^{1/3}\right]^2 dm_p. \end{aligned} \tag{20}$$

The integration limits in the first and second terms of the right-hand side correspond to the mass ranges of α- and β-meteoroids, respectively. Accordingly, the integrands and integration limits – functions $K(r,m_p,m)$ and $m_c(m)$ – are calculated with the impact velocities v_{imp} and $v_{\beta imp}$, respectively.

Non-linear integral equation (20) is solved iteratively with respect to $n_\alpha(r,m)$. We therefore get the number density of α-meteoroids that allows for both their mutual collisions and their collisions with β-meteoroids. However, the distribution of the latter was obtained in the first approximation. Thus, when a new density $n_\alpha(r,m)$ is found, we re-compute $n_\beta(r,m)$ with the aid of Eq. (8). After that we re-calculate $n_\alpha(r,m)$ by iterative solution of (20) again, and so on. This double-iterative procedure finally gives us the distributions of both α- and β-meteoroids which are in a collisional balance with each other.

4.2. Results

The size distribution at several distances from the star, computed in the second approximation, is depicted in Fig. 5. Comparing

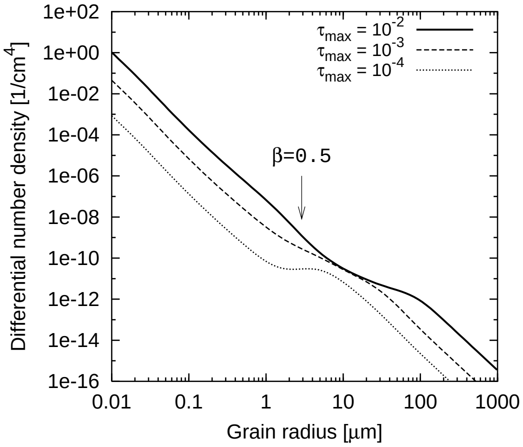

Fig. 6. Size distribution at 100 AU for different values of the maximum optical depth τ_m.

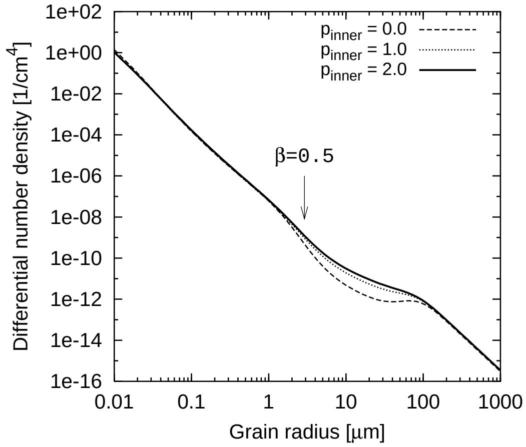

Fig. 7. Dependence of the size distribution at 100 AU on the inner radial slope p_{inner}.

the results with those found in the first approximation (Fig. 2), we see that the distribution of α-meteoroids in the zone adjacent to the blow-out limit slopes more gently. This displaces the sizes of cross section-dominating grains from several micrometres to several tens of micrometres. Interestingly, one general feature of the overall size distribution remains unchanged: small particles, especially β-meteoroids, are by about two orders of magnitude depleted with respect to the distribution of large grains extrapolated to smaller sizes. Another "stable" feature of the distribution is that the relative contribution of smaller particles grows with the distance from the star.

For the distance of 100 AU, Fig. 6 illustrates the dependence of the resulting size distribution on τ_m — in fact, on the disc dustiness. Actual values of τ_m given by different authors differ markedly – from about 5×10^{-3} (Burrows et al., 1995) or 7.6×10^{-3} (Artymowicz & Clampin, 1997) to $\sim (1 \text{ to } 2) \times 10^{-2}$ Pantin et al. (1997). This is particularly explained by different definitions of τ adopted by different workers. Our definition of τ_m (the extinction efficiency factor multiplied by the cross section area of grains along the line of sight) is the same as that of (Pantin et al., 1997), which justifies using the value 10^{-2} as a standard one (Eq. 16). Another motivation for us to vary the optical depth is applications to discs of some other stars, which are typically less dense than β Pic. For instance, 55 Cnc, ϵ Eri and α PsA, being by about one order of magnitude less dusty than β Pic (see Krivova, 2000 and references therein), have $\tau_m \sim 10^{-3}$. Fig. 6 shows the response of the size distribution to a change in τ_m. For $\tau_m = 10^{-3}$, the distribution is smoother (closer to a single power law) than for $\tau_m = 10^{-2}$, because the hyperbolic particles are less abundant and hence less efficient in depleting the population of micrometre-sized α-meteoroids. For yet smaller $\tau_m = 10^{-4}$ (closer to the disc of Vega), the hyperbolic grains are nearly incapable of reducing the density of α-meteoroids. Thus the curve, depressed at the lower size end, describes the almost intact population of α-meteoroids and the population of β-meteoroids in equilibrium between their collisional production and radiation pressure blow-out.

