Scaling of coronal emission with activity

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MPS
SGS, 29.7.2014

solar eclipse, 11.8.1999, Wendy Carlos and John Kern
Magnetohydrodynamics (MHD)

\[ \nabla \times B = \mu j \quad \nabla \cdot B = 0 \]
\[ \nabla \times E = -\partial_t B \quad \nabla \cdot E = \frac{1}{\varepsilon} \rho_e \]
\[ j = \sigma (E + \mathbf{v} \times B) \]
\[ \dot{\mathbf{j}} \times \mathbf{B} = \frac{1}{\mu} (\nabla \times \mathbf{B}) \times \mathbf{B} \]

\text{induction eq.}
\[ \partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B}) \]

\text{continuity eq.}
\[ \partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \]

\text{momentum eq.}
\[ \rho \partial_t \mathbf{u} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \rho \mathbf{g} + \dot{j} \times \mathbf{B} + \nabla \cdot \mathbf{\tau} \]

\text{viscous stress tensor } \mathbf{\tau}:
\[ \nabla \cdot \mathbf{\tau} = \rho \nu \left( \Delta \mathbf{u} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{u}) \right) \]

\text{energy eq.}
\[ (\partial_t + \mathbf{u} \cdot \nabla) e + \frac{5}{2} \rho \nabla \cdot \mathbf{u} = -\nabla \cdot \mathbf{q} - L_{\text{rad}} + \eta \dot{j}^2 + Q_{\text{visc}} \]

\text{internal energy:} \quad e = n \frac{3}{2} k_B T

\rightarrow \text{for coronal diagnostics it is essential to get energy equation right}
3D MHD coronal model including spectral synthesis

3D MHD model: \( T, \rho, v, B \)

- full energy equation
  (heat conduction, radiative losses)
- up to 1024 \( \times \) 1024 \( \times \) 512 grid
- horizontally periodic, open top
- non-uniform mesh

Pencil Code
Brandenburg & Dobler (2002)
Comp Phys Comm 147, 471
- efficient parallelization (MPI)

synthesized coronal emission \( \text{Mg} \times 625 \text{ Å} \)

intensity map (inv.)
Doppler map

compare

real observations
Hinode / EIS Fe xv 284 Å

160"x125"

Horizontally averaged heating rate (per particle)

- heating concentrated in low atmosphere
- maximum heating/particle in transition region
- but there is still heating needed in corona!

→ is because of scale heights:
  - $p$ chromosphere: < 1 Mm
  - Ohmic heating: ≈ 10 Mm
  - $p$ corona: > 50 Mm

already hinted at by Galsgaard & Nordlund (1996)
Coronal evolution

Mg X (625 Å) ~10^6 K

- large coronal loops connecting active regions
- gradual evolution in line intensity ("wriggling tail")
- higher spatial structure and dynamics in Doppler shift signal

→ it is important to have full spectral information!

DEM inversion using CHIANTI:

1 – using synthetic spectra derived from 3D MHD model

2 – using solar observations (SUMER, same lines)

\[ DEM = n_e^2 \frac{dh}{dT} \]

Supporting suggestions that numerous cool structures cause increase of DEM to low \( T \)

AND:

velocities reduce \( \nabla T \)!
Doppler shifts

quiet Sun:

active region:
Bourdin, Bingert, hp (2014) in prep.

- QS: Peter & Judge (1999)
- AR: Teriaca et al. (1999)
- Gaussian fit of local maximum
- Thresholded spatial average
- Moment of total spectrum
- Mean of the three estimators

Constant cross section loops

horizontally integrated through computational box

loop has cross section of ~2x AIA PSF width
(PSF ≈ 1.3' ≈ 2.5 pxl)

synthetic AIA 171 Å (10^6 K)

Hot cores of active regions

Bourdin, Bingert, hp (2014) PASJ, in press
Coronal model driven by emerging flux simulation

- Loops form at different places at different times after the sunspots have formed.
- Loop footpoints are in sunspot periphery (penumbra).
- Loops form where Poynting flux in the photosphere peaks → where / when flux is pushed into sunspot.

34 min out of 10 hrs

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- loops form where Poynting flux in the photosphere peaks
  → where / when flux is pushed into sunspot

The same, but for higher resolution.
Zoom of middle part only.

