Solar Turbulence
What is it? Why do we care? And what can we do about it?

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What is turbulence?

- Nonlinear
- Chaotic
- Large range of interacting scales
Nonlinearity & the Reynolds number

Incompressible MHD equations

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho_0} \nabla P + \mathbf{j} \times \mathbf{b} + \mathbf{F} + \nu \nabla^2 \mathbf{v}$$

$$\partial_t \mathbf{b} + \mathbf{v} \cdot \nabla \mathbf{b} = \mathbf{b} \cdot \nabla \mathbf{v} + \eta \nabla^2 \mathbf{b}$$

$$\nabla \cdot \mathbf{v} = 0, \quad \mathbf{\omega} = \nabla \times \mathbf{v}, \quad \nabla \cdot \mathbf{b} = 0, \quad \mathbf{j} = \nabla \times \mathbf{b}$$

$$\mathbf{b} = \mathbf{B}/\sqrt{\mu_0 \rho_0}, \quad \partial_t \rho = 0$$

Reynolds numbers

$$\frac{[V]^2[L]^{-1}}{\nu[L]^2[V]} \sim Re \equiv \frac{v_{r.m.s.} L}{\nu}$$

$$Re_M \equiv \frac{v_{r.m.s.} L}{\eta}$$
Cascade to small scales

e.g. $\partial_t \nu + \nu \partial_x \nu = \nu \partial_{xx} \nu$

[Graphs showing speed vs. position for different times: 0 minutes, 1.25 minutes, and 2.5 minutes]
Kolmogorov picture of the “direct” cascade (K41 theory)

**Assumptions**
- spectral locality
  - no forcing
  - no dissipation
  - → “inertial” range
- homogeneity
- isotropy
- self-similarity
  → $k^{-5/3}$ spectrum
Nonlinear
Large range of scales
Further difficulties

Kolmogorov picture of the “direct” cascade (K41 theory)

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MHD problems
- 2 time scales: turn-over time, Alfvén time
- $k^{-3/2}$ Iroshnikov-Kraichnan (IK theory)
- $k^{-5/3}$ again (Goldreich & Sridhar 1995)
- Which spectrum anyhow?
Incomplete picture

- Self-organization: “inverse cascade”
  - Quasi 2D nonconducting fluid - inverse cascade of energy
  - MHD - inverse cascade of magnetic helicity, $\int a \cdot b \, dV$
- Violations of Kolmogorov’s assumptions
  - Nonlocal transfers (Alekaxis, Mininni, & Pouquet, 2005, 2006)
  - Intermittency
**Parameterized turbulence**

- Random stretching $\rightarrow$ small-scale dynamo (Batchelor 1950)
- Stratification & rotation $\rightarrow$ large-scale dynamo ($\alpha$-effect)
- Interaction of rising flux tubes with turbulence
Other effects

- Does intermittency & nano-flares play a role in coronal heating? (Parker 1994; Galtier & Pouquet 1998)
- Did intermittency play a role in the Maunder minimum? (Charbonneau 2001)
- What more happens at very high Reynolds numbers? (Mininni et al 2006)
But we can’t simulate solar turbulence

K41: \( \text{dof} \propto \text{Re}^{9/4} \)

- Photospheric
  \( \eta \sim 10^8 \text{cm}^2 \text{s}^{-1} \), supergranules: \( L \sim 30 \text{Mm} \), \( v \sim 1 \text{km/s} \)
  \( \text{Re}_M = \frac{vL}{\eta} \sim 3 \cdot 10^6 \), \( l_\eta \sim 100 \text{m} \rightarrow 300,000^3 \) simulation
  4096\(^3\) Earth Simulator (Kaneda et al 2003) \( \rightarrow \) year 2040 to resolve magnetic field
  \( P_M = \frac{v}{\eta} \sim 10^{-5} \), \( \text{Re} \sim 10^{11} \), \( l_\nu \sim \text{cm} \rightarrow \text{year 2080} \) to resolve velocity field

- Chromospheric
  \( \eta \sim 2 \cdot 10^7 \text{cm}^2 \text{s}^{-1} \), \( L \sim 5 \text{Mm} \), \( v \sim 8 \text{km/s} \)
  \( \text{Re}_M \sim 2 \cdot 10^7 \rightarrow \text{year 2055} \) (magnetic)

- Coronal
  \( \eta \sim m^2 \text{s}^{-1} \), \( L \sim 700 \text{Mm} \), \( v \sim 100 \text{km/s} \)
  \( \text{Re}_M \sim 7 \cdot 10^{13} \rightarrow \text{year 2110} \) (magnetic)
What can we do about it?

