Magnetic Field Extrapolation

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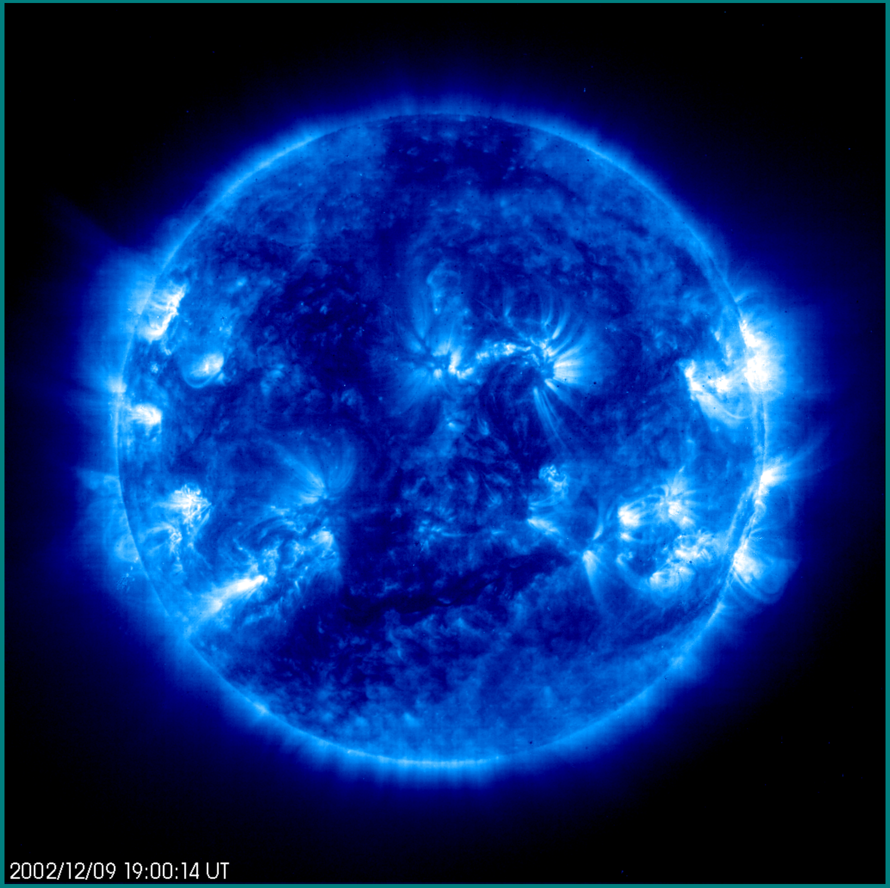
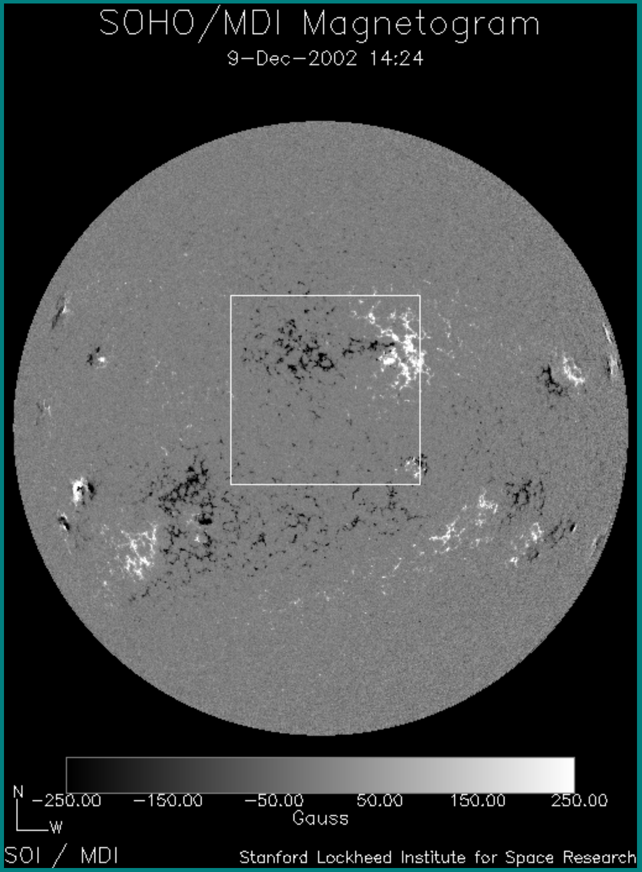
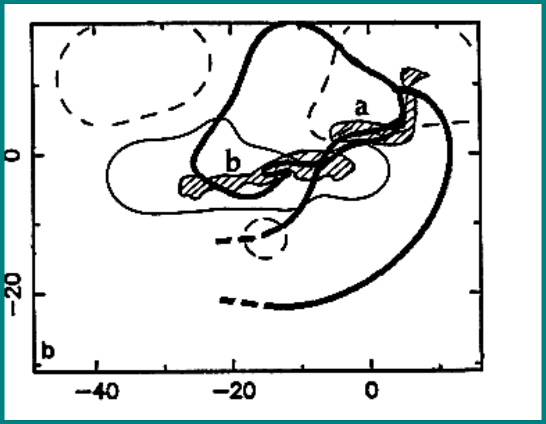
B-field structures corona