The computed size distribution at 100 AU for several choices of other model parameters is shown in Figs. 7–10. The bold line in each figure corresponds to the standard model (16) – the same as shown in Fig. 5. It should be noted that for a particular system, β Pic in our case, some of these parameters are not independent. For example, modifying the slopes p_{inner} or p_{outer} requires changes in τ_m – to keep the observed luminosity of the disc. However, we choose to vary the parameters separately. There are two reasons for that. One is that we would like to see clearly the role of each parameter in the model. Another one is possible applications to discs of stars other than β Pic, which vary in optical depth and steepness of the radial distribution of dust.

Fig. 7 depicts the size distribution at 100 AU for several choices of the slope of the optical depth's radial distribution in the inner zone, p_{inner}. The reason for us to vary it is that some authors report a less pronounced inner depletion zone with p_{inner} down to zero, which corresponds to $n \propto r^{-1}$ (Golimowski et al., 1993; Kalas & Jewitt, 1995; Mouillet et al., 1997; Heap et al., 2000). A flatter distribution in the inner part of the disc leads to a more pronounced depletion of grains above the blow-out limit, which is because in this case more β-meteoroids are generated close to the star, and the destruction of α-meteoroids by them is more intensive.

Fig. 8 shows the dependence of the size distribution at two distances – 100 and 500 AU – on the outer slope p_{outer}. Smith & Terrile (1984) found $p_{outer} = 2.3$. Later on, values in the range from 1.5 to 2.2 were reported by many authors (Artymowicz et al., 1989; Golimowski et al., 1993; Lecavelier des Etangs et al., 1993; Kalas & Jewitt, 1995; Mouillet et al., 1997). The most recent results suggest a steeper radial dependence, $p_{outer} \approx 2.8$ to 3.5 (Heap et al., 2000). Radial distribution that steep is also typical of many other discs of Vega-type stars (see, e.g., Krivova 2000 for a review). As is seen from Fig. 8, a steeper radial distribution leads to a more pronounced dip in the size range from micrometres to tens of micrometres farther out from the star. This can easily be explained. Since in these runs we keep τ_m and p_{inner} unchanged and vary p_{outer} only, the amount of β-meteoroids

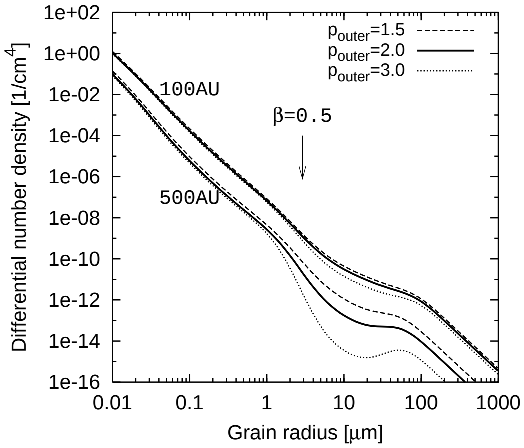

Fig. 8. Dependence of the size distribution on the outer radial slope p_{outer}.

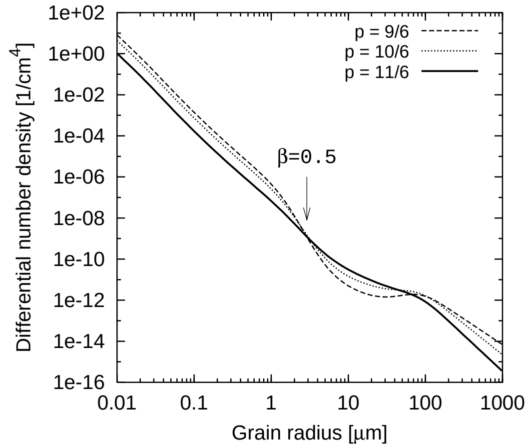

Fig. 10. Dependence of the size distribution at 100 AU on the parameter p (Eq. 13).

