Stellar Dynamos

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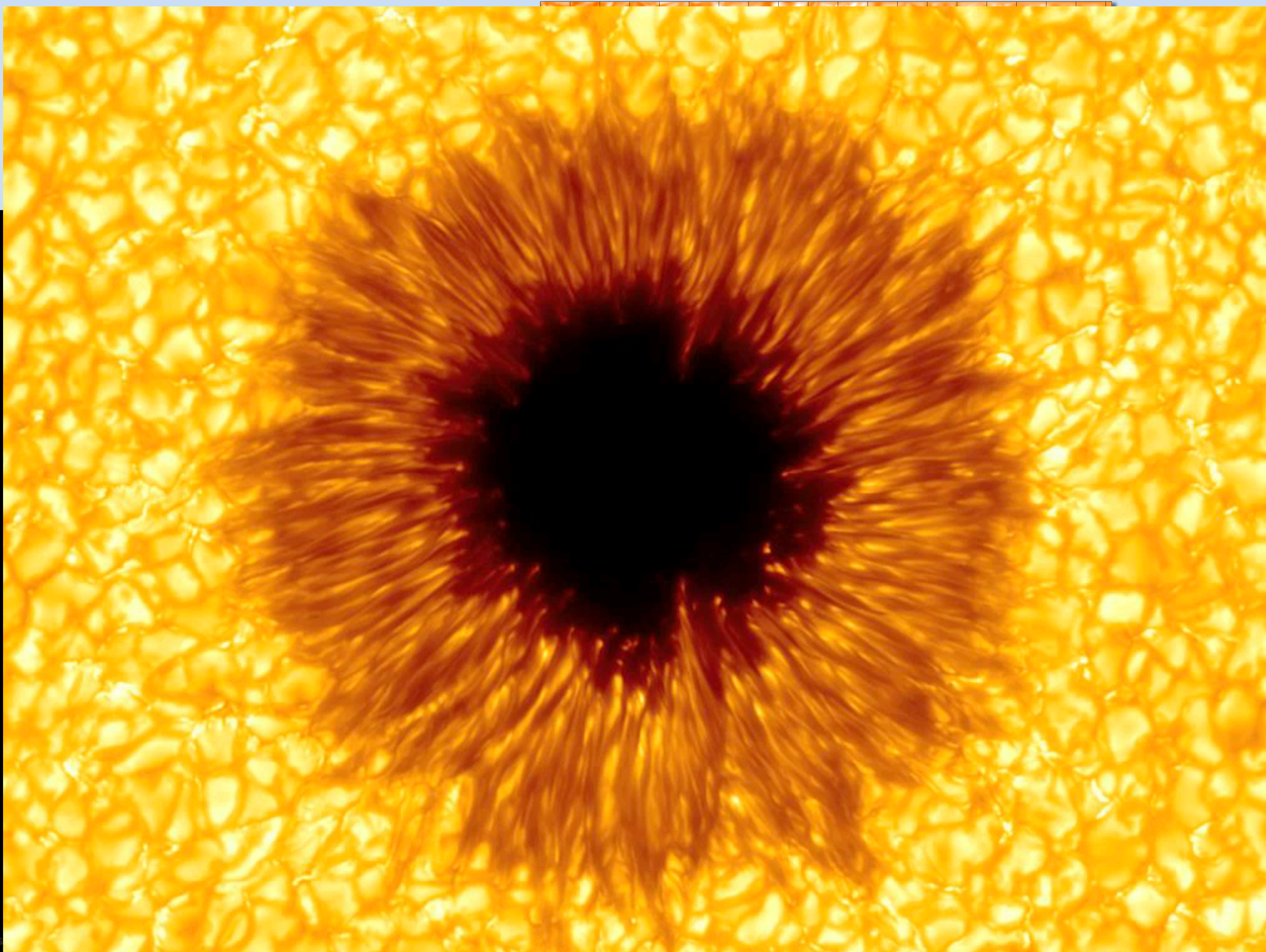
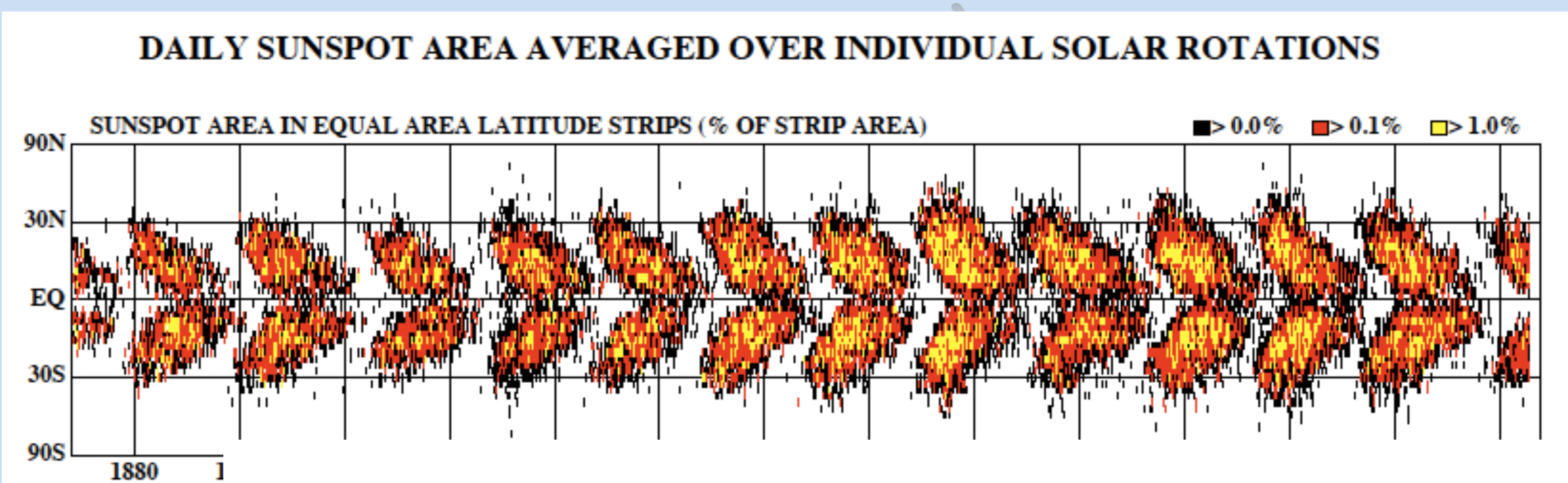
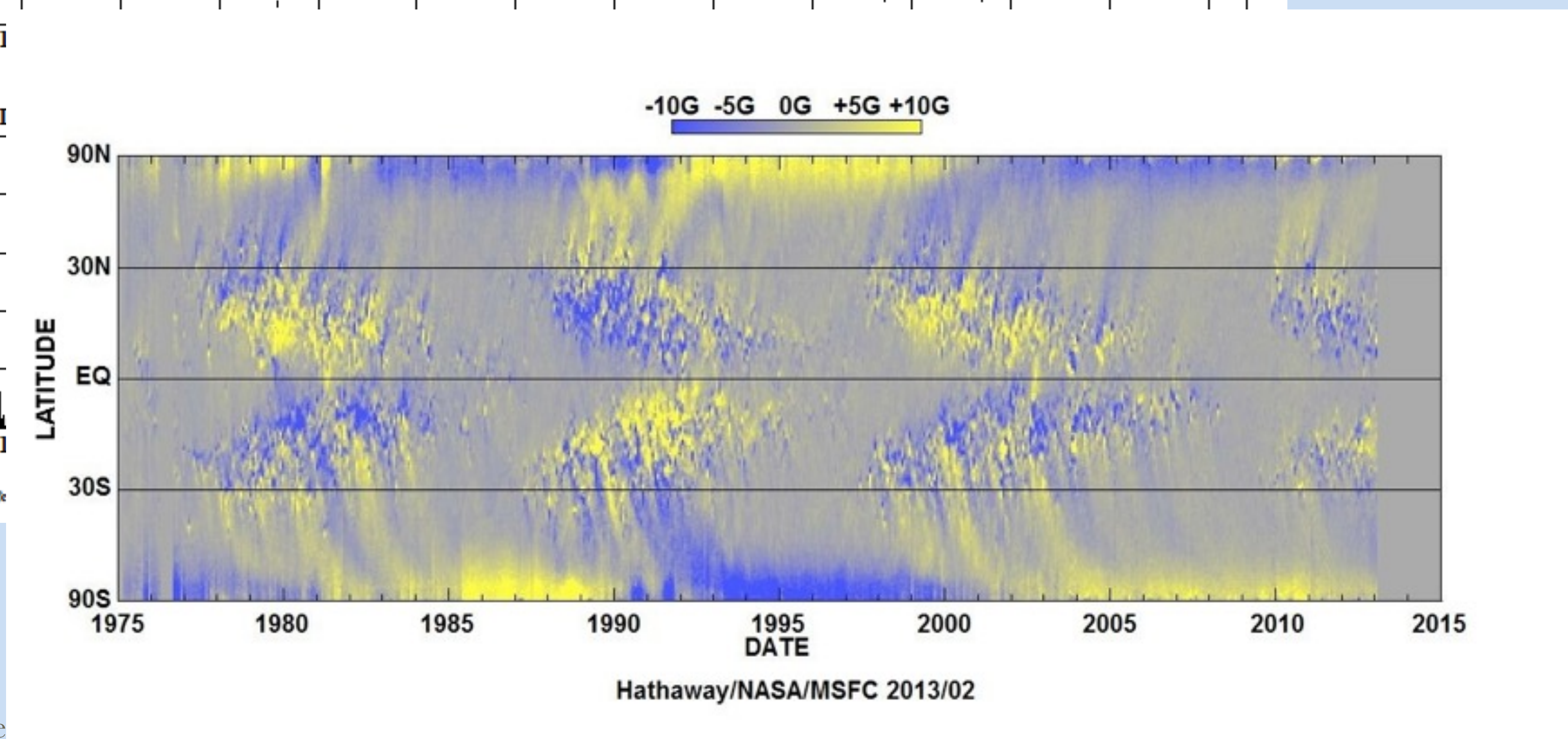
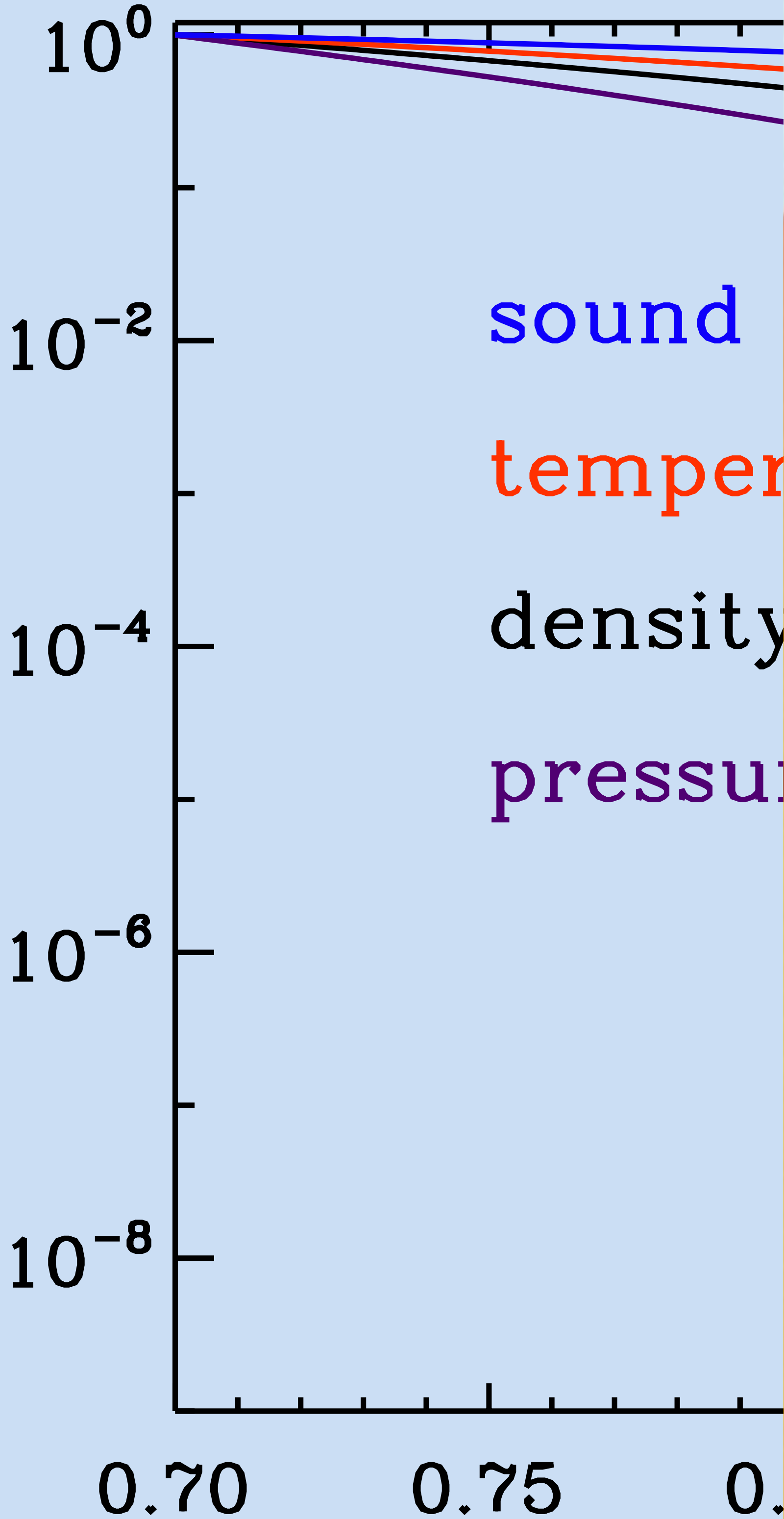


