

Inversion of the two-point velocity correlations on the Sun's surface

Damien Fournier

Institut für Numerische und Angewandte Mathematik, Göttingen
d.fournier@math.uni-goettingen.de

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Helioseismology and Applications

Context

- ▶ The two-point velocity correlations $R_{ij}(\mathbf{x}_1, \mathbf{x}_2)$ is a measure of convection. It converges to the Reynolds stresses when \mathbf{x}_1 tends to \mathbf{x}_2 .

Context

Observations

The forward problem

Noise model

The inverse problem

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where v_0 is the deterministic mean part of the velocity and V the fluctuating (turbulent) part.

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- ▶ Supposing that the turbulent part is horizontally spatially homogeneous, the two-point velocity correlations can be written as

$$R_{ij}(\mathbf{d}, z_1, z_2) = \langle V_i(\mathbf{x}, z_1) V_j(\mathbf{x} + \mathbf{d}, z_2) \rangle.$$

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- ▶ Aim: recover R_{ij} from SDO/HMI observations.

Context

Inversion possibilities

$$\tau^{\Delta}(\mathbf{x}) = \int_V \mathbf{K}^{\Delta}(\mathbf{x}' - \mathbf{x}, z) \cdot v(\mathbf{x}', z) d^2\mathbf{x}' dz + \Lambda^{\Delta\Delta'}(\mathbf{x}')$$

$$\tau_1, \dots, \tau_N \xrightarrow{\text{Standard inversion}} v_1, \dots, v_N$$

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The forward problem

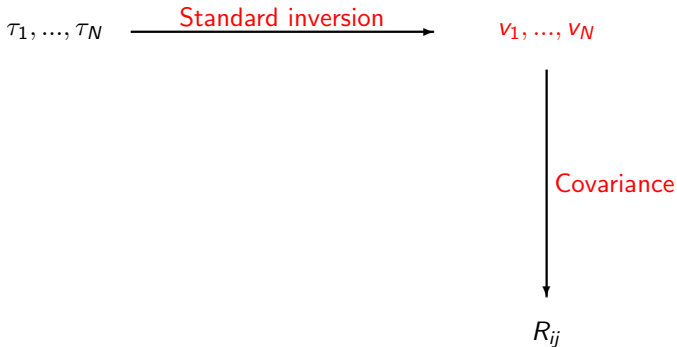
Noise model

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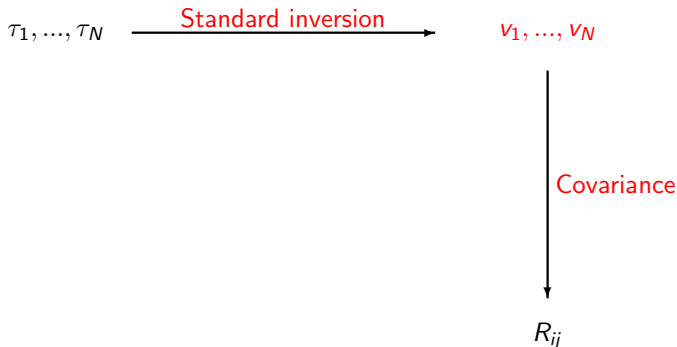
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Advantage: We know how to do it!

