IS THE SMALL-SCALE MAGNETIC FIELD CORRELATED WITH THE DYNAMO CYCLE?

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Draft version October 19, 2015

(arXiv:1505.06632)

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Large-scale magnetic field of Sun
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Generated from the (global) large-scale dynamo operating in the solar convection zone.

**Rotation:** Makes the convective motion helical (mirror-asymmetric)

**Non-uniform rotation:** Stretch the magnetic field lines.
Small-scale magnetic field of Sun

Vertical flux density map, calculated from the MDI line-of-sight magnetogram recorded on 2002 April 26. Bipolar magnetic regions are enclosed in rectangular boxes. The gray-scale cuts are set at $+100$ G (white) and $-100$ G (black).

- Known for long time e.g., Howard & Stenflo 1972; Frazier & Stenflo 1972

Stenflo & Kosovichov (2012)
MDI magnetogram:

Solar maximum

Solar minimum

Jin et al. (2011)
Recent results from Sunrise (Anusha Bhasari et al.) find even extended spectrum on the left side!

Figure 2: Log-log plot of the frequency of emergence against emergence event flux. The fit (dashed line) is a power-law distribution with an index of $-2.7$. Distributions plotted are from Kitt Peak (HZ93 – Harvey and Zwaan, 1993), MDI (HST03 – Hagenaar, Schrijver and Title, 2003) and SOT (TP10_{bcd}/TP10_{tbd} – Thornton and Parnell, 2011). (Based on Thornton and Parnell, 2011.)
Interestingly, even the internetwork field—the weakest component of solar magnetism—having an unsigned flux of $\approx 10^{15} - 10^{18} \text{ Mx}$, contributes at least four orders of magnitude larger flux than that of the bipolar-active regions during solar maximum.

Figure 2: Log-log plot of the frequency of emergence again (dashed line) is a power-law distribution with an index $c$ from Kitt Peak (HZ93 – Harvey and Zwaan, 1993), ML and Title, 2003) and SOT (TP10bcd/TP10tbd – Thornton and Parnell, 2011). (Based on Thornton and Parnell, 2011.)
Origin of the small-scale magnetic field

A. Result of a large-scale dynamo: the shredding of large-scale magnetic field and the decay of active regions. Then this small-scale field should be correlated with the sunspot cycle!

Small-scale magnetic field does not have solar cycle dependence, and it does not have any latitudinal dependence (Hagenaar et al. 2003; Sanchez Almeida 2003; Lites et al. 2008; Lites 2011; Buehler et al. 2013).

Jin et al. (2011)


Faurobert & Ricort (2015)
Does the variation of solar inter-network horizontal field follow sunspot cycle? Chunlan Jin, Jingxiu Wang

The ubiquitousness of solar inter-network horizontal magnetic field has been revealed by the space-borne observations with high spatial resolution and polarization sensitivity. However, no consensus has been achieved on the origin of the horizontal field among solar physicists. For a better understanding, in this study we analyze the cyclic variation of inter-network horizontal field by using the spectro-polarimeter observations provided by Solar Optical Telescope on board Hinode, covering the interval from 2008 April to 2015 February. The method of wavelength integration is adopted to achieve a high signal-to-noise ratio. It is found that from 2008 to 2015 the inter-network horizontal field does not vary when solar activity increases, and the average flux density of inter-network horizontal field is 87$\pm$1 G. In addition, the imbalance between horizontal and vertical field also keeps invariant within the scope of deviation, i.e., 8.7$\pm$0.5, from the solar minimum to maximum of solar cycle 24. This result confirms that the inter-network horizontal field is independent of sunspot cycle. The revelation favors the idea that a local dynamo is creating and maintaining the solar inter-network horizontal field.

Cyclic behavior of solar inter-network magnetic field Chunlan Jin, Jingxiu Wang

Solar inter-network magnetic field is the weakest component of solar magnetism, but contributes most of the solar surface magnetic flux. The observations suggest that the inter-network magnetic field does not arise from the flux diffusion or flux recycling of solar active regions, thereby indicating the existence of a locally small-scale dynamo.