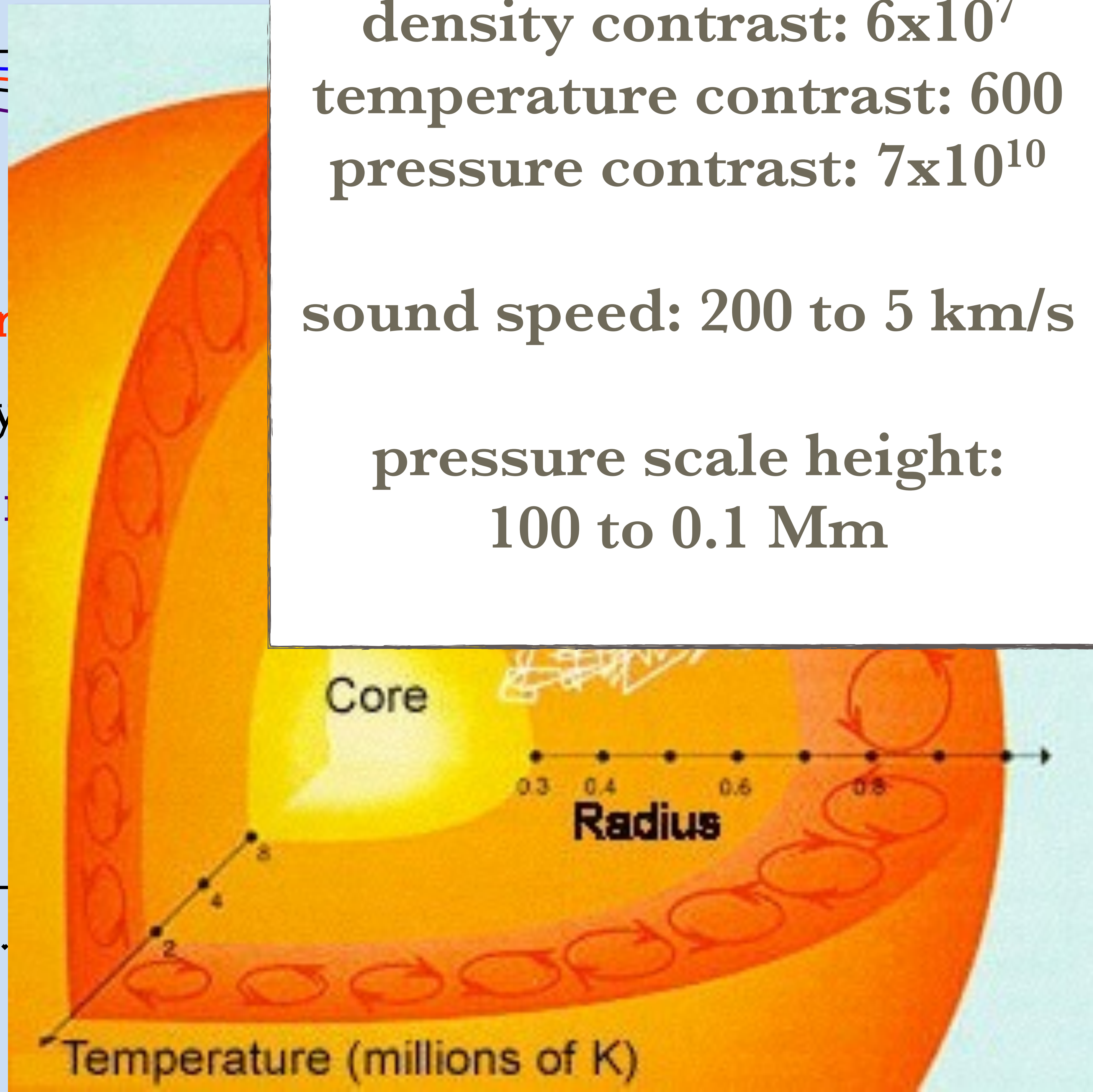
Solar Cycle


DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS
SUNSPOT AREA IN EQUAL AREA LATITUDE STRIPS (% OF STRIP AREA)
> 0.0%
> 0.1%
> 1.0%
90N
30N
EQ
30S
90S
1880
0.5
0.4
0.3
0.2
0.1
0.0
12
1880


-10G -5G 0G +5G +10G
90N
30N
EQ
30S
90S
LATITUDE
1975
1980
1985
1990
1995
2000
2005
2010
2015
DATE
Hathaway/NASA/MSFC 2013/02

Number	Symbol	Formular	The Sun	Earth Core
Ekman number	E	$\nu/\Omega d$	10^{-15}	10^{-15}
Taylor number	Ta	$\frac{4}{\mathrm{E}^2}$	10^{31}	10^{31}
Rayleigh number	Ra	$\frac{GM_\odot d^4}{\nu\chi R_\odot^2}\left(-\frac{1}{c_\mathrm{P}}\frac{\mathrm{d}s_\mathrm{hs}}{\mathrm{d}r}\right)_{r_\mathrm{m}}$	10^{24}	10^{27}
Prandtl number	Pr	ν/χ	10^{-6}	10^{-1}
mag. Prandtl number	Pr_M	ν/η	10^{-3}	10^{-6}
Rossby number	Ro	$u/\Omega d$	1	10^{2}
Coriolis number	Co	$2/\mathrm{Ro}$	1	10^{-2}
Reynolds number	Re	$u\,d/\nu$	10^{12}	10^{9}
mag. Reynolds number	Re_M	$u\,d/\eta$	10^{9}	10^{5}
Pecley number	Pe	$u\,d/\chi$	10^{6}	10^{8}

