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Solar Magnetic Fields: Triple Arcade Structures

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Abstract

Recent observation with the LASCO coronagraph on board the SOHO spacecraft have shown that helmet streamers may have an internal triple arcade structure. The observations suggest that this triple structure might be important for the initiation of coronal mass ejections (CME's). In this paper we undertake a first step towards the theoretical description of such triple structures by calculating self-consistent solutions of the magnetohydrostatic equations in two dimensions. For simplicity, we start with the investigations of current-free structures, leading to potential magnetic fields. We then go on to investigate the case of linear currents. These two cases have the disadvantage of being linearly stable in the framework of MHD, which makes it difficult to take them as starting-points for further investigations. Therefore, we also investigate special exact solutions of a nonlinear case and apply the method of asymptotic expansion to another nonlinear problem to obtain approximate elongated solutions, which might be useful for linear or nonlinear stability checks.

1. Introduction

Arcade type structures play an import role in solar flare physics. Two dimensional magnetohydrostatic models assuming a photospheric dipole field and corresponding single arcade structures have been studied for example in [1, 2, 3]. Studies about the existence, uniqueness and stability of magnetohydrostatic equilibria can be found for example in [4, 5].

Recent observations (LASCO telescope on board the SOHO spacecraft, [6, 7]) of coronal helmet streamers show a triple structure. This suggests that both a dipolar and a quadrupolar component of the global solar magnetic field are important inside these structures. The observations also show that these structures can open and eject material into the interplanetary medium. It is therefore important to investigate the properties of such structures theoretically. We take a first step in this direction and try to calculate equilibria with a triple arcade structure.

We present four different methods to calculate solutions with a triple arcade structure. First we present three method leading to exact solutions of two-dimensional magnetostatic equilibria which may be useful to describe arcade structures near the sun.

For simplicity, we start with the investigations of current free structures and structures with a linear current. In both cases one can find exact solutions in the form of Fourier series. These structures a linearly stable in the framework of MHD. Therefore, we also investigate a special nonlinear case which has exact solutions. To describe strongly stretched structures we use the method of asymptotic expansion.

2. Basic equations

We use the equations of magnetohydrostatics to describe the coronal plasma:

$$-\nabla P + \frac{1}{\mu_0}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} - \rho \nabla \Psi = 0, \tag{1}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{2}$$

$$P = \rho R T \tag{3}$$

where P is the pressure, $\boldsymbol{B}$ the magnetic field, ρ the mass density, Ψ the gravitation potential, R the gas constant, T the temperature and μ_0 the permeability of free space. As a first step to describe the observed triple arcade structures we use a Cartesian geometry. For simplicity the configurations considered are two-dimensional ($\partial/\partial y = 0$). In the 2D case the magnetic field may be written as:

$$\boldsymbol{B}(x, z) = \nabla A(x, z) \times \boldsymbol{e}_y + B_y(A)\boldsymbol{e}_y. \tag{4}$$

A is the y-component of the vector potential $\boldsymbol{A}$ and $B_y(A)$ is the magnetic field component in the invariant direction. Equation (2) is then identically fulfilled.

Using eq. (4) one can find from eq. (1)

$$-\Delta A = \frac{\partial}{\partial A}\left(\mu_0 P(A, \Psi) + \frac{B_y^2(A)}{2}\right). \tag{5}$$

With a homogeneous gravitational field g and the assumption of constant temperature, one finds (see for example [1, 4])

$$P(A, \Psi) = P(A, z) = p(A) \cdot \exp\left(-\frac{z}{H}\right) \tag{6}$$

where $H = RT/g$ is the scale height. If one assumes a hydrogen plasma and a coronal temperature $T = 3 \times 10^6 K$ one finds $H \approx 9 \times 10^7$ m. Since this is a first attempt to describe triple structures and since gravitation has no influence on the lateral structure of the magnetic field, we will make the assumption that we can approximate the pressure as:

$$P(A, \Psi) \simeq p(A). \tag{7}$$

We include gravitation in section "Asymptotic Expansion" because the radial length scale of the equilibria are comparable or larger than the scale height in this case. We ignore gravitation in all other cases, because we assume $H \gg z$ there. In the present contribution, we do not explicitely take a magnetic shear component (B_y) into account. In those cases in which we neglect the dependence of the pressure on the gravitational field, all results apply as well if one replaces the pressure $p(A)$ by $B_y(A)^2/2\mu_0$ or a combination of both.

