On the extrapolation of magneto-hydro-static equilibria on the sun

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Introduction

- While the solar corona can be modeled by the force-free assumption, we need to take care about the pressure gradient and gravity to model the magnetic field in the photosphere and chromosphere.
- In this work, we generalize the optimization method to solve the magneto-hydro-static (MHS) equations which is a better description of the solar lower atmosphere.

Optimization method for MHS extrapolation

- MHS equations: $\frac{1}{\mu_0}(\nabla \times B) \times B - \nabla p - \rho g \hat{z} = 0,$

$$\nabla \cdot B = 0$$

- Define $L = \int (B^2\Omega_a^2 + B^2\Omega_b^2) dV$

with $\Omega_a = B^{-2}\left[\frac{1}{\mu_0}(\nabla \times B) \times B - \nabla p - \rho g \hat{z}\right]$

$\Omega_b = B^{-2}[(\nabla \cdot B)B]$

- Using following variable transformation to ensure the positive pressure and density:

$$p = Q^2, \rho = \frac{R^2}{gH_s}, (H_s: height\ scale)$$

- MHS equations can be solved by: $Minimize\ L(B, Q, R)$
- Steepest descent: $\delta B = -\frac{\partial L}{\partial B}, \delta Q = -\frac{\partial L}{\partial Q}, \delta R = -\frac{\partial L}{\partial R}$

Pressure and density on the photosphere

- On the photosphere, no pressure data available, we try to use the Lorentz force to derive pressure consistently.
- For the Lorentz force f_{ph} on the photosphere:

$$f_{ph} = \nabla \times A + \nabla \varphi \implies \nabla \cdot f_{ph} = \nabla^2 \varphi \implies p = \varphi + const$$

- The density on the photosphere is uniform and fixed during optimization.

Numerical implementation

1. Calculate potential field using LOS magnetogram.
2. Replace the magnetic field on the photosphere with vector magnetogram, iterate for B by NLFFF approach.
3. Insert a isothermal gravity stratified atmosphere.
4. Iterate for B, Q, R by steepest descet until L reaches its minimum. (update bottom p during minimization)

Linear MHS model

- Low (1991) presented a class of analytical MHS solution by assuming: $\nabla \times B = \alpha B + f(z)\nabla B_z \times \hat{z},\ f(z) = ae^{-\kappa z}$
- Solution: B can be solved by FFT, $p = p_0(z) - \frac{1}{2\mu} f(z) B_z^2$

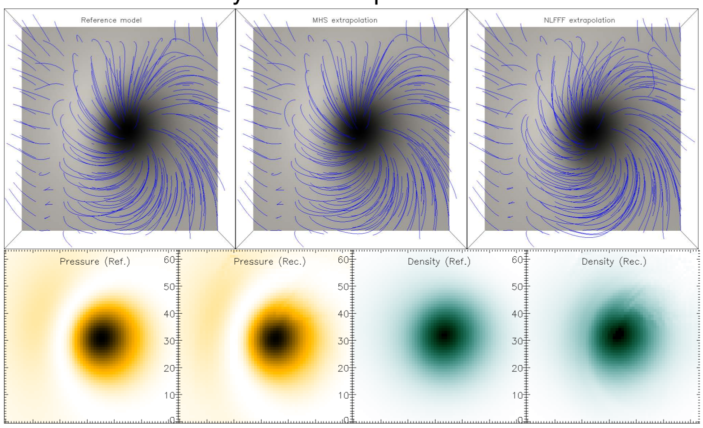
$$\rho = -\frac{1}{g}\frac{dp_0}{dz} + \frac{1}{\mu g}\left[\frac{df}{dz}\frac{B_z^2}{2} + f(z)(B \cdot \nabla)B_z\right]$$

- Parameters setting:

1. Low & Lou (1990) LOS magnetogram $|B| = 380G, B_{min} = -2380G, B_{max} = 726G$
2. Linear current $\alpha = -3.0$, Lorentz force strength $a = 0.5$
3. Sun-like background atmosphere
4. Computation box: $x, y(Mm) \in [-1.6, 1.6], z \in [0, 1.28]$
5. Grids: 80*80*32 with 40km/grid

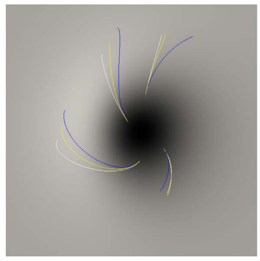
Analytical tests

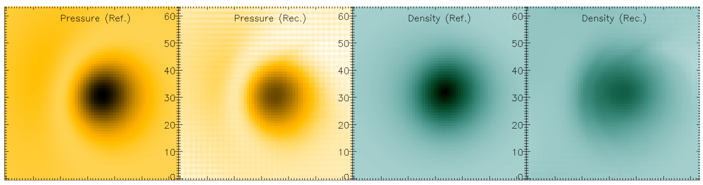
- Test I: all boundary conditions provided



- Test II: bottom vector magnetogram provided

Field line comparison:
- white: reference model
- yellow: MHS extrapolation
- blue: NLFFF extrapolation





Quantitative comparisons: (Initial state consists of a NLFFF and an isothermal gravity stratied atmosphere)

Model	C_{vec}	C_{cs}	$1 - E_m$	$1 - E_n$	$corr2D.p$	$corr2D.\rho$
Initial state	0.988	0.968	0.805	0.754	0.000	0.000
MHS extra.	0.992	0.973	0.860	0.798	0.983	0.968

Discussion

- The MHS equilibria is reconstructed accurately by the generalized optimization method.
- Although no p data on the photosphere available, we design a method to derive bottom p consistently.
- NLFFF is a better initial state than potential field.
- Future extrapolation would be comprised of MHS extrapolation in the lower atmosphere and computational less expensive NLFFF extrapolation in the corona.

References

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Poster

1. Fundamental physical processes and modeling

An optimization principle to reconstructing magneto-hydro-static equilibria in the lower solar atmosphere

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Modelling the interface region between solar photosphere and corona is challenging, because the relative importance of magnetic and plasma forces changes by several orders of magnitude. While the solar corona can be modeled by the force-free assumption, we need to take care about plasma-force (pressure and gravity) in photosphere and chromosphere, here within the magneto-hydro-static (MHS) model. We solve the MHS-equations with the help of an optimization principle and use photospheric measurements as boundary condition. Positive pressure and density are ensured by reformulating the MHS-euqations by introducing two new basic variables which are derived by a transformation from pressure and density. Furthermore, we use the Lorentz force information during optimization to update the plasma pressure on the bottom boundary. Our code is tested by application to a linear MHS solution. Although the code works in ideal conditions, there are obstacles still to be overcome before the code can be applied to real data.