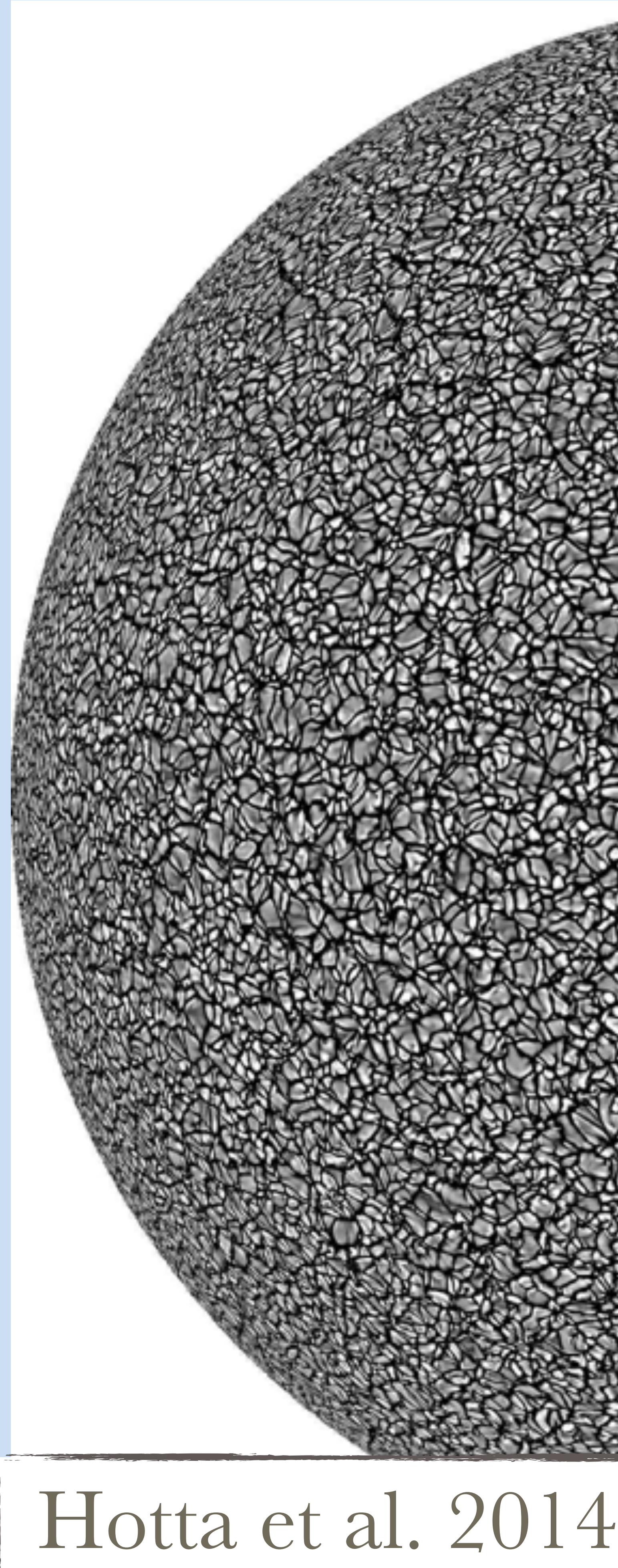


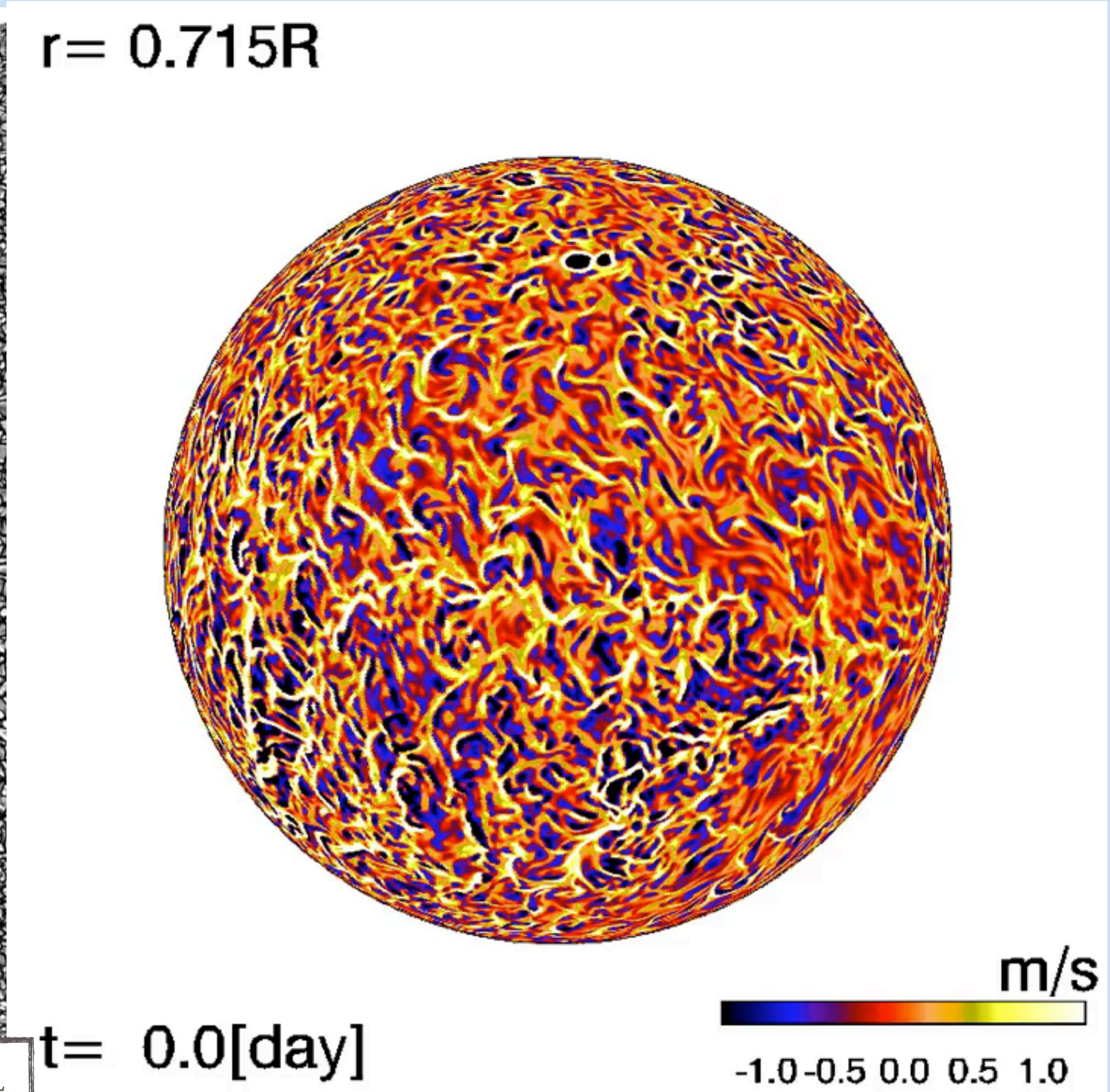


density contrast: 6×10^7
temperature contrast: 600
pressure contrast: 7×10^{10}

sound speed: 200 to 5 km/s

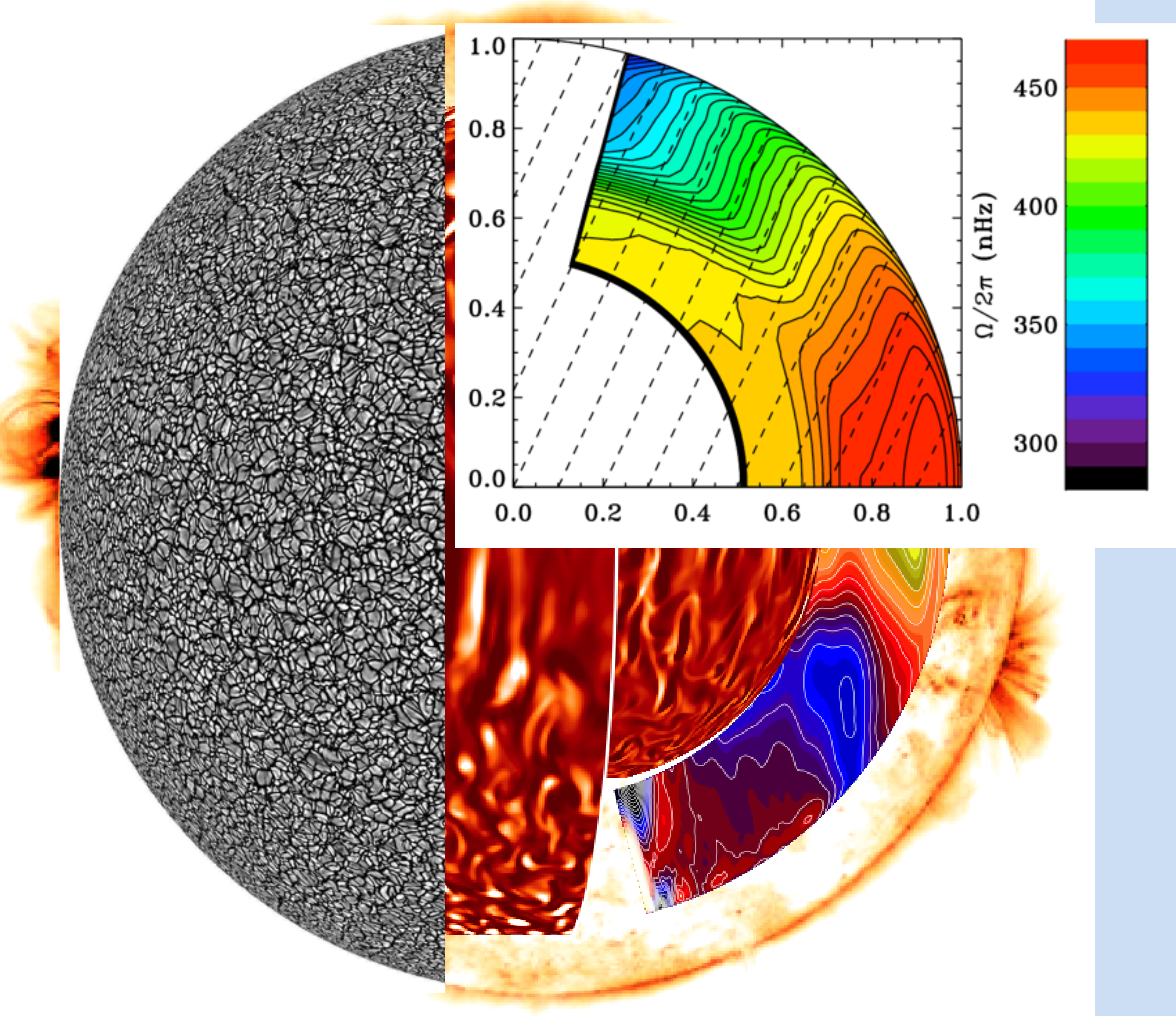
pressure scale height:
100 to 0.1 Mm





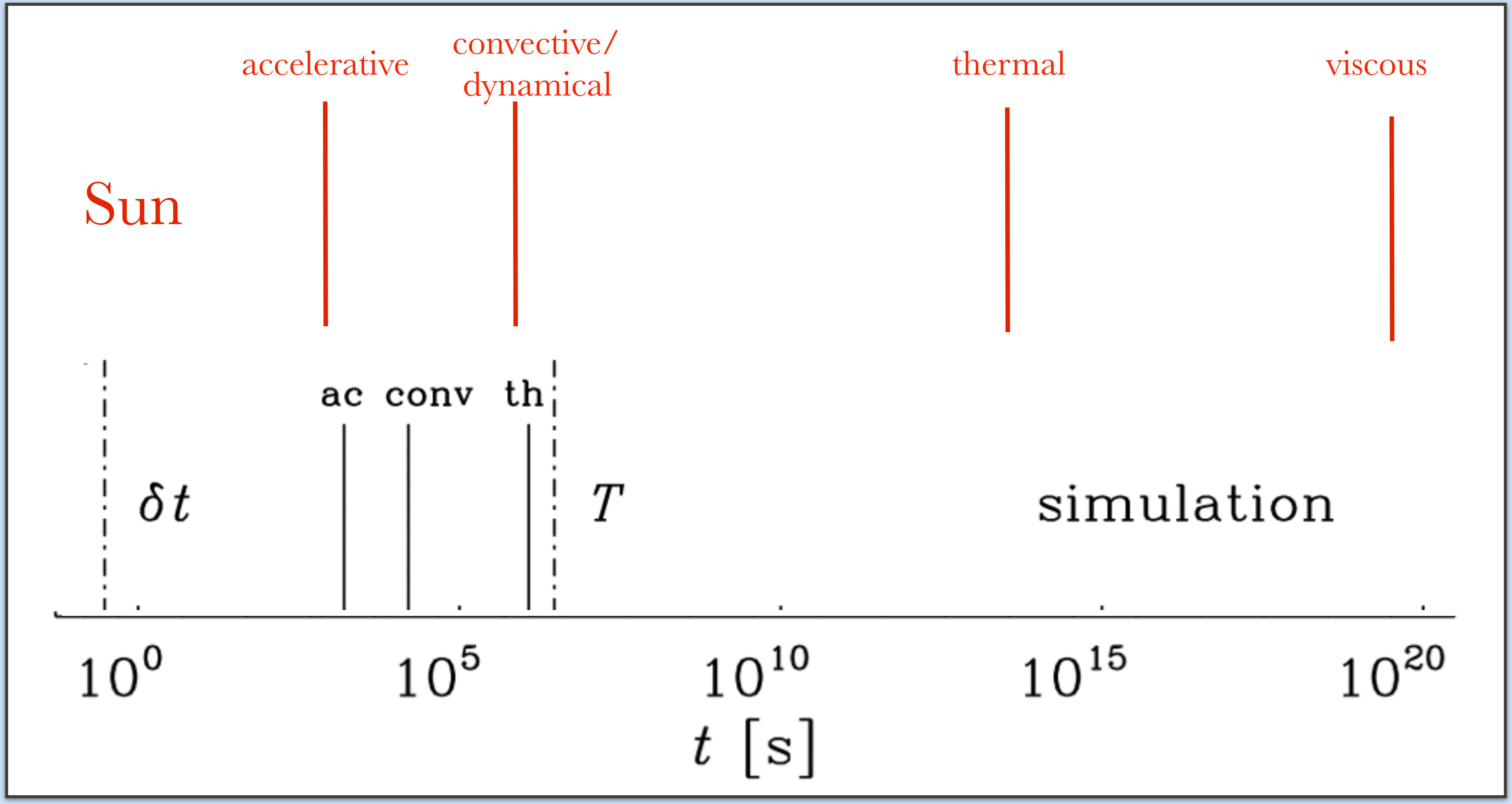
Hotta et al. 2014

Number	Symbol	The Sun [bot]	The Sun [top]
turnover time	τ	months	mins
velocity	u	1 km/s	0.1 km/s
pressure scale height	H_p	100 Mm	0.1 Mm
Ekman number	E	10^{-16}	10^{-12}
Taylor number	Ta	10^{32}	10^{24}
Rayleigh number	Ra	10^{24}	10^{25}
Prandtl number	Pr	10^{-3}	10^{-10}
mag. Prandtl number	Pr_M	1	10^{-7}
Rossby number	Ro	1	10^{3}
Coriolis number	Co	1	10^{-3}
Reynolds number	Re	10^{13}	10^{10}
mag. Reynolds number	Re_M	10^{10}	10^{6}
Pecley number	Pe	10^{3}	10^{7}



1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Ω/2π (nHz)
450
400
350
300

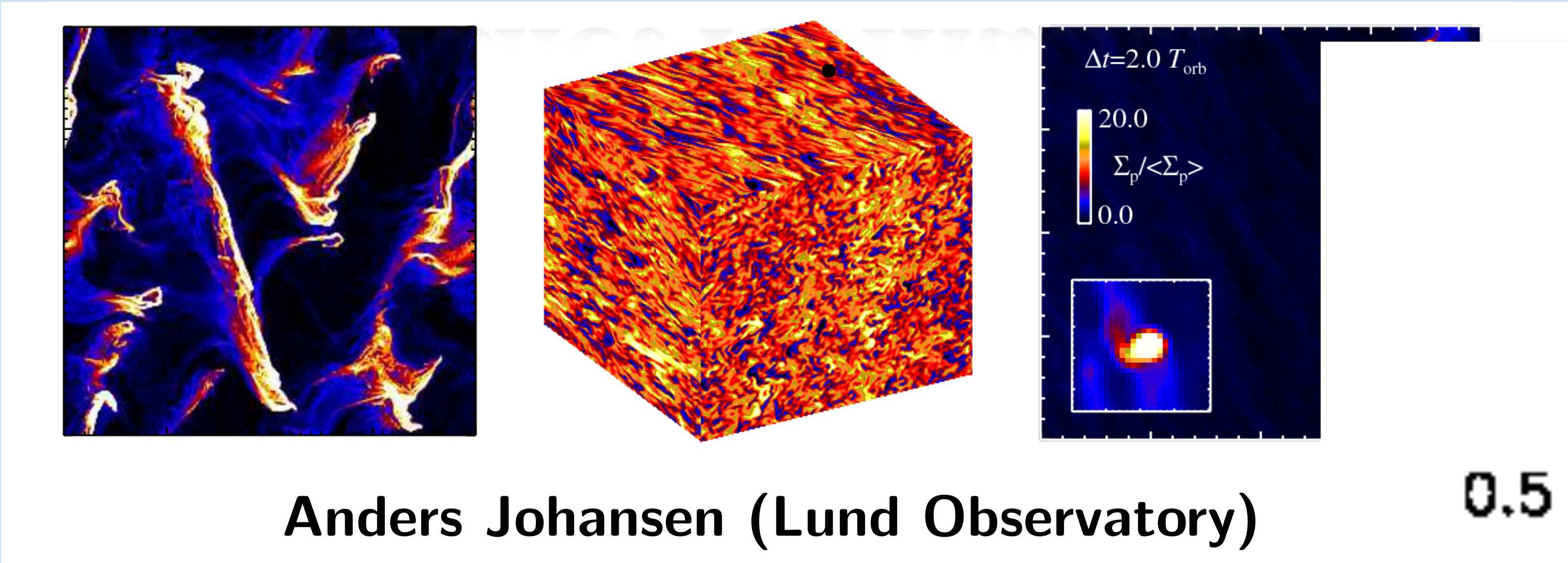
Time scales



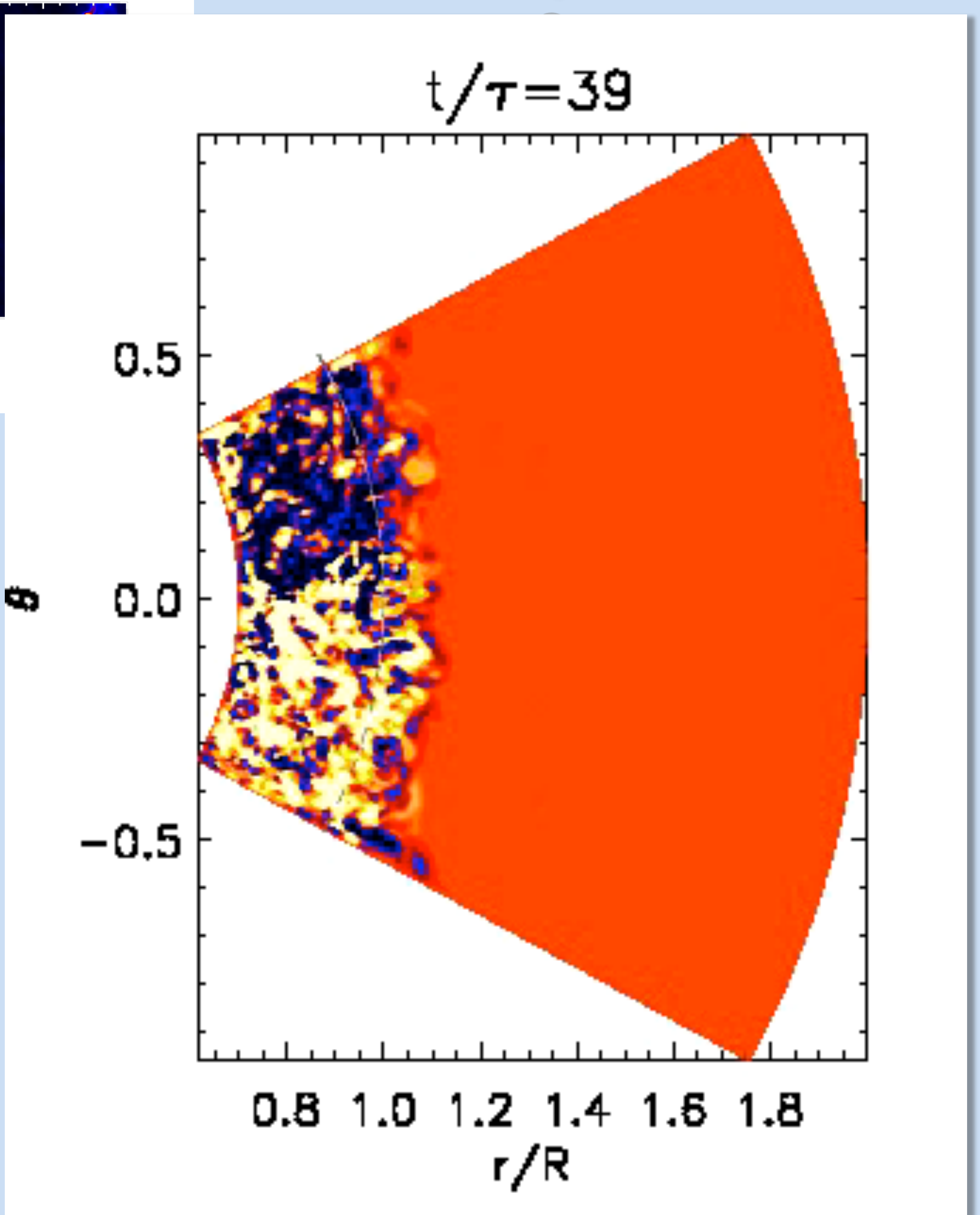
acclerative
convective/
dynamical
thermal
viscous
Sun
ac
conv
th
δt
T
simulation
10^0
10^5
10^{10}
10^{15}
10^{20}
t [s]