3. Linear models

3.1. *Potential fields*

If we assume that there are no currents, eq. (5) simplifies to:

$$\Delta A = 0. \tag{8}$$

We can solve eq. (8) easily by using a separation method (see for example [8]) $A(x, z) = A_x(x) \times A_z(z)$ and get:

$$A_x(x) = a \cos(kx) + b \sin(kx), \tag{9}$$

$$A_z(z) = c_1 \exp(-kz) + c_2 \exp(+kz). \tag{10}$$

We discard the exponentially growing solution because we demand the magnetic field to be bounded for $z \to \infty$. We can write the solution of (8) in Fourier-Series:

$$A(x, z) = \sum_{k=0}^{\infty} \exp(-kz)[a_k \cos(kx) + b_k \sin(kx)]. \tag{11}$$

3.2. *Linear current solutions*

In the absence of gravity and assuming the current to be linear in A,

$$j(A) = \frac{\partial}{\partial A}\left(\mu_0 p(A) + \frac{B_y^2(A)}{2}\right) = c^2 A,$$

we get from (5)

$$-\Delta A = c^2 \cdot A. \tag{12}$$

Equation (12) includes the linear force free case ($\partial p(A)/\partial A = 0$, $B_y(A) = c \cdot A$) and the quadratic pressure case without magnetic shear ($p(A) = c^2/2 \cdot A^2$, $B_y(A) = 0$).

To solve eq. (12) we can use a similar separation method as in the potential case and get:

$$A_x(x) = a \cos(kx) + b \sin(kx),$$

$$A_z(z) = c_1 \exp(-\nu z) + c_2 \exp(+\nu z), \qquad k > c,$$

$$A_z(z) = d_1 \cos(\omega z) + d_2 \sin(\omega z), \qquad k < c,$$

$$A_z(z) = e_1 z + e_2, \qquad k = c, \tag{13}$$

($\nu^2 = k^2 - c^2 > 0$; $\omega^2 = c^2 - k^2 > 0$). We discard the exponentially growing solution and the linearly growing solution because we demand the magnetic field to be bounded for $z \to \infty$. Furthermore we do not consider the solutions with $k < c$ because these solutions are periodic in z and do not lead us to arcade structures. As a result we get a solution of eq. (12) in Fourier-Series:

$$A(x, z) = \sum_{k=1}^{\infty} \exp(-\nu z)[a_k \cos(kx) + b_k \sin(kx)]. \tag{14}$$

3.3. *Examples*

In this section we use eq. (11) and eq. (14) to get solutions which have triple arcade structures. We prescribe photospheric boundary conditions to calculate the coefficients a_k and b_k in eq. (11) and eq. (14). We want to get solutions that are symmetric with respect to $x = 0$ and we set $A(-(\pi/2), z) = A(+/\pi/2), z) = 0$ from which we get: $b_k = 0 \forall k \geqslant 1$. (We remark that a_0 and b_0 add only constants to the vector potential A and have no influence on the magnetic field.) We prescribe the normal component of the magnetic field as the photospheric boundary condition. The chosen photospheric boundary conditions are $a_k = 0$ for almost all k; the non-vanishing a_k are given explicitely in the following. In Fig. 1(a)–(f) we present solutions of the potential eq.(8) with the photospheric magnetic field as:

(a) dipole-like field ($a_1 = 1$),
(b) quadrupole-like field ($a_3 = 1$)

In (c)–(f) we present solutions with combinations of a photospheric dipole-like and quadrupole-like field:

(c) $a_1 = 1$, $a_3 = 1$,
(d) $a_1 = 1$, $a_3 = -1$,
(e) $a_1 = 1$, $a_3 = -2$,
(f) $a_1 = 1$, $a_3 = -0.5$.