Drawback: Costly! Requires N inversions then averaging



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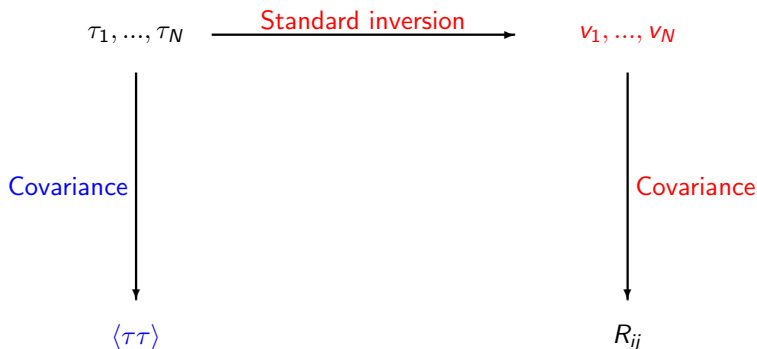
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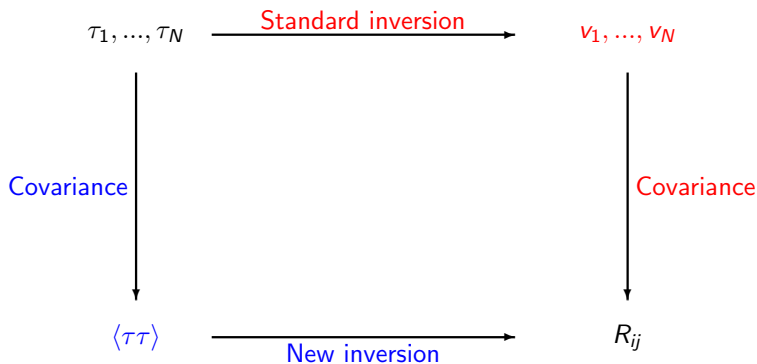
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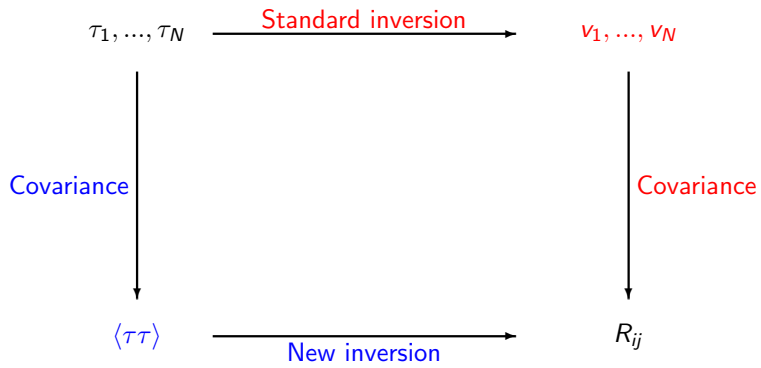
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Advantage: Only one inversion required. Should be fast! Improve the signal-to-noise ratio

Drawback: We need kernels, a noise model



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Relation between products of travel times and two-points velocity correlations

- ▶ Relation between travel times and velocity

$$\tau^{\Delta}(\mathbf{x}) = \int_V \mathbf{K}^{\Delta}(\mathbf{x}' - \mathbf{x}, z) \cdot v(\mathbf{x}', z) d^2\mathbf{x}' dz + \Lambda^{\Delta\Delta'}(\mathbf{x}')$$

where $\Lambda^{\Delta\Delta'}(\mathbf{x}') = \text{Cov}[\tau^{\Delta}(\mathbf{x}), \tau^{\Delta'}(\mathbf{x} + \mathbf{x}')]]$.

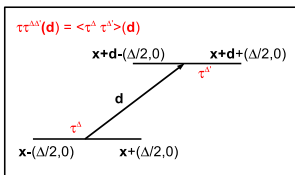
- ▶ Similarly, the two-point velocity correlations are linked to a product of travel times

$$\underbrace{\langle \tau^{\Delta}(\mathbf{x}), \tau^{\Delta'}(\mathbf{x} + \mathbf{d}) \rangle}_{\text{new observables } O^{\Delta\Delta'}} - \underbrace{\Lambda^{\Delta\Delta'}(\mathbf{d})}_{\text{noise for } \tau} = \int_{z'} \int_z \underbrace{K_i^{\Delta}(\mathbf{x}; z) * K_j^{\Delta'}(\mathbf{x} + \mathbf{d}; z')}_{\text{new kernels } K_{ij}^{\Delta\Delta'}} * R_{ij}(\mathbf{d}, z, z') dz dz' + \underbrace{\Gamma^{\Delta\Delta'} \delta \delta'(\mathbf{d}, \mathbf{d}')}_{\text{new noise for } \tau^{\Delta} \tau^{\Delta'}}$$

where

$$\Gamma^{\Delta\Delta'} \delta \delta'(\mathbf{d}, \mathbf{d}') = \text{Cov}[\langle \tau^{\Delta}(\mathbf{x}), \tau^{\Delta'}(\mathbf{x} + \mathbf{d}) \rangle, \langle \tau^{\Delta}(\mathbf{x}), \tau^{\Delta'}(\mathbf{x} + \mathbf{d}') \rangle]$$

