Frequency shifts induced by near-surface convective scattering of waves

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Abstract

We have shown that waves propagating through a rapidly fluctuating medium feel an averaged effect of the fluctuations. We compute an effective wave speed using spatial homogenization. We apply flow speed corrections to solar oscillation equations, and show that this improves modelled frequency estimates.

Introduction

Modelled solar oscillation frequencies display a systematic drift from the observed frequencies. There are several possible origins of this difference:

- Inaccurate modelling of stellar structure.
- Convective scattering not being incorporated in wave equation.
- Non-adiabaticity in outer layers of the Sun.
- Non-linearity in the wave equation.

Remedial measures exist in the form of power law fits [Kjeldsen et al., 2008, Ball and Gizon, 2014], which however do not get to the root of the problem - what is the reason behind the observed difference? It is likely that the real reason is a combination of all of the above, so a better approach than power law fits is to correct for each effect systematically.

Spatial Homogenization

- Two length scales - wavelength scale λ and flow scale L. Scale ratio ε is a small parameter about which we perturbatively expand the wave field $\boldsymbol{\xi}$.
- Leading order field $\boldsymbol{\xi}_0$ satisfies the wave equation

$$-\omega^2 \boldsymbol{\xi}_0 = \boldsymbol{\nabla}_x \cdot (\mathbb{C} : \boldsymbol{\nabla}_x \boldsymbol{\xi}_0) \quad (1)$$

where $\mathbb{C}$ is a fourth order tensor.

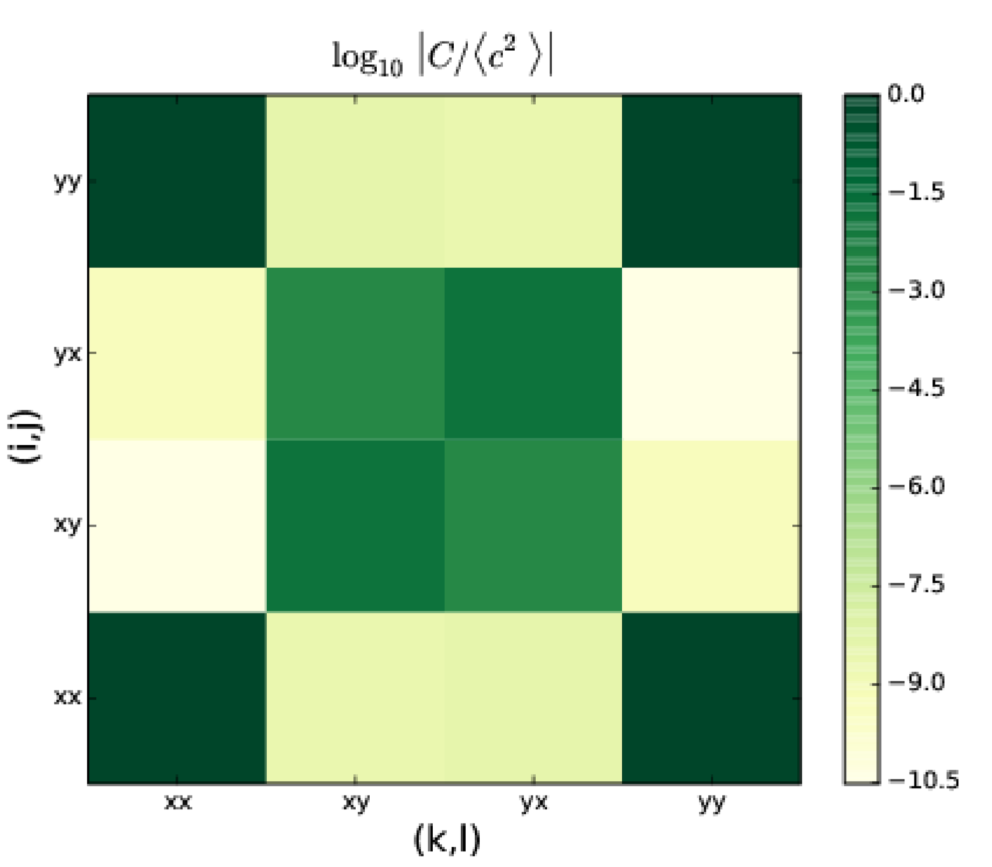

Figure 1: *Components of the tensor $\mathbb{C}$. Dominant diagonal components of $\mathbb{C}$ behave as $c^2 - u^2$ corresponding to a flow speed u.*

