Extension to Spherical Geometry - Sensitivity Kernels for Flows in Time-Distance Helioseismology

Vincent Böning

Kiepeneuer-Institut für Sonnenphysik, Freiburg

04.09.2014

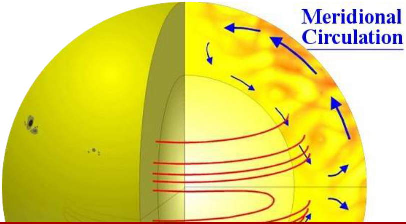
Collaborators:
M. Roth & W. Zima, KIS Freiburg

contact: vboening@kis.uni-freiburg.de

Motivation: We need spherical Kernels.

- They are already used in the current scientific debate, e.g. :
  - Meridional flow measurements: e.g. Zhao et al. (2013), Kholikov et al. (2014).
  - Studies on supergranules: e.g. Duvall & Hanasoge (2013), Duvall et al. (2014).
  - → Both perform modelling with ray approximation kernels.
- Born kernels not yet available in spherical geometry.
- Cartesian Born kernels used in HMI pipeline for subsurface flow inversions (e.g. Zhao et al., 2012).

Graphics: R. Arlt, AIP



How to calculate Born Kernels?

- Using general recipe of Gizon and Birch (2002), and for flows Birch and Gizon (2007) = BG2007.
  - Solve zero and first order damped and stochastically driven wave equation.
  - Via Green's functions, using Model S eigenfunctions.
  - Find expression for perturbed cross-correlation.
  - Find travel-time difference shift as a function of flow: $\delta\tau_{\text{diff}} = \int \mathbf{K} \cdot \mathbf{v} \, \mathrm{d}^3 r$.

How to calculate Born Kernels?

- Using general recipe of Gizon and Birch (2002), and for flows Birch and Gizon (2007) = BG2007.
  - Solve zero and first order damped and stochastically driven wave equation.
  - Via Green's functions, using Model S eigenfunctions.
  - Find expression for perturbed cross-correlation.
  - Find travel-time difference shift as a function of flow: $\delta\tau_{\text{diff}} = \int \mathbf{K} \cdot \mathbf{v} \, \mathrm{d}^3 r$.

→ And how to do spherical?

How to calculate Born Kernels?

- Using general recipe of Gizon and Birch (2002), and for flows Birch and Gizon (2007) = BG2007.
  - Solve zero and first order damped and stochastically driven wave equation.
  - Via Green's functions, using Model S eigenfunctions.
  - Find expression for perturbed cross-correlation.
  - Find travel-time difference shift as a function of flow: $\delta\tau_{\text{diff}} = \int \mathbf{K} \cdot \mathbf{v} \, \mathrm{d}^3 r$.

→ And how to do spherical?

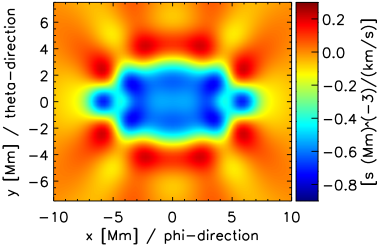
  - First attempts by Roth, Gizon & Birch (2006).
  - Expand Green's functions in spherical harmonics.
  - Find a formula that can actually be calculated numerically.
  - Validate the method.

Sanity Check: The Cartesian Limit

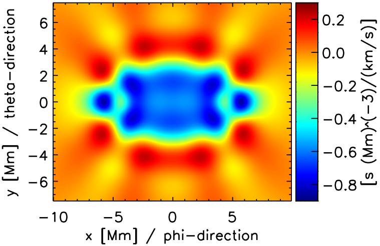
Sanity Check: The Cartesian Limit

- With A. C. Birch & L. Gizon.
- Setup: point-to-point travel-times on equator, $\Delta = 10\,\mathrm{Mm}$
- $K_\phi(r, \theta, \phi) = K_x(x, y, z)$, sensitivity for zonal flows, horizontal cuts.
- Line asymmetry not taken into account: Both results only using f-mode ridge in computation.

Spherical:



Cartesian (BG2007 code):

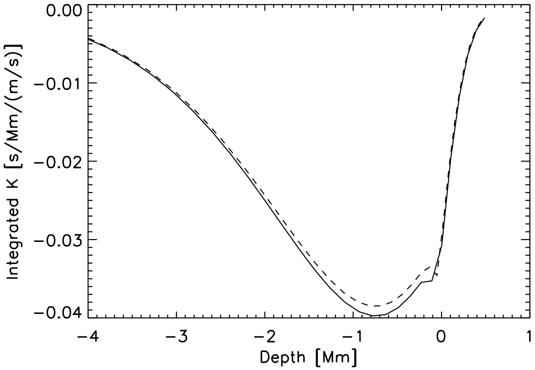


Maximum value: 8 % off.