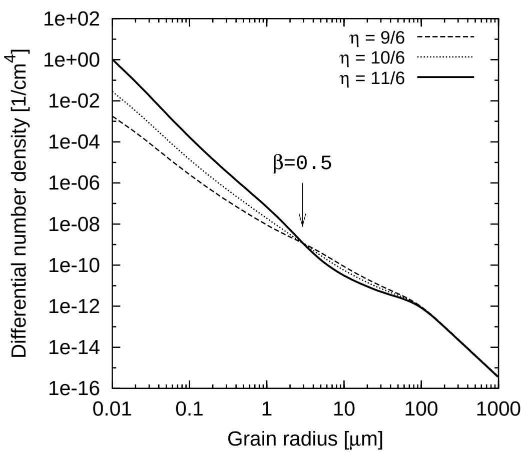

Fig. 9. Dependence of the size distribution at 100 AU on the slope of the fragment mass distribution η.

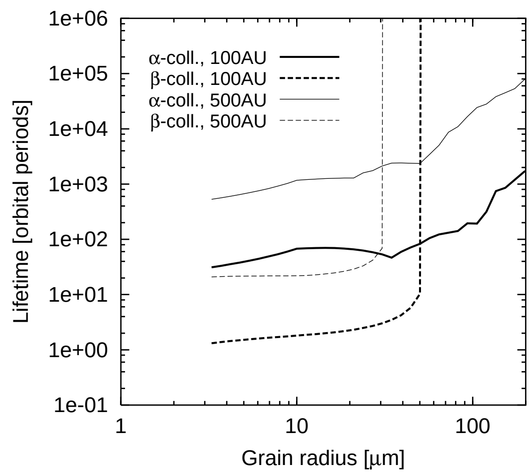

Fig. 11. The same as in Fig. 3, but in the second approximation.

that are produced mostly in the inner parts of the discs remains nearly the same. When p_{outer} is larger, the densities of α-meteoroids far from the star are lower, and they are more efficiently destroyed by collisions of β-meteoroids.

Figs. 9 and 10 show the dependence of the size distribution on the exponent of the size distribution of a single impact, η (Eq. 5) and on the exponent of the mass distribution of large grains, p (Eq. 13), respectively. As noted above, the former is known from the impact experiments with a large uncertainty, while the latter comes from the modelling (Dohnanyi, 1969) and cannot be constrained from observations. As one would expect, η largely determines the slope of the size distribution of small particles, β-meteoroids. On the contrary, p determines the shape of the distribution of larger grains, α-meteoroids.

In Fig. 11, the lifetimes of α-meteoroids are shown. It is clearly seen that when the collisional balance is reached, the grains are mostly destroyed by impacts of hyperbolic particles. For all sizes in the range from ~ 3 to several tens of micrometres, the lifetimes against collisions with β-meteoroids are nearly by two orders of magnitude shorter that those against mutual collisions between α-meteoroids. The former range from several to several tens of orbital periods of dust grains around the star, depending on the distance.

5. Conclusions and discussion

In this paper we have constructed a model for the size distribution of dust in a "collision-dominated" dust disc, in which the collisional lifetimes of grains are much shorter than their Poynting-Robertson time scales. This is the case for a number of circumstellar discs of main-sequence and old pre-main-sequence stars currently observed, including the β Pic system.

5.1. The disc of β Pic

The main result of this paper is the derivation of a general shape of the size distribution in the β Pic disc. We have shown that the size distribution is continuous and broad, extending down to tiny submicrometre-sized particles. *The overall distribution shows essentially three different slopes:* steeper ones for both small and large grains and a gentler one for intermediate-sized particles from a few micrometres to a few tens of micrometre in size (Fig. 5). This resembles the size distribution of interplanetary particles in the Solar system (cf. Fig. 1) which, however, is shaped by different mechanisms.

An important result is that the disc sustains *a considerable population of "small" grains below the radiation blow-out limit*. The presence of a substantial amount of grains not larger than a few micrometres in size was expected from the IR data, including the presence and the profile of the silicate emission at $10\,\mu$m (Telesco et al., 1988; Telesco & Knacke, 1991; Backman et al., 1992; Knacke et al., 1993; Aitken et al., 1993; Li & Greenberg, 1998), as well as from the analysis of the polarimetric and colorimetric data (Krivova et al., 2000a). For all plausible grain models, the particles of such sizes must be streaming out through the disc in hyperbolic trajectories, but they are continuously replenished by collisions of larger grains. The steady-state size distribution of hyperbolic grains deviates from the distribution of large grains (i.e., those which move in bound orbits), extrapolated to smaller sizes. On the one hand, the depletion, by two orders of magnitude on the average, is not as large as one might expect from the fact that small grains continuously leave the disc in hyperbolic orbits. Consequently, describing the overall distribution by a power law with a single exponent (which should be lower than 3.5) may be a rough first approximation. On the other hand, such a description is too crude to be used for fitting of the observational data. Krivova et al. (2000a) have made calculations of the observed polarization and colours, using the size distribution computed on the base of a simpler dynamical model than the one presented here (not taking into account the breakup of α-meteoroids by impact of β-meteoroids). And indeed, the computed overall size distribution agrees with the polarimetric and colorimetric observations much better than the commonly adopted power law with a single exponent over the whole range of sizes.