Observations



$$\tau\tau_{xx}(\mathbf{d}) = \frac{1}{V} \int_V \tau^{\Delta}(\mathbf{x}) \tau^{\Delta'}(\mathbf{x} + \mathbf{d}) d\mathbf{x}$$

$$\Lambda_{xx}(\mathbf{d}) = \frac{1}{V} \int_V n^{\Delta}(\mathbf{x}) n^{\Delta'}(\mathbf{x} + \mathbf{d}) d\mathbf{x}$$

$$\mathbf{O}_{xx}(\mathbf{d}) = \tau\tau^{\Delta\Delta'}(\mathbf{d}) - \Lambda^{\Delta\Delta'}(\mathbf{d})$$

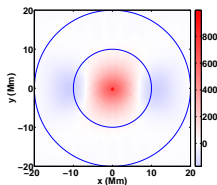
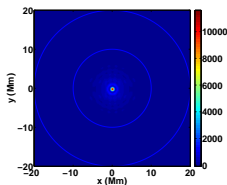
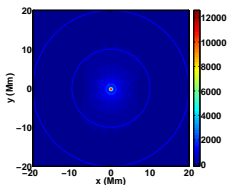
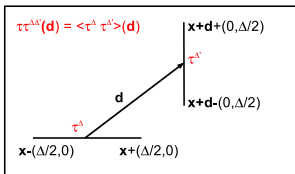


Figure: $\tau\tau_{xx}(\mathbf{d})$ (left), $\Lambda_{xx}(\mathbf{d})$ (middle) and $\tau\tau_{xx}(\mathbf{d}) - \Lambda_{xx}(\mathbf{d})$ (right) in s^2 in the real space as a function of \mathbf{d} at a latitude of $+20^\circ$. All units are s^2 .

Observations



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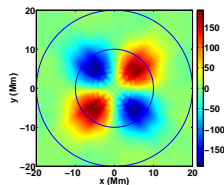
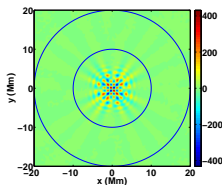
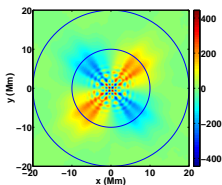


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The forward problem

Observations	Kernels			
O_{xx}	K_{xx}^{xx}	K_{xx}^{xy}	K_{xx}^{yy}	R_{xx}
O_{xy}		K_{xy}^{xy}	K_{xy}^{yy}	R_{xy}
O_{yy}			K_{yy}^{yy}	R_{yy}

Context

Observations

The forward problem

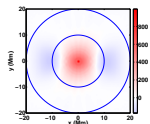
Noise model

The inverse problem

The forward problem

Observations

Kernels

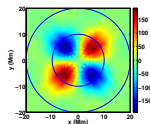


$$K_{xx}^{xx}$$

$$K_{xx}^{xy}$$

$$K_{xx}^{yy}$$

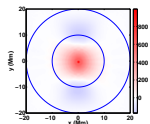
$$R_{xx}$$



$$K_{xy}^{xy}$$

$$K_{xy}^{yy}$$

$$R_{xy}$$



$$K_{yy}^{yy}$$

$$R_{yy}$$

Context

Observations

The forward problem

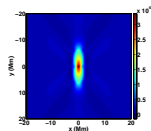
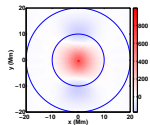
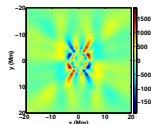
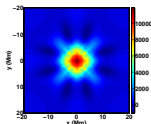
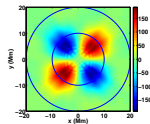
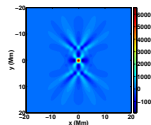
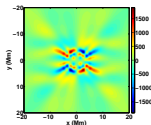
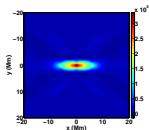
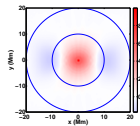
Noise model

The inverse problem

The forward problem

Observations

Kernels



R_{xx}

R_{xy}

R_{yy}

Context

Observations

The forward problem

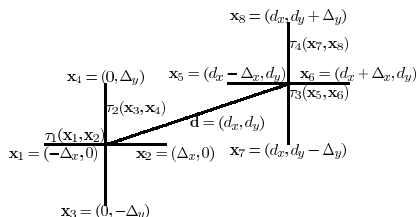
Noise model

The inverse problem

Covariance matrix for products of travel times

A good approximation of the noise covariance matrix for products of travel times is given by [Fournier 2014]