The Pencil Code

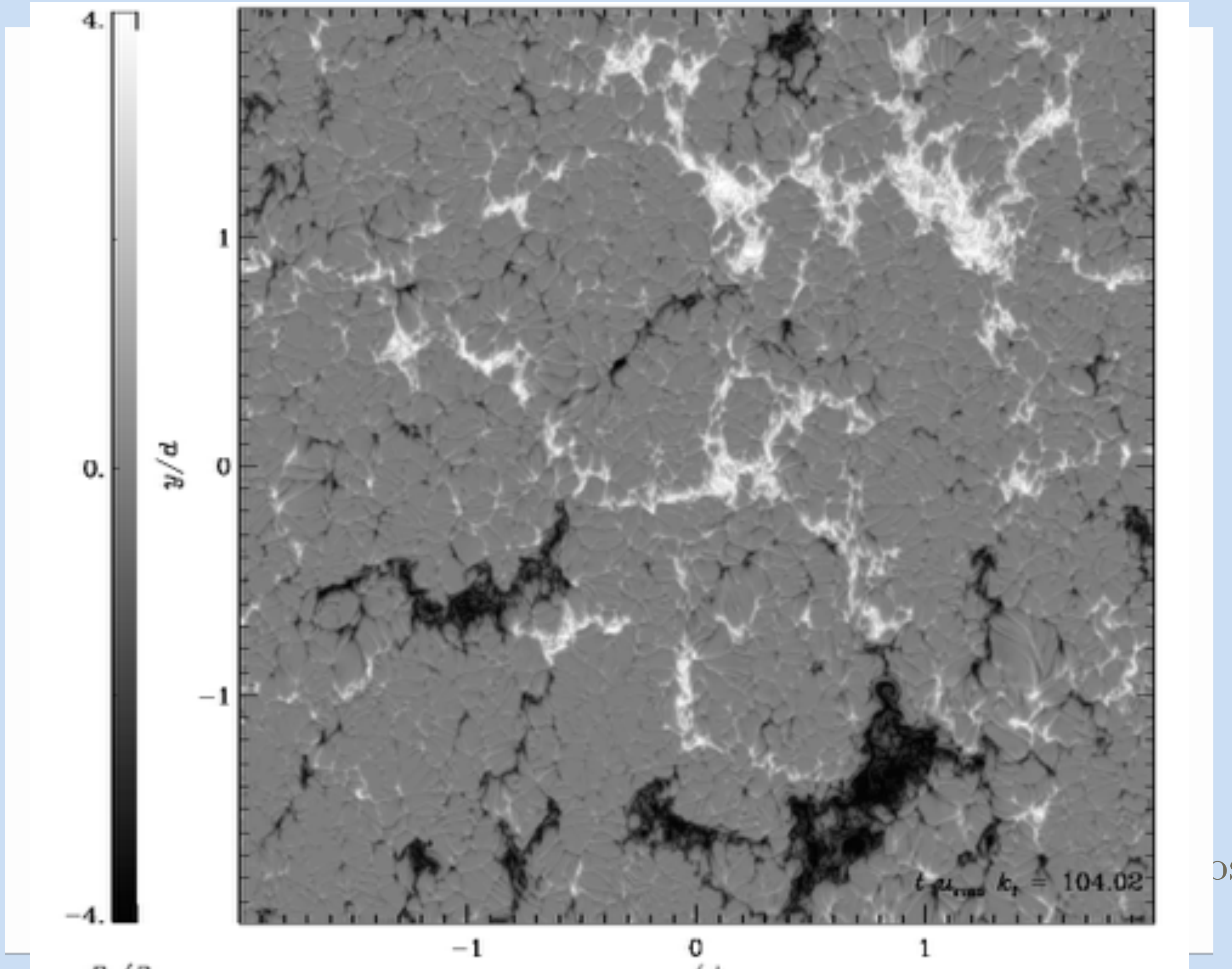
Planet formation



Coronal ejection



Flux concentrations



Global convective dynamo simulations

$$\frac{\partial A}{\partial t} = u \times B + \eta \nabla^2 A$$

$$\frac{D \ln \rho}{Dt} = -\nabla \cdot u$$

$$\frac{Du}{Dt} = g - 2\Omega_0 \times u + \frac{1}{\rho}\left(J \times B - \nabla p + \nabla \cdot 2\nu\rho S\right)$$

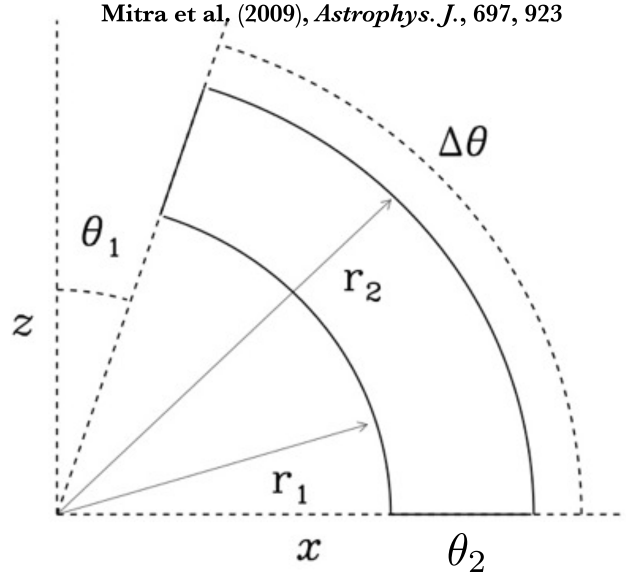
$$T\frac{Ds}{Dt} = \frac{1}{\rho}\nabla \cdot (K\nabla T + \chi_t \rho T \nabla s) + 2\nu S^2 + \frac{\mu_0 \eta}{\rho} J^2 - \Gamma_{\mathrm{cool}}(r),$$

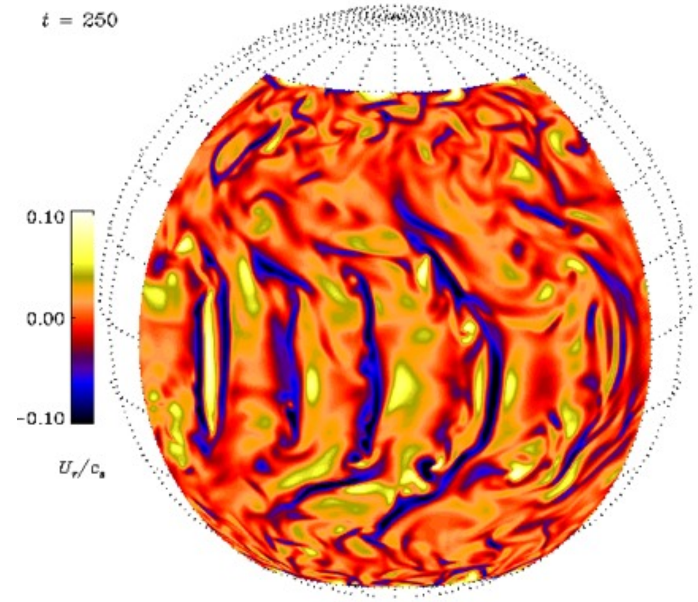
- high-order finite-difference code
- scales up efficiently to over 60.000 cores
- compressible MHD

https://github.com/pencil-code/pencil-code/

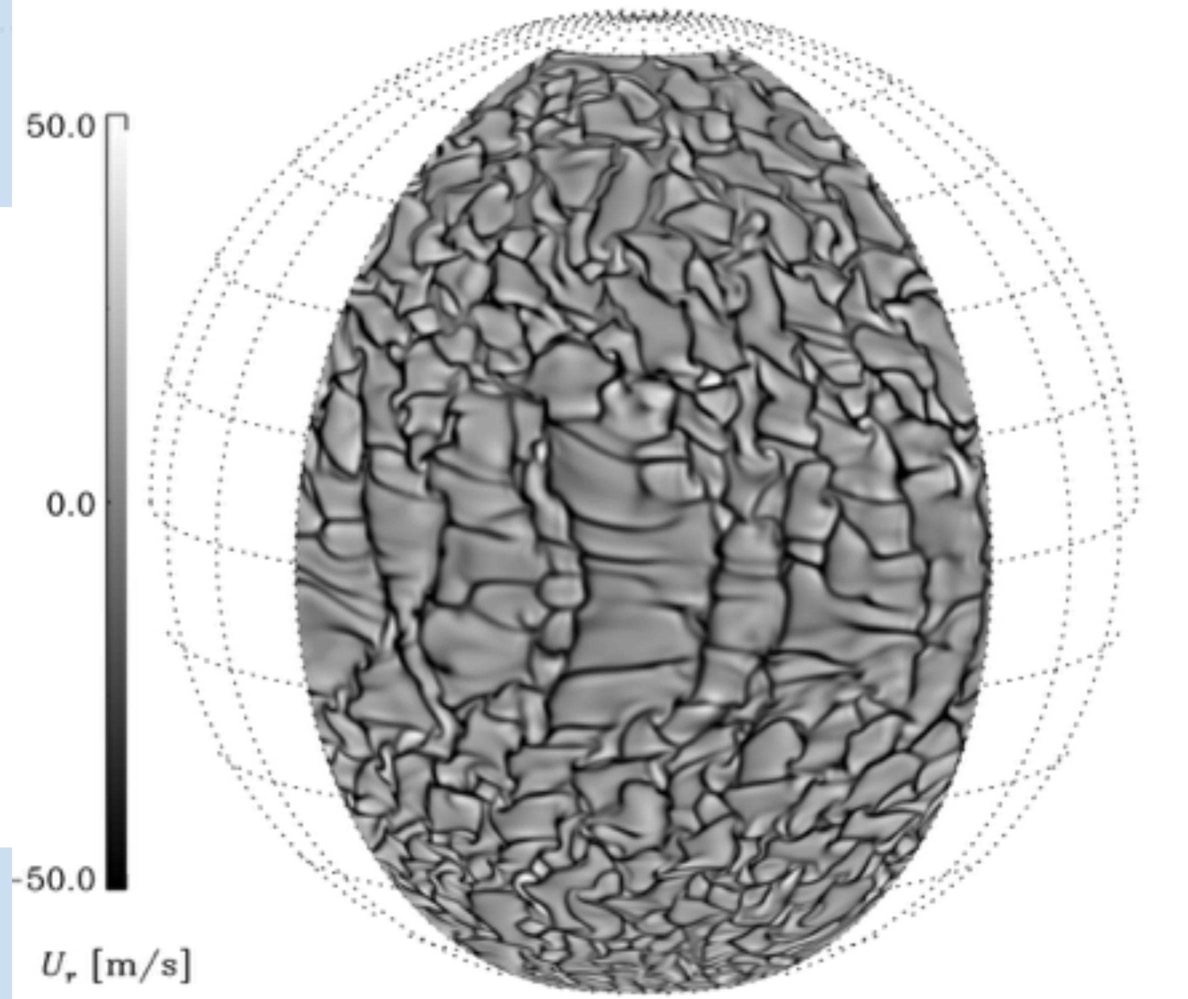
Global convective dynamo simulations

Mitra et al, (2009), *Astrophys. J.*, 697, 923





Käpylä et al. (2010b), *Astron. Nachr.*, 331, 73

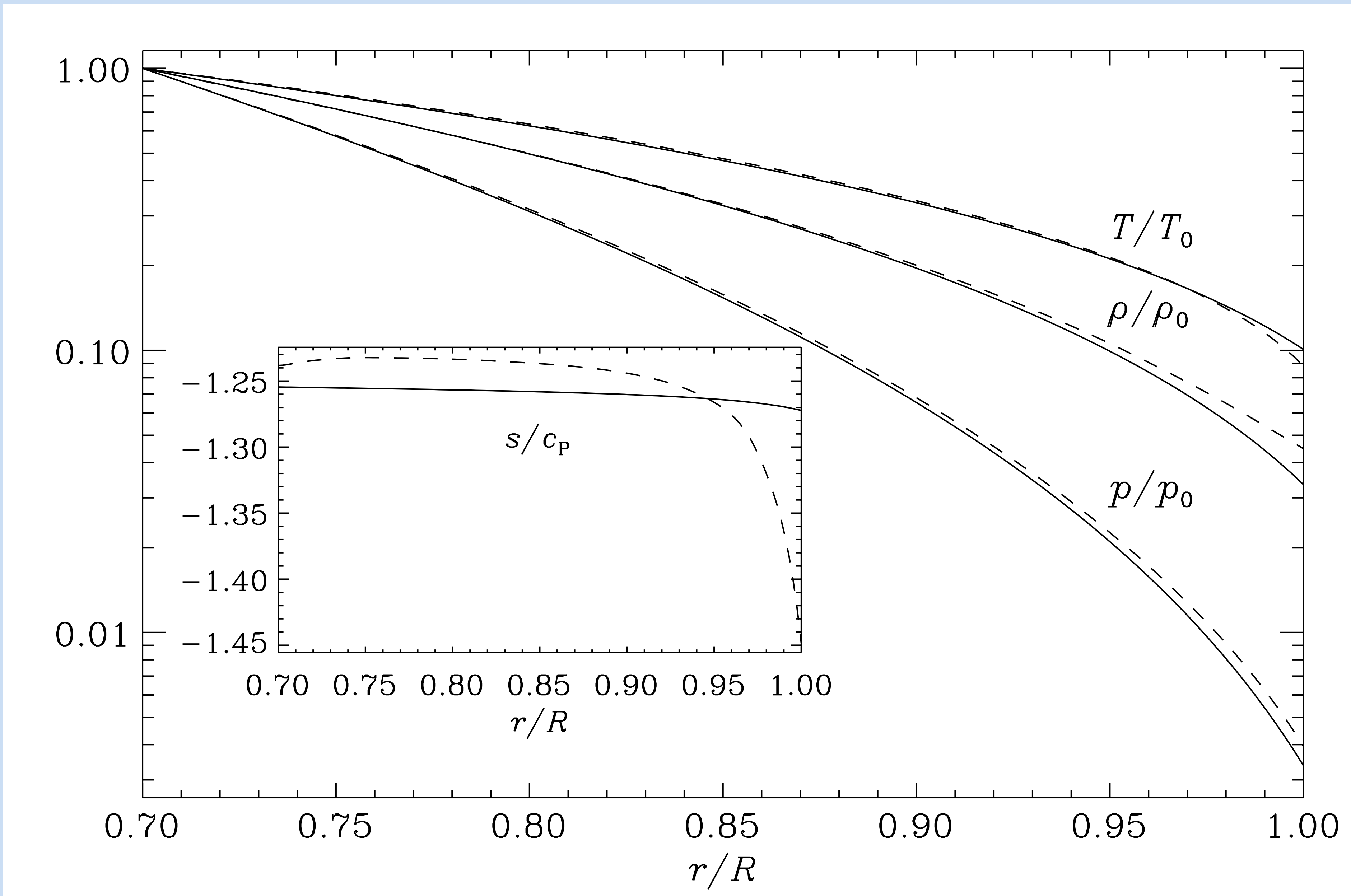


$0.7R < r < R \qquad \theta_1 < \theta < \theta_2 \qquad 0 < \phi < \Delta\phi \qquad k_{\rm f} = 2\pi/\Delta R$