In (g) and (h) we present solutions of eq. (12) and use as

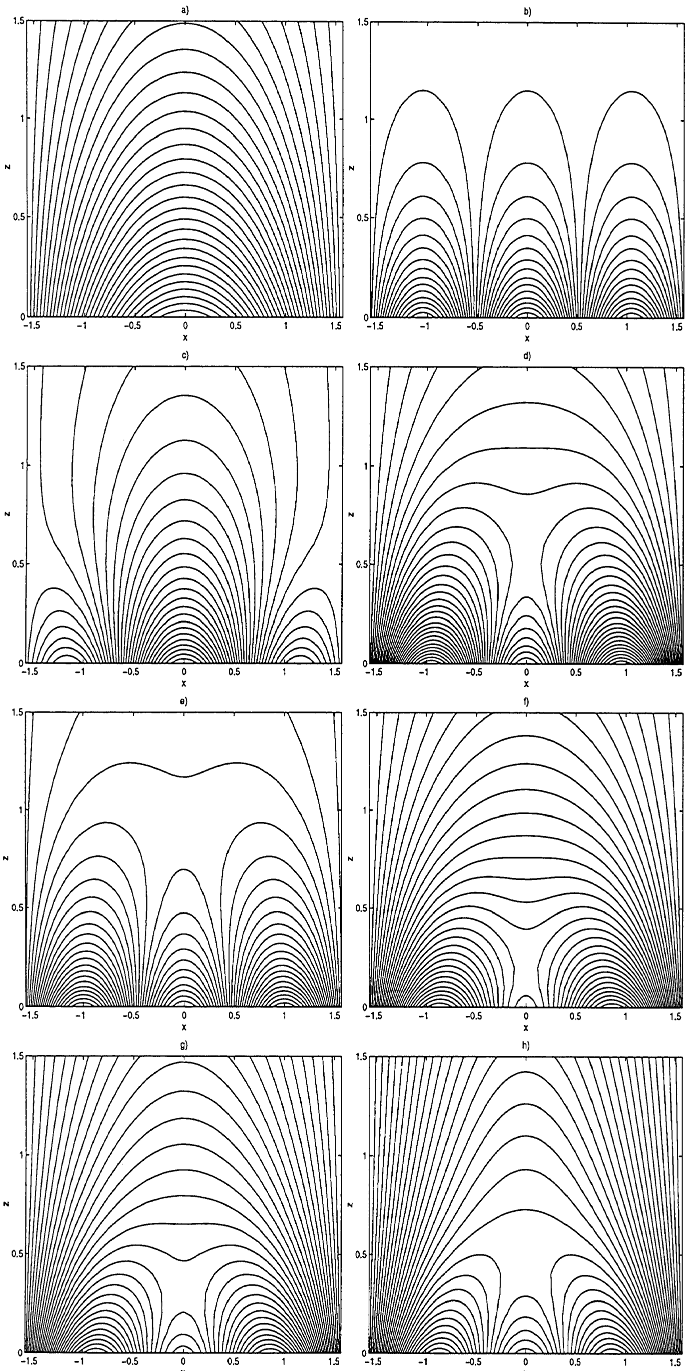

Fig. 1. Magnetic field lines (contourplots of $A(x, z)$) for different boundary conditions in the linear case (see text).

boundary conditions the same values as in (f), but different values of c^2:
(g) $c^2 = 0.5$,
(h) $c^2 = 0.8$.

As one can see the dipole-like boundary condition (a) results in a single arcade-structure and the quadrupole-like boundary condition (b) results in a periodic triple structure. The structure (c) seems to be unrealistic because the observations do not show a large arcade structure between two small arcade structures, but (d)–(h) show a qualitative agreement with the structures observed with the LASCO-Coronagraph.