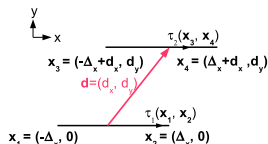
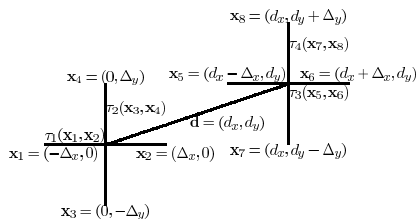
$$\text{Cov}[\tau_1\tau_2, \tau_3\tau_4] \approx \text{Cov}[\tau_1, \tau_3]\text{Cov}[\tau_2, \tau_4] + \text{Cov}[\tau_1, \tau_4]\text{Cov}[\tau_2, \tau_3].$$



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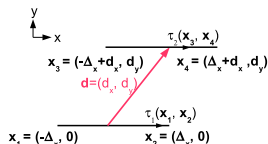
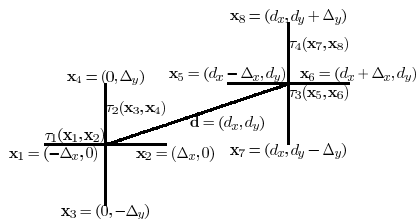
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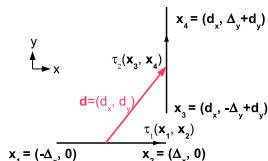
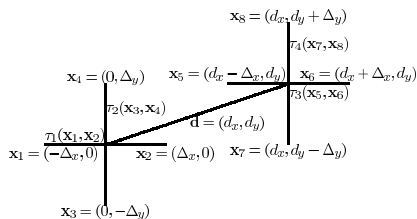
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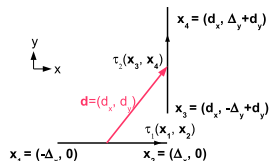
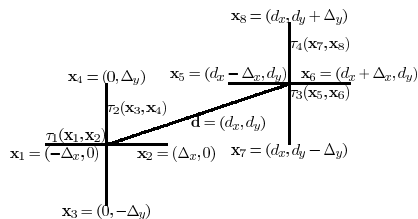
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Inversion

Two-point velocity correlations on the Sun's surface

Damien Fournier

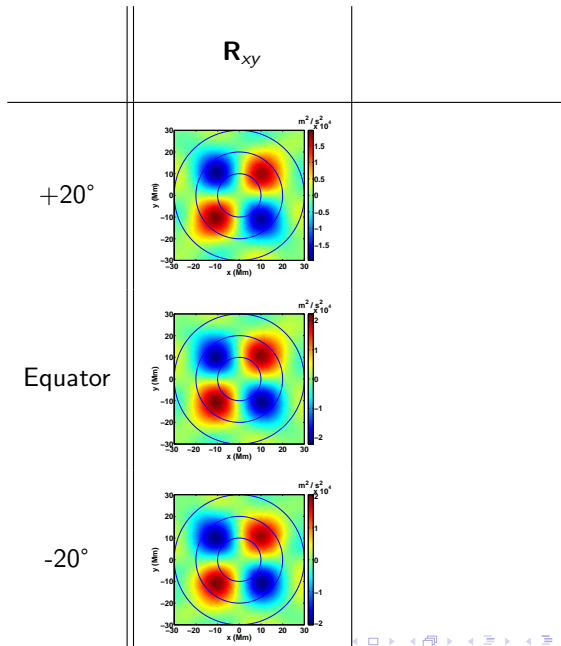
Context

Observations

The forward problem

Noise model

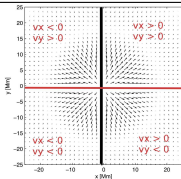
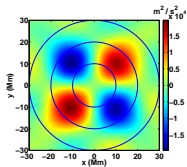
The inverse problem



