Techniques for stellar magnetic field measurements
From the **Sun** to the **stars**
From the Sun to the stars

- Going from Sun to stars means **losing**
  - spatial resolution
  - photons and hence sensitivity
- & **gaining** in diversity of stars & parameters
  - Hot stars: different magnetic structure
  - Cool stars: how usual or unusual is today’s Sun
  - Probe non-solar parameter regimes
- Depending on the type of star different measurement techniques have to be applied
Stars with large-scale field: e.g. Ap stars

Field in early-type stars is dominated by low-order multipoles, e.g. dipoles.

A tilted dipole produces a roughly sinusoidal variation of Stokes V.
Complex fields of cool stars and missing spatial resolution!

- Solar magnetogram
- Note complexity of the magnetic signal: magnetic polarities are mixed often on small scales!
- Average over the whole solar disk gives extremely small Stokes signals
Cancellation of magnetic polarity

unresolved star with flux is distributed on small scales

- positive polarity magnetic field
- negative polarity magnetic field
Measuring B on Sun-like stars

- For slowly rotating stars, the polarisation signal is strongly reduced by mixture of magnetic polarities on the stellar surface. Detect field from its weak influence on intensity spectra.

- Example: Even ε Eri with $fB \approx 160$ G (outside starspots) needs high S/N for field to be visible.

Rüedi et al. 1997
Rapid rotation: boon and bane

+ Rapid rotation produces more activity and larger magnetic flux (lecture 9) ➔ easier to measure

+ Larger activity ➔ larger magnetic features ➔ less mixing on small scales?

+ Zeeman degeneracy is reduced by rapid rotation: Zeeman Doppler Imaging can be used. Works for \( v \sin i = 10-100 \, \text{km/s} \) and \( i = 20-70^\circ \)

- With increasing vsini, S/N is reduced as line gets weakened. Reason for 100 km/s limit on ZDI
Doppler Imaging: the principle

- Brightness structures on surface of rapidly rotating star map onto shape of line profile & its variation with time
Doppler Imaging: does it work?

- Aim: recreate 2-D image of stellar surface
- Data: spectrum (1-D) + its variation (1-D)
- Ill-posed inverse problem. Soluble, but needs regularization (e.g. maximum entropy)
- Tests using synthetic stars have been successful
Zeeman Doppler Imaging

Use Stokes spectra to determine distribution of field (Semel 1989)

Radial field
Latitude of B: 30°

Azimuthal field
30°

Radial field
60°
Limitations of ZDI

- Determining 2-D maps of full magnetic vector (3x2 = 6-dimensional data set) from just 2 Stokes parameters I and V is not trivial (Q and U are not measurable on cool stars: in Ap stars all 4 Stokes params can be used, Piskunov et al.)

- Misses a significant, in cool stars even dominant fraction of the field (since it is ordered on small scales)

- Is not sensitive to fields in dark features, e.g. starspots: strongest field regions in cool stars are not well covered

- S/N is an issue

- All limitations inherent to Doppler Imaging also apply
Least Squares Deconvolution (LSD)

Part of observed spectrum. Stokes V: red, Stokes I: black

LSD V and I profiles

Proposed by Semel & Li (1992) named by Donati et al. (1997). Basically averages signal from 1000s of lines. Brings out signal hidden in noise. LSD $V$, but not $Q$ & $U$, may be modelled as single line!
Magnetic field regimes: stronger fields

\[ H = -\frac{\hbar}{2m} \nabla^2 + V(r) + \xi(r) \mathbf{L} \cdot \mathbf{S} + \left( -\frac{e}{2mc} \mathbf{B} \cdot (\mathbf{L} + 2\mathbf{S}) + \frac{e^2}{8mc^2} (Br \sin \theta)^2 \right) \]

- Perturbation theory regimes:
  - Quadratic magnetic term << linear term << spin-orbit term: (linear) Zeeman effect
  - Quadratic magnetic term << spin-orbit term << linear term: Paschen-Back effect
  - Spin-orbit term << linear term << quadratic magnetic term: quadratic Zeeman effect
  - Electronic binding term << quadratic magnetic term: needle atoms

Following J. Landstreet
$B$ at which different regimes are reached

- May estimate size of magnetic terms by taking $L \sim \hbar$, $r \sim$ Bohr radius $a_0$, $V \sim Ze/r$. We find

- For normal atoms and $B < 50$ kG (5 T), most atomic lines are in linear Zeeman regime

- Above about 100 kG quadratic term becomes important. Quadratic Zeeman effect is observed in lines of H

- Above about 10 MG magnetic terms become comparable to Coulomb term, perturbation methods no longer work. Must solve structure of atom in combined (external and internal) field