Another conclusion of the paper is that the amount of hyperbolic particles – β-meteoroids – in the β Pic disc is so large that breakups of α-meteoroids not only by mutual collisions, but also by β-meteoroids become significant. The lifetimes of α-meteoroids with sizes just above the blow-out limit are determined by catastrophic collisions of β-meteoroids, rather than of other α-meteoroids. It *flattens the size distribution of α-meteoroids in the size regime from the blow-out limit ($\sim 3\,\mu$m for β Pic) up to tens of micrometres*. As a consequence, *the cross section-dominating size shifts from several to tens of micrometres*. This result falls in agreement with the observations as well: for instance, the grains of up to $20\,\mu$m are required to explain the mid-IR and visual images (Artymowicz et al., 1989). It also agrees with the polarimetric data (Krivova et al., 2000b).

Of course, our model calculations were made under a number of simplifying assumptions, of which at least two deserve a special discussion. One is the assumption that the disc grains are moving in low-eccentricity orbits. This, in turn, implies that (i) parent bodies (comets, planetesimals) are on nearly-circular orbits and (ii) the radiation pressure does not induce appreciable eccentricities on the daughter grains. While the former can be true, at least for planetesimals or comets on low-eccentric orbits (orbiting-evaporating bodies, OEBs, see Lecavelier des Etangs et al., 1996b), the latter holds true only for larger grains and is certainly a poor assumption for grains adjacent to the blow-out limit. Indeed, a grain produced by a parent body moving in a circular orbit acquires an eccentricity $e \approx \beta/(1-\beta)$ (e.g., Burns et al. 1979; Artymowicz & Clampin, 1997). Therefore, α-meteoroids several micrometres in size are likely to have quite eccentric orbits. Although there is no easy way to incorporate non-zero eccentricities into the present model, we can speculate that larger eccentricities would allow such grains to spend much time at larger distances, where their removal by β-meteoroids is significantly decreased, and simultaneously the production of β-meteoroids by these α-meteoroids is decreased. As a result, the "dip" in the size distribution at $a \gtrsim a_0$ could be less pronounced than is predicted by our model.

Another assumption is that we neglected the direct injection of material by the source bodies into the size range covered by our model ($a \lesssim 1\,$mm) and assumed that all grains are supplied in this range indirectly, by collisional cascade of larger aggregates, which is not always true (Lecavelier des Etangs et al., 1996b). Mathematically speaking, it means that we omitted the first term in Eq. (3). Furthermore, we neglected the fact that comets in very eccentric orbits (falling-evaporating bodies, FEBs) can produce most of the material in the form of β-meteoroids, no matter what the size distribution of generated dust is (Lagrange et al., 1989; Ferlet et al., 1987; Vidal-Madjar et al., 1994). These effects can still be treated with the method presented here by solving Eq. (3) with a properly written first term. However, this would require a separate modelling to describe the source functions and would increase drastically the computational difficulties. Thus we leave this task for future studies.