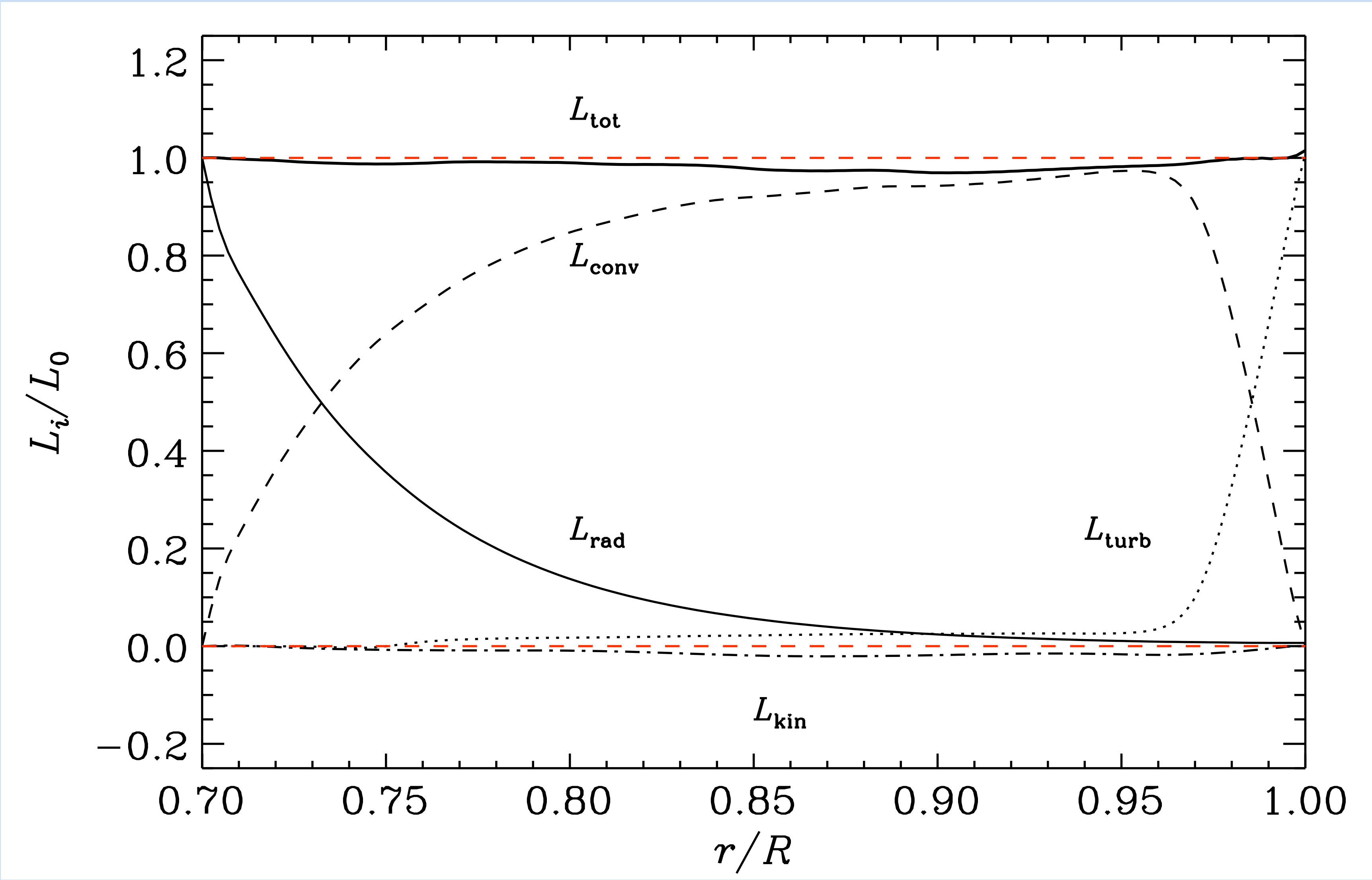
We model a spherical sector (`wedge') where only parts of the latitudinal and longitudinal extents are taken into account.

Normal field condition for *B* at the outer radial boundary and perfect conductor at all other boundaries. Impenetrable stress-free boundaries on all boundaries.