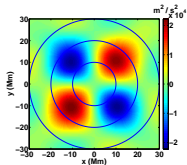
Inversion

R_{xy}

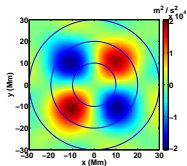
+20°



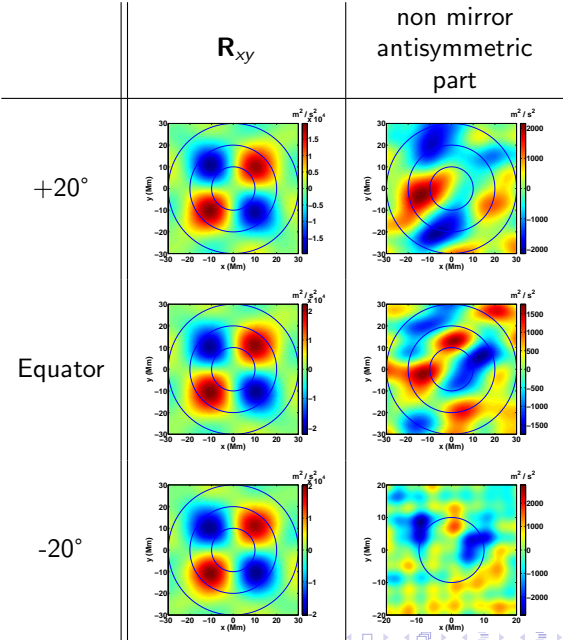
Equator



-20°



Inversion



Additional averaging

- ▶ Without rotation \mathbf{O}_{xy} should be mirror antisymmetric

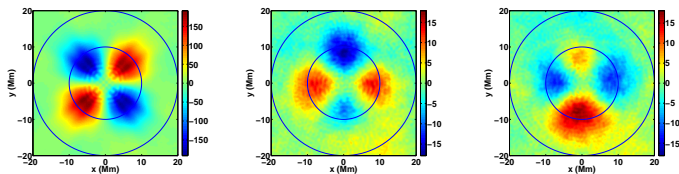
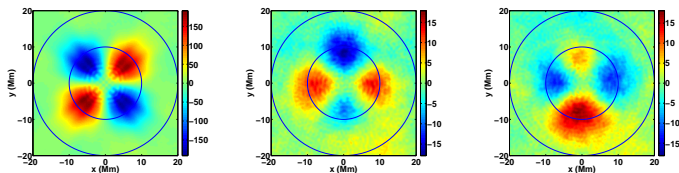


Figure: Observations $\mathbf{O}_{xy}(\mathbf{d})$ (left) and the non mirror antisymmetric part at a latitude of $+20^\circ$ (middle) and -20° (right).

Additional averaging

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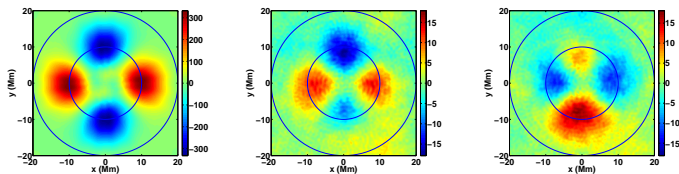
- ▶ With rotation, following [Kitchatinov 1986]

$$\mathbf{O}_{xy}(\mathbf{k}) = \beta \left(\mathbf{O}_{yy}^{(0)}(\mathbf{k}) - \mathbf{O}_{xx}^{(0)}(\mathbf{k}) \right) + \beta_{xy} \mathbf{O}_{xy}^{(0)}(\mathbf{k}) + \beta_{yx} \mathbf{O}_{yx}^{(0)}(\mathbf{k})$$

where $\mathbf{O}_{ij}^{(0)}(\mathbf{k})$ denotes $\mathbf{O}_{ij}(\mathbf{k})$ in the absence of rotation.

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where $\mathbf{O}_{ij}^{(0)}(\mathbf{k})$ denotes $\mathbf{O}_{ij}(\mathbf{k})$ in the absence of rotation.

- ▶ As $\mathbf{O}_{xy}^{(0)}$ is mirror antisymmetric, we observed a quantity related to $\mathbf{O}_{yy}^{(0)} - \mathbf{O}_{xx}^{(0)}$ as plotted in the mixing length approximation [Rüdiger 2005].

Additional averaging

- ▶ Without rotation \mathbf{O}_{xx} should be mirror symmetric

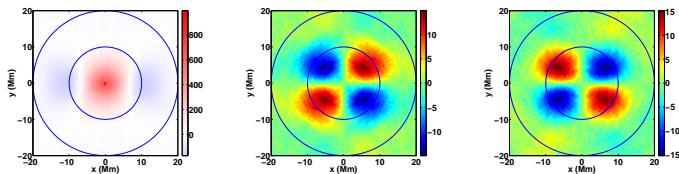


Figure: Observations $\mathbf{O}_{xx}(\mathbf{d})$ (left) and the non mirror symmetric part at a latitude of $+20^\circ$ (middle) and -20° (right). All units are s^2 .