Sanity Check: The Cartesian Limit

Horizontal integrals: $K_\phi = K_x$, sensitivity for zonal flows.

From Cartesian BG2007 code (solid) and from spherical code (dashed).



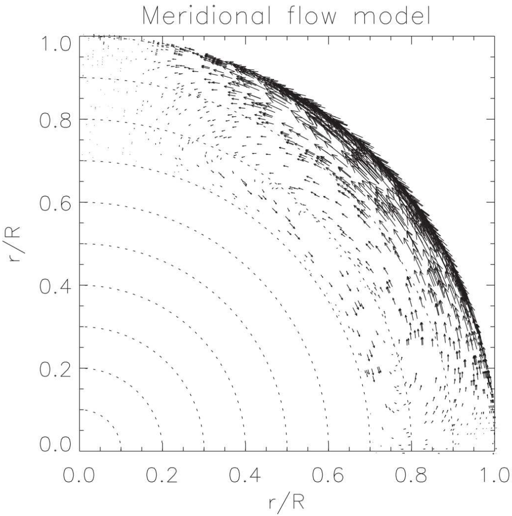
Maximum value: 3 % off. Similar to additional consistency tests.

Validation of Method with Simulated Data

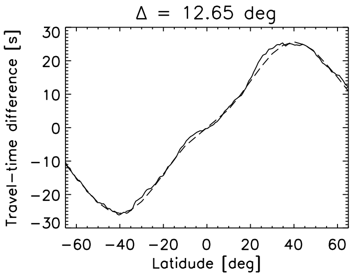
- Data and flow model (right) from Hartlep et al. (2013).
- Original flow model from Rempel (2006), amplified by factor of 36: $v_{\max} = 500\,\mathrm{m/s}$ at the surface.

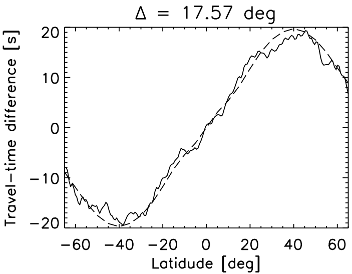
→ Do simulated and forward-modelled travel-times agree?

- Analysis done without filters, proceeding similarly to Hartlep et al. (2013).

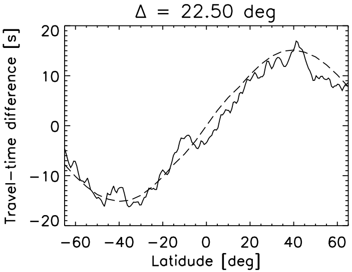


Validation of Method with Simulated Data





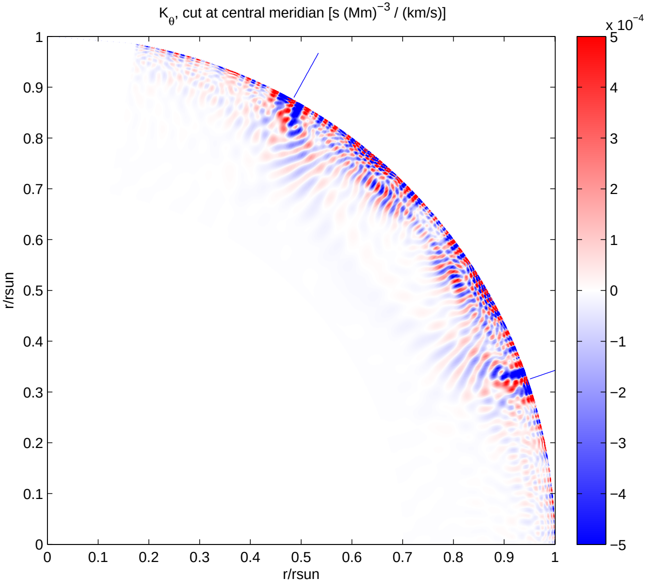
- Travel-times fitted with Gizon and Birch (2004) procedure to simulated data (solid line).
- Travel-times from forward-model, i.e. kernel integrated over flow model (dashed line).
- Travel-time differences: S-N!



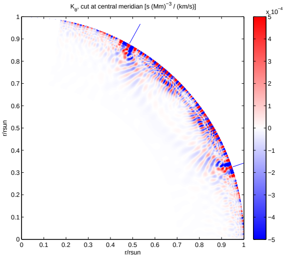
Deep Meridional Flow Kernels: First results