Initial conditions



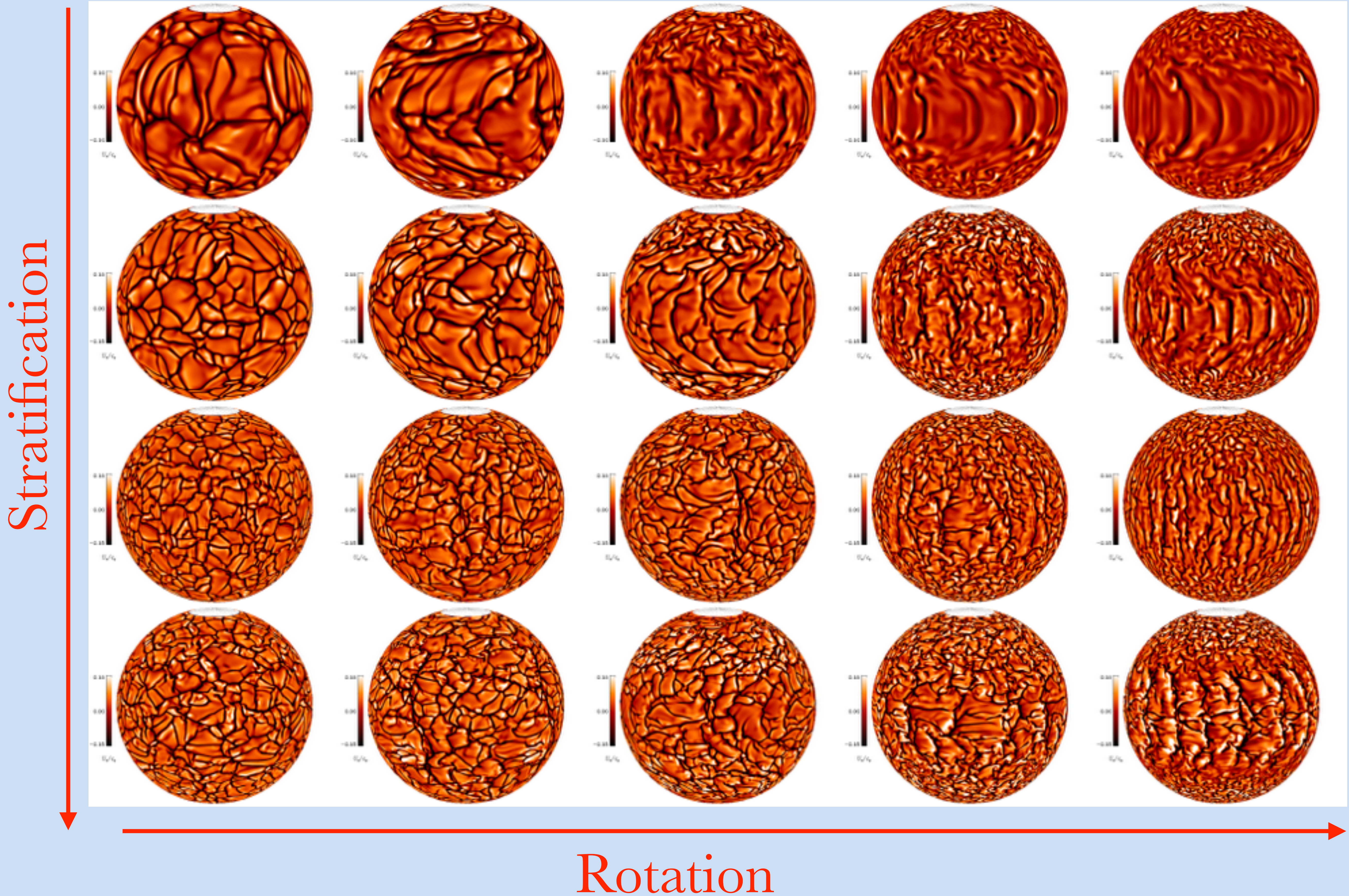
Characterization of the model



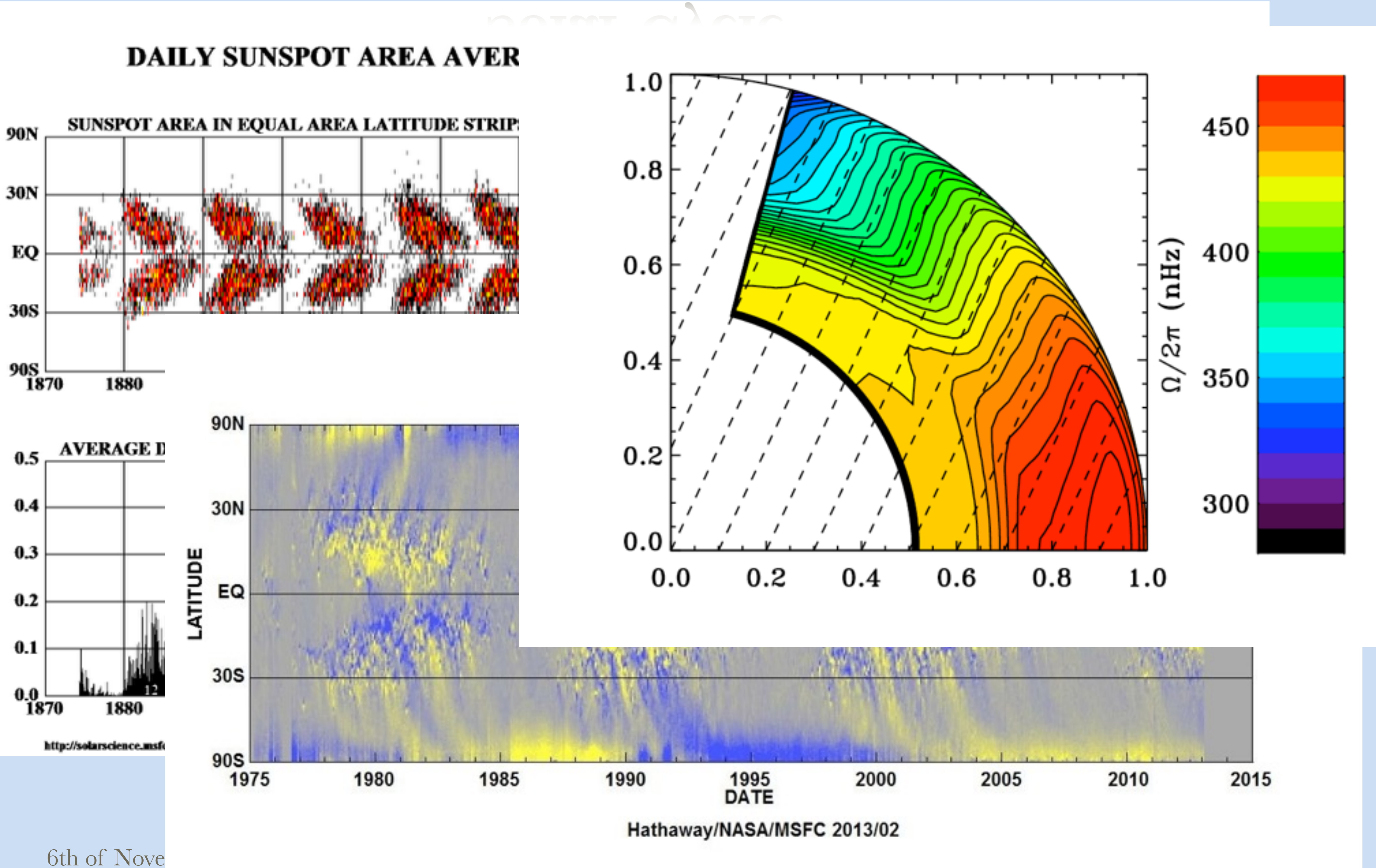
Pr=2
Pm=1

Ta=1.5 x 10^{10}

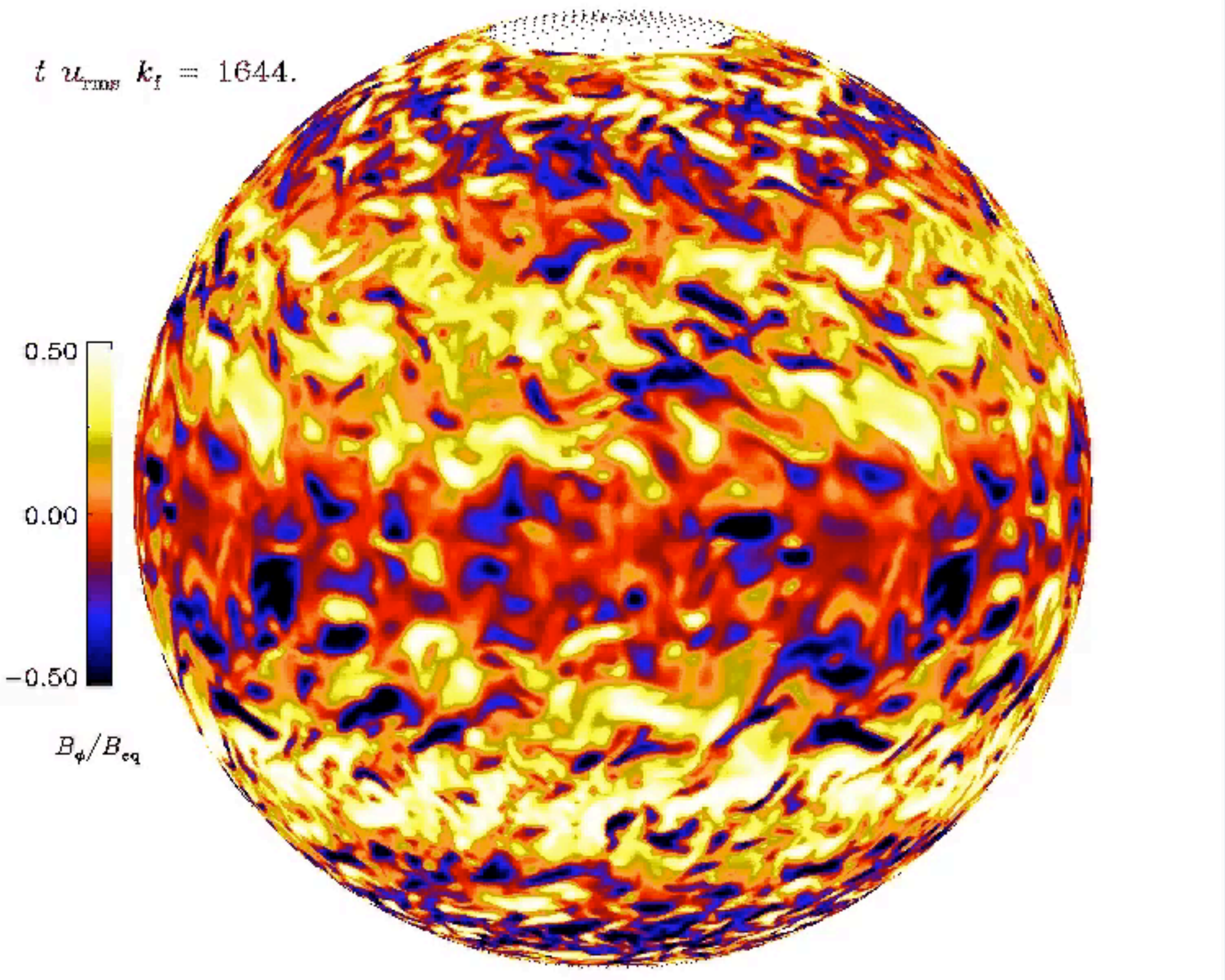
Ra=4 x 10^{7}


Stratification
Rotation

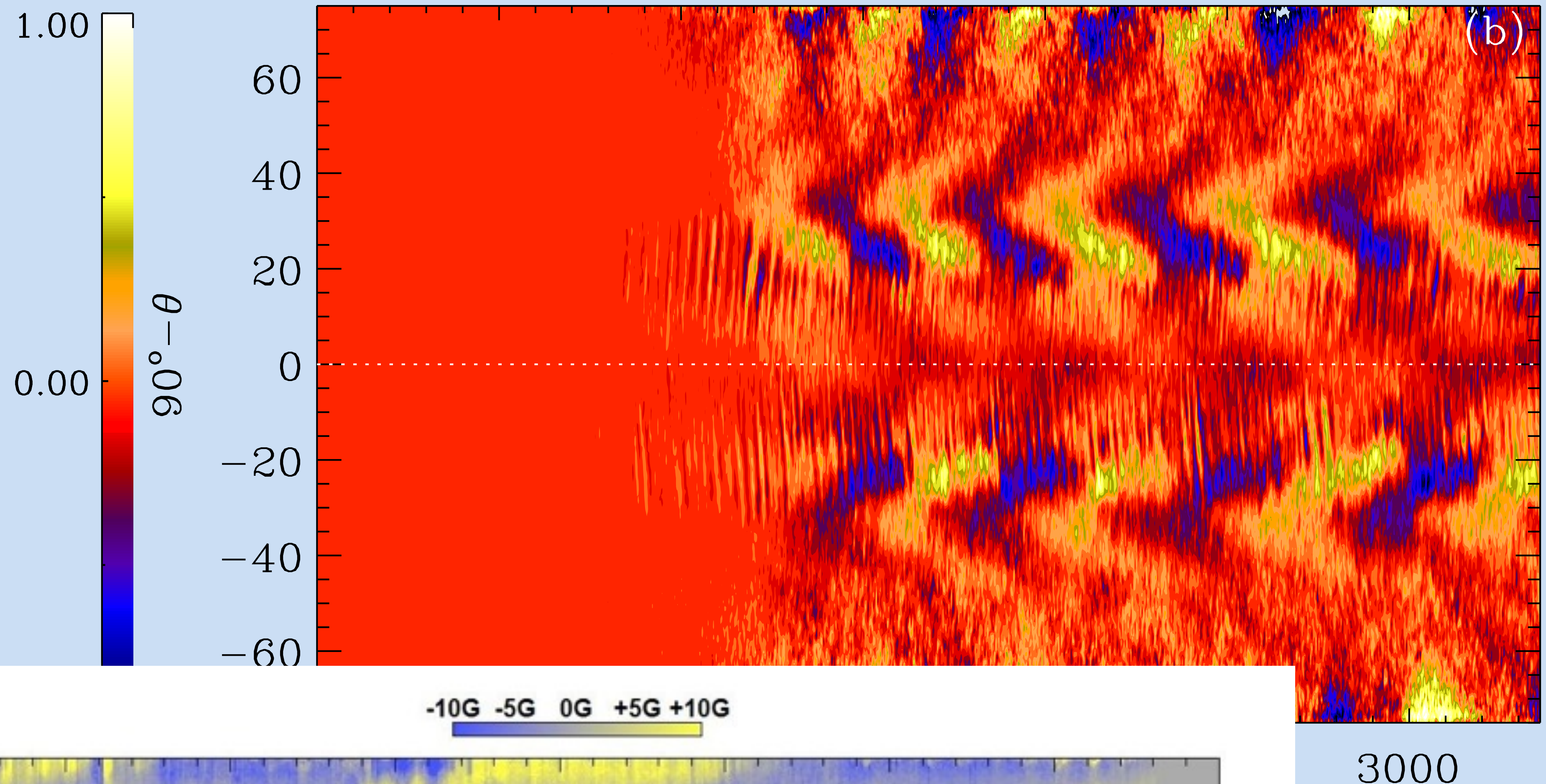
Solar Cycle

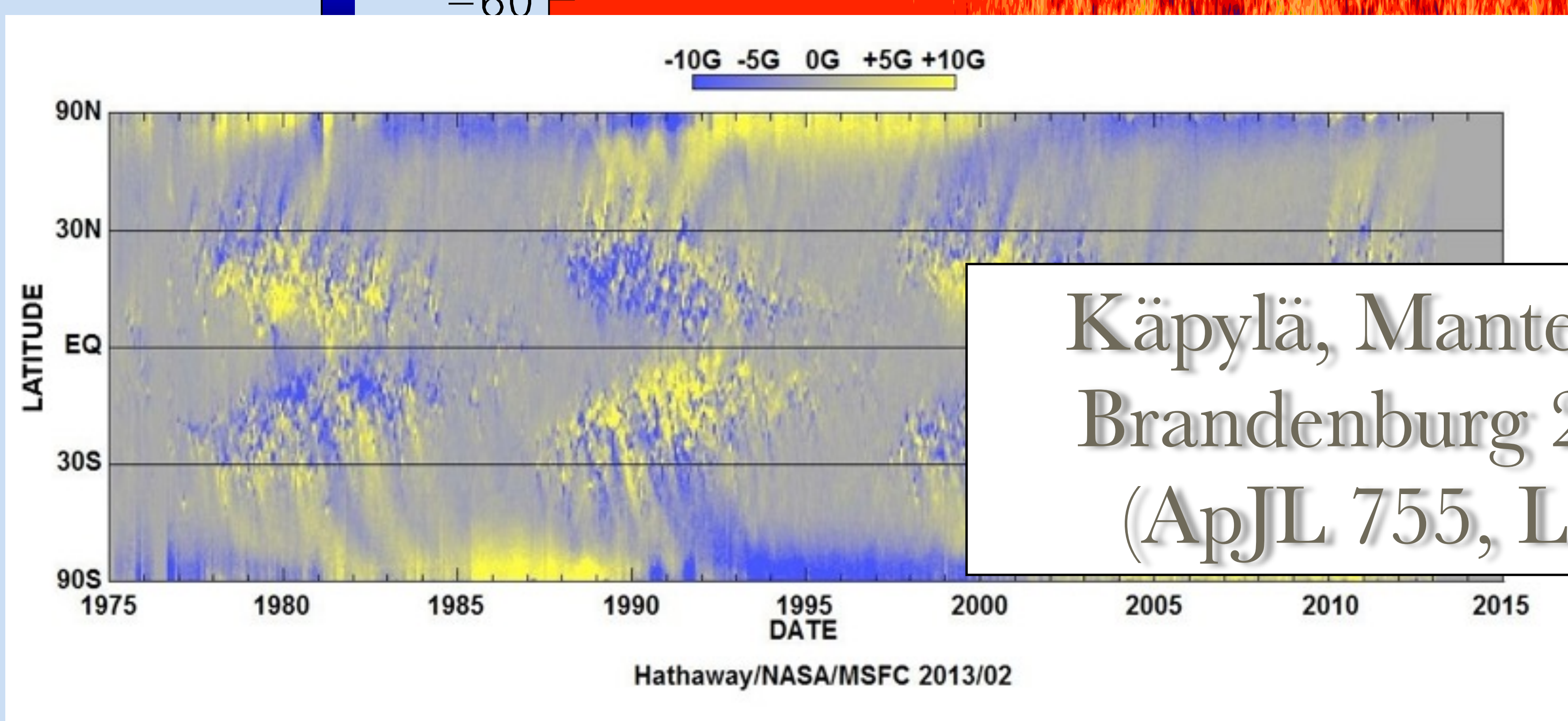


DAILY SUNSPOT AREA AVER
SUNSPOT AREA IN EQUAL AREA LATITUDE STRIP
90N
30N
EQ
30S
90S
1870
1880
AVERAGE D
0.5
0.4
0.3
0.2
0.1
0.0
1870
1880
http://solarscience.msf
90N
30N
EQ
30S
90S
LATITUDE
1975
1980
1985
1990
1995
2000
2005
2010
2015
DATE
Hathaway/NASA/MSFC 2013/02
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Ω/2π (nHz)
450
400
350
300


$t\ u_{rms}\ k_f\ =\ 1644.$
0.50
0.00
−0.50
B_ϕ/B_{eq}

Equatorward Migration I





Käpylä, Mantere & Brandenburg 2012 (ApJL 755, L22)

