Understanding the Solar Dynamo from Global Convective Dynamo Simulations

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Dynamos

\[ \frac{\partial B}{\partial t} = \nabla \times (u \times B) - \nabla \times \eta J \]

\[ B = \bar{B} + b' \quad u = \bar{U} + u' \]

\[ \frac{\partial \bar{B}}{\partial t} = \nabla \times (\bar{U} \times \bar{B} + u' \times b') - \nabla \times \eta \bar{J} \]

\[ \nabla \times (\bar{U} \times \bar{B}) = (\bar{B} \cdot \nabla)\bar{U} - \bar{B}(\nabla \cdot \bar{U}) - (\bar{U} \cdot \nabla)\bar{B} \]
Electromotive force

\[ \bar{E} = \bar{u}' \times \bar{b}' \]

\[ = \alpha \bar{B} + \gamma \times \bar{B} + \beta \nabla \times \bar{B} + \delta \times (\nabla \times \bar{B}) + \kappa \nabla \bar{B} \]

Simplifications:

\[ \alpha = \frac{\tau_c}{3} \left( -\omega \cdot \bar{u} + \frac{j \cdot \bar{b}}{\bar{\rho}} \right) \quad \text{Pouquet et al. 1976} \]

\[ \frac{\partial \bar{B}_{\text{pol}}}{\partial t} = \alpha \nabla \times \bar{B}_{\text{tor}} + \eta_T \Delta \bar{B}_{\text{pol}} \]

\[ \frac{\partial \bar{B}_{\text{tor}}}{\partial t} = (\bar{B}_{\text{pol}} \cdot \nabla) \bar{u}_{\text{tor}} + \alpha \nabla \times \bar{B}_{\text{pol}} + \eta_T \Delta \bar{B}_{\text{tor}} \]
How to solve the problem I

1. Observational constraints

   a) Surface observations + solar cycle evolution
   b) Helioseismology
   c) Other stars

Tail of a dog?
Not everything, but more than we think!
2. Mean-field modeling

- Put physics in as you want.
- Control experiments.
- Tunable parameters.
- Good parameters come from observations or fundamental theories.

Reproducing the Sun’s magnetic field is possible with many different approaches.
How to solve the problem III

2. Global convective dynamo simulations

- Numerical experiments.
- Parameters far from realistic.
- Testing mean-field theories.
- Testing observational contrains.

Interpreting this simulations helps us to understand the solar dynamo!
\[
\begin{align*}
\frac{\partial A}{\partial t} &= u \times B + \eta \nabla^2 A \\
\frac{D \ln \rho}{Dt} &= -\nabla \cdot u \\
\frac{Du}{Dt} &= g - 2\Omega_0 \times u + \frac{1}{\rho} \left( J \times B - \nabla p + \nabla \cdot 2\nu \rho S \right) \\
T \frac{Ds}{Dt} &= \frac{1}{\rho} \nabla \cdot \left( K \nabla T + \chi_\nu \rho T \nabla s \right) + 2\nu S^2 + \frac{\mu_0 \eta}{\rho} J^2 - \Gamma_{\text{cool}}(r),
\end{align*}
\]
Equatorward Migration

Käpylä, Mantere & Brandenburg 2012
(ApJL 755, L22)
Cause of Equatorward Migration

\[ \text{Pr} = \nu / \eta = 2.5 \]
\[ \text{Pr} = 0.5 \]
\[ \text{Pm} = \nu / \eta = 1 \]
\[ \text{Pm} = 0.5 \]
Parker—Yoshimura—Rule

\[ s_{\text{mig}}(r, \theta) = -\alpha \hat{e}_\phi \times \nabla \Omega, \]

Parker 1955

Warnecke et al. 2014a (ApJL 796, L12)
Radial and polar field are connected

Figure 17: Time evolution of the mean magnetic field in the convection zone. The dashed horizontal lines show the location of the equator at $\theta = \pi/2$. The magnetic field is normalized by its equipartition value, $B_{eq}$.
Torsional oscillations and inflows?

Figure 7: Time evolution of the surface velocity fields. The left panel shows the differential rotation \( \frac{u_\phi}{r \sin \theta} + \Omega_0 / \Omega_0 \). The right panel shows the meridional circulations \( \frac{u_\theta}{\Omega_0 R} \). In both panels, the time average in the saturated stage (starting point is indicated by the white vertical line) is subtracted.

Figure 9: Time evolution of the mean magnetic field in the convection zone. The first column shows the \( B_\phi \) and the second \( B_r \). In the upper two rows, the mean magnetic field is plotted as a function of turnover time \( t/\tau \) and latitude at \( r/R = 0.98 \) and \( r/R = 0.84 \). The last row shows the dependence with radius. Dark blue shades represent negative values and light yellow shades positive values. The dashed horizontal lines show the location of the equator at \( \theta = \pi/2 \). The magnetic field is normalized by its equipartition value, \( B_{eq} \).
the convection zone above its base. This is somewhat deceptive. The wreaths appear rooted at the base of the convection zone, which is averaged over a single energy cycle or 3.1 years.

The structure of the wreaths and caps is apparent in Figure 1(d), where the evolution of the longitudinally-averaged radial and longitudinal magnetic fields over an extended interval of the K3S simulation is shown. The equatorward propagation of magnetic features observed in Figure 3(b), arises through two mechanisms. The primary process here is the nonlinear feedback of the Lorentz force, wherein further equatorial propagation is prohibited by a spatially local effect. The Lorentz force alone, however, does not explain how the Lorentz forces of the wreaths locally weaken the shear, while poloidal field is built at both the polar and equatorial edges of the wreath. However, the shear on the poleward side of the edge has already been much reduced, leading to a greater generation of longitudinal field. Therefore, the appearance of equatorward motion in K3S could be considered the product of such a nonlinear dynamo wave.

