Astron. Nachr. / AN **326**, No. 3/4, 227–230 (2005) / **DOI** 10.1002/asna.200410381

Stochastic resonance in a bistable geodynamo model

S. LORITO[1,2], D. SCHMITT[3], G. CONSOLINI[4], and P. DE MICHELIS[1]

[1] Istituto Nazionale Geofisica e Vulcanologia, 00143 Roma, Italy
[2] Dipartimento di Scienze Fisiche, Università Federico II, 80126 Napoli, Italy
[3] Max-Planck-Institut für Sonnensystemforschung, 37191 Katlenburg-Lindau, Germany
[4] Istituto di Fisica dello Spazio Interplanetario, CNR, 00133 Roma, Italy

Received 6 December 2004; accepted 17 December 2004; published online 11 March 2005

Abstract. Recently a signal with a period of 100 kyr in the distribution of residence times between reversals of the geomagnetic field has been suggested as signature of stochastic resonance. Here we test this suggestion by applying periodic modulations to a model of the geodynamo as a bistable oscillator, where stochastic fluctuations of the induction effect (multiplicative noise) lead to random transitions between the two polarity states of the supercritically excited fundamental axial dipole mode. By adding a weak periodic component either to the dynamo effect (multiplicative periodic term) or as a source term to the dynamo equation (additive periodic term) we demonstrate stochastic resonance. Depending on the multiplicative (additive) character of the periodic term, we find peaks at integer (half-integer) values of the applied period, superimposed on the otherwise Poissonian distribution of residence times. Especially the optimum resonance conditions for various mean times between reversals and various periodicities are derived. The periodic terms need to be about 0.1 in amplitude compared to the other terms in the dynamo equation to show the observed signatures of the magnetic field of the Earth. A sharp peak at the forcing frequency in the power spectrum of the dipole amplitude is only found in the additive case. As such a peak is absent in the Earth data, this rules for the multiplicative case. It is yet unclear what may cause such an effect to the geodynamo.

Key words: geomagnetic field – geodynamo – stochastic resonance

1. Stochastic resonance and geomagnetic reversals

Stochastic resonance provides an example of a noise-induced transition in a nonlinear system driven simultaneously by noise and an information signal (Gammaitoni et al. 1998; Anishchenko et al. 1999). Consider, for example, a heavily damped particle moving in a symmetric bistable potential. The particle is subject to fluctuational forces which cause random transitions between the potential wells with a mean rate given by the Kramers rate (Fig. 2). The residence times between transitions obey a Poissonian distribution with an exponential decay time equal to the mean residence time. If we apply a weak periodic forcing, either the potential wells are tilted asymmetrically up and down (for an additive periodic source) or the potential barrier is periodically raised and lowered (in the case of a multiplicative periodic signal) (Fig. 2). Although the periodic forcing is too weak to let the particle roll from one potential well into the other, the noise-induced hopping between the potential wells can become partly synchronized with the periodic forcing. This leads to a periodic modulation of the otherwise Poissonian distribution of residence times with peaks separated by the period of the modulating force (analogous to Figs. 3, 5 or 6).

Such a signal was recently found in the distribution of polarity times between geomagnetic reversals (Consolini & De Michelis 2003). One of the most spectacular phenomenon of geomagnetism is that the Earth has reversed the polarity of its almost dipolar magnetic field many times in the past at irregular intervals of 10^5 to 10^7 yr (Jacobs 1994; Merrill et al. 1996). The mean time between reversals is approximately 300 kyr, whereas reversals are fast events lasting a few kyr. The distribution of polarity intervals is mainly Poissonian, with a weak periodic component superimposed with a period of 100 kyr (Fig. 6). Consolini & De Michelis (2003) interpreted this as a signature of stochastic resonance. A weak periodicity of 100 kyr is also marginally seen in paleomagnetic records of geomagnetic intensity, declination and inclination (Channell et al. 1998; Yamazaki & Oda 2002). As 100 kyr co-