Additional averaging

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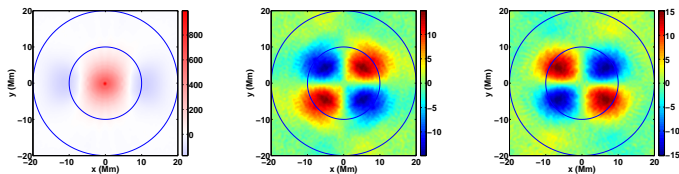


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$$\mathbf{O}_{xx}(\mathbf{k}) = \alpha_{xx} \mathbf{O}_{xx}^{(0)}(\mathbf{k}) + \alpha_{xy} \mathbf{O}_{xy}^{(0)}(\mathbf{k}) + \alpha_{yx} \mathbf{O}_{yx}^{(0)}(\mathbf{k}) + \alpha_{yy} \mathbf{O}_{yy}^{(0)}(\mathbf{k})$$

where $\mathbf{O}_{ij}^{(0)}(\mathbf{k})$ denotes $\mathbf{O}_{ij}(\mathbf{k})$ in the absence of rotation.

Context

Observations

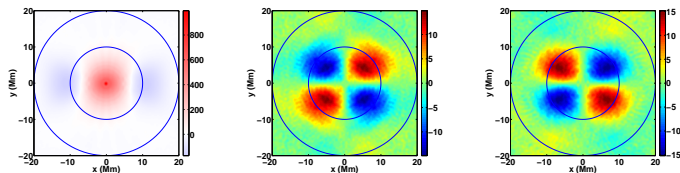
The forward problem

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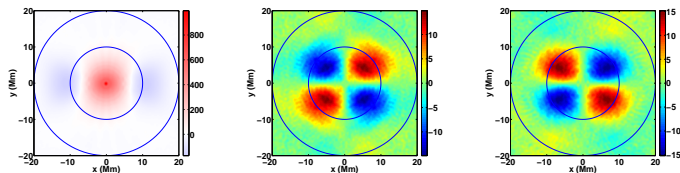
$$\mathbf{O}_{xx}(\mathbf{k}) = \alpha_{xx} \mathbf{O}_{xx}^{(0)}(\mathbf{k}) + \alpha_{xy} \mathbf{O}_{xy}^{(0)}(\mathbf{k}) + \alpha_{yx} \mathbf{O}_{yx}^{(0)}(\mathbf{k}) + \alpha_{yy} \mathbf{O}_{yy}^{(0)}(\mathbf{k})$$

where $\mathbf{O}_{ij}^{(0)}(\mathbf{k})$ denotes $\mathbf{O}_{ij}(\mathbf{k})$ in the absence of rotation.

- ▶ As $\mathbf{O}_{xx}^{(0)}$ and $\mathbf{O}_{yy}^{(0)}$ are mirror symmetric, we observed a quantity related to $\mathbf{O}_{yx}^{(0)}$ as plotted in the mixing length approximation [Rüdiger 2005].

Additional averaging

- ▶ Without rotation \mathbf{O}_{xx} should be mirror symmetric



Context

Observations

The forward problem

Noise model

The inverse problem

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- ▶ As $\mathbf{O}_{xx}^{(0)}$ and $\mathbf{O}_{yy}^{(0)}$ are mirror symmetric, we observed a quantity related to $\mathbf{O}_{yx}^{(0)}$ as plotted in the mixing length approximation [Rüdiger 2005].
- ▶ The term α_{xy} contains $\sin(\text{latitude})$ which explains the change of sign between -20° and $+20^\circ$.

Conclusions and perspectives

- ▶ We have linked the two-point velocity correlations to products of travel times and derived the forward and noise models. A first inversion showing the concept has been performed.

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- ▶ Averaging over angles shows interesting features on the data that can be mainly explained by Kitchatinov and Rüdiger theory of rotating turbulence.
- ▶ Inversions with all the data must now be performed in order to have a precise reconstruction and recover the Reynolds stress.

