Magnetic helicity simulation
in solar atmosphere

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Introduction to Magnetic helicity
Magnetic helicity

Helicity is an integral that measures some topological properties of vector fields,

\[ H = \int \vec{F} \cdot \nabla \times \vec{F} d^3x \]

Examples:

Fluid velocity helicity \[ H_v = \int \vec{v} \cdot \nabla \times \vec{v} d^3x \]

Current helicity \[ H_c = \int \vec{B} \cdot \left( \nabla \times \vec{B} \right) d^3x \]

Magnetic helicity \[ H_m = \int \vec{A} \cdot \left( \nabla \times \vec{A} \right) d^3x \]

\[ = \int \vec{A} \cdot \vec{B} d^3x \]
Magnetic helicity is conserved in ideal magneto hydrodynamics (Woltjer 1958)

It conserves approximately if magnetic Reynolds number is large enough (Taylor 1974)

... even for any fast reconnection!
Invariance of magnetic helicity despite of different ways to determine the vector potential

\[ H_m = \int \vec{A} \cdot \vec{B} d^3x \]

\[ \vec{A}' = \vec{A} + \nabla \psi \]

\[ H_m' = \int_v (\vec{A} + \nabla \psi) \cdot \vec{B} d^3x \]  

There should not be different magnetic helicity for different gauges!

Magnetic helicity is determined for definite boundary conditions

\[ \Delta H = \oint \oint \psi \vec{B} \cdot \hat{n} d^2x \]

\[ \vec{B} \cdot \hat{n} \bigg|_{s} = 0 \]

\[ \Delta H = 0 \]

Unfortunately, these boundary conditions are usually not satisfied
Relative magnetic helicity

\[ \mathbf{B}_1 = B(x \in V) \]
\[ \mathbf{B}_2 = \mathbf{P}(x \in V) \]
\[ \mathbf{B} = \nabla \times \mathbf{A} \]
\[ \mathbf{P} = \nabla \times \mathbf{A}_p \]
\[ \nabla \cdot \mathbf{A} = 0 \]
\[ \nabla \cdot \mathbf{A}_p = 0 \]
\[ \mathbf{B} \cdot \mathbf{n} \big|_s = \mathbf{P} \cdot \mathbf{n} \big|_s \]

Gravitational potential energy

\[ E = mgh \]
\[ E = mgz \quad h = z \]
\[ E = 0 \quad h = 0 \]
The relative magnetic helicity is gauge invariant.
Magnetic helicity change rate

Magnetic helicity change rate in a volume $V$:

$$\frac{dH_R(V)}{dt} = -2 \int_V \left( \bar{E} \cdot \bar{B} \right) dV + 2 \oint_S \left( \bar{A}_p \times \bar{E} \right) \cdot d\bar{S}$$

$$\nabla \cdot \bar{A}_p = 0 \quad \bar{P} = \nabla \times \bar{A}_p$$

$$\bar{A}_p \cdot \hat{n} \bigg|_S = 0 \quad \bar{B} \cdot \hat{n} \bigg|_S = P \cdot \hat{n} \bigg|_S$$

The first term represents helicity dissipation inside the volume.

The second term represents the helicity flux across the boundary.
\[ \frac{dH_R(V_a)}{dt} = -2 \int_{V_a} (\vec{E} \cdot \vec{B}) dV + 2 \oint_S (\vec{A}_p \times \vec{E}) \cdot d\vec{S} \]

Ideal conducting fluid  \[ \vec{E} = -\vec{V} \times \vec{B} \]

\[ \frac{dH_R}{dt} = 2 \oint_S \left( \left( \vec{A}_p \cdot \vec{V} \right) \vec{B} - \left( \vec{A}_p \cdot \vec{B} \right) \vec{V} \right) \cdot d\vec{S} \]

1. The first term is due to footpoint motion of the flux.

2. The second term is due to emergence of new magnetic flux.
Deducing $V$

\[
\frac{dH_R}{dt} = -2 \int_S (\vec{A}_P \cdot \vec{B}) \vec{V} \cdot d\vec{S}
\]

Local Correlation Tracking can be used to calculate the first term

Chae et al. (2001)

The two terms in the formula can combine together under the ideal magneto hydrodynamics if we use LCT method to deduce $V$

Demoulin et al. (2003)
Magnetic helicity at a certain time $t$

\[
\begin{align*}
    \frac{dH_R}{dt} &= -2 \int_S \left( \vec{A}_P \cdot \vec{U}_{LCT} \right) B_n d\vec{S} \\
    H_R(t) &= \int_0^t \frac{dH_R(t)}{dt} \, dt
\end{align*}
\]