Correspondence to: schmitt@mps.mpg.de

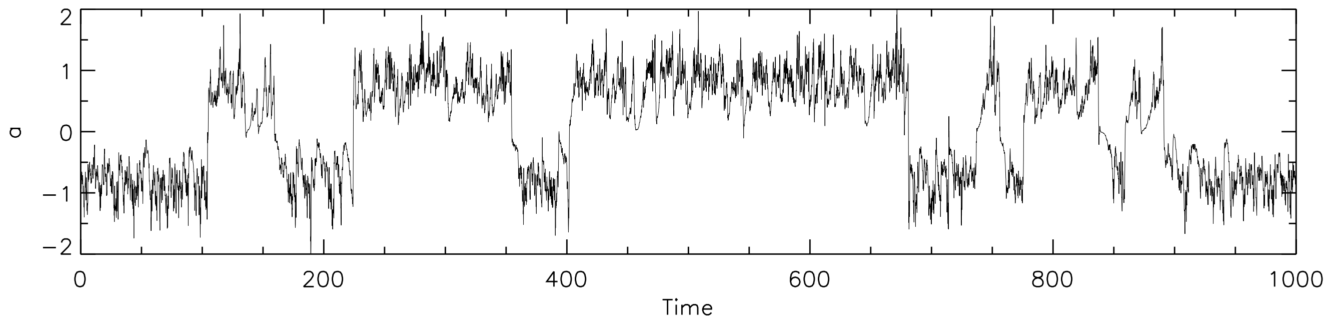

Fig. 1. Amplitude a of the fundamental dipolar dynamo mode. The unit of time is the turbulent diffusion time which is estimated as 5 kyr. Only a short interval of a much longer run is displayed. For the particular strength of the random forcing in this example, about 5000 reversals occurred in 300 000 diffusion times. This yields a mean time between reversals T_r of 60 diffusion times or 300 kyr.

incidences with the typical scale of the Earth's orbit variation, it has been speculated that the orbital eccentricity variations might affect the geodynamo.

2. The geodynamo as a bistable oscillator

The statistics of reversals has recently been addressed by Hoyng et al. (2001) and Schmitt et al. (2001) with the help of an axisymmetric mean-field $\alpha\Omega$-dynamo model where the fundamental mode is a supercritically excited non-oscillatory dipolar mode. Fluctuations in the helicity of the turbulent convection perturb the fundamental dynamo mode and lead to the stochastic excitation of otherwise damped higher modes. This results in stochastic oscillations of the dipole field amplitude (secular variation) and occasional fast polarity changes (reversals) (Fig. 1). The dipole amplitude behaves like the position of a stochastically forced, heavily damped particle in a bistable potential with minima representing normal and reversed polarity, and occasional jumps between them (Fig. 2). The shape of the potential is determined by supercritical dynamo excitation (central hill) and nonlinear limitation of field growth (side walls).

The solution of the model is controlled by the dynamo number C which is the product of the Reynolds numbers of the α-effect and of differential rotation. It is chosen such that the fundamental mode is a supercritically excited non-oscillatory dipole.

The effect of the stochastic helicity fluctuations is determined by a parameter involving the mean relative amplitude, the correlation length and the correlation time of the convective eddies. This parameter is chosen such that the mean time between reversals T_r corresponds to the Earth case. In the mean-field description the fluctuations manifest in the α-effect. Since this is a parameter in the mean-field dynamo equation, multiplied by the magnetic field, we speak of multiplicative noise.

The model accounts for the large variation of polarity intervals by only slight changes in the strength of the fluctuations and for the observed relation between the secular variation and the reversal rate of the geomagnetic field (Schmitt et al. 2001). It further reproduces the amplitude distribution of the dipolar field inferred from the Sint-800 record (Guyodo & Valet 1999; Hoyng et al. 2002).

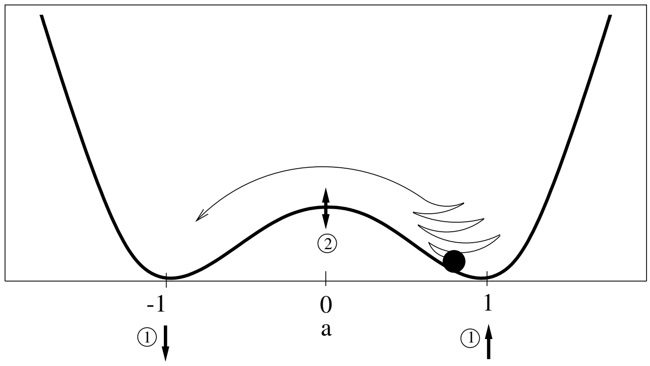

Fig. 2. The amplitude of the dipole mode a behaves as the position of a heavily damped particle, subject to random forcing, in a bistable potential. The shape of the potential is determined by the properties of the dynamo model. Indicated by arrows are the periodic asymmetric up- and down-tilting of the potential wells (case 1) as well as the periodic variation of the height of the potential barrier (case 2).