Is the small-scale magnetic field correlated with the dynamo cycle? Bidya Binay Karak, Axel Brandenburg
Origin of the small-scale magnetic field

B. Small-scale dynamo/local dynamo:

*Three-dimensional* velocity fields sufficiently random in space and/or time can amplify small-scale magnetic fluctuations via random stretching of the field lines (Batchelor 1950; Zel'dovich et al. 1984; Childress & Gilbert 1995).

No *net helicity* is required!
Direct numerical simulation (DNS) => very challenging!
(because of large spatial and the temporal scale and smaller values of fluid and magnetic diffusivity in the solar convection zone.)
The magnetic Prandtl number in SCZ is extremely small (∼ $10^{-5}$). At small Pm, the critical magnetic Reynolds number, $R_{mC}$ needed to excite the small-scale dynamo increases at first linearly with deceasing Pm (Rogachevskii & Kleeorin 1997; Boldyrev & Cattaneo 2004; Schekochihin et al. 2005) then it saturates for Pm << 1 (Iskakov et al. 2007; Brandenburg 2011).

**Early MHD simulations of small-scale dynamo**
Cattaneo (1999); Emonet & Cattaneo (2001); Cattaneo et al. (2003); Voegler et al. (2005); Voegler & Schuessler (2007); Rempel (2014);

Hotta, Rempel & Yokoyama (2015) → Small-scale dynamo!
Kapyla et al., Warneck et al., Karak et al. → Large-scale dynamo in the solar convection zone!

However, in Sun both dynamos are operating in the same plasma!
A simple setup for dynamo simulations
(excites both large-scale and small-scale dynamos)

\[
\frac{DU}{Dt} = -SU_x \hat{y} - c_s^2 \nabla \ln \rho + \rho^{-1} [J \times B + \nabla \cdot (2\rho \nu S)] + f, \quad (2)
\]

\[
\frac{D \ln \rho}{Dt} = -\nabla \cdot U, \quad (3)
\]

\[
\frac{\partial A}{\partial t} + \overline{U}^{(S)} \cdot \nabla A = -SA_y \hat{x} + U \times B + \eta \nabla^2 A. \quad (4)
\]

\[f(x, t) = \text{Re}\{N f_{k(t)} \exp[i k(t) \cdot x + i \phi(t)]\}\]

\[-\pi < \phi(t) \leq \pi \]

\[N = f_0 c_s (|k| c_s / \delta t)^{1/2}\]

\[f_k = R \cdot f_k^{(nohel)} \quad \text{with} \quad R_{ij} = \frac{\delta_{ij} - i \sigma \epsilon_{ijk} \hat{k}_k}{\sqrt{1 + \sigma^2}}\]

Here \(D/Dt = \partial/\partial t + (U + \overline{U}^{(S)}) \cdot \nabla\) is the advective time derivative, \(\overline{U}^{(S)} = (0, Sx, 0)\) with \(S = \text{const}\) is the Isothermal & compressible gas.

Periodic box, imposed large-scale shear,
turbulence is generated by helically forced flow.
Dynamo number:

\[ D = C_\alpha C_\Omega \]

\[ C_\alpha = \frac{\alpha_0}{\eta_{T0} k_1} \]

\[ C_\Omega = \frac{|S|}{\eta_{T0} k_1^2} \]

\[ \alpha_0 = -\frac{\tau \langle \omega \cdot u \rangle}{3} \]

\[ \tau = \left( u_{\text{rms}} k_f \right)^{-1} \]

\[ \eta_{T0} = \frac{\tau \langle u^2 \rangle}{3} \]

\[ R_m = \frac{u_{\text{rms}}}{\eta k_f} \]

\[ \text{Re} = \frac{u_{\text{rms}}}{\nu k_f} \]

(Large \( R_m \), small \( D \))

small-scale dynamo!

(Large \( R_m \), large \( D \))

Both dynamos!

(Small \( R_m \), small \( D \))

No dynamo!

(Small \( R_m \), large \( D \))

large-scale dynamo!
Results from: only large-scale dynamo and, no small-scale dynamo

The small-scale field is produced from the tangling of the large-scale field!
When there is only a small-scale dynamo
(Large $R_m$, small $D$)
Why this anti-correlation?
Quenching of small-scale dynamo!

Results from a simulation when both dynamos are operating
(Large $R_m$, large $D$)
Results from a simulation when both dynamos are operating (Large $R_m$, large $D$).