- Seismic reciprocity is maintained as the flow has zero mean over a cell.

Toy problem: 2D fluctuations

- We study wave propagation through a 2-dimensional surface which is periodically tiled with cellular flows.
- The analysis can be readily extended from periodic media to random media. (Papanicolaou and Varadhan, 1982).
- Stepping stone towards the Sun.

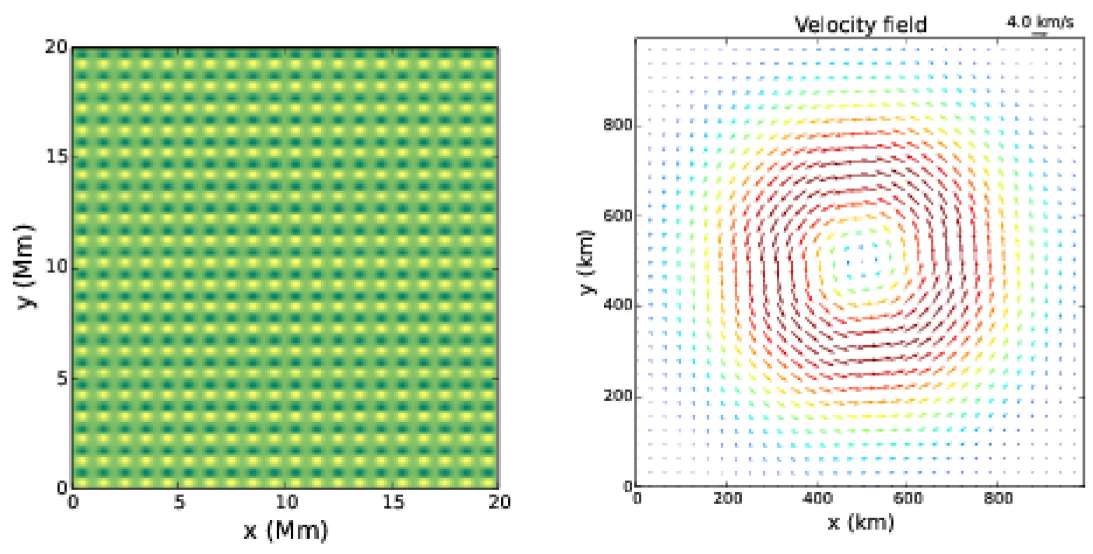

Figure 2: *Periodically fluctuating background (left). Each lattice cell has a vortical flow (right).*