3. Stochastic resonance in the geodynamo model

In this paper we address the question whether a weak periodic modulation of the fore-mentioned dynamo model leads to stochastic resonance. Since the nature of this variation is yet unclear, we alternatively apply (i) an additive periodic source to the magnetic field $\partial \boldsymbol{B}/\partial t = ... + \delta \boldsymbol{B}/\tau \cos(2\pi t/T_\omega)$ which leads to an antisymmetric variation of the two potential wells, or (ii) a weak periodic component in the dynamo number $C = C_0[1 + \delta_\omega \cos(2\pi t/T_\omega)]$, which is a multiplicative signal and leads to a varying height of the central potential hill (Fig. 2). The period of the signal is denoted by T_ω and set to 20 diffusion times or 100 kyr, with an estimated diffusion time of 5 kyr. Typical relative amplitudes $\delta \boldsymbol{B}/\boldsymbol{B}$ and δ_ω of the periodic signal are of order 0.1.

3.1. Additive periodic source – asymmetric modulation

When we apply an additive periodic external source term with a period $T_\omega = 20$ of a third of the mean time between reversals $T_r = 60$, we indeed find an oscillatory signal superimposed on the Poissonian distribution of polarity time intervals (Fig. 3) which is very similar to the observed one by Con-

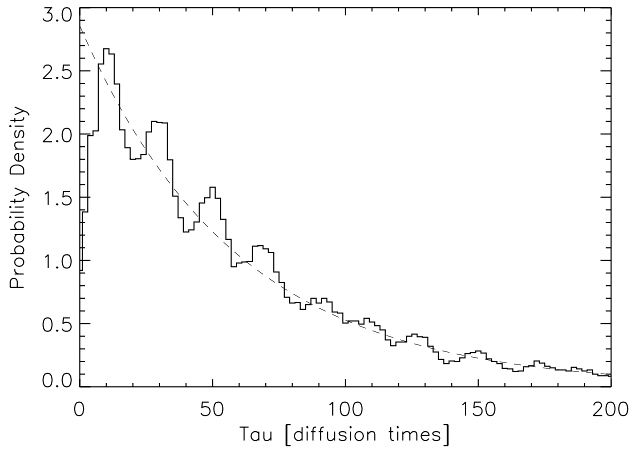

Fig. 3. Distribution function of polarity time intervals in the case of an additive periodic source with a period of 20 diffusion times and a mean time between reversals of 60 diffusion times.

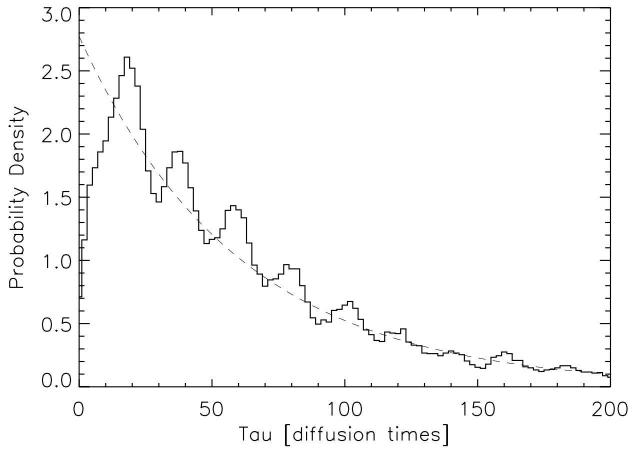

Fig. 5. Distribution function of polarity time intervals in the case of a multiplicative signal of relative strength of 0.1

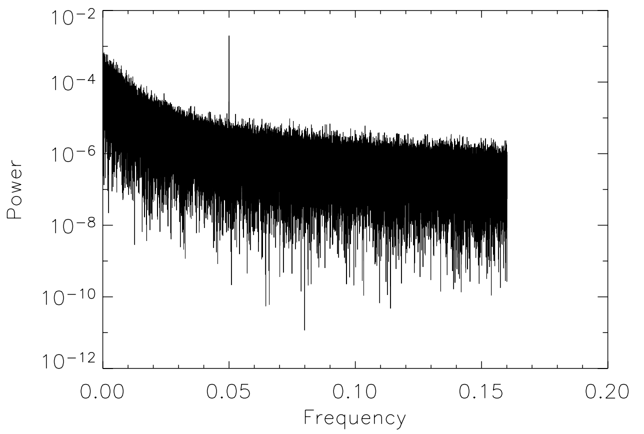

Fig. 4. The power spectrum of the dipole amplitude shows a sharp and prominent peak at the frequency $1/T_\omega = 1/20 = 0.05$ of the additive periodic source.