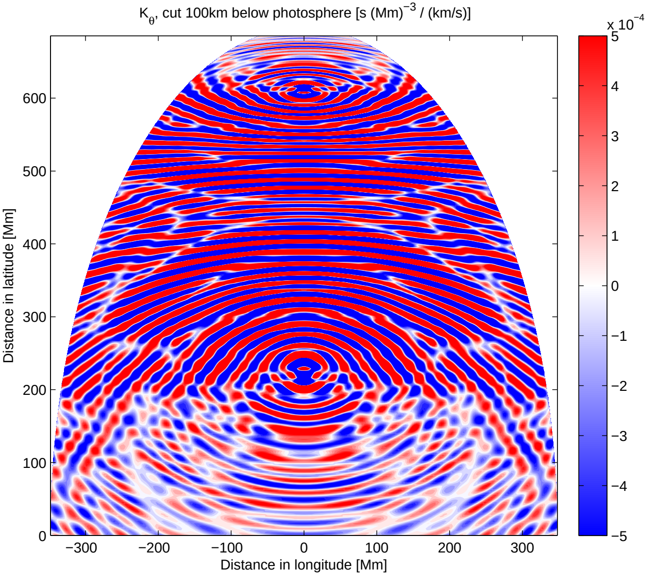
- We compare kernels with different filters (no filter, low-pass in l, phase-speed).
- K_θ, i.e. sensitivity to southward flow,
- cuts at central meridian and just below photosphere,
- $\Delta = 42.19\,\text{deg}$ in N-S-direction,
- centered at latitude 40 degrees north,
- computation uses $l \leq 170$, same as simulation in Hartlep et al. (2013),
- modelling radial component of wavefield.

Unfiltered, $\Delta = 42.19\,\mathrm{deg}$

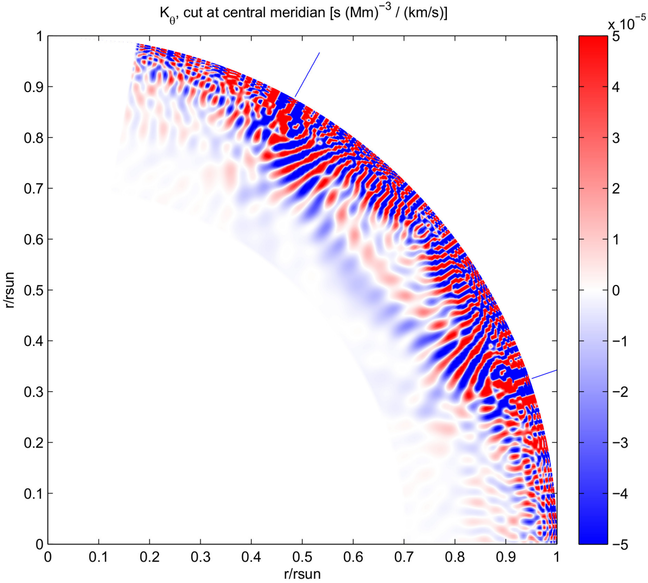


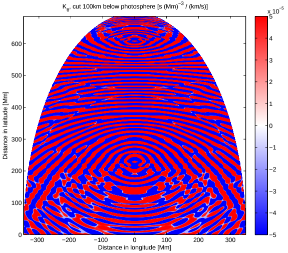
Unfiltered, $\Delta = 42.19\,\mathrm{deg}$



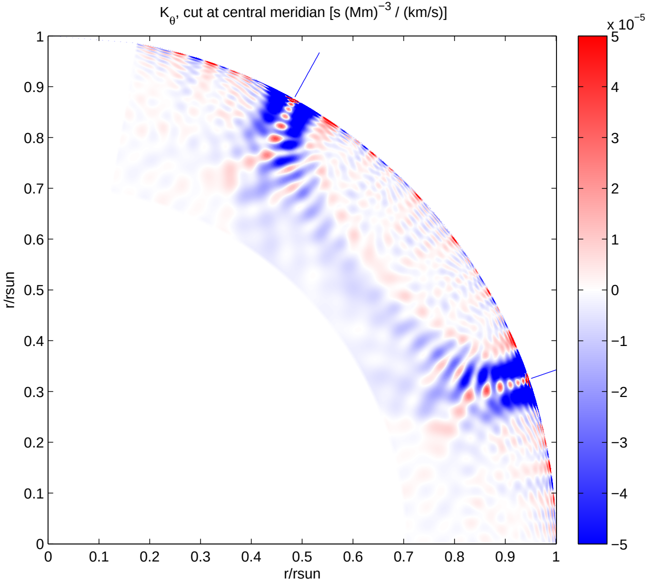


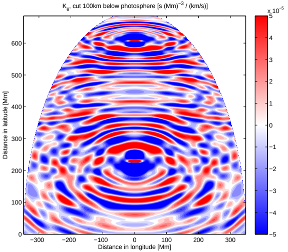
Unfiltered, $\Delta = 42.19\,\mathrm{deg}$