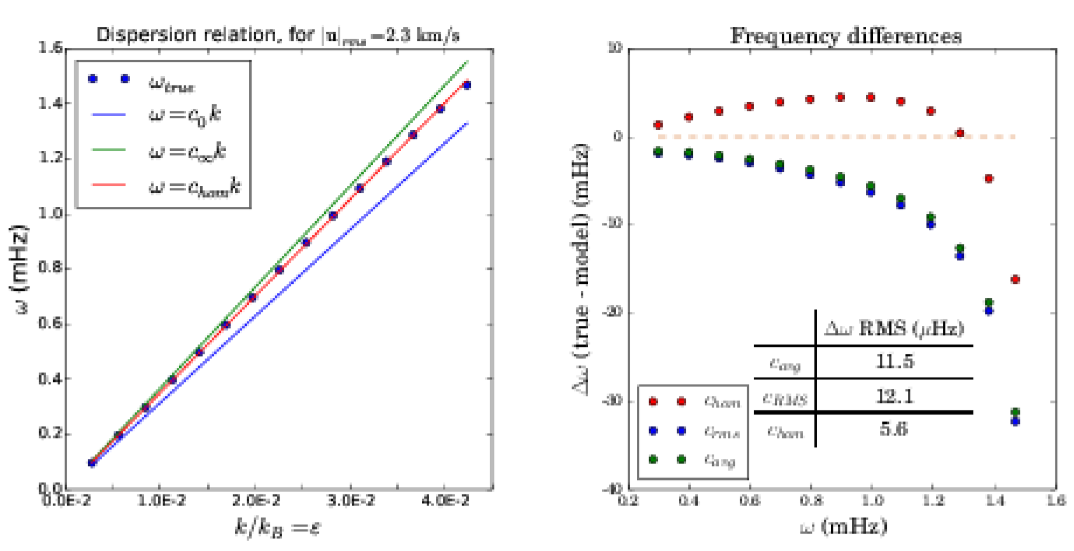

Figure 3: *Dispersion relations of the form $\omega = ck$ corresponding to different values of wave speed (left). The homogenized dispersion relation (red) is a better estimate than averaged sound speeds (right).*

This simple example shows us that a homogenized description of of the background medium is sufficient to reproduce the leading order characteristics of the wave field. Encouraged by the results, we apply a similar approach to analyze solar frequencies.

Flows on the sun

- Modified wave speed a implies $\delta p = a^2 \delta \rho$. Brünt-Väisälä frequency and Lamb frequency are affected.
- We start with a solar model which uses the Canuto-Mazzitelli formulation of convective flux (Basu and Antia, 1994). We call this Model A.
- Construct homogenized wave speed by introducing flow profile (Model Hom)
- We solve for eigenfrequencies in Model A and Model Hom, and compare them with frequencies observed by BiSON.