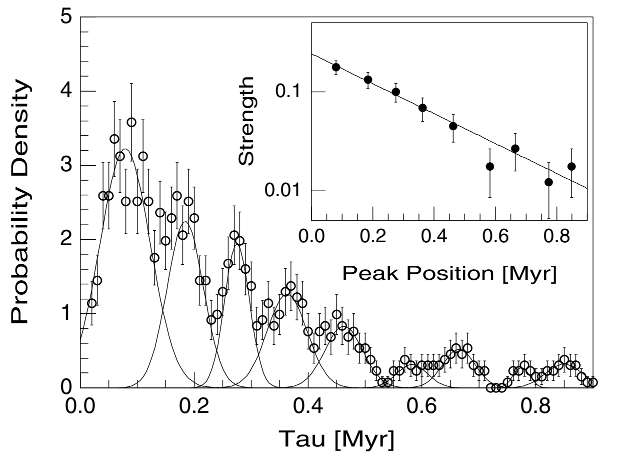

Fig. 6. Observed distribution of polarity chrons as evaluated using the technique described in Consolini & De Michelis (2003). Solid lines are Gaussian functions located at peak positions which are separated by about 100 ± 10 kyr. The inset shows the scaling of the peak strengths which fall off exponentially with a typical time scale of 300 ± 30 kyr.

solini & De Michelis (2003) (Fig. 6). The peaks are located at half-integer values of T_ω, i.e. $T_n = (n - 1/2)T_\omega, n = 1, 2, 3, \ldots$ which is a classical result of stochastic resonance with additive periodic sources (e.g. Gammaitoni et al. 1998). The two wells of the potential are tilted asymmetrically up and down in this case (Fig. 2). The power spectrum of the dipole amplitude shows a sharp peak at the frequency of the source with a large signal-to-noise ratio (e.g. Anishchenko et al. 1999) (Fig. 4).

3.2. Multiplicative periodic signal – symmetric modulation

When the dynamo number is slightly periodic we find a similar oscillatory signal in the distribution of residence times where the peaks now however are located at integer values of T_ω, i.e. $T_n = nT_\omega$ (Gammaitoni et al. 1994) (Fig. 5). As only the height of the central hill varies (Fig. 2), the symmetry of the potential is not broken in this case. In the power spectrum of the dipole amplitude there is no peak at the frequency of the periodic signal. This is because transitions across the central hill when it is low are equally easy from both sides, in phase and in anti-phase with the periodic signal.

3.3. The Earth case

Figure 6 shows the actual probability distribution of the geomagnetic polarity time intervals as evaluated using the two polarity reversal time scales compiled by Cande and Kent (1992, 1995) and by Ogg (1995). The inset shows the peak strength s_n defined as $s_n = \int_{T_n - T_\omega/4}^{T_n + T_\omega/4} P(\tau) d\tau$ where T_n is the position of the n-th peak, T_ω is taken as the characteristic 100 kyr periodic modulation, $P(\tau)$ is the probability density function of the chrons. The positions of the distribution peaks seem to be not very decisive to distinguish between additive or multiplicative periodic modulation. As shown in Fig. 7, these positions, indeed, scale linearly both to half-integer as well as integer numbers with almost equal ease.

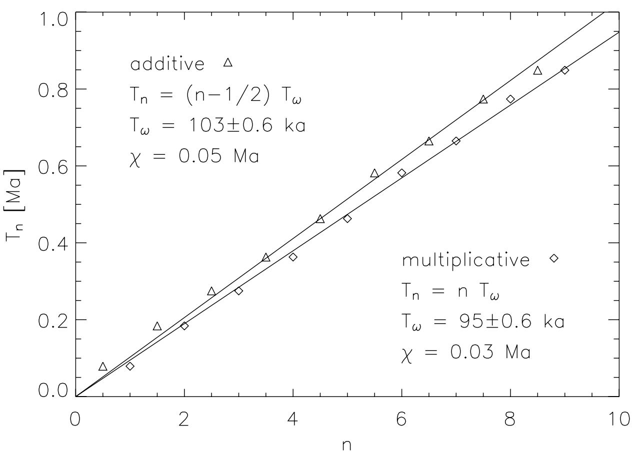

Fig. 7. The positions of the peaks in Fig. 6 versus half-integer (in the additive case, triangles) and integer numbers (in the multiplicative case, diamonds).