4. Non-linear models

If one assumes local thermodynamic equilibrium one can find the pressure in the form $p(A) = \lambda/c \exp(-cA)$. Thus we get from eq. (5), if we use $B_y = 0$ for simplicity:

$$\Delta A = \lambda \exp(-cA). \tag{15}$$

We use a method, developed by Liouville to solve this equation [4, 9].

With $u = x + iz$ and $\bar{u} = x - iz$ the general solution of eq. (15) is

$$A(u, \bar{u}) = \frac{2}{c} \log \frac{1 + \frac{c\lambda}{8} |\Psi(u)^2|}{\left|\frac{\partial \Psi}{\partial u}\right|}. \tag{16}$$

With eq. (16) every analytic function $\Psi(u)$ specifies a solution of eq. (15).

4.1. *Examples*

We use the Liouville method to produce some illustrative solutions. To specify solutions of (15) which could have relevance for coronal triple structures we use $\Psi(u) = -a_1 \cos(a_2 u) + a_3 u^2$ (with real parameters a_i) and get

$$\begin{aligned} A(x, z) = {} & \frac{2}{c} \log \left(1 + \frac{c\lambda}{8} \left[(-a_1 \cos(a_2 x) \cosh(a_2 z) \right.\right. \\ & + a_3(x^2 - z^2))^2 \\ & \left.\left. + (a_1 \sin(a_2 x) \sinh(a_2 z) + 2a_3 xz)^2 \right] \right) \\ & - \frac{2}{c} \log ([(a_1 a_2 \sin(a_2 x) \cosh(a_2 z) + 2a_3 x)^2 \\ & + (a_1 a_2 \cos(a_2 x) \sinh(a_2 z) + 2a_3 z)^2]^{1/2}). \end{aligned} \tag{17}$$

Figure 2 shows the magnetic field lines (contourplots of $A(x, z)$) for this solutions with $c = 2$. We present solutions of the form (17) with $a_1 = 3$, $a_2 = 2$, $a_3 = 5$ but with different values of λ (a) $\lambda = 0.1$; (b) $\lambda = 0.9$.

These parameters are chosen to illustrate that one can find exact triple arcade solutions within a nonlinear model. As one can see with increasing λ the arcade structure in the middle decreases. The observations also show differences in the height of the single arcade structures within a triple arcade structure, but we cannot relate our parameters directly to the observations yet. We remark that it is necessary for the Liouville method that $B_x = -(\partial A/\partial z)$ and $B_z = \partial A/\partial x$ are of the same order of magnitude to obtain triple structures. If $B_z \gg B_x$ one cannot find triple arcade structures with this method because the current cannot change its sign if one assumes $j(A) \propto \exp(-cA)$. A necessary condition for the formation of triple structures in all described methods is $\partial^2 A/\partial x^2 = 0$ between the single arcade structures. If one assumes $B_z \gg B_x$ one obtains $j \approx -(\partial^2 A/\partial x^2)$. Thus the Liouville method cannot lead to stretched triple structures.

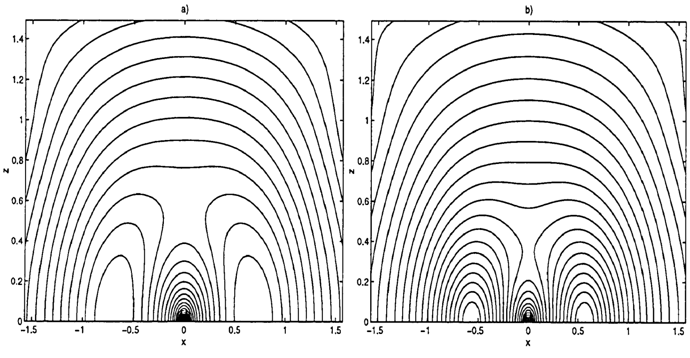

Fig. 2. Magnetic field lines (contourplots of $A(x, z)$) of solutions calculated with the Liouville method (see text).