Although the disc of β Pic was treated as axisymmetric throughout the paper, our results on the population of small grains, β-meteoroids, may help explaining the global difference between two wings observed in the disc (e.g., Kalas & Jewitt, 1995). While some other asymmetries, namely local warps in the brightness distribution, are usually attributed to the presence of a planet orbiting the star at several tens of astronomical units, it is not yet clear whether the global asymmetry in the brightness of the SW- and NE-wings is physically connected with these local warps. One conceivable hypothesis is that the difference of the wings is caused by the ISM bombardment of the disc grains that produces fine collisional debris in the upstream wind, presumably NE one (Krivova et al., 2000a). Another possibility is that the global difference is physically connected with the inner warps, and in turn, are due to the gravity of yet unknown planet(s). Lecavelier des Etangs (1998), for instance, pointed out that already a moderate orbital eccentricity of an alleged planet would produce alignment of periastrons of the dust sources (comets, planetesimals), causing, in turn, a noticeable global asymmetry between the wings of the dust disc. Not ruling out this scenario, we could suggest yet another possible mechanism. The planet creates arcs of particles trapped in low-order mean-motion resonances (Scholl et al., 1993; Roques et al., 1994; Lazzaro et al., 1994); the same features caused by Neptune have been predicted for the EKB dust in our Solar system (Liou & Zook, 1999). Lecavelier des Etangs et al. (1996a) have shown that for the actual optical depth of the β Pic disc, mutual collisions still do not destroy the zones of enhanced dust density. Because the number density of grains in the resonant features is higher than outside, whereas the relative velocities of the grains are nearly the same as elsewhere in the disc, we can expect that collisions are more frequent there. This should lead to an enhanced production of β-meteoroids in the "resonant" regions. Furthermore, larger bodies in the disc may also concentrate there, like plutinos in the Solar system, further increasing the dust concentration there through mutual collisions (with the consequences just described) and also producing β-meteoroids directly. Since tiny grains are streaming outward from their birthplace, they should be over-abundant in the outer parts of the disc, adjacent to the locations of the resonant "clusters". Krivova et al. (2000a) have shown that 20% to 30% of extra small grains would be sufficient to account for the observed difference in both brightness and polarization of the two wings. From our Eqs. (5) and (8), this can be insured by just a 10% to 15% number density excess in the resonant swarms with respect to the "smooth" disc. Although this idea still needs to be verified by dedicated simulations, this seems to be another possible physical link between a presumed planet and the global asymmetry between the two sides of the disc. Besides, it is, in principle, testable observationally. Obviously, the resonant clumps of dust and associated streams of β-meteoroids should rotate with the planet. For a planet at 50 AU from the star, the orbital period is about 300 years, so that in about ten to several tens of years the observed asymmetry should start to change. This time span is, however, still less than the actual time interval, on which the system is observed with sufficient resolution and sensitivity.

5.2. The discs of other stars

Although particular calculations were made in this paper for the β Pic disc, our basic findings should well apply to the debris discs around some other Vega-type stars – namely, to those which are gas-poor and have similar or somewhat lower optical depths. The absence of a marked gaseous component is required to keep high relative velocities of the disc particles and therefore to maintain the dominant role of catastrophic collisions. Besides, the gaseous component would influence the dynamics of the hyperbolic grains, modifying their spatial distribution and, for the discs where such grains are very abundant (such as for instance BD+31°643), producing pronounced effects in the observed brightness (Lecavelier des Etangs et al., 1998). As far as the optical depths are concerned, they must be above a certain level, on the order of 10^{-4}, to ensure that the collisional lifetimes are much shorter than the P-R time scales. The upper limit for the optical depth is less certain. However, as noted by Artymowicz (1997), the maximum dustiness of gas-poor discs cannot be much higher than β Pic's, because it is self-limited by the dust avalanche processes. Dust avalanches imply that the catastrophic collisions of β-meteoroids with α-meteoroids produce new β-meteoroids which, in turn, break up other α-meteoroids, and so on. This mechanism is able to clear up denser discs, but may already be of importance for β Pic. Although the collisional model presented in this paper does not include avalanches, we argue that this process would not change the gross features of the dust distribution drastically. Indeed, the β Pic disc represents a typical self-balanced system: a depletion of the α-meteoroid population caused by streams of β-meteoroids would lead to a decreased production of the latter, resulting in the enhanced lifetimes of the α-meteoroids and restoring their population, and so on. Still, generalizing the model to allow for the avalanching process is a challenging task for the future studies.

Which of our conclusions could be expected to hold for other moderately dusty, gas-poor discs? We argue that the general shape of the size distribution is preserved. What only changes from one star to another (and is strongly affected by model assumptions) is the particular slopes featured in the size distribution and the location of the "transitional" sizes between different dust populations. Indeed, though the presence of large grains ($\gtrsim 5$–$20\,\mu$m) is inferred by observational data for many stars (see, e.g., Chini et al., 1990; Backman & Paresce, 1993; Zuckerman & Becklin, 1993; Greaves et al., 1998; Jayawardhana et al., 2000), some evidence of smaller grains, with sizes below the blow-out limits, also exists (e.g., Skinner et al., 1992; 1995; Fajardo-Acosta et al., 1993; Sylvester et al., 1996; Sylvester & Skinner, 1996). This conclusion has particularly been drawn for HR4796A and HD141569 – interesting and intensively studied objects with the disc fractional luminosity similar to that of β Pic (Augereau et al., 1999; Wyatt et al., 1999; Telesco et al., 2000; Fisher et al., 2000). Directions of future work may include specific calculations of dynamically-expected size distributions for several stars, for which radial profiles of optical depths have been retrieved from observations.

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