The mean deviation of the peak positions T_n from the linear fit is smaller by a factor of about 2 in the multiplicative case. By weightening more to the prominent first peaks would give an even better χ in this case. In addition, the position of the first peak $T_1 = 80$ kyr is closer to $T_\omega = 95$ kyr (multiplicative case) than to $T_\omega/2 = 103/2 = 51.5$ kyr (additive case). This seems to favor the multiplicative case, resulting in a forcing period of about 95 kyr.

By allocating a dipole amplitude of $+1$ for normal and -1 for reversed polarity with discontinuous jumps at times of reversals one can derive a power spectrum of the geomagnetic dipole amplitude (data not shown). This spectrum displays no peaks, certainly not at the frequency corresponding to a 100 kyr periodicity. This clearly speaks in favor of the multiplicative case with symmetric potential modulation. The deviation of the peaks from a pure Poissonian fall-off in the distribution function requires a relative amplitude variation in the dynamo number of the order of 0.1.

4. Optimal resonance condition

The optimal resonance condition in the sense that most of the transitions occur in the first peak of the probability distribution is $T_r = T_\omega/2$ in the case of an asymmetric periodic modulation of the bistable potential and is characterized by a maximal synchronization of the hopping mechanism with the periodic forcing (Gammaitoni et al. 1995). In the case of a symmetric periodic modulation this condition is met at $T_r = T_\omega$ (Lorito et al. 2005). The Earth case with $T_r \approx 3T_\omega$ is far away from the optimal condition.

5. Conclusion

We have shown that the findings of Consolini & De Michelis (2003) of a peaky structure in the distribution function of geomagnetic polarity chrons can be reproduced by allowing for a weak periodic modulation in the dynamo number of the geodynamo model as a bistable oscillator by Hoyng et al. (2001) and leads to stochastic resonance without symmetry breaking. The cause of such an effect to the geodynamo is yet unclear. The observed period of 100 kyr, which is characteristic for orbital eccentricity variations, raised speculations of an orbital forcing. This could be the case if precession plays a role as driving force of the flows that generate the Earth's magnetic field (Malkus 1968; Tilgner 1999).

Acknowledgements. Enlightening discussions with P. Hoyng, O. Preuss, P. Reimann and A. Tilgner are greatly acknowledged.

References

Anishchenko, V.S., Neiman, A.B., Moss, F., Schimansky-Geier, L.: 1999, SvPhU 42, 7
Cande, S., Kent, D.V.: 1992, JGR 97, 13917
Cande, S., Kent, D.V.: 1995, JGR 100, 6093
Channell, J.E.T., Hoddel, D.A., McManus, J., Lehman, B.: 1998, Nature 394, 464
Consolini, G., De Michelis, P.: 2003, PhRvL 90, 058501-1/4
Gammaitoni, L., Marchesoni, F., Santucci, S.: 1994, PhLA 195, 116
Gammaitoni, L., Marchesoni, F., Santucci, S.: 1995, PhRvL 74, 1052
Gammaitoni, L., Hänggi, P., Jung, P., Marchesoni, F.: 1998, RvMP 70, 223
Guyodo, Y., Valet, J.P.: 1999, Nature, 399, 249
Hoyng, P., Ossendrijver, M.A.J.H., Schmitt, D.: 2001, GApFD 94, 263
Hoyng, P., Schmitt, D., Ossendrijver, M.A.J.H.: 2002, PEPI 130, 143
Jacobs, J.A.: 1994, *Reversals of the Earth's Magnetic Field*, Cambridge University Press
Lorito, S., Consolini, G., Schmitt, D., De Michelis, P.: 2005, in preparation
Malkus, W.V.R.: 1968, Sci 160, 259
Merrill, R.T., McElhinny, M.W., McFadden, P.L.: 1996, *The Magnetic Field of the Earth, Paleomagnetism, the Core, and the Deep Mantle*, Academic Press, San Diego
Ogg, J.G.: 1995, in: T.J. Ahrens (ed.), *Global Earth Physics – A Handbook of Physical Constants*, AGU, Washington, p. 240
Schmitt, D., Ossendrijver, M.A.J.H., Hoyng, P.: 2001, PEPI 125, 119
Tilgner, A.: 1999, JFM 379, 303
Yamazaki, T., Oda, H.: 2002, Sci 295, 2435