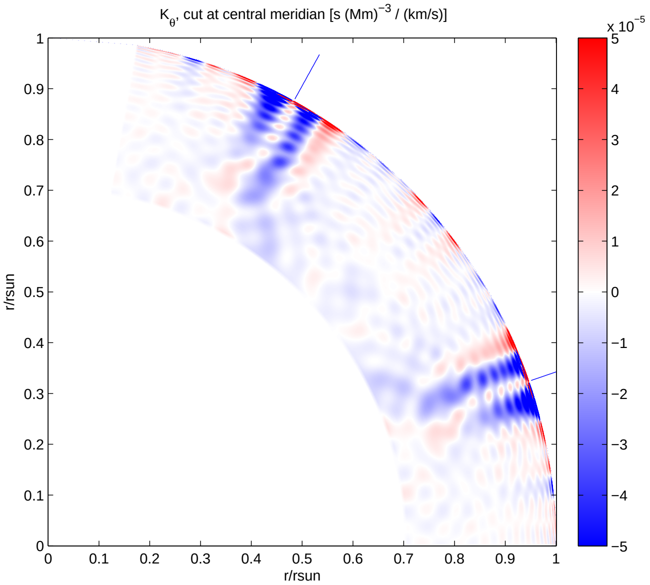


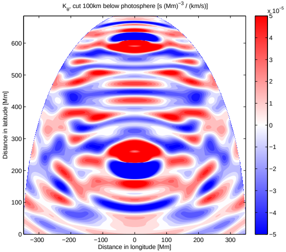
$\Delta = 42.19\,\mathrm{deg}$, filtered: $F(l,\omega) = \exp(-\frac{l^2}{2\,\delta l^2})$, $\delta l = 50$



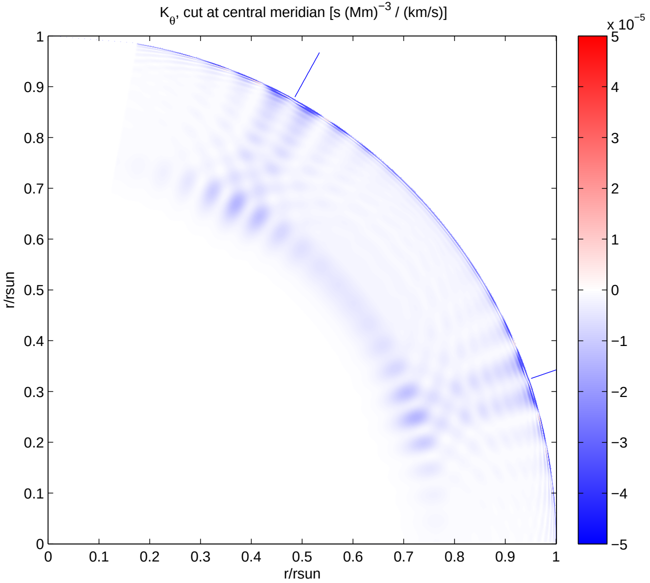


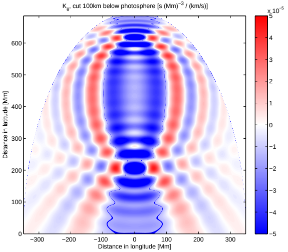
$\Delta = 42.19\,\text{deg}$, filtered: $F(l,\omega) = \exp\left(-\frac{l^2}{2\,\delta l^2}\right)$, $\delta l = 20$



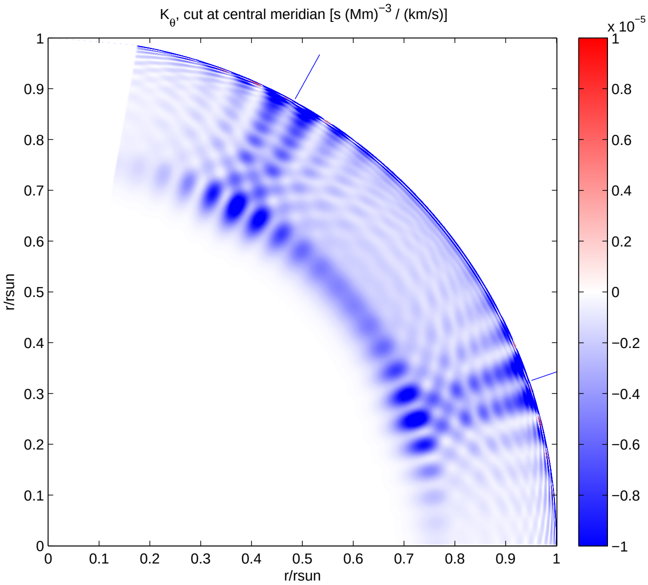


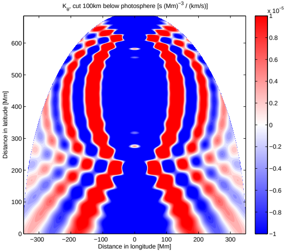
Phase-speed filt., $v_0 = 284.3\,\mathrm{km/s}, \delta v = 7\,\mathrm{km/s}$ (Kholikov et al., 2014)





Phase-speed filt., $v_0 = 284.3\,\mathrm{km/s}$, $\delta v = 7\,\mathrm{km/s}$, $\Delta = 42.19\,\mathrm{deg}$





How big is the sensitivity to the return flow? ($\Delta = 42\,\mathrm{deg}$)

- Kernel integrated over Hartlep et al. (2013) meridional flow profile.