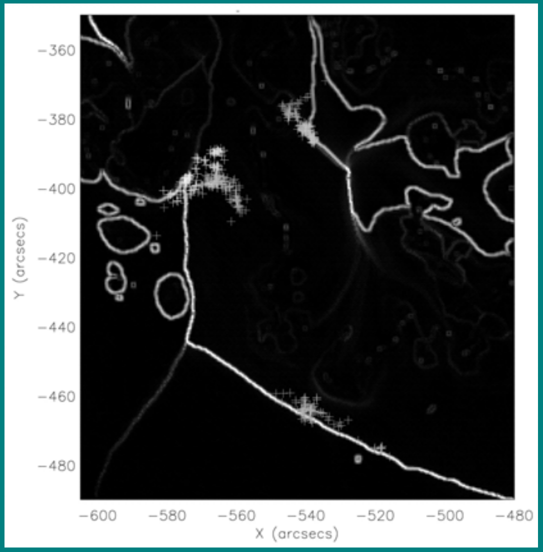


Why determine the B-field in the corona ?

- provides a direct link between theoretical models and observations
- comparison of B-field structure with observed plasma structures
- determination of magnetic topology (null points, separatrix surfaces, bald patches) or geometry (quasi-separatrix layers); see figures
- Determination of (free) magnetic energy in active regions
- ...



Mandrini et al. 1995



Metcalf et al. 2004

How ?

- Routine measurements of magnetic field with sufficient accuracy and spatial resolution only possible in photosphere (chromosphere)
- Observed fields have to be extrapolated into the corona
- Coronal magnetic field is calculated using model assumptions for electric current density and using measured B-field as boundary condition

Why force-free ?

- Coronal plasma tenuous, > 10^6 K hot
- Plasma pressure << magnetic pressure

$$p \ll \frac{B^2}{2\mu_0}$$

(Plasma $\beta \ll 1$)

- Assumption: can neglect all forces except Lorentz force ($\mathbf{j} \times \mathbf{B}$) in momentum balance equation (correct in photosphere and chromosphere ?)