5. Asymptotic expansion

In the preceding sections we presented fully two dimensional solutions. If the configuration is strongly stretched in one dimension, which means:
$\partial/\partial x = O(1) \Rightarrow B_z = O(1)$, $\partial/\partial z = O(\epsilon) \ll 1 \Rightarrow B_x = O(\epsilon)$, $B_y = O(\epsilon) \ll 1$ or, for simplicity, $B_y = 0$, we can ignore $\partial^2/\partial z^2 = O(\epsilon^2)$. This method is well known in the theory of the Earth's magnetotail [1, 11]. In this section we investigate also the effect of gravity because the radial length scale of the equilibria are comparable or larger then the scale height in this case. Therefore we use $P(z, A)$ as defined in eq. (6). With these approximations and without magnetic shear we get from eq. (5):

$$-\frac{\partial^2 A}{\partial x^2} = \mu_0 \exp\left(-\frac{z}{H}\right) \frac{\partial p(A)}{\partial A}. \tag{18}$$

After one integration with respect to x we get:

$$\frac{\partial A}{\partial x} = \pm \sqrt{2\mu_0 \left(p_0(z) - \exp\left(-\frac{z}{H}\right) p(A) \right)}. \tag{19}$$

$p_0(z)$ is a constant of integration which depends parametrically on z. For some special forms of $p(A)$ one can find analytical solutions [10, 11].

To get triple arcade solutions in stretched configurations the current density has to change its sign between the single arcade structures. The current density is defined as $j = \partial p/\partial A$. Thus the pressure function must have a minimum or we have to allow a discontinuity in $j(A)$. In this paper we do not present solutions with a discontinuity in in the current function. We remark that changing the sign in (19) is equivalent to changing the direction of the z component of magnetic field $B_z = \partial A/\partial x$ which is the only relevant component for the magnetic pressure $B^2/2\mu_0$ in the tail approximation. To get triple arcade structures it is necessary to change the sign of (19) if $\partial A/\partial x = 0$.

In principle every smooth pressure function $p(A)$ with a minimum leads to triple arcade structures. To illustrate this we present some example solutions with two different pressure profiles and solve (19) numerically. As a boundary condition we use the total pressure at $x = 0$ and choose:

$$p_0(z) = \frac{z_0^4}{(z_0 + z)^4} + p_0^*,$$

(with $z_0 = 300$, $p_0^* = 0.3$). A motivation for this choice is that the magnetic field outside the whole configuration, in the solar wind, varies $\propto 1/z^2$. Thus the magnetic pressure $p_m = B^2/2\mu_0$ varies $\propto 1/z^4$. We assume that the total pressure $p_0(z)$ is a sum of the magnetic pressure outside the configuration and p_0^* which we interpret as the asymptotic solar wind pressure for $z \to \infty$. We remark that the exact form of $p_0(z)$ is not important because it is only an integration constant in this model. One may use any monotonously decreasing function $p_0(z)$ and get similar structures.

In Fig. 3 we present the magnetic field lines for the solutions of eq. (19). In Fig. 3(a) and (b) we present solutions to the pressure function $p_1(A) = \exp(-2A) + 0.5A$. In (a) we ignore gravity ($H \to \infty$) and in (b) we consider a homogeneous gravitation field ($H = 500$). In Fig. 3(c) and (d) we present solutions to another pressurefunction $p_2(A) = \exp(-2A) + 0.5A^2$. In (c) we ignore gravity and in (d) we consider a homogeneous gravitation field ($H = 500$). Note the difference in horizontal scale between the upper and lower panel in this figure and note also that the scales differ from the scales in the previous figures. We show a plot of the two pressure functions in Fig. 4. We remark that the chosen pressure functions are only examples for functions which can be used to get triple arcade structures with the method of asymptotic expansion. One cannot calculate the exact form of $p(A)$ or derive it directly from the observations.