Kernel	$\delta\tau$ for $r/R_\odot \leq 0.79$	% of total $\delta\tau$	ray kernels *
unfiltered	-0.446 s	10.4 %	$\approx$ 20 %
$\delta l = 50$	-0.489 s	13.4 %	
$\delta l = 20$	-0.467 s	14.8 %	
phase-sp.	-0.503 s	11.4 %	

Table: * Ray kernel value from Hartlep et al. (2013).

How big is the sensitivity to the return flow? ($\Delta = 42\,\mathrm{deg}$)

- Kernel integrated over Hartlep et al. (2013) meridional flow profile.

Kernel	$\delta\tau$ for $r/R_\odot \leq 0.79$	% of total $\delta\tau$	ray kernels *
unfiltered	-0.446 s	10.4 %	$\approx$ 20 %
$\delta l = 50$	-0.489 s	13.4 %	
$\delta l = 20$	-0.467 s	14.8 %	
phase-sp.	-0.503 s	11.4 %	

Table: * Ray kernel value from Hartlep et al. (2013).

- Divide $\delta\tau$ by ≈ 25 to get realistic numbers: $\delta\tau_{\leq 0.79} \approx 0.02\,\mathrm{s}$!
- The sensitivity is always concentrated in the upper convection zone.
- Ray and Born kernel values are quite different. Is that a problem?

How big is the sensitivity to the return flow? ($\Delta = 42\,\mathrm{deg}$)

- Kernel integrated over Hartlep et al. (2013) meridional flow profile.

Kernel	$\delta\tau$ for $r/R_\odot \leq 0.79$	% of total $\delta\tau$	ray kernels *
unfiltered	-0.446 s	10.4 %	$\approx$ 20 %
$\delta l = 50$	-0.489 s	13.4 %	
$\delta l = 20$	-0.467 s	14.8 %	
phase-sp.	-0.503 s	11.4 %	

Table: * Ray kernel value from Hartlep et al. (2013).

- Divide $\delta\tau$ by ≈ 25 to get realistic numbers: $\delta\tau_{\leq 0.79} \approx 0.02\,\mathrm{s}$!
- The sensitivity is always concentrated in the upper convection zone.
- Ray and Born kernel values are quite different. Is that a problem?

⇒ Unfiltered kernel has smallest sensitivity to return flow.

⇒ Low-pass filtering in l gives the strongest relative sensitivity to return flow.

⇒ Phase-speed filtered kernels are best localised at the target depth.

Summary

1. We can adequately calculate spherical Born kernels:
   - ✓ Results from Cartesian geometry (BG2007) reproduced.
   - ✓ Effect of meridional flow correctly modelled (Hartlep et al., 2013).

Summary

1. We can adequately calculate spherical Born kernels:
   - ✓ Results from Cartesian geometry (BG2007) reproduced.
   - ✓ Effect of meridional flow correctly modelled (Hartlep et al., 2013).
2. Example kernels for meridional flow measurements:
   - 10-15% of the total sensitivity is due to the return flow for a standard meridional flow profile.
   - Ray and Born kernels have different sensitivity to return flow by a factor of 2.
   - Low-pass filtering in l gives the strongest relative sensitivity to return flow.
   - Phase-speed filtered kernels are best localised at target depth.

Summary

1. We can adequately calculate spherical Born kernels:
   - ✓ Results from Cartesian geometry (BG2007) reproduced.
   - ✓ Effect of meridional flow correctly modelled (Hartlep et al., 2013).
2. Example kernels for meridional flow measurements:
   - 10-15% of the total sensitivity is due to the return flow for a standard meridional flow profile.
   - Ray and Born kernels have different sensitivity to return flow by a factor of 2.
   - Low-pass filtering in l gives the strongest relative sensitivity to return flow.
   - Phase-speed filtered kernels are best localised at target depth.

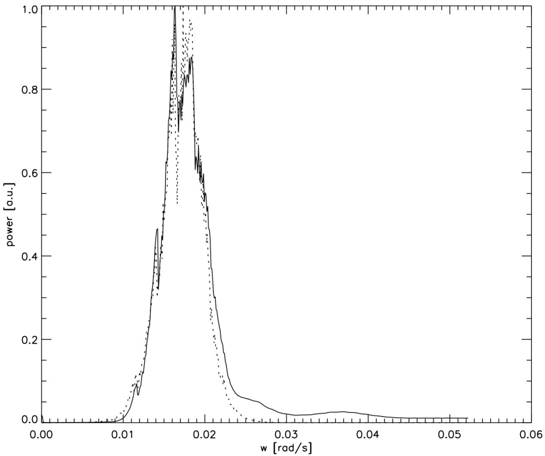
Thank you very much!