- From small-scale dynamo + tangling
- From small-scale dynamo

Graph showing $B_y^2 / B_{eq}^2$ vs. $t / \tau_{diff}$.
Why this anti-correlation?
Quenching of small-scale dynamo!

Results from a simulation when both dynamos are operating
(Large $R_m$, large $D$)

Energy spectrum
Results from a simulation when both dynamos are operating (Large $R_m$, large $D$)

Same as earlier but here large-scale dynamo is weaker.
Conclusion

We have considered a simple setup of turbulent dynamo: It captures the essential mechanism of an alpha-Omega dynamo model. It can also excite the small-scale dynamo when $Rm$ is sufficiently large.

When both dynamos are operating, the small-scale field is produced from both the small-scale dynamo and the tangling of the large-scale field.

In this scenario, when the large-scale field $< B_{eq}$, the small-scale field is almost uncorrelated with the large-scale magnetic cycle.

However, when the large-scale field $> B_{eq}$, we observe an anticorrelation. This could be due to the suppression of small-scale dynamo through Lorentz force.
Next step: Convective dynamo simulation in spherical geometry

\[
\frac{\partial A}{\partial t} = u \times B - \mu_0 \eta J,
\]

\[
\frac{D \ln \rho}{Dt} = - \nabla \cdot u,
\]

\[
\frac{D u}{Dt} = g - 2\Omega_0 \times u + \frac{1}{\rho} \left( J \times B + \nabla \cdot 2\nu \rho \mathbf{S} - \nabla p \right)
\]

\[
T \frac{Ds}{Dt} = - \frac{1}{\rho} \nabla \cdot (F^{\text{rad}} + F^{\text{SGS}}) + 2\nu S^2 + \frac{\eta \mu_0}{\rho} J^2,
\]

\[
p = (\gamma - 1)\rho e, \quad e = c_v T
\]

\[
F^{\text{rad}} = -K \nabla T \quad \text{and} \quad F^{\text{SGS}} = -\chi_{\text{SGS}} \rho T \nabla s
\]

\[
K(r) = K_0 [n(r) + 1]
\]

\[
n(r) = \delta n (r/r_0)^{-15} + n_{\text{ad}} - \delta n
\]

\[
n_{\text{ad}} = 1.5
\]

\[
K_0 = (L/4\pi)c_v(\gamma - 1)(n_{\text{ad}} + 1)\rho_0 \sqrt{GM_\odot R_\odot}
\]

Thank you!

Kapyla et al. (2012, 2013, 2014)
Fig. 1 Kinetic and net helicity (bottom) pronounced peak of $E_M(k)$ at $k = k_1$ in the case with helicity. The energy input wavenumbers are $k_1 = 1.0 k_1$ in the non-helical case (upper panel, $Re_M = 600$, $Pr_M = 1$) and $k_f / k_1 = 4$ in the helical case (lower plot, $Re_M = 450$, $Pr_M = 1$). Adapted from Brandenburg and Subramanian (2005a) and Brandenburg et al. (2008)
Fig. 11.—Composite distribution function of Sun per day, per flux interval of $10^{18}$ Mx. The data (defined as regions larger than $2.5 \text{ deg}^2$) taken together account for about a factor of 8 through a typical cycle; these data bound the dark shaded area on the right. The number of the smallest ephemeral regions—studied here—shading—is likely weakly in antiphase with the solar rotations. Histograms are shown for 1997 October (black) and turnover below $10^{18}$ Mx likely reflects the detection of a lightly shaded area between the smallest ephemeral regions is an approximation that still awaits confirmation.

Hagenaar et al. (2003)
Azimuthally averaged **toroidal field** near the bottom of SCZ

5 times rotating than Sun
Warnecke et al. (2014)