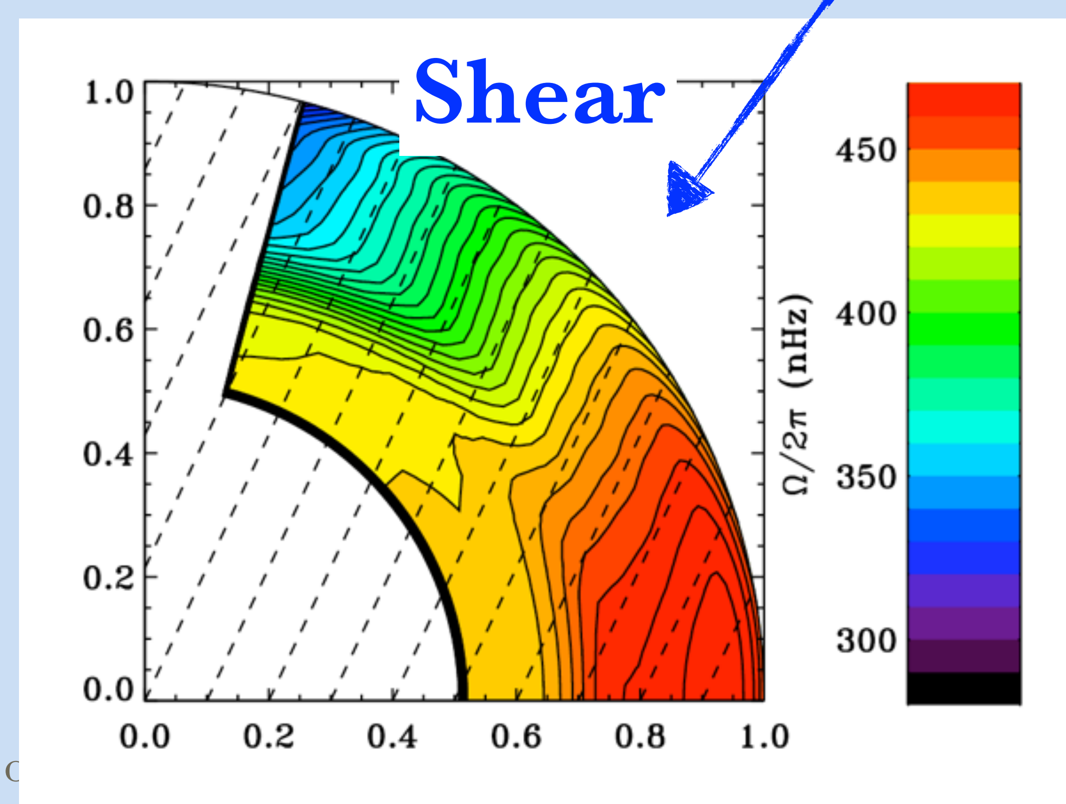
Dynamos

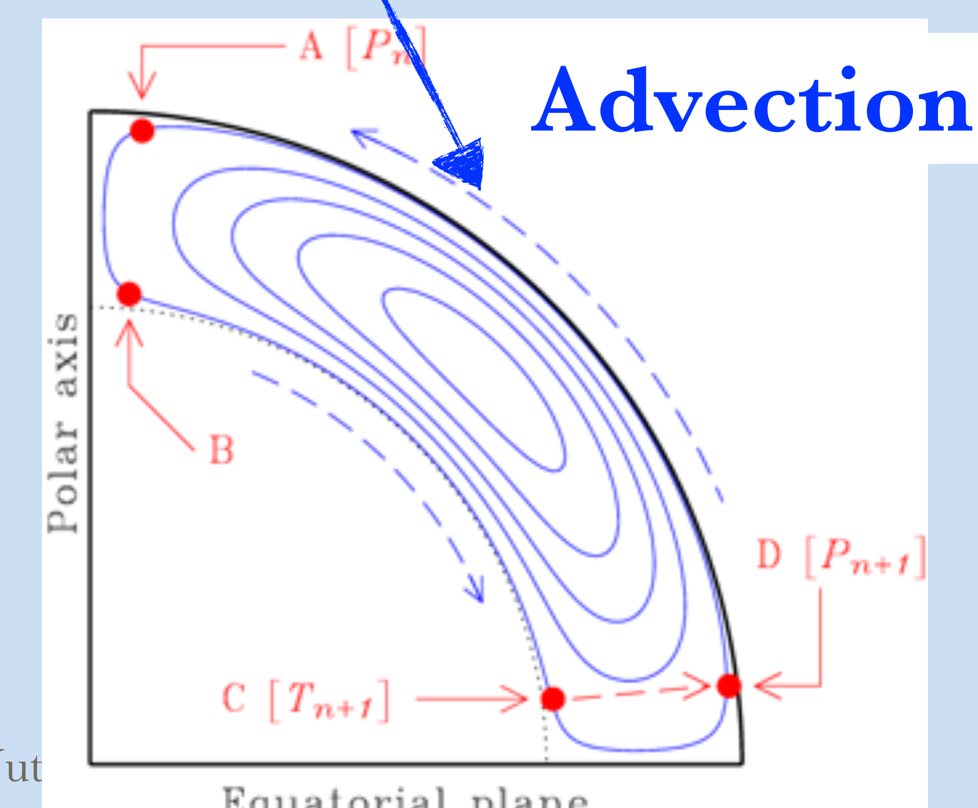
$$\frac{\partial B}{\partial t} = \nabla \times (u \times B) - \nabla \times \eta J$$

$$B = \overline{B} + b' \qquad u = \overline{U} + u'$$

$$\frac{\partial \overline{B}}{\partial t} = \nabla \times (\overline{U} \times \overline{B} + \overline{u' \times b'}) - \nabla \times \eta \overline{J}$$

$$\nabla \times (\overline{U} \times \overline{B}) = (\overline{B} \cdot \nabla)\overline{U} - \overline{B}(\nabla \cdot \overline{U}) - (\overline{U} \cdot \nabla)\overline{B}$$





Electromotive force

$$\overline{\mathcal{E}} = \overline{u' \times b'}$$

$$= \alpha \overline{B} + \gamma \times \overline{B} + \beta \nabla \times \overline{B} + \delta \times (\nabla \times \overline{B}) + \kappa \nabla \overline{B}$$

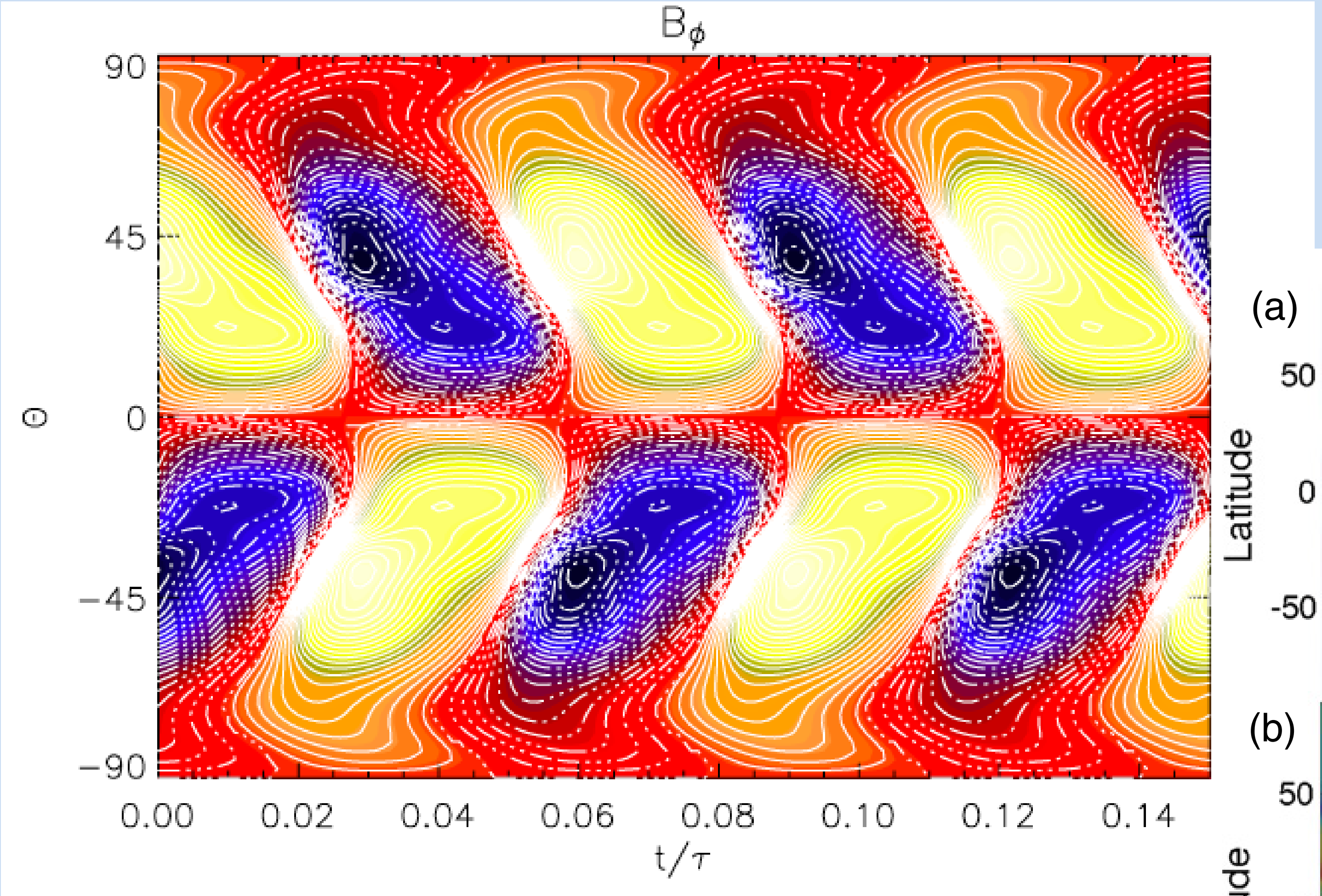
Simplifications:

$$\frac{\partial \overline{B}_{\text{pol}}}{\partial t} = \alpha \nabla \times \overline{B}_{\text{tor}} + \eta_T \Delta \overline{B}_{\text{pol}} \qquad \overline{B} = \overline{B}_{\text{pol}} + \overline{B}_{\text{tor}}$$

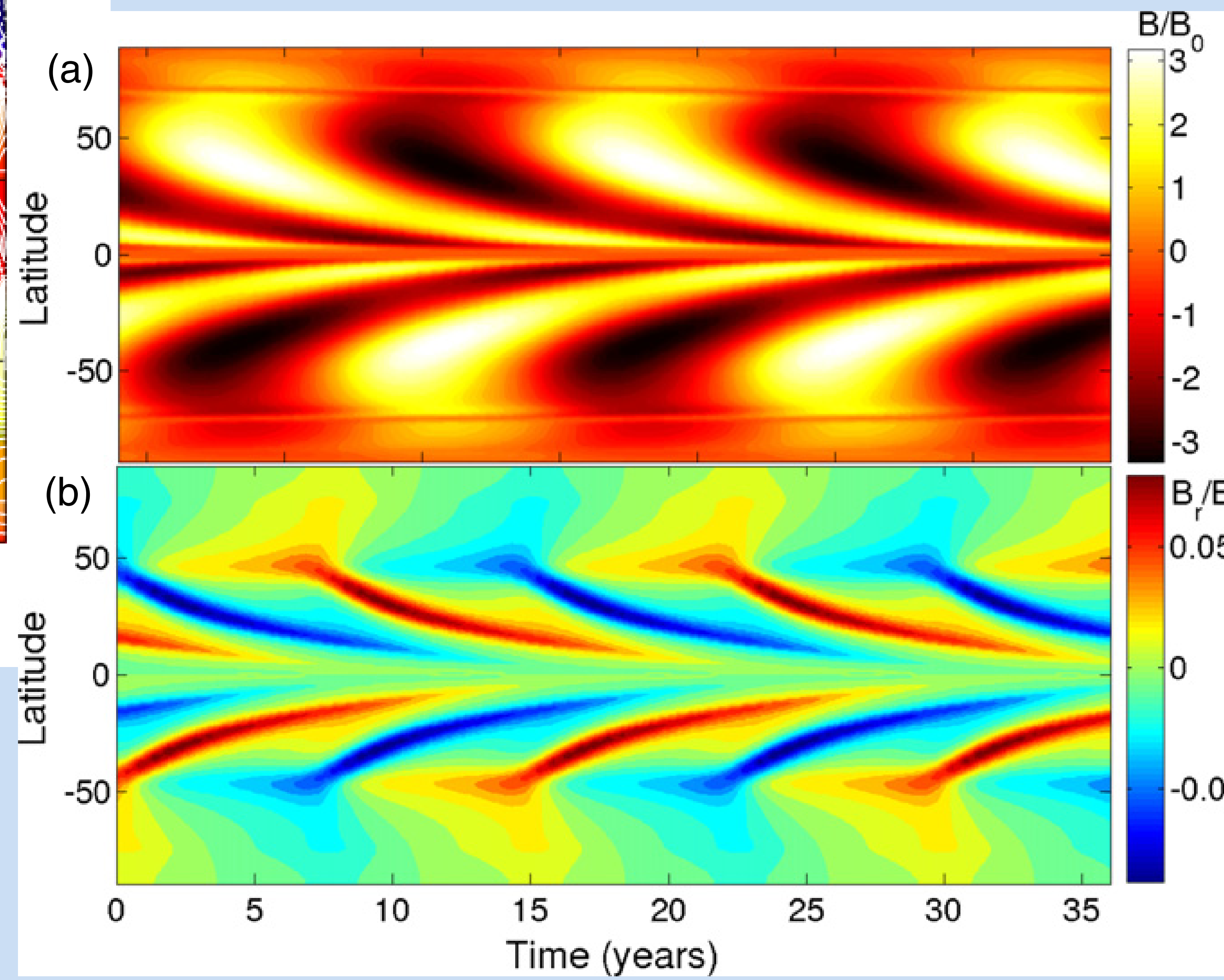
$$\frac{\partial \overline{B}_{\text{tor}}}{\partial t} = (\overline{B}_{\text{pol}} \cdot \nabla) \overline{u}_{\text{tor}} + \alpha \nabla \times \overline{B}_{\text{pol}} + \eta_T \Delta \overline{B}_{\text{tor}}$$