30 min out of 10 hrs
High-resolution model of emerging active region

Comparison of 3D model to scaling laws

RTV scaling laws: \( T \sim H^{2/7} L^{4/7} \)

Scaling laws
Magnetohydrodynamics (MHD)

\[
\begin{align*}
\nabla \times B &= \mu j \\
\nabla \cdot B &= 0 \\
\nabla \times E &= -\partial_t B \\
\nabla \cdot E &= \frac{1}{\varepsilon} \rho_e \\
\n\rho j &= \sigma (E + \mathbf{v} \times B) \\
\n\frac{\partial}{\partial t} \rho + \nabla \cdot (\rho \mathbf{u}) &= 0 \\
\rho \frac{\partial}{\partial t} \mathbf{u} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} &= -\nabla p + \rho \mathbf{g} + j \times B + \nabla \cdot \mathbf{\tau} \\
\n\mathbf{\tau} &= \rho \nu \left( \Delta \mathbf{u} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{u}) \right) \\
\frac{\partial}{\partial t} e + \frac{5}{2} \rho \nabla \cdot \mathbf{u} &= -\nabla \cdot \mathbf{q} - L_{rad} + \eta j^2 + Q_{visc} \\
\text{internal energy: } &e = n \frac{3}{2} k_B T
\end{align*}
\]
Heat conduction and radiative losses

**Heat conduction** ($\parallel B$)

$$q = -\kappa_0 T^{5/2} \frac{\partial T}{\partial s}$$

$\rightarrow$ important at high $T$

**Radiative losses**

in an optically thin atmosphere

$$L_{\text{rad}} = P_{\text{rad}} n_e^2$$

approximation through power laws:

$$P_{\text{rad}} \propto T^{-1/2}$$

$$L_{\text{rad}} \propto T^{-1/2} n_e^2$$

$\rightarrow$ radiative losses important at low $T$
Radiation vs heat conduction

simple 1D semi-empirical atmosphere

$\text{div } q$

$L_{\text{rad}}$

$H$

$q$

$L_{\text{rad}}$
Order of magnitude considerations

\[ \frac{\partial e}{\partial t} = 0 = -\nabla \cdot q - L_{\text{rad}} + H \]

**top:** heating \( \sim \) conduction

\[ H \sim \nabla \cdot q \quad \text{with} \quad q \propto T^{5/2} \nabla T \]

\[ \rightarrow T \sim H^{2/7} L^{4/7} \]

**bottom:** conduction \( \sim \) radiative losses

\[ \nabla \cdot q \sim L_{\text{rad}} \quad \text{with} \quad L_{\text{rad}} \propto T^{-1/2} n^2 \quad \text{and} \quad n \propto p/T \]

\[ \rightarrow T^3 \sim p L \]

\[ \rightarrow p \sim H^{6/7} L^{5/7} \]

identical to **RTV scaling laws**


radiative loss in corona \( \sim n^2 \)

\[ L_X \sim n^2 \sim H^{8/7} L^{2/7} \sim p^{4/3} L^{-2/3} \]

\[ \rightarrow \text{coronal emission is roughly proportional to energy input} \]
Models for different stellar activity
Stellar activity and rotation

Rossby number: \( Ro = \frac{t_{\text{rot}}}{\tau_{\text{conv}}} = \frac{\text{rotation time}}{\text{convective time}} \)

Pizzolato et al. (2003) A&A 397, 147

usual line of argument:
- rotation speed
- dynamo
- surface flux
- coronal heating
- X-ray output

how does this work?

clear connection between rotation and activity
X-ray emission and magnetic field

stellar soft X-ray flux vs. hemisphere-averaged absolute magnetic flux density

\[ F_X \sim B^{0.9\pm0.1} \]

**X-ray emission and magnetic field**

Series of 3D MHD models with different surface $B$

Surface $B$ increases by factor of $\sim4$
Magnetic field and energy input

Energy input scales roughly linear with surface magnetic field (van Wettum, Bingert, hp)
Appearance of coronae with different surface $B$

Surface $B$ increases by factor of $\sim 4$

Stronger surface field leads to more extended coronae

(van Wettum, Bingert, hp)
Doppler shift patterns and activity

Different Doppler patterns for models with different surface magnetic field (van Wettum, Bingert, hp)

