A mechanism for the dependence of sunspot group tilt angles on the strength of the solar cycle

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The magnetically variable Sun

The 11-year solar cycle

Solar magnetic activity varies with a period of roughly 11 years. Long-term variations are superposed upon this cycle.

“Maunder Minimum”

Graph showing solar activity from 1600 to 2000.
Tilt angle: why important?

- Latitudinal separation of +/-
- Magnetic flux crossing the equator
- Polar fields (N) — toroidal field (N+1)

Jiang et al. (2010)
Tilt angles change with cycles.

- Anti-correlation observed
- Self-sustained mechanism?

**Fig. 1.** Corrected Fig. 4a of Dasi-Espuig et al. (2010). Cycle averaged tilt angle normalised by the mean latitude vs. the strength of the same cycle based on MW data ($r_c = -0.79$). The error bars represent 1σ error and the dashed line a linear fit to the points. Cycles 15 and 21 are shown as squares with dashed lines for the error bars because of their incompleteness.
Inflows towards active regions/belts

- Gizon et al. (2001)
- Gonzalez-Hernandez et al. (2008)
- Zhao et al. (2014)
- ...

Possibly related to enhanced cooling

Could it suppress the effective tilt angle?

Sun et al. 2015
Effect of the meridional component

- **BMR simulations**: limits on the generation of polar magnetic flux
  - Jiang, Işık, Cameron, Schmitt, Schüssler (2010)
- **Cycle simulations**: improved correlations in SFT models
  - Cameron & Schüssler (2012)
Indications to an alternative...

- Cycle-dependent changes in the internal structure?
- Theoretical: Radiative heating of a magnetised convective overshoot layer [Rempel 2003]
- Helioseismology: sound speed reduced [Baldner & Basu 2008]
Helioseismic evidence

SOLAR CYCLE RELATED CHANGES AT THE BASE OF THE CONVECTION ZONE

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Sound speed is reduced near the base (Cyc 23 min to max)

Reduction pattern correlated with the butterfly diagram

\[ \frac{\delta c^2}{c^2} = (7.23 \pm 2.08) \times 10^{-5} \]
Radiative heating of a magnetic layer

Thermal properties of magnetic flux tubes

II. Storage of flux in the solar overshoot region

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Rempel (2003)

- Convective heat flux quenched by B
- …cooling the overshoot region
- …stabilising the overshoot region
Perturbing the stratification

\[ T_1 = T_m \exp \left[ -\frac{(r - r_p)^2}{\sigma^2} \right], \]

... Perturbation in temperature...

\[ \rho_1 = \rho_0 \left( \frac{p_1}{p_0} - \frac{T_1}{T_0} \right), \]

... density

\[ \frac{dp_1}{dr} = -\frac{p_1}{H_p 0} + \rho_0 g \frac{T_1}{T_0}, \]

... pressure

\[ s_1 = c_p \left( \frac{T_1}{T_0} - \nabla_{ad} \frac{p_1}{p_0} \right), \]

... specific entropy

\[ \delta_1 = -\frac{H_p 0}{c_p} \frac{ds_1}{dr}. \]

... superadiabaticity

Non-local mixing length model of solar CZ

stabilised!
Overshoot region ‘stabilised’


Implications for magnetic flux tubes?
Magnetic flux tubes stabilised

- Magnetic flux tubes stabilised

\[
B_p \leq c \frac{QH}{E} \left( \frac{\delta}{\beta} \right) \approx 3.6 \times 10^5 \text{ G},
\]

using the sound speed perturbation from the helioseismic result, and the local gas pressure from the structure model used here.

- From another perspective, Rempel (2003) found that when a magnetic field of \(10^5\) G quenches the convective heat conductivity by a factor of 100, a local cooling

- Figure 3. Instability maps of a thin flux tube as a function of latitude and field strength in the middle of the overshoot region, for \(T_0\) and \(T_5\) (upper panels); \(T_{10}\) and \(T_{20}\) (lower panels). The contours show growth times from the linear stability analysis. The dots clustered along the densely packed contours (growth times 40–60 days with 5-day intervals) show the nonlinear simulations performed. The light (dark) shaded regions denote the wavenumber of the fastest-growing mode \(m = 1\) (\(m = 2\)). It is noticeable that the instability threshold field strength shifts to larger values as the thermal perturbation is increased. Note that the range of field strength is different on each plot.

- Figure 4. Latitude dependence of the tilt angle (Joy’s law) for simulations \(T_0\) to \(T_{50}\) with different amplitudes of local cooling. The tilt angles are averages over 5° bins (continuous lines). The dotted lines show the sinusoidal fits (Equation 10). The average tilt angle and the steepness of the dependence decrease with increasing temperature perturbation.

<table>
<thead>
<tr>
<th>(T_m) (K)</th>
<th>(\delta \times 10^{-5})</th>
<th>(B_\sim)</th>
<th>(\beta)</th>
<th>(\gamma)</th>
<th>(\nu)</th>
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<td>0.21</td>
<td>0.23</td>
<td>13.7</td>
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<tr>
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<td>0.17</td>
<td>0.19</td>
<td>11.2</td>
<td>1.03</td>
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<tr>
<td>(-20)</td>
<td>3.63</td>
<td>0.14</td>
<td>0.15</td>
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<td>0.86</td>
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<tr>
<td>(-50)</td>
<td>2.91</td>
<td>0.11</td>
<td>0.13</td>
<td>7.7</td>
<td>0.72</td>
</tr>
</tbody>
</table>


Magnetic flux tubes stabilised

- Instability threshold shifts to stronger field strengths

Linear stability analysis for thin flux tubes

cf. Ferriz Mas & Schüssler (1995)
Joy’s law & the observed anti-correlation

- Stronger cycles, lower tilt angles
- 5-20 K of cooling sufficient
- Observed min-max variation: -140 K

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<th>$T_m$ (K)</th>
<th>$\delta \times 10^{-5}$</th>
<th>$\langle \alpha \rangle$</th>
<th>$\langle \alpha \rangle/\langle \lambda \rangle$</th>
<th>$a$</th>
<th>$\gamma_0$</th>
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Mean Tilt Angles and Joy’s Law Parameters

![Graph showing Joy’s law and observed anti-correlation.](image)
Implications

Nonlinear Babcock-Leighton dynamo with stochastic sources
(Işık, Cameron, Schüssler 2016, ongoing work)

❖ A potential nonlinearity for the solar cycle.
❖ It can account for the observed range of cycle-averaged tilts.
❖ It will be used in BL dynamo models.