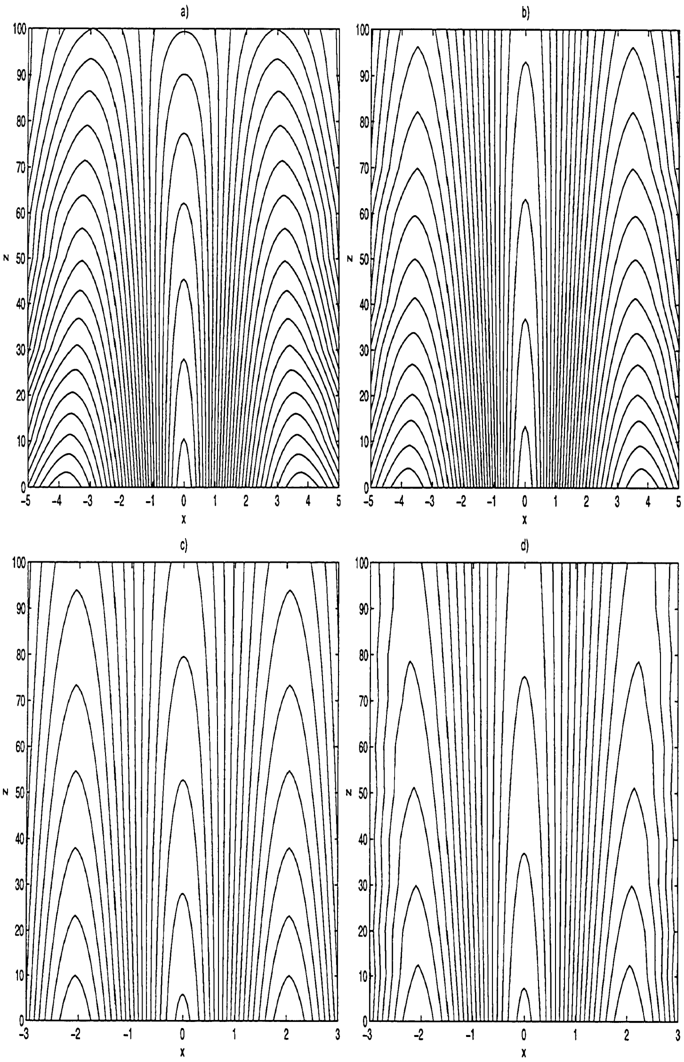

Fig. 3. Contourplots of $A(x, z)$ calculated with the method of asymptotic expansion for different pressure functions. The configurations on the left hand side are calculated without gravity. The configurations on the right hand side include a homogeneous gravitation field (see text). Note the different in horizontal scales in the upper and lower panels. Note also the different scaling compared with the previous figures.

One can see that the configuration calculated with the function $p_1(A)$ is wider than those calculated with $p_2(A)$. Especially between the outer arcade structures there is a factor of more then two. To obtain the reason for this fact look at Fig. 4. The increasing part of the pressure function p_2 is much steeper as the one of p_1. As one can see from eq. (18) this means that $B_z = \partial A/\partial x$ varies more quickly with respect to x if one uses p_2. Consequently the configuration with p_2 is thinner than the configuration with p_1.

The panels on the right hand side show the same calculation as the one on the left hand side but include a homogeneous gravitation field. The effect of gravity is that the configurations become more stretched and that the outer arcade structures bend towards the boundary. The second effect is more clear in the lower panels. In the middle ($x = 0$) of our configurations we have $P(A, z) = p_0(z)$ because B_z and consequently the magnetic pressure vanishes here. If we use $P(A, z)$ as defined in eq. (6) we get $p(A) = p_0(z) \exp(z/H)$. Consequently in a configuration with gravity, $p(A)$ has the same values at a lower value of z than in a configuration without gravity. The effect increases with increasing z. Thus the configurations with gravity become more stretched.