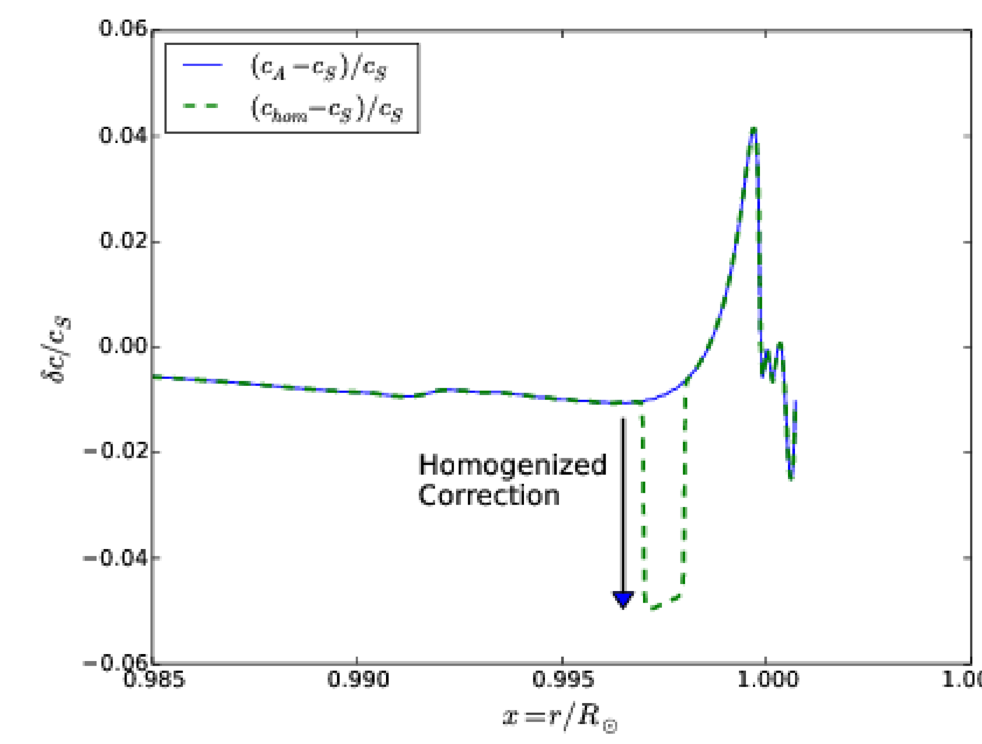

Figure-4: *Difference between wave speeds in the two models A and Hom, and the wave speed in Model S (Christensen-Dalsgaard, 1996). In Model Hom, the wave speed is reduced by 4% over a 700 km shell at a depth of 2 Mm below the solar surface.*

Solar frequencies

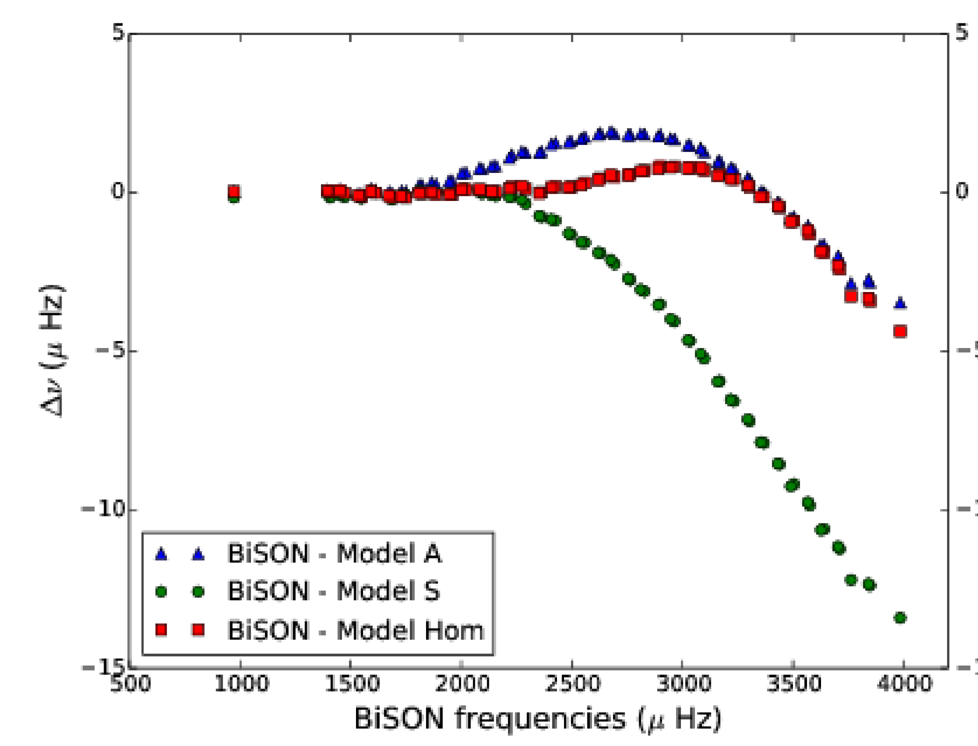

Figure-5: *Differences between modelled and BiSON frequencies for low degree modes ($\ell = 0-3$). A combination of Canuto-Mazzitelli model of convective flux and homogenized flow correction improves frequency estimates markedly.*

Conclusions

- Large-wavelength waves propagating through rapidly fluctuating media feel an average effect of the fluctuations through a modified wave speed.
- A reduction in wave speed brings modelled solar oscillation frequencies closer to observed ones.
- Caveat: Result is sensitive to background model; works better for Canuto-Mazzitelli formulation than mixing length.
- Consistency check on solar models - accounting for convection should bring oscillation frequencies closer to the observed ones.

Further work

- Effect of convective scattering on high-ℓ modes
- Combined impact of non-adiabaticity and flows.
- Improved models of homogenization.