(NLFF) Extrapolation Basics

From

$\mathbf{j} \times \mathbf{B} = \mathbf{0}$

$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$

$\nabla \cdot \mathbf{B} = 0$

follows that

$\mu_0 \, \mathbf{j} = \alpha(\mathbf{x}) \, \mathbf{B}$

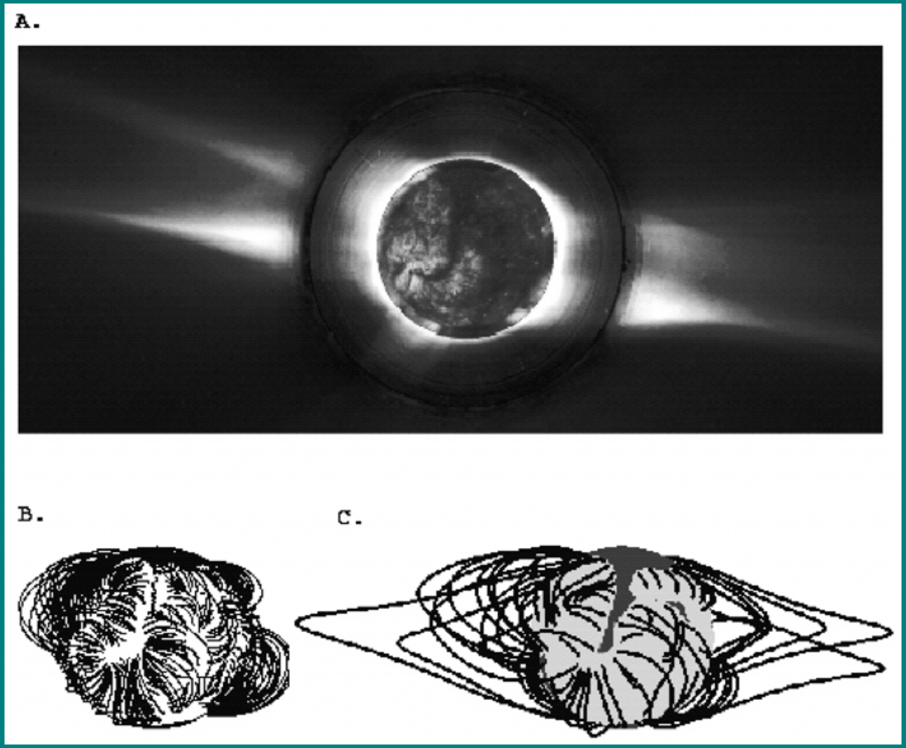
$\mathbf{B} \cdot \nabla \alpha = 0$

Different Extrapolation Methods

- potential fields **j = 0,** α=0
- linear force free fields **j=**α **B**, α=constant
- nonlinear force-free fields **j=**α(**x**) **B**, **B·**$\nabla\alpha$**=**0
- non-force-free fields (e.g. Aulanier & Démoulin,1998; Petrie & Neukirch, 2000; Wiegelmann & Inhester, 2004)

Global Extrapolation

- usually potential fields with source surface
- synoptic data for whole solar surface (for alternative see Rudenko, 2001)
- non-force-free fields possible (e.g. Bogdan & Low, 1986; Neukirch, 1995; Zhao & Hoeksema, 1994; Gibson et al. 1996)
- full MHD models by e.g. J. Linker, Z. Mikic

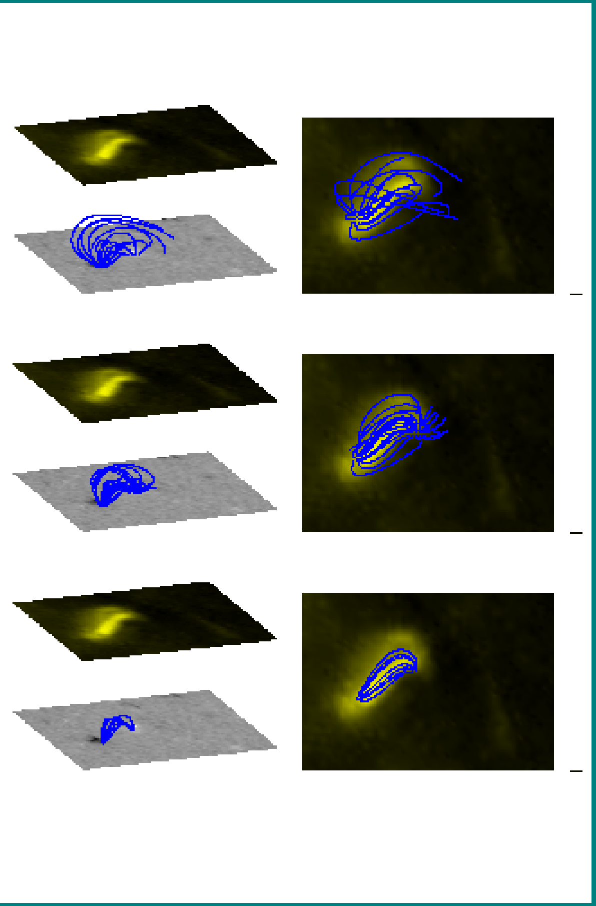


Gibson et al., 1999

Local extrapolation

Petrie, 2000

- standard methods at present : potential and linear force-free extrapolation
- reasons : availability of line-of-sight magnetograms and relative simplicity of methods
- able to give first overview over magnetic field topology etc. (see e.g. Schmieder & Aulanier, 2003)
- not suitable for more sophisticated problems, e.g. active regions with strongly localized current density, determination of energy budgets, etc