Surface B increases by factor of ~3

Observation of solar-mass stars with different activity levels

Flux-flux relations: scaling of magnetic activity

\[ F_X \sim F_{\text{Ca}}^{1.5} \]

similar relations exist for other wavelength bands

relation steeper for hotter emission

Schrijver et al. (1992) A&A 258, 432
Flux-flux relation

stellar observations: relation between flux in different emission bands is close to power law

\[ F_X \sim (F_{\text{C IV}})^{1.4} \]

(Schrijver et al. 1989, 90; Saar 2001)

► 3D MHD models give some slope close to 1.4 slope seems to increase with temperature

(van Wettum, Bingert, hp)
Surface temperature and buried coronae
Stellar coronae in the HRD

Hertzsprung-Russell diagram  Hipparcos: 41.704 stars

- almost all cool stars (main sequence) show X-ray emission
- young stars are very X-ray active e.g. T-Tauri
- giants: coronal dividing line


ROSAT X-ray detections
The coronal graveyard

- giants with strong winds: why do they not have coronae?
  - does magnetic field play a role? → wind driven by luminosity…
  - magnetic configuration → mainly open magnetic field?
  - low g → stretched chromospheres → "buried" magnetic loops

Ayres (2004) ESA SP-575
Chromospheric pressure and corona heat input

Hammer, Linsky, Endler (1982)
in Advances in Ultraviolet Astronomy: Four Years of IUE Research
eds. Kondo, Mead & Chapman, NASA-CP 2238, p. 268
Heat input into the corona

- 3D MHD models consistently give an exponential decrease of volumetric heat input with height (on average, but also along individual fieldlines)

- This has been used extensively in 1D loop models since ~1982

\[ H_{\text{corona}} \propto \exp(-h/\lambda) \]

Based on 3D model of Bingert & hp (2011) A&A 530, A112

\[ H_{\text{Ohm}} = \mu_0 \eta j^2 \]
Pressure stratification

\[ L = 4\pi R^2 \sigma T^4 \]
\[ R \propto M^{1} \quad (M < 1.4 M_\odot) \]
\[ L \propto M^{3.8} \]

\( g \propto \frac{M}{R^2} \propto T^{-2.2} \quad \rightarrow \quad H_p = \frac{k_B T}{m g} \propto T^{3.2} \)

Surface pressure and temperature

\[ p = p_0 \exp(-z/H_p) \]

3D simulation of Beeck et al. for stars on ZAMS (except for M2V)
Crossing of gas pressure and coronal pressure

\[ p = p_0 \exp(-z/H_p) \]

\[ H_p \propto T^{3.2} \]

\[ p_0 \propto T^{-4.2} \]

Barometric pressure stratification

\[ T_{\text{eff}} = 5787 \]

Coronal pressure exponentially dropping with scale height 7 Mm

Top of the chromosphere / base of corona
Crossing of gas pressure and coronal pressure

\[ p = p_0 \exp\left(-\frac{z}{H_p}\right) \]

\[ H_p \propto T^{3.2} \]

\[ p_0 \propto T^{-4.2} \]

For \( T > 10,000 \, K \) the corona starts only above \( \sim 10 \, \text{Mm} \)

→ part of magnetic field will be “buried” in chromosphere

similar to idea of T. Ayres for giants
Dependence of coronal pressure on surface temperature

\[ L_X \sim p^{4/3} \]

→ in the range of stars of Beeck et al
we can expect the X-ray luminosity
to change by almost a factor of 2
solely due to the surface temperature
Scatter in $L_X$ plot

scatter will be due to
► stellar variability
► measurement error
► and surface temperature?

$\log L_X = (30.71 \pm 0.05) - (2.01 \pm 0.05) \log P$

factor $\sim 2$
Conclusions
Conclusions

- 3D MHD models give good representation of the observed solar corona
- small parameter study shows path to extent solar coronal models to star
  → already good representation of flux-flux relations
- basic considerations show how surface temperature might control coronal emission
  → pressure profile depends on surface temperature
  → equilibrium of chromospheric pressure and coronal requirements
    sets coronal pressure and thus coronal emission
- could this explain some of the scatter in the $L_X$ plots?
- does this give a quantitative explanation of “buried” coronae?
thanks...