$\alpha\Omega$ dynamo

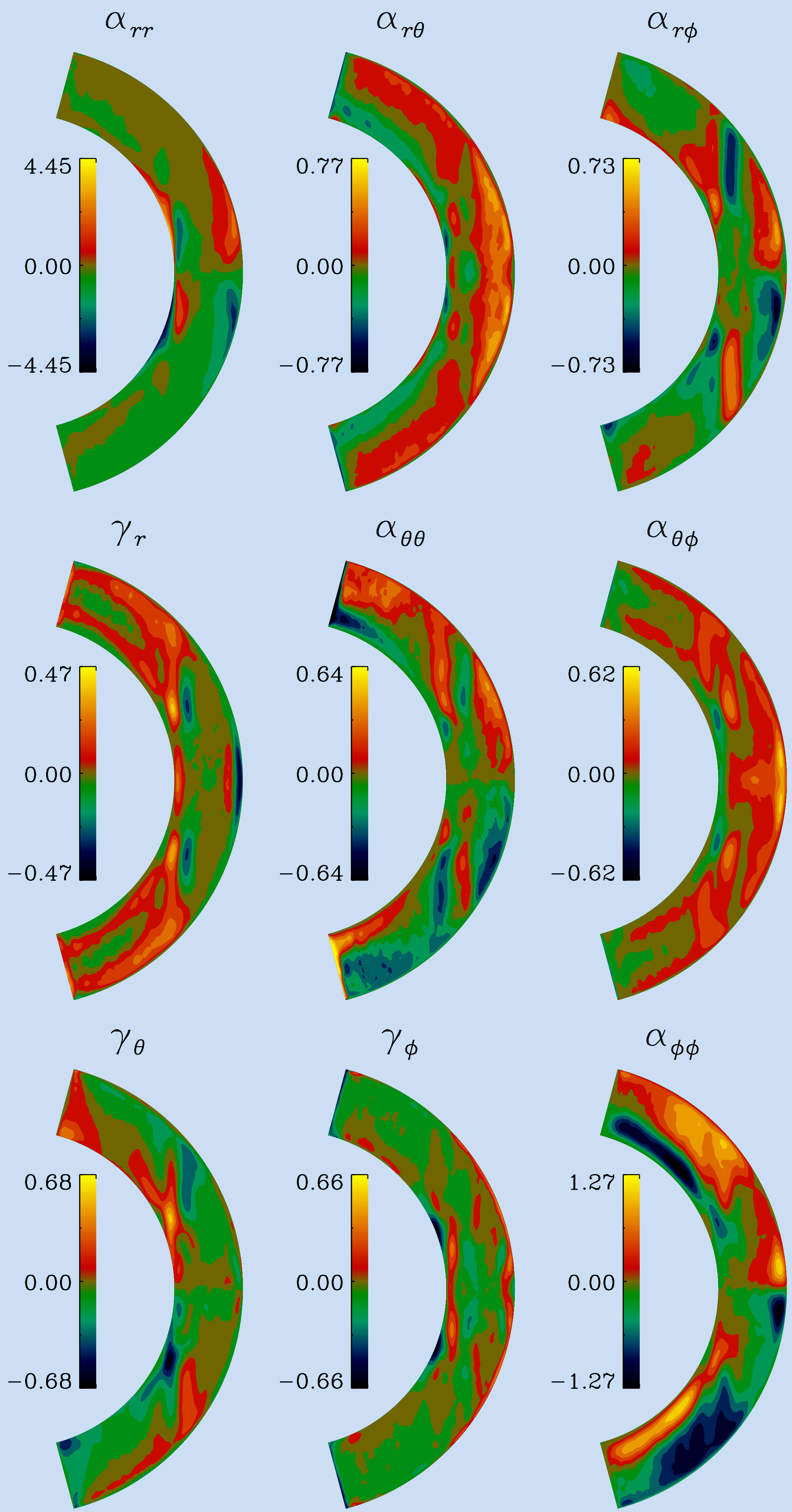
Mean field models

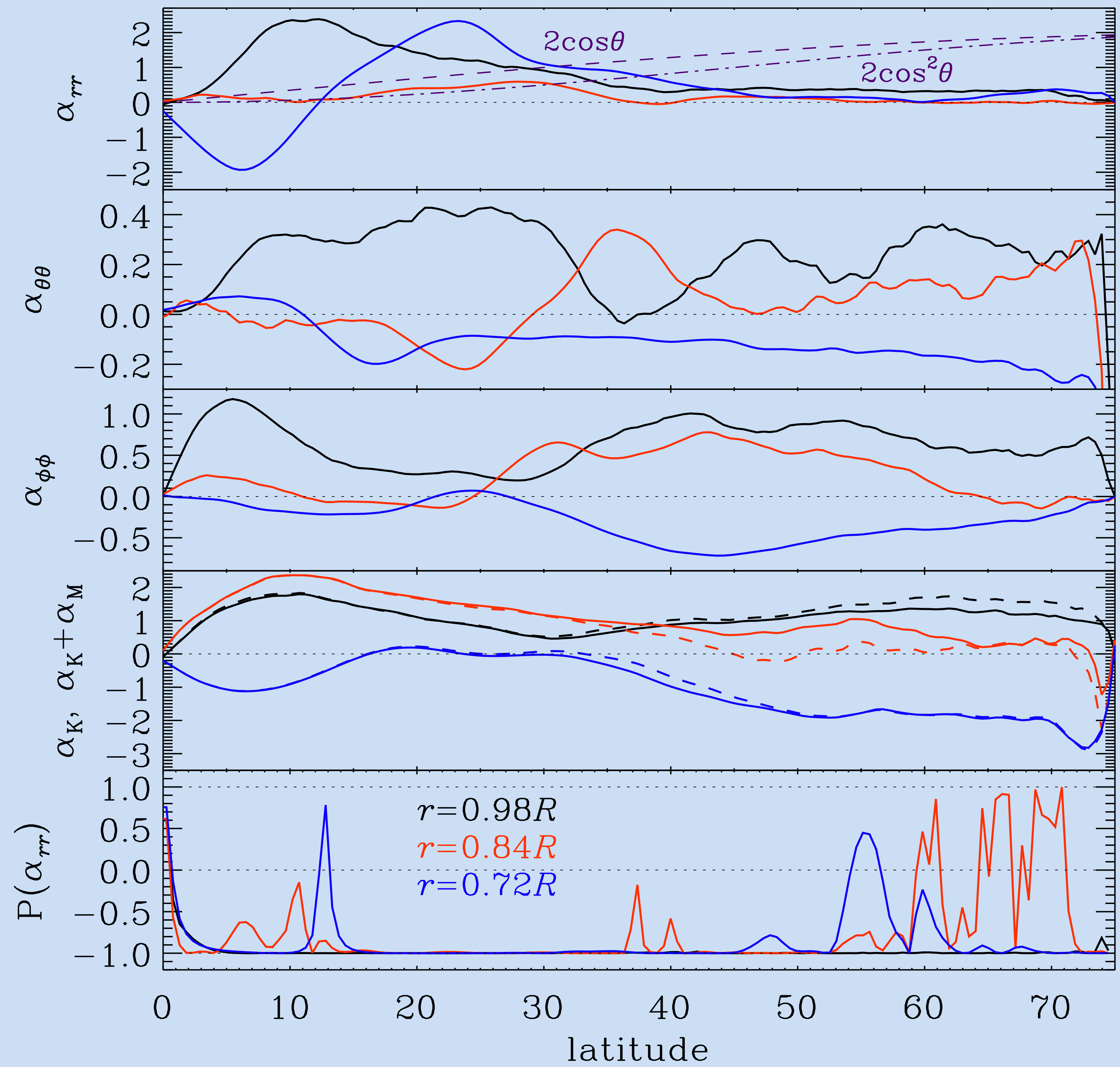


Käpylä et al. 2006

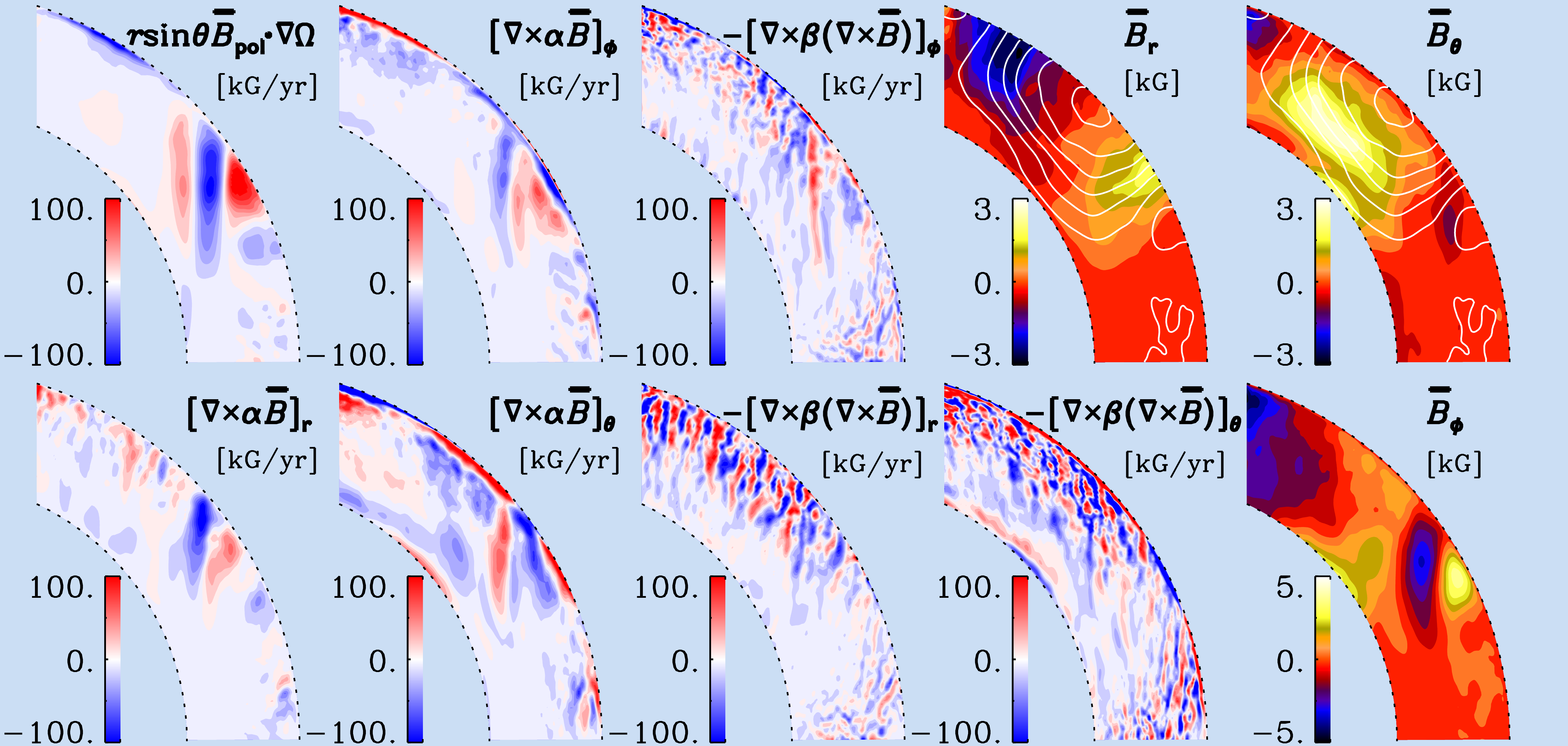


Karak et al. 2013

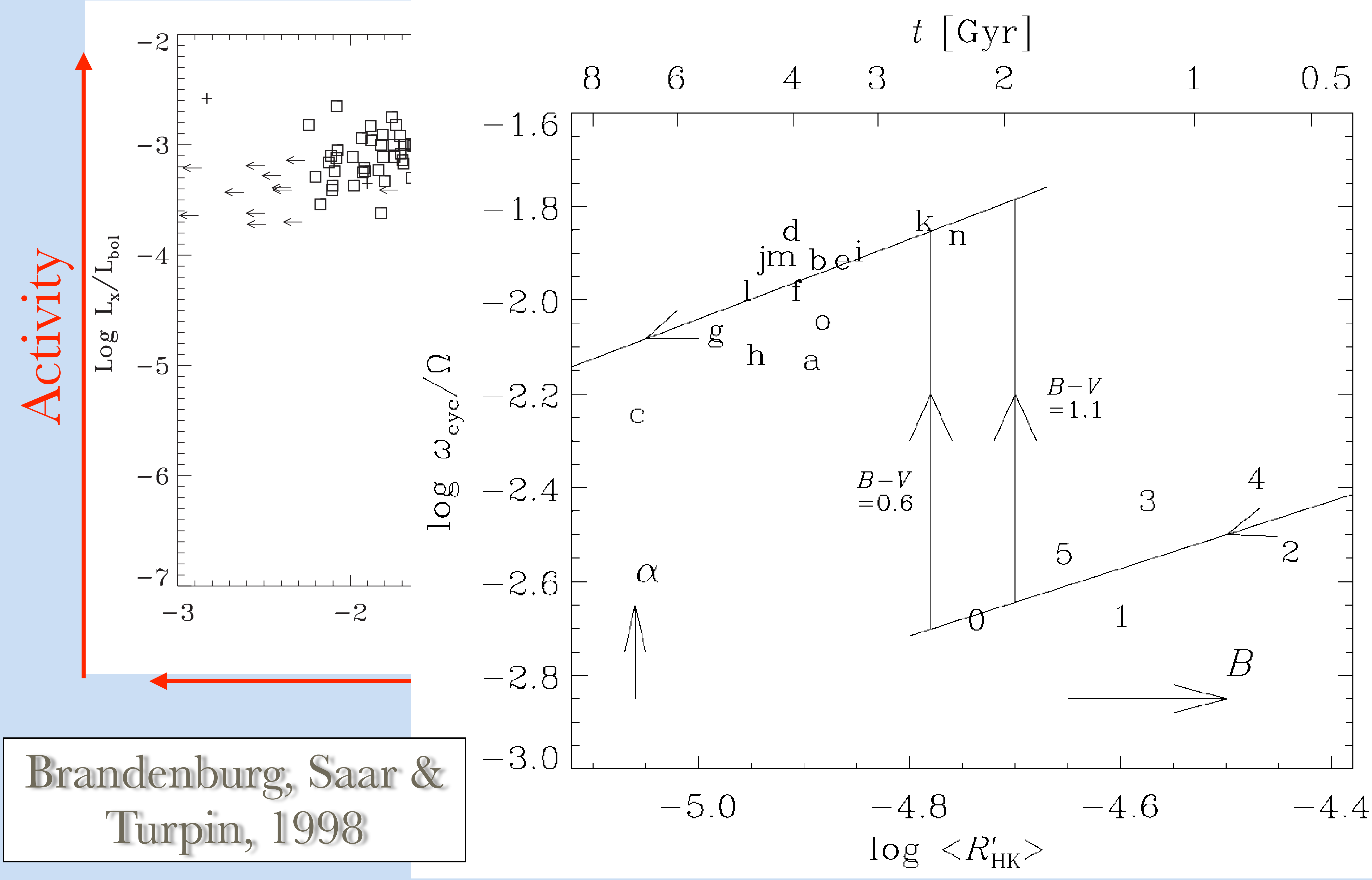

α_{rr}
4.45
0.00
−4.45
$\alpha_{r\theta}$
0.77
0.00
−0.77
$\alpha_{r\phi}$
0.73
0.00
−0.73
γ_r
0.47
0.00
−0.47
$\alpha_{\theta\theta}$
0.64
0.00
−0.64
$\alpha_{\theta\phi}$
0.62
0.00
−0.62
γ_θ
0.68
0.00
−0.68
γ_ϕ
0.66
0.00
−0.66
$\alpha_{\phi\phi}$
1.27
0.00
−1.27


α_{rr}
$2\cos\theta$
$2\cos^2\theta$
$\alpha_{\theta\theta}$
$\alpha_{\phi\phi}$
$\alpha_K, \; \alpha_K+\alpha_M$
$P(\alpha_{rr})$
$r=0.98R$
$r=0.84R$
$r=0.72R$
0 10 20 30 40 50 60 70
latitude

Magnetic field generation



There is Hope !



Brandenburg, Saar & Turpin, 1998

Conclusions

- Solar dynamo produce an cyclic magnetic field.
- Large density and pressure contrast in convection zone.
- Pressure scale height varies from 100 Mm to 0.1 Mm.
- Pencil code is great!
- Mean field theory is a good approach.
- Solar/Stellar dynamos might be complicated.
- Observations gives as hope, that there is a simple picture.