DIRECTIONAL TIME-DISTANCE PROBING OF SOLAR MAGNETIC REGIONS

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with

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Motivation: Helioseismic Implications of MHD Mode Conversion

• Using local helioseismology we have observed travel-time shifts in active regions

• These shifts have typically been interpreted as arising predominantly from subsurface inhomogeneities

• Question: What is the role played by MHD mode conversion in these observed travel-time shifts?

![Diagram of wave interactions]

• Method: conduct linear forward modelling & “directional helioseismology” to look for a correspondence between

  • i) the upward acoustic and magnetic wave losses
  • ii) time distance travel-time shifts

• If a correspondence exists, then what’s happening in the atmosphere is affecting the helioseismology and we want to try and quantify it
Linking wave energy losses in the atmosphere and travel times  
(Cally & Moradi, 2013, MNRAS)

- **Forward Modelling**
  - Linear MHD wave propagation using SPARC
  - 3D computational box: 140 Mm x 140 Mm x 27 Mm
  - Model S + uniform inclined field (0° ≤ θ ≤ 90°)
  - Random stochastic wave sources

- **Analysis**
  - Filtering:
    - Directional “ball” filter to isolate \( k_h \) and azimuthal direction \( \phi \) (0° ≤ \( \phi \) ≤ 180°)
    - Frequency filters also applied
  - Wave Energy Flux
    - Wave vector energy flux: \( \mathbf{F} = \mathbf{F}_{ac} + \mathbf{F}_{mag} = \text{Re}[\rho \mathbf{v}^* + \mathbf{e} \times \mathbf{b}^*] \)
    - Vertical fluxes measured at \( z = 1.37 \) Mm (fast wave evanescent at these heights → no flux contribution)
  - Time-Distance Travel Times
    - (Phase) travel time perturbations (\( \delta \tau \)) extracted from Gabor wavelet fits to the cross-correlations
    - \( \delta \tau \) calculated for each \( k_h \), \( \theta \) and \( \phi \) and at \( z = 0.3 \) Mm
Directionally Filtered Wave Energy Fluxes

• Negligible acoustic power at low $\theta$ (acoustic cutoff frequency $\sim 5$ mHz)

• Once $\omega > \omega_c \cos \theta \Rightarrow$ ramp effect kicks in and substantial acoustic flux is recorded

• Magnetic flux generally peaks at higher $\Phi$ than acoustic flux

• Results in good agreement with previous studies of fast-to-Alfvén mode conversion (e.g., Cally & Goossens 2008; Khomenko & Cally 2011, 2012)

$k_h = 1.0 \text{ Mm}^{-1}$

$k_h = 0.75 \text{ Mm}^{-1}$

$k_h = 0.5 \text{ Mm}^{-1}$

acoustic

magnetic

acoustic

magnetic
Directionally Filtered Travel Times

• Strong correspondence with wave energy flux contours:
  • Clear manifestations of acoustic cutoff & directional ($\phi$) dependence at 3 and 5 mHz
  • At low $\theta$ (below acoustic cutoff) $\Rightarrow$ $\delta\tau$ is small, primarily negative
  • At high $\theta$ (above acoustic cutoff) $\Rightarrow$ substantial negative $\delta\tau$ evident
  • Above acoustic cutoff and at $\phi$ associated with magnetic losses $\Rightarrow$ significant “slow down”/increases in $\delta\tau$
Comparison of Filtered Fluxes and Travel Times with BVP Calculations (Cally & Goossens 2008, *Sol. Phys.*)

1 kG, \( k_h = 1.0 \text{ Mm}^{-1} \), 5 mHz

- As seen in Paul’s talk \( \rightarrow \) fluxes and travel times consistent with SPARC calculations
- Results confirm a direct correspondence between wave energy losses and “travel time” perturbations
- What about a “realistic” sunspot atmosphere? \( \rightarrow \) large field strengths, low density on axis
Numerical Issue: Alfvén Wave Speed ($c_a$)

- Exponential increase in $c_a$ above the surface introduces a significant CFL time-step constraint for explicit numerical codes ($\Delta t \approx \Delta z/c_a$)

- Results in the need for very small $\Delta t$ when simulating even moderate magnetic field strengths

**Solutions?:**

- i) Live with a very small $\Delta t$ → not practical

- ii) Employ a Lorentz Force or $c_a$ “limiter” → popular in computational helioseismology
  - Scale the Lorentz Force by a factor when $c_a/c_s$ becomes too big, effectively caps $c_a$ at a particular value (e.g., Rempel et al. 2009; Cameron et al. 2011; Braun et al. 2012)
  - Scale the magnetic field by a factor so $c_a$ does not exceed a predetermined value (e.g., Hanasoge et al. 2012)

**Helioseismic Implications?:**

- Fast waves: reflect off the $c_a$ gradient back to the surface at height where $c_a \sim \omega/k_h$

- What are the effects on travel times if $c_a$ limiter is too close to $\omega/k_h$ being studied?