References

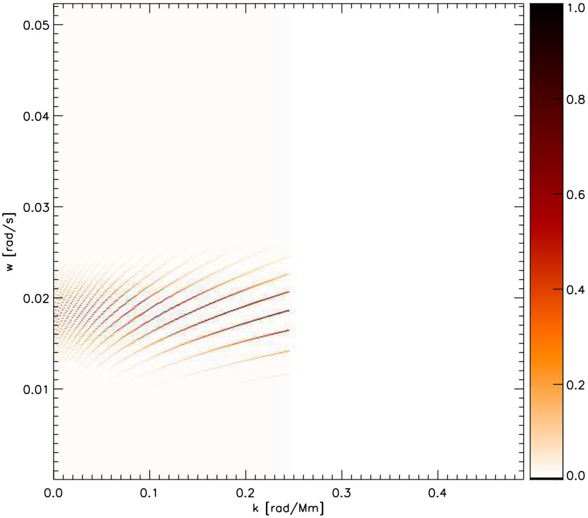
- Zhao et al. (2013)
- Kholikov et al. (2014)
- Duvall & Hanasoge (2013)
- Duvall et al. (2014).
- Zhao et al. (2012)
- Gizon and Birch (2002)
- Birch and Gizon (2007, BG2007)
- Roth, Gizon & Birch (2006)
- Hartlep et al. (2013)
- Rempel (2006)
- Gizon and Birch (2004)

Power: kernel vs simulated data

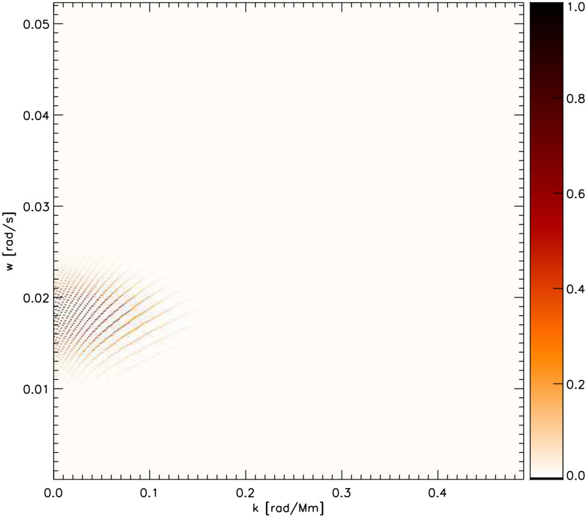
Unfiltered power integrated over k, normalized, for Hartlep et al. 2013 (solid) and spherical kernels (dashed), using $l \leq 169$:



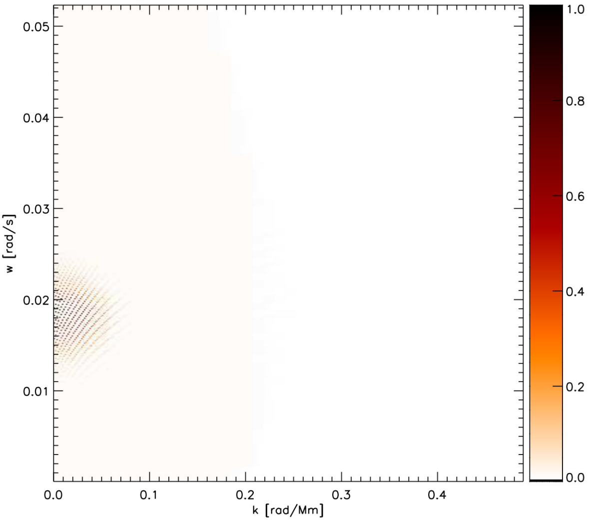
Unfiltered Kernel



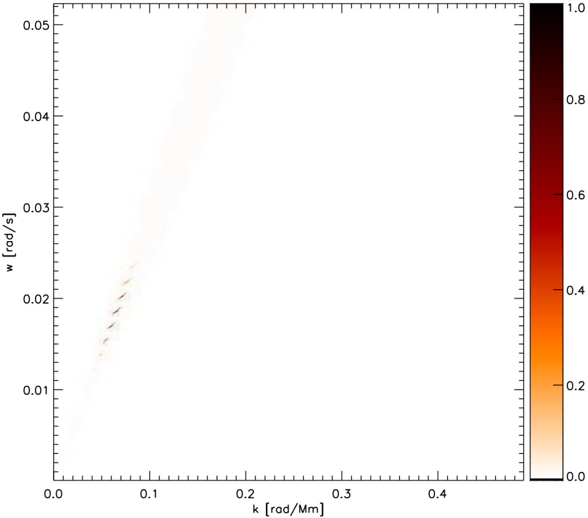
Low-pass I, $\delta l = 50$



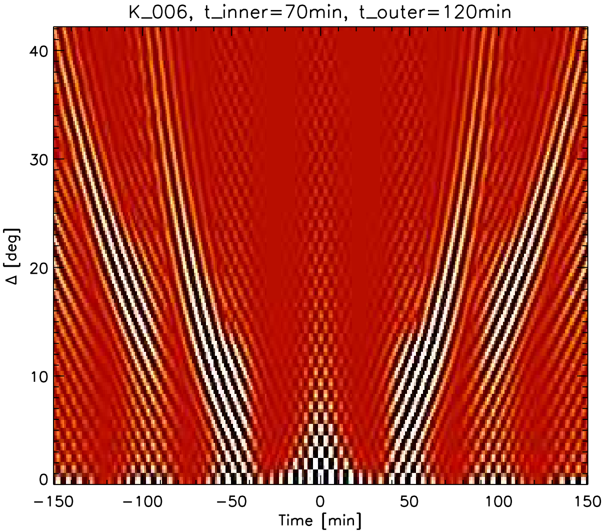
Low-pass I, $\delta l = 20$



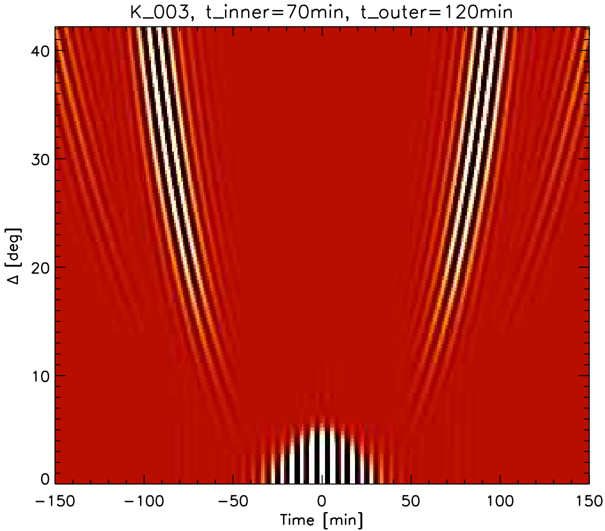
Phase-speed



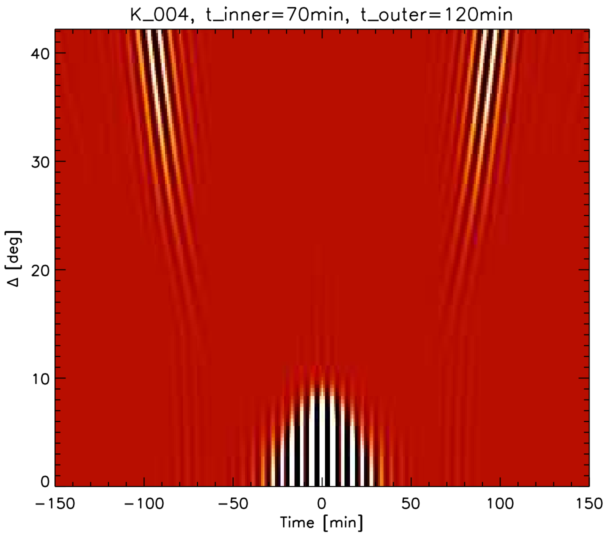
Unfiltered Kernel



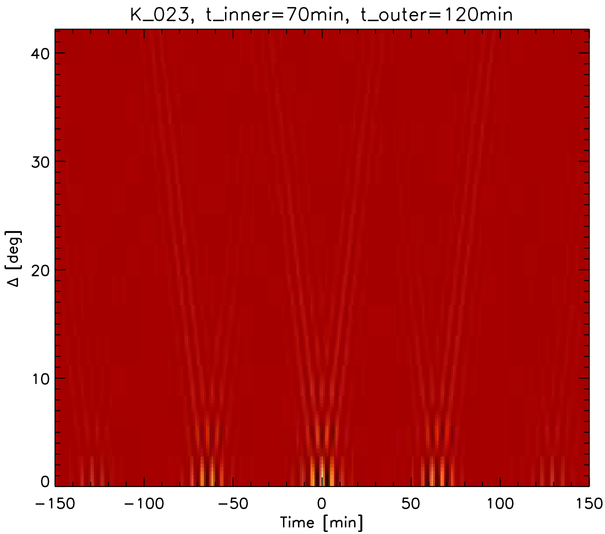
Low-pass l, $\delta l = 50$



Low-pass I, $\delta l = 20$

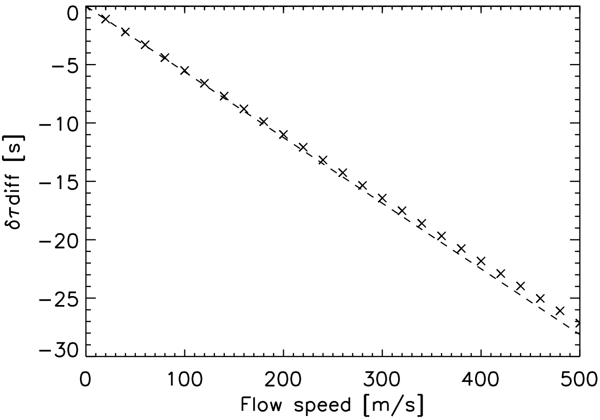


Phase-speed


K_023, t_inner=70min, t_outer=120min
Δ [deg]
0
10
20
30
40
−150
−100
−50
0
50
100
150
Time [min]

$v_{\mathrm{max}} = 500\,\mathrm{m/s}$ still in linear regime?

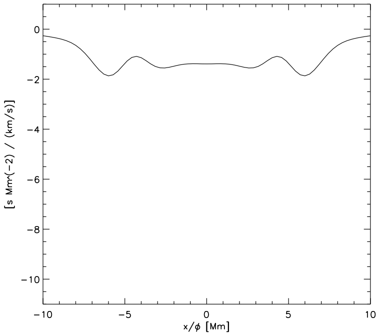
For $\Delta = 22.5\,\mathrm{deg}$, travel-times from E-W-kernel (dashed) at equator and *exact* perturbed cross-correlation (crosses, method see Jackiewicz et al., 2007) for a solid body rotation corresponding to a certain equatorial zonal flow speed (x-axis): Linear regime extends to these flow speeds.