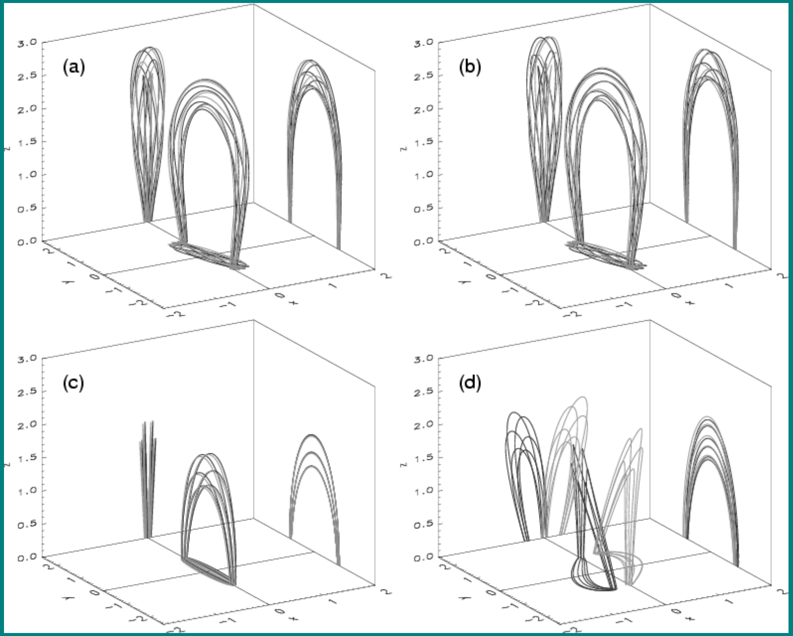


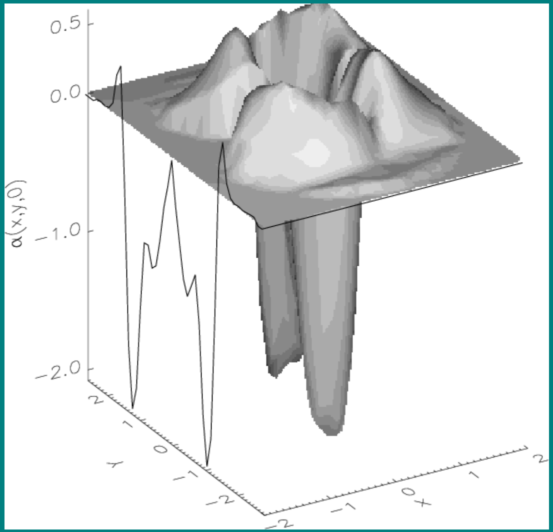
An illustrative example

- twisted flux rope model of Török & Kliem (2003)
- potential field twisted by strongly localized “vortices” on lower boundary
- system has no net current ➔ α must change sign (return current) !
- Valori et al. (2004) show that lff extrapolation does not properly represent the field, even if using e..g. the α_{best} method
- Kliem & Valori (in prep.) show that in some cases α_{best} can have sign opposite to α in core of loop

- Highly desirable :

  a (or several) fast, reliable and robust method(s) for nonlinear force-free field extrapolation





Methods for NLFF Extrapolation

- direct upward integration (e.g. Wu et al., Cuperman et al., ...)
- Grad-Rubin-type methods (e.g. Sakurai; Amari, Régnier et al.; Wheatland,...)
- Optimization methods (e.g. Wheatland et al., Wiegelmann, McTiernan, ...)
- MHD evolution/relaxation methods (McClymont, Jiao & Mikic; Roumeliotis; Valori, Kliem & Keppens)
- Boundary integral method (Yan, Sakurai, et al)
- ....