Modeling

- Implicit
  - Dissipative numerical techniques
  - Moderate $Re$ models high $Re$?
- Explicit
  - Reynolds averaging
  - Large Eddy Simulations (LES)
- Combinations?
LES
Modeling the effect of unresolved scales

What is it?
Why do we care?
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Modeling - LES
Lagrangian-averaged $\alpha$ model
Validation MHD — $\alpha$: 2D MHD

$$L : \mathbf{z} \rightarrow \tilde{\mathbf{z}}$$
$$\partial_j \tau_{ij} = \partial_j \left( \bar{v}_i \bar{v}_j - \bar{v}_i \bar{v}_j \right)$$

Advantage
- Reduced resolution → reduced cost

Challenges
- “Back scatter” from unresolved scales
- Nonlocal interactions with unresolved scales
No general LES for MHD

- Dissipative LES (Theobald et al 1994)
  - Ignore sub-filter scale energy exchanges
  - Assumes energy spectra of non-conserved quantities
- Dissipative LES (Zhou et al 2002)
  - non-helical, stationary MHD
  - $k^{-5/3}$ and fixed ratio of energies
- Cross-helicity model (Müller & Carati 2002)
  - Assumes alignment between the fields
  - Reduced intermittency
- Low $Re_M$ LES (Ponty et al 2004)
- Hyper-resistivity (not LES - Haugen & Brandenburg 2006)
  - Requires recalibration of length scales to known DNS
Lagrangian-averaged $\alpha$ modeling

**Advantages**

- Conservation of energy, helicities, etc. ($H^1_\alpha$ not $L^2$)
- Preserves Alfvén's theorem
- No assumptions on inertial range spectrum

(Andrews & McIntyre 1978)
Model Validation

**Validation Steps**
- 2D MHD
- 3D NS
- 3D MHD
- Compressibility
- Solar applications...

**2D MHD approximation**

Solar corona
\( \beta < 1 \)

Granulation

Solar photosphere
\( \beta > 1 \)

\( L_0 \sim 10^7 m \)
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2D MHD Free Decay

Energy

\[ v_x(x, 0) = \sum_{\sqrt{k_x^2 + k_y^2} \leq \alpha} \sin(k_x x + \phi_{k_x}) \sin(k_y y + \phi_{k_y}) \]

2048^2 DNS \( Re \approx 10^4 \) 10 min 32 POWER4 1.3GHz

1024^2 MHD\(-\alpha\)

512^2 MHD\(-\alpha\) 1/16 computational cost
Does what a subgrid model should do

**DNS $2048^2$**

- $\Psi$

- $a_z$

**$\alpha^{512^2}$**

- $\Psi$

- $a_z$
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Modeling - LES
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Validation MHD — $\alpha$: 2D MHD

$\text{DNS}_{2048^2}$

$\omega$

$j$

$\alpha_{512^2}$
Captures spectra
Without scaling assumption
Measuring arbitrarily small oscillations

**Cancellation exponent**

\[ \mu_i(l) = \frac{\int_{Q_i(l)} d\mathbf{x} j_z(x)}{\int_{Q(L)} d\mathbf{x} |j_z(x)|} \]

\[ \chi(l) = \sum Q_i(l) |\mu_i(l)| \]

\[ \chi(l) \sim l^{-\kappa} \]

\[ \kappa = (d - D)/2 \]

Prelude to flares
(Sorriso-Valvo et al 2004)
Captures time evolution of cancellation
Completed studies

- MHD–$\alpha$ reproduces large scale behavior (Mininni et al 2005).
- Reproduces scalings & intermittency of MHD $\rightarrow$ can model phenomena with strong impulsive events (Pietarila Graham et al 2005, 2006).
- Has been applied to incompressible, simple geometry small scale dynamo (Ponty et al 2005).
- $N/4 \rightarrow$ factor of 256 for 3D = 12 years!!
Problems with Navier-Stokes $- \alpha$

![Graph showing $E(k)$ vs $k$ with lines $k'$ and $k^{-1}$]
But MHD$-\alpha$ looks OK
Open questions

- Combination models: $\alpha +$ dissipative model?
- MHD$-\alpha$: how small $b$ to suppress rigid bodies?
- Derive compressible MHD$-\alpha$
Incompressible MHD$-\alpha$ model
(Holm 2002)

\[ \partial_t \mathbf{v} + \mathbf{\omega} \times \mathbf{v} + \nabla \pi - \mathbf{j} \times \mathbf{b} = \nu \nabla^2 \mathbf{v} \]

\[ \partial_t \mathbf{b} + \mathbf{\bar{v}} \cdot \nabla \mathbf{b} - \mathbf{b} \cdot \nabla \mathbf{\bar{v}} = \eta \nabla^2 \mathbf{b} \]

\[ \nabla \cdot \mathbf{\bar{v}} = 0, \quad \nabla \cdot \mathbf{b} = 0 \]

\[ \mathbf{v} = (1 - \alpha^2 \nabla^2)\mathbf{\bar{v}}, \]

\[ \mathbf{b} = (1 - \alpha_M^2 \nabla^2)\mathbf{\bar{b}} \]