We use the two formulas to calculate the magnetic helicity in the observation.
Counterclockwise Vortex

$H<0$

Clockwise Vortex

$H>0$
Motivation to Magnetic helicity simulation study
Helicity hemisphere Rule:

Southern hemisphere  $H > 0$
Northern hemisphere  $H < 0$

This result is based on analyzing the current helicity $h_c = \vec{B}_\parallel \cdot (\nabla \times \vec{B})_\parallel$
or the force free parameter $\alpha$ ($\nabla \times \vec{B} = \alpha \vec{B}$)

If the magnetic helicity flow accumulated in active regions (ARs) is calculated then, however, the hemispheric rule is not fulfilled very well as shown for 393 ARs in Labonte et al. (2007) and for 58 newly emerging ARs in Yang et al. (2009).

What is the more important phase of an AR evolution for magnetic helicity injection into the solar corona above an active region (a) the emerging phase (Nindos and Zhang 2002) or (b) the shearing phase related to flows in the photosphere, e.g. differential rotation (Devore 2000)?
Moreover, Devore (2000) found that the input of magnetic helicity to corona by differential rotation depends both on time and on the initial orientation of the bipole.

- Redistribution problem of magnetic helicity in the Corona

Corona  →  Ideal conducting fluid  →  Magnetic helicity is conserved

How does the magnetic helicity added to an AR evolve?
How does the magnetic helicity redistribute in the Corona?
Interaction of active regions from the point view of magnetic helicity exchange.

Magnetic helicity accumulation in the two active regions

-> strongly implies that magnetic helicity exchange had happened.

Based on the three points above, it’s appropriate to investigate the magnetic helicity evolution by using 3D MHD simulation method especially we don’t have relative accurate magnetic field measurement in corona by now.
MHD simulation model
Data-Driven MHD model

Buchner et al. 2004ab; Buchner 2006; Santos and Buchner 2007
Santos et al. 2008

\[
\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \vec{u}
\]

\[
\frac{\partial \rho \vec{u}}{\partial t} = -\nabla \cdot \rho \vec{u} \vec{u} - \nabla p + \vec{j} \times \vec{B} - \nu \rho (\vec{u} - \vec{u}_0)
\]

\[
\frac{\partial \vec{B}}{\partial t} = -\nabla \times (\vec{u} \times \vec{B} - \eta \vec{j})
\]

\[
\frac{\partial p}{\partial t} = -\nabla \cdot p \vec{u} - (\gamma - 1) p \nabla \cdot \vec{u} + (\gamma - 1) \eta \vec{j}^2
\]

Ohm Laws: \[\vec{E} = \vec{u} \times \vec{B} - \eta \vec{j}\]

Ampere equation: \[\nabla \times \vec{B} = \mu_0 \vec{j}\]

State equation: \[p = 2nk_B T\]
Magnetic helicity calculation process
\[ \Delta H = H(\vec{B}) - H(\vec{P}) \]
\[ = \int_{V_a} \left( \vec{A} \cdot \vec{B} - \vec{A}_p \cdot \vec{P} \right) dV \]

\[ \vec{B} = \nabla \times \vec{A} \quad \vec{P} = \nabla \times \vec{A}_p \]

\[ \nabla \cdot \vec{A} = 0 \quad \nabla \cdot \vec{A}_p = 0 \]

\[ \vec{B} \cdot \hat{n} \big|_s = \vec{P} \cdot \hat{n} \big|_s \quad \vec{A}_p \cdot \hat{n} \big|_s = 0 \]
 Boundary Value

At the six boundary:

\[ \vec{B} \cdot \hat{n} \bigg|_s = \vec{P} \cdot \hat{n} \bigg|_s \quad \vec{A}_p \cdot \hat{n} \bigg|_s = 0 \quad \vec{P} = \nabla \times \vec{A}_p \]