- **Forward Modelling**
  - Linear MHD wave propagation using SPARC
  - 3D computational box: 140 Mm X 140 Mm X 27 Mm
  - Model S + uniform inclined field (500 G, $0^\circ \leq \theta \leq 90^\circ$)
  - Single source wave excitation
  - 50 x 1 hour simulations: no $c_a$ cap, with various $c_a$ caps, quiet sun reference

- **Analysis**
  - Using $v_z$ at constant geometrical height ($z = 0.3$ Mm)
  - Selecting a receiver point away from central axis in $xy$-plane allows us to isolate wave propagation direction ($\phi$)
  - Frequency filtered point-to-point directional $\delta \tau$ calculated for various travel distances $\Delta$ (and $\omega/k_h$)

<table>
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<tr>
<th>$c_a$ cap (km/s)</th>
<th>$\Delta t$ (s)</th>
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<tr>
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<table>
<thead>
<tr>
<th>$\Delta$ (Mm)</th>
<th>$\omega/k_h$ (km/s)</th>
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<tbody>
<tr>
<td>11.6</td>
<td>16.3</td>
</tr>
<tr>
<td>24.35</td>
<td>34.8</td>
</tr>
<tr>
<td>42.95</td>
<td>46.8</td>
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</table>
• “No limiter” case consistent with random sources results

• Significant $\delta \tau$ discrepancies arise when $c_a \text{ cap} < \omega/k_h$

• Situation improves as $c_a \text{ cap}$ is raised above $\omega/k_h$

• Even with $c_a \text{ cap}$ at 160 km/s there are still $\delta \tau$ discrepancies of ~ 1–2 s $\Rightarrow$ can live with that?

• Conclusion: Keep $c_a \text{ cap}$ well above the $\omega/k_h$ you are intending to analyse!
Directional Time-Distance Probing of a Realistic Sunspot Atmosphere
(Moradi & Cally, *in prep*)

- **Forward Modelling**
  - Linear MHD wave propagation using SPARC
  - 3D computational box: 140 Mm \times 140 Mm \times 12 Mm
  - MHS sunspot models from Khomenko & Collados (2008) + some improvements \(\rightarrow\) see Damien Przybylski’s poster
  - Free parameters: radius, photospheric field strength (\(B_{\text{phot}}\)), inclination, Wilson depression (\(z_{\text{wd}}\)) etc.
  - Single source(s) wave excitation along \(-x, y = 0\)
  - \(c_a\) cap at 80 km/s \(\rightarrow\) only small \(\Delta/\text{waves with } \omega/k_h\)
    well below \(c_a\) limit considered for the analysis

- **Analysis**
  - Data analysed at optical depth \(\log \tau_5 = -1.6\) (\(\sim\) height where typical photospheric spectral lines formed)
  - Point-to-point directional \(\delta \tau\) calculated as function of source position (\(\theta\)) and receiver direction (\(\phi\))
Directional Travel Times

$B_{\text{phot}} = 1.5 \text{ kG}$

$\Delta = 6.2 \text{ Mm}$

$z_{\text{Wd}} = -300 \text{ km}$

$z_{\text{Wd}} = -400 \text{ km}$

$z_{\text{Wd}} = -500 \text{ km}$

$\delta \tau (\text{sec})$

3 mHz

5 mHz
Summary

1. Using forward modelling & directional helioseismology we found substantial wave “travel time” discrepancies of several tens of seconds related to phase changes resulting from mode conversion, and not “actual” travel time changes.

2. These results were also separately verified using BVP methods.

3. In a related study, we also found that employing a Lorentz Force/$c_a$ “limiter” severely impacts the reflection of fast waves in the atmosphere (and the $\delta \tau$ as a result) unless it is placed well above $\omega/k_h$ associated with the wave travel distances being studied.

4. Results from directional time-distance probing of model sunspot atmospheres are consistent with uniform magnetic field + horizontally invariant atmosphere results.

5. Overall our results indicate that processes occurring higher up in the atmosphere can strongly influence the helioseismology, and argue strongly for the viability of directional time-distance probing of real solar magnetic regions.