The explanation for the second effect is that with increasing z the pressure-function $P(z, A)$ and $\partial P(z, A)/\partial A$ decrease for the same values of A. We explained above that a flatter pressure function leads to a wider configuration. Thus in a configuration with gravity the configuration becomes wider with increasing z. In a Cartesian geometry the configuration gets bend. We used $z_{max}/H = 0.2$ to investigate the effect of gravity. Although this value is unrealistically low one may use it to investigate the effect of gravity. To describe huge structures with $z_{max} > H$, which are more realistic, one

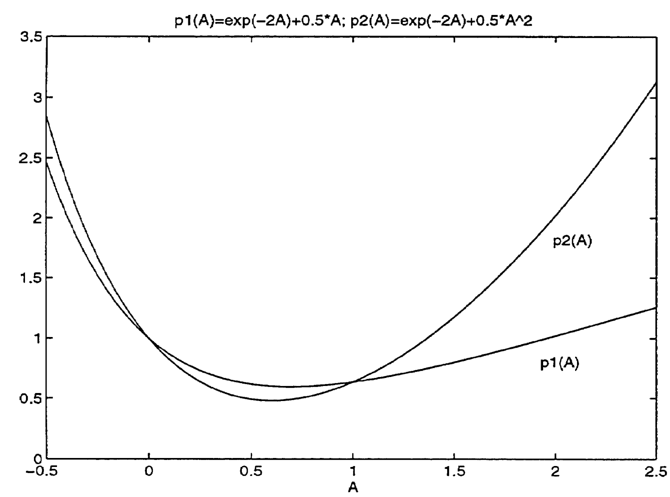

Fig. 4. Pressure functions $p_1(A) = \exp(-2A) + 0.5A$ and $p_2(A) = \exp(-2A) + 0.5A^2$.

should use spherical coordinates and a realistic gravitation field (not a homogeneous one).

6. Conclusions

In this paper we presented different possibilities to calculate solutions which could have relevance for the observed triple structures associated with helmet streamers. To describe triple arcade structures near the sun we used 2D solutions of the Grad Shafranov equation. In a first attempt we used the linear theory which allows solutions in the form of Fourier series. Thus we derived triple arcade structures by prescribing the Fourier coefficients.

As a next step we presented analytical solutions of one type of the non linear Grad Shafranov equation (Liouville equation ($\Delta A = \lambda \exp(-cA)$)). We showed that one can find exact triple arcade solutions of this equation. We remark that it is necessary for this method that $B_x = -(\partial A/\partial z)$ and $B_z = \partial A/\partial x$ are of the same order of magnitude to obtain triple structures. If $B_z \gg B_x$ one cannot find triple structures with this method because the current density $j(A) = -\lambda \exp(-cA)$ does not change its sign. Thus this method cannot be used to describe stretched triple structures. This is a disadvantage of the Liouville method because unstable stretched structures are assumed to play an important role in the theory of coronal mass ejections.

Special methods to describe stretched magnetic fields have been developed in [10] and [11] with applications to the Earth's magnetotail. We illustrated that one can use this methods also to get triple arcade structures if a special form of the pressure profile, which must have a minimum, is used. Analytical solutions are not yet available for this case.

It is not clear, which of these methods will turn out to be the most useful one. The linear theory allows to describe structures near the sun and stretched structures but it is well known (not proofed in this paper) that these structures are (linearly) stable. With the Liouville method one can calculate exact solutions of one special nonlinear case, but one cannot use this method to describe stretched triple structures. The method of asymptotic expansion allows us to calculate stretched triple structures in a nonlinear case. The observations suggest that stretched arcade structures could have relevance for coronal mass ejections. Thus these configurations may be used as a start equilibrium for MHD simulations related to coronal mass ejections.

An important question for future work is whether triple structures make the occurrence of eruptions more likely and also more energetic. To answer this question, investigations and comparisons of the stability and the time evolution of single and multiple arcade structures are necessary. This will be done in the future with the use of MHD simulations.

Acknowledgements

The author dedicates this work to his academic teacher Prof. Karl Schindler.

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