Grad-Rubin-type Methods

$\mathbf{B}^n \cdot \nabla \alpha^n = 0$,

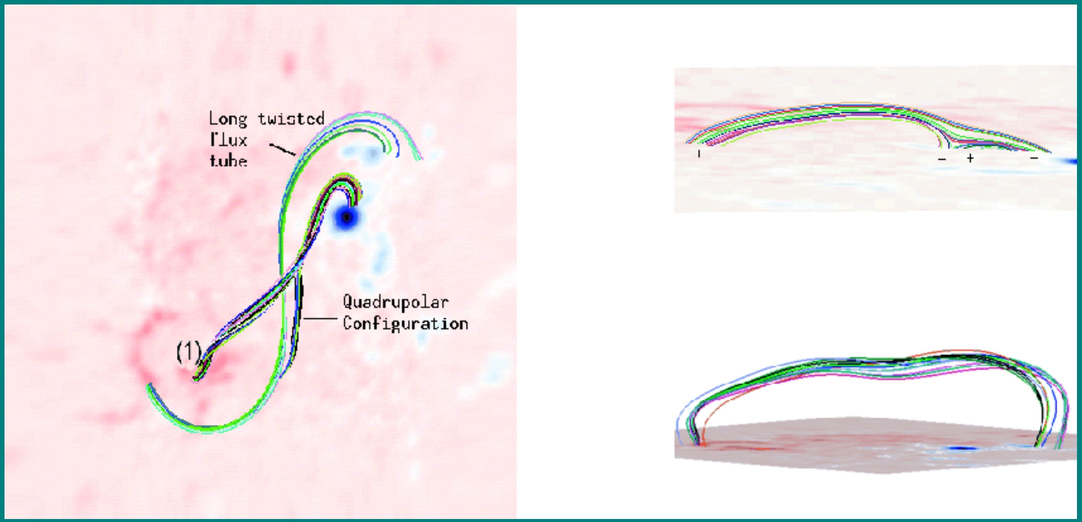
α_{bdry} from data,

$\nabla \times \mathbf{B}^{n+1} = \alpha^n \mathbf{B}^n$

Method by Amari, Régnier, et al. uses vector potential to keep

$\nabla \cdot \mathbf{B} = 0$

Régnier & Amari,2004

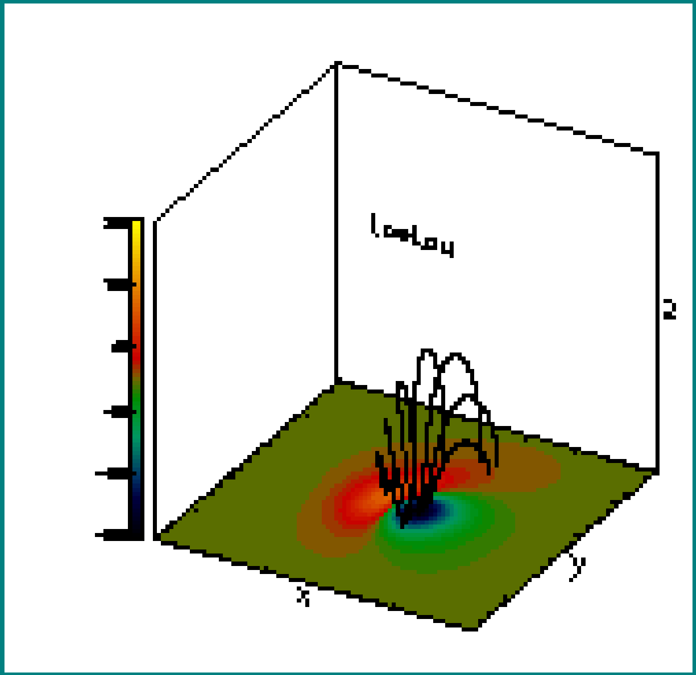


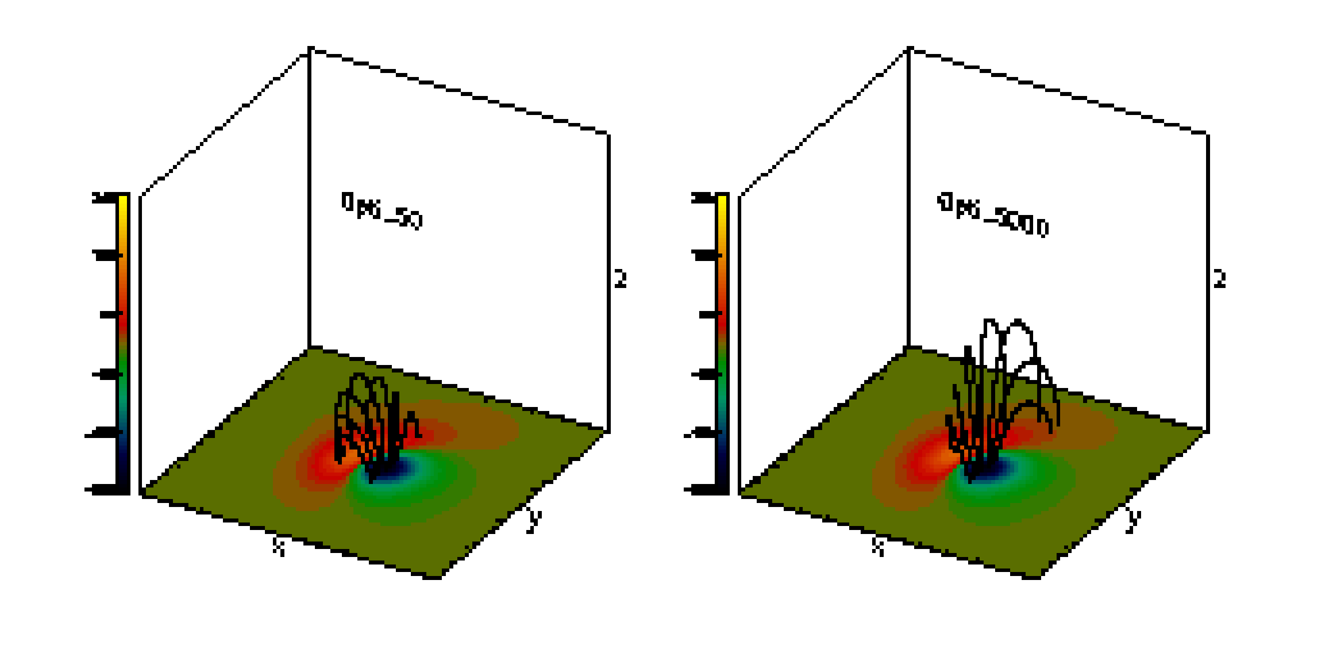
Method successfully applied to vector magnetograms of active regions (see talk by Stéphane Régnier)

Optimization Methods

- Minimize
  $L=\int B^{-2}|(\nabla\times\mathbf{B})\times\mathbf{B}|^2+|\nabla\cdot\mathbf{B}|^2\, d^3x/V$
  (or variant)
- L=0 implies that **B** force-free and divergence free
- L decreases monotonically during calculation
- It is not guaranteed that either L=0 or $\nabla\cdot\mathbf{B}=0$ is achieved
- no real problem in test cases so far, although Wiegelmann & Neukirch (2003) found that for noisy boundary conditions convergence is less good

- Test with Low & Lou (1990) nonlinear force solution
- Known b.c.ˢ on all six boundaries of computational box
- Start from potential field
- Convergence in about 5000 steps
- Comparison with other codes ("NLFF Consortium") indicates that optimization is best performing method for test case





Wiegelmann & Neukirch, 2003

Problems

- Problems with methods

  Convergence, speed, numerical effort, boundary conditions (lateral and upper boundaries),...