Hence the appearance of equatorward motion in K3S could be considered the product of such a nonlinear dynamo wave. An analysis of the Parker-Yoshimura mechanism for case K3S indicates that near and poleward of the edge of the low-latitude wreaths the sign of the differential rotation seen in Figure 2(a).

Thus, a second possible mechanism should be considered. This mechanism is the Parker-Yoshimura mechanism. In this mechanism, the simple effect to consider is the Parker-Yoshimura mechanism for case K3S. In this mechanism, the interpretation of the field as a vertically aligned dipole.

Such a mechanism explains the periodic modifications of the magnetic energy generation balancing the production of shear, as the Lorentz forces of the wreaths locally weaken the shear, while the wreaths approach this equilibrium distance. One relation that must be considered is the Parker-Yoshimura mechanism for case K3S, as in Figure 2(b) and the broad panorama of Figure 3.

As with ASH and EULAG, 3D simulations in spherical projection, elucidating the poleward propagation of mid and high-latitude magnetic field and the equatorward propagation of features at lower latitudes. These caps act to moderate the polar vortices. The average of the energy corresponds to that of the greatest shear. So the regime of Figure 3 might expect an equilibrium to be established with the magnetic energy generation balancing the production of shear, as the Lorentz forces of the wreaths locally weaken the shear, while the equatorial propagation is initiated and sustained. Instead, one might expect an equilibrium to be established with the magnetic energy generation balancing the production of shear, as the Lorentz forces of the wreaths locally weaken the shear, while poloidal field is built at both the polar and equatorial edges of the wreath.
Adding a coronal envelope

Influence of a coronal envelope on global convective dynamo simulations

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ABSTRACT

Aims.

Methods.

Results.

Key words. Magnetohydrodynamics (MHD) – turbulence – Sun: dynamo – Sun: activity – stars: magnetic fields

1. Introduction

Warnecke et al. (2013)

2. Model and Setup

The setups are basically the same as the one layer model of Käpylä et al. (2012, 2013) and the two-layer model of Warnecke et al. (2013), which have been also used recently in Warnecke et al. (2014). We use a wedge in spherical polar coordinates \((r, \theta, \phi)\), where the layer below the surface \((r_0 \leq r \leq R)\) represents the convection zone, where \(R\) is the solar radius, and \(r_0\) corresponds the bottom of the convection zone \(r = 0\).

The layer above the surface represents a simplified coronal envelope, which extends to different outer radii \((R \leq r \leq R_C)\). The domain extends in colatitudes to \(15^\circ \leq \theta \leq 65^\circ\) and in longitude to \(0^\circ \leq \phi \leq 90^\circ\) (a quarter of a sphere). We solve the following equations of compressible magnetohydrodynamics,

\[
\frac{\partial A}{\partial t} = u \times B - \mu_0 \eta J,
\]

\[
\frac{D \ln \rho}{Dt} = -\nabla \cdot u,
\]

\[
\frac{Du}{Dt} = g - 2\Omega_0 \times u + \frac{1}{\rho}(J \times B - \nabla p + \nabla \cdot 2\nu \rho S),
\]

\[
\frac{D\mathbf{s}}{Dt} = -\frac{1}{\rho} \nabla \cdot (F_{\text{rad}} + F_{\text{SGS}}) + 2\nu S^2 + \mu_0 \eta \rho J^2 - \Gamma_{\text{cool}},
\]

where the magnetic field is defined via the vector potential \(B = \nabla \times A\), taking care of the divergence free at all time.

\(J = \nabla \times B\) is the current density with \(\mu_0\) being the vacuum permeability, \(\eta\) is the magnetic diffusivity and \(\nu\) the kinematic viscosity.

\(u\) the plasma velocity field, \(\rho\) the mass density and \(s\) the specific entropy.

![Graphs showing radial profiles of azimuthally and latitudinally averaged temperature and density normalized by their values at the bottom of the domain.](image)
Differential rotation zoomed in the northern hemisphere in the near-surface shear layer.

\[ Co = \frac{2\Omega_0}{\kappa f u_{rms}} \approx 2 \frac{\text{Rotation rate}}{\text{Turnover time}} \]

\( Co = 11 \)

\( Co = 5.2 \)
The Baroclinic Term

\[
\frac{\partial \omega_\phi}{\partial t} = r \sin \theta \frac{\partial \Omega^2}{\partial z} + (\nabla T \times \nabla s)_\phi + \ldots
\]

\[
R \nabla_\theta \bar{s} / c_P
\]

Comparison

Warnecke et al. 2015b
(in preparation)
Reynolds stresses and Lambda-effect

\[
Q_{r\phi} = \frac{\tau_{r\phi}}{\frac{\nu}{\Lambda}}\]

\[
Q_{\theta\phi} = \frac{\tau_{\theta\phi}}{\frac{\nu}{\Lambda}}\]

Rüdiger 1989
Determine the turbulent transport coefficients from global convective simulations

\[
\bar{E} = \bar{u}' \times \bar{b}' = \alpha \bar{B} + \gamma \times \bar{B} + \beta \nabla \times \bar{B} + \delta \times (\nabla \times \bar{B}) + \kappa \nabla \bar{B}
\]

Solving 27 equations for \(b',\) with 27 independent test-fields
Conclusions

- Migration of mean magnetic field can be entirely explained by an alpha-omega-dynamo wave.
- Radial fields seem to be important for polar fields.
- A coronal envelope helps for Spoke-like rotation profile.
- Self-consistent produced by a baroclinic term.
- Temperature distribution important for the Lambda-effect and differential rotation.
- Test-field method is the way to go!