Following J. Landstreet
Zeeman and Paschen-Back effects

- In Paschen-Back regime, \( L \) and \( S \) decouple, so \( J \) is not a good quantum number. Now \( M_L \) and \( M_S \) good quantum numbers \( \Rightarrow \) perturbation energy \( (e/2mc) B(M_L+2M_S) \hbar \)

- all lines are split by same amount. Only three line components \( (\Delta M = -1,0,1) \)

- Atomic PBE: main application WDs. Only few lines in non-degenerate stars. Molecular PBE: common, also in cool stars

Follow J. Landstreet
Molecular Zeeman & PB effect

- Molecular lines are interesting for cool stars: cool stars or starspots (and sunspot umbrae) show strong molecular absorption features.
- Spectral lines of many diatomic molecules display Zeeman splitting. Molecular energy levels often lie close together, PBE takes place already at low field strengths (often a few 100 G) and must be included.
- Full theory for arbitrary molecular electronic states
Molecular Zeeman & PB effect

- Peculiarities due to the PBE ⇒ New diagnostics and higher sensitivity
  - Stokes profile asymmetries ⇒ Net polarization across line profiles
  - Wavelength shifts and polarization sign changes depending on B
Quadratic Zeeman effect

- The effect of the quadratic term in the Hamiltonian of an atom in a magnetic field is to shift all spectral line components in H to shorter wavelengths by about

\[ \Delta \lambda_Q \approx \left( -\frac{e^2}{8mc^3h} \right) \lambda^2 n^4 \left( 1 + M_L^2 \right) B^2 \]

where \( \lambda \) is in Å, \( a_0 \) is the Bohr radius, and \( n \) and \( M_L \) are the principal and magnetic quantum numbers of the upper level.

- Quadratic effect dominates for hydrogen H10 for \( B > 10 \) kG.

- At 1 MG, H8 is shifted by about 350 km/s relative to H\( \alpha \), an easily detectable effect (Preston 1970, ApJ 160, L143).

- Polarisation effects are similar to those of Zeeman effect, but components are not split symmetrically about unsplit line.
Atomic structure in huge fields

- For fields above 10 MG the magnetic terms in the Hamiltonian are comparable to the Coulomb terms, and the structure of the combined system must be solved consistently.

- Has been solved for H, and to a large extent for He (review: e.g. Becken & Schmelcher 2002, Phys Rev A, 65, 033416).

- Basically, each line component decouples from the others and moves about (in $\lambda$) in a dramatic way.

- Absorption lines in stellar spectra for fields over about 50 MG are affected by fact that the line positions vary rapidly with B. If B is not constant over the stellar surface. Lines occur at wavelengths where for some range of B the absorption wavelength does not change rapidly.
Splitting of H lines in strong fields

- Plotted are the \( \lambda \) of the Zeeman components of the lowest Ly, H, Paschen and Brackett lines of hydrogen vs. \( \beta = 4.7 \cdot 10^9 \) G.

- Components move over large parts of spectrum.

⇒ Identifying them can be quite adventurous.

Wunner 1990
Splitting of H lines in strong fields (contd.)

- For large $B$ values, the $\sigma$-components of spectral lines vary rapidly with wavelength. They are almost undetectable on stars where $B$ varies by a factor of two.

- Some $\pi$-like transitions vary little over a range of $B$ ("stationary components"). Such transitions can produce useful lines over a range of field strengths in the range of hundreds of MG.

Techniques for measuring white dwarf magnetic fields

Fields of white dwarfs are observed using several detection methods based on the behaviour of atoms & electrons in increasingly strong fields

- For $B$ below about 100 kG, the normal Zeeman effect (and perhaps the Paschen-Back effect in H) are used, as in non-degenerate stars
- From 100 kG to about 10 MG, the linear Zeeman effect is overtaken by the quadratic Zeeman effect
- Above 10 MG, even the spectrum of H is no longer easily recognised. It is greatly distorted, and continuum polarisation (circular and then linear) becomes detectable
- In polars $e^-$ cyclotron radiation is observed & employed
Measurement of field on Grw +70 8247

- Top panel: computed hydrogen line positions vs. B
- Middle panel: observed spectrum
- Bottom panel: H line positions computed by another group
Continuum polarisation of white dwarf radiation in MG fields

- Free $e^-$ spiral around field lines $\Rightarrow$ continuum absorption is *dichroic* (cyclotron radiation). Right & left circularly polarised light is absorbed *differently* $\Rightarrow$ continuum becomes circularly polarised by field with comp. along line of sight. In visible range this happens for $B > 10$ MG.

- For $B \geq 100$ MG a similar effect gives continuum linear polarisation.