Solar-like rotation
Karak et al. (2015)
| Regime | Run  | grid | $k_f$ | $\nu$ | $P_m$ | $|S|$ | $u_{rms}/c_s$ | $R_m$ | $\bar{B}_x$ | $\bar{B}_y$ | $\bar{B}$ | $\bar{b}$ | $C_\alpha$ | $C_\Omega$ | $D$ | $\text{Corr}(\bar{B}_y^2, \bar{b}^2)$ | Dynamic |
|--------|------|------|------|------|------|------|-------------|------|-------------|-------------|---------|---------|-----------|----------|-----|----------------|----------|
| I      | R1   | $48^2$ | 5.1  | $10^{-2}$ | 1.25 | 0.05 | 0.023 | 0.55 | 0.000 | 1.40 | 1.41 | 0.274 | 0.75 | 10.6 | 7.9 | -- | -- | No dynamic |
| II     | R2a  | $48^3$ | 5.1  | $10^{-2}$ | 1.25 | 0.1  | 0.022 | 0.55 | 0.165 | 1.40 | 1.41 | 0.274 | 0.75 | 10.6 | 7.9 | 0.77 | LSD |
| II     | R2b  | $96^3$ | 3.1  | $5 \times 10^{-4}$ | 0.42 | 0.04 | 0.066 | 17.7 | 0.246 | 1.34 | 1.38 | 0.662 | 1.38 | 6.1 | 8.4 | 0.42 | LSD |
| II     | R2c  | $96^3$ | 3.1  | $5 \times 10^{-4}$ | 0.50 | 0.04 | 0.063 | 20.1 | 0.298 | 1.50 | 1.54 | 0.711 | 1.58 | 5.2 | 8.2 | 0.48 | LSD |
| II     | R2d  | $48^3$ | 5.1  | $10^{-2}$ | 1.25 | 0.15 | 0.022 | 0.54 | 0.202 | 2.57 | 2.61 | 0.418 | 0.71 | 15.9 | 11.4 | 0.91 | LSD |
| II     | R2e  | $48^3$ | 5.1  | $10^{-2}$ | 1.25 | 0.2  | 0.039 | 0.97 | 0.294 | 4.56 | 4.57 | 0.671 | 1.14 | 18.9 | 21.6 | 0.76 | LSD |
| II     | R2f  | $48^3$ | 5.1  | $10^{-2}$ | 1.25 | 0.25 | 0.039 | 0.97 | 0.271 | 5.17 | 5.18 | 0.699 | 1.11 | 23.6 | 26.2 | 0.70 | LSD |
| II     | R2g  | $48^3$ | 5.1  | $10^{-2}$ | 5.00 | 0.08 | 0.021 | 2.06 | 0.308 | 7.17 | 7.18 | 0.862 | 2.00 | 23.7 | 47.4 | 0.62 | LSD |
| II     | R2h  | $48^3$ | 5.1  | $10^{-2}$ | 5.00 | 0.08 | 0.021 | 2.06 | 0.282 | 8.06 | 8.07 | 0.884 | 1.98 | 29.7 | 58.8 | 0.50 | LSD |
| III    | R3   | $144^3$ | 3.1  | $10^{-3}$ | 5.00 | 0.0  | 0.054 | 86.1 | 0.076 | 0.11 | 0.15 | 0.526 | 0.00 | 0.0 | 0.0 | -- | SSD |
| III    | R3'  | $288^3$ | 3.1  | $10^{-3}$ | 5.00 | 0.0  | 0.054 | 86.1 | 0.076 | 0.11 | 0.15 | 0.528 | 0.00 | 0.0 | 0.0 | -- | SSD |
| IV     | R4a  | $144^3$ | 3.1  | $10^{-3}$ | 5.00 | 0.05 | 0.057 | 90.7 | 0.216 | 1.83 | 1.88 | 0.882 | 2.07 | 8.0 | 16.6 | -- | SSD+LSD |
| IV     | R4a' | $288^3$ | 3.1  | $10^{-3}$ | 5.00 | 0.05 | 0.057 | 90.6 | 0.235 | 1.88 | 1.90 | 0.937 | 2.14 | 8.0 | 17.2 | -- | SSD+LSD |
| IV     | R4b  | $144^3$ | 3.1  | $10^{-3}$ | 5.00 | 0.1  | 0.065 | 104 | 0.155 | 1.42 | 1.45 | 0.835 | 1.41 | 14.1 | 19.8 | -- | SSD+LSD |
| IV     | R4c  | $144^3$ | 3.1  | $4 \times 10^{-4}$ | 5.00 | 0.05 | 0.059 | 234 | 0.226 | 1.72 | 1.74 | 0.983 | 2.38 | 7.9 | 18.8 | -- | SSD+LSD |
| IV     | R4d  | $144^3$ | 3.1  | $2 \times 10^{-4}$ | 1.00 | 0.05 | 0.069 | 110 | 0.231 | 1.65 | 1.68 | 0.831 | 1.87 | 6.7 | 12.5 | -- | SSD+LSD |
| IV     | R4e  | $144^3$ | 3.1  | $10^{-4}$ | 0.50 | 0.05 | 0.069 | 110 | 0.230 | 1.56 | 1.58 | 0.828 | 1.92 | 6.6 | 12.7 | -- | SSD+LSD |
| IV     | R4f  | $144^3$ | 3.1  | $4 \times 10^{-4}$ | 5.00 | 0.05 | 0.056 | 225 | 0.211 | 1.44 | 1.47 | 1.005 | 1.99 | 8.23 | 16.4 | -- | SSD+LSD |
| IV     | R4g  | $144^3$ | 3.1  | $10^{-3}$ | 5.00 | 0.3  | 0.170 | 271 | 0.059 | 0.53 | 0.53 | 0.486 | 0.14 | 16.4 | 2.3 | 0.01 | SSD+LSD |
| IV     | R4h  | $144^3$ | 3.1  | $10^{-3}$ | 5.00 | 0.2  | 0.162 | 258 | 0.062 | 0.39 | 0.40 | 0.456 | 0.16 | 11.5 | 1.9 | 0.14 | SSD+LSD |