- Problems with data

  inaccuracy of transverse field measurements, 180° ambiguity, data B force-free ?, flux in magnetogram balanced ?, ...

- Both influence each other

  e.g. boundary conditions and flux balanced magnetograms (field lines/current allowed to leave box or not)

- Force-free approximation justified in photosphere/chromosphere ?

Force-free vs. non-force-free

- Force-free assumption based on small plasma β
- However, large β does not necessarily mean that force-free assumption fails
- Extreme example: high β plasma in hydrostatic balance ($-\nabla p = \rho\, \mathbf{g}$; $\mathbf{j}\times\mathbf{B}=\mathbf{0}$); remember that solar atmosphere is stratified to a reasonable degree
- More important than β: relative influence of perpendicular currents to parallel currents

- Speculation:

  expansion possible with lowest order force-free fields plus stratified background atmosphere, next order non-force-free corrections ?

Outlook

- Magnetic field extrapolation is an essential tool for solar physics
- Potential and linear force-free extrapolation are most used; OK if limitations are borne in mind
- Nonlinear force-free methods have to be further developed to become applicable on a routine basis (e.g. to be able to use huge vector magnetogram data sets expected from SOLIS or SOLAR-B)
- Other tests than Low & Lou (e.g. Titov & Démoulin, 1999); ultimate test : real data