Such as Bottom boundary

\[ \vec{A}_{px} = -\frac{\partial \varphi}{\partial y} \quad \frac{\partial \vec{A}_{px}}{\partial x} + \frac{\partial \vec{A}_{py}}{\partial y} = 0 \]

\[ \vec{A}_{py} = \frac{\partial \varphi}{\partial x} \quad \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = \Delta \varphi = \vec{B} \cdot \hat{n} \bigg|_s \]
Solving of Poisson equation

\[ \nabla \times \vec{P} = 0 \quad \iff \quad \nabla \times \left( \nabla \times \vec{A}_p \right) = \nabla \left( \nabla \cdot \vec{A}_p \right) - \Delta \vec{A}_p = 0 \]

\[ \nabla \times \vec{B} = \vec{J} \quad \iff \quad \nabla \times \left( \nabla \times \vec{A} \right) = \nabla \left( \nabla \cdot \vec{A} \right) - \Delta \vec{A} = \vec{J} \]

(a) \[ \begin{cases} \Delta \vec{A}_p = 0 \\ \nabla \cdot \vec{A}_p = 0 \end{cases} \]

(b) \[ \begin{cases} \Delta \vec{A} = -\vec{J} \\ \nabla \cdot \vec{A} = 0 \end{cases} \]

We use a fast Poisson solver based on the HODIE finite-difference scheme (Boisvert, R. 1984) to resolve of the two Poisson equation.

We also clean the divergence of \( \vec{A}_p \) and \( \vec{A} \)
Magnetic helicity information from the data

- Magnetic helicity in the simulation box

\[ H_R = H(\vec{B}) - H(\vec{P}) = \int_{V_a} \left( \vec{A} \cdot \vec{B} - \vec{A}_p \cdot \vec{P} \right) dV \]

- Magnetic helicity dissipation in the simulation box

\[ \frac{dH_R}{dt} \bigg|_{\text{dissipation}} = -2 \int_{V_a} (\vec{E} \cdot \vec{B}) dV \]

- Magnetic helicity flux across the boundary

\[ \frac{dH_R}{dt} \bigg|_{\text{Boundary}} = \sum_{i=1}^{n=6} \left( \vec{A}_p \times \vec{E} \right) \cdot d\vec{S} \]

For one data with grid points $131 \times 131 \times 131$. It takes about 5 minutes to get one magnetic helicity result.
Results

- Test case AR 8210
AR 8210
Testing of magnetic helicity calculation 1

\[ H_R(V) = H_R(V_1) + H_R(V_2) \ldots + H_R(V_n) \]

\[ V = V_1 + V_2 \ldots + V_n \]

Data Courtesy of Dr. Santos
Testing of magnetic helicity calculation 2

(Ideal conducting fluid without resistivity)

\[ \frac{dH_R}{dt} = 2 \int_S \left( (A_p \cdot \vec{V}) \vec{B} - (A_p \cdot \vec{B}) \vec{V} \right) \cdot d\vec{S} \]

- Helicity from Calculation
- Helicity across boundary

Data Courtesy of Dr. Santos
Testing of magnetic helicity calculation 3
(conducting fluid with resistivity)

\[
\frac{dH_R(V)}{dt} = -2 \int_{V_a} (\vec{E} \cdot \vec{B}) dV + 2 \oint_{S} (\vec{A}_p \times \vec{E}) \cdot d\vec{S}
\]
Case of the interacting ARs
(AR 9188 and AR 9192)
MDI data

NOAA 9188

NOAA 9192
Spatial Fourier filter to select the first 8 modes for the data

L (AR9192)  R (AR9188)
Adding Vortex at the boundary  (step 1)

Clockwise
Adding Vortex at the boundary (step 2)

Counterclockwise
Helicity evolution for L and R in step 1

Left region \( H > 0 \)

Right region \( H > 0 \)
Helicity evolution for $L$ and $R$ in step 2

Left region  from $H>0$ to $H<0$

Right region  $H>0$ while decreasing
Negative magnetic helicity
Accumulated in this region
Conclusion and Discussion
We have developed a scheme for calculating magnetic helicity of active regions from simulation data and verified for the case of AR 8210.

The helicity transport calculations also verified the physical correctness of the 3D MHD simulation.

Initial results for two interacting ARs (9188 and 9192) show that the magnetic helicity exchange process between active regions can be confirmed and appropriately described by the simulation.

So far the imposed boundary conditions (velocity vortex) are simple (clock or counterclockwise). A real velocity pattern will be applied in the future.

Further detailed analysis of helicity accumulation and exchange processes in ARs will be carried out using the MHD simulation.
Thank you!
\[ H_m = \int \vec{A} \cdot \vec{B} d^3 x = 5\Phi^2 = Twist + Writhe \]

\( \Phi \) Magnetic flux

\( H_k = +\Phi^2 \)

\( H_k = -\Phi^2 \)

\( \Phi \) Magnetic flux

Simulation box $V$