Note. — The $k_f$ is in unit of $k_1 = 2\pi/L_x = 1$. Except for Run R3 for which $\sigma = 0$, for all runs we have $\sigma = 1$. For Runs R2e and R2f we have $f_0 = 0.01$, while for all other runs $f_0 = 0.01$. The $\bar{B}$ is the temporal mean in the statistically stationary state of the large-scale magnetic field over the whole domain. $\bar{B}_{rms} = \langle (B_x)^2 + (B_y)^2 + (B_z)^2 \rangle^{1/2}$, normalized by $B_{eq}$. Similarly, $\bar{B}_i = \langle B_i^2 \rangle^{1/2}/B_{eq}$ for $i = x$ and $y$, and $\bar{b} = \langle b^2 \rangle^{1/2}/B_{eq}$. Here, $\text{Corr}(\bar{B}_y^2, \bar{b}^2)$ is the linear correlation coefficient $r$ between $\bar{B}_y^2$ and $\bar{b}^2$. SSD and LSD stand for small-scale and large-scale dynamos. Bold fonts in the left column indicate useful regimes of our primary interest.
To interpret our results in terms of a mean-field picture, we define the mean-field dynamo number \( D = C_\alpha C_\Omega \) with

\[
C_\alpha = \frac{\alpha_0}{\eta T_0 k_1} \quad \text{and} \quad C_\Omega = \frac{|S|}{\eta T_0 k_1^2},
\]

where \( \eta T_0 = \eta t_0 + \eta \) is the theoretically expected total (turbulent plus microphysical) magnetic diffusivity, \( \eta t_0 = \tau \langle u^2 \rangle / 3 \), \( \alpha_0 = -\tau \langle \omega \cdot u \rangle / 3 \), \( \tau = (u_{\text{rms}} k_f)^{-1} \), and \( k_f \) is the mean forcing wavenumber (Blackman & Brandenburg 2002). The other usual diagnostic parameters, \( R_m, \text{Re} \) (fluid Reynolds number), and \( P_m \) are defined as

\[
R_m = u_{\text{rms}} / \eta k_f, \quad \text{Re} = u_{\text{rms}} / \nu k_f, \quad P_m = \nu / \eta,
\]

where \( u_{\text{rms}} = \langle u^2 \rangle^{1/2} \) is the rms value of the velocity in the statistically stationary state with \( \langle \cdot \rangle \) denoting the average over the whole domain and over time, and \( B_{\text{eq}} = u_{\text{rms}} \) is the volume-averaged equipartition field (we recall that in our units, \( \mu_0 \langle \rho \rangle = 1 \)). Furthermore, the large-scale (mean) field

\[
\overline{b^2} = \overline{B^2} - \overline{B}^2
\]
(Large $R_m$, small $D$) Only small-scale dynamo!

(Small $R_m$, small $D$) No dynamo!

(Large $R_m$, large $D$) Only large-scale dynamo!

Small $R_m$, large $D$) Both dynamos!
When there is only a small-scale dynamo
(Large $R_m$, small $D$)