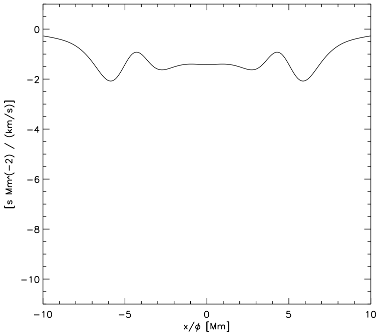
Sanity Check: The Cartesian Limit

$K_\phi = K_x$, sensitivity for zonal flows, integrated wrt depth, cut along $y = 0$:

Spherical:



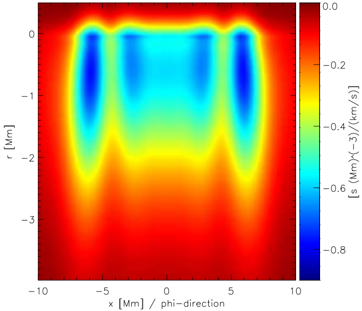
BG2007-Code (Cartesian)



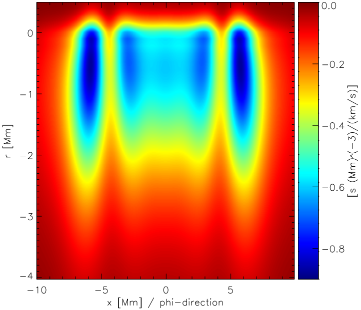
Sanity Check: The Cartesian Limit

$K_\phi = K_x$, sensitivity for zonal flows.

Spherical:



BG2007-Code (Cartesian)

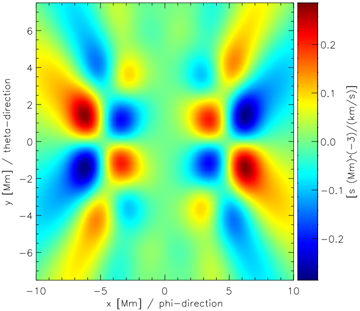


Maximum value: 8 % off.

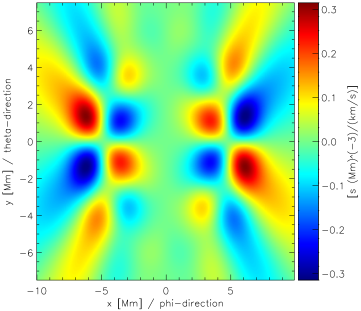
Sanity Check: The Cartesian Limit

$K_\theta = -K_y$, sensitivity for meridional flows.

Spherical:



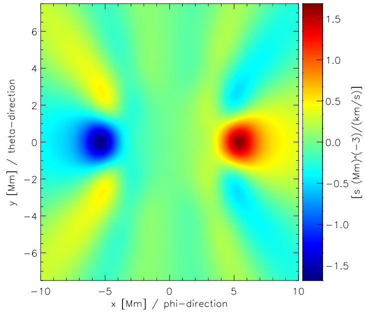
BG2007-Code (Cartesian)



Sanity Check: The Cartesian Limit

$K_r = K_z$, sensitivity for convective flows.

Spherical:



BG2007-Code (Cartesian)