Letter to the Editor

New IRIS constraints on the solar core rotation

L. Gizon[1,2], **E. Fossat**[3], **M. Lazrek**[4,5], **A. Cacciani**[6], **S. Ehgamberdiev**[7], **B. Gelly**[3], **G. Grec**[5], **J.T. Hoeksema**[8], **S. Khalikov**[7], **P.L. Pallé**[9], **A. Pantel**[3], **C. Régulo**[9], **F.-X. Schmider**[3], and **P.R. Wilson**[1]

[1] School of Mathematics and Statistics, University of Sydney, Sydney, NSW 2006, Australia
[2] ESA Space Science Department, ESTEC, 2200 AG Noordwijk, The Netherlands
[3] Département d'Astrophysique, URA CNRS 709, Université de Nice-Sophia Antipolis, F-06108 Nice Cedex 2, France
[4] Laboratoire d'Astronomie et de Géophysique, CNCPRST, BP 8027, Rabat, Morocco
[5] Département Cassini, URA CNRS 1362, Observatoire de la Côte d'Azur, BP 229, F-06304 Nice Cedex 4, France
[6] Dipartimento di Fisica, Università di Roma "La Sapienza", Piazza Aldo Moro, Roma, Italy
[7] Astronomical Institute of the Uzbek Academy of Sciences, Astronomicheskaya 33, Tashkent, Uzbekistan
[8] Center for Space Science and Astrophysics, Stanford University, Stanford, CA 94305, USA
[9] Instituto de Astrofisica de Canarias, E-38205 La Laguna, Tenerife, Spain

Abstract. Four time series of IRIS data (4 to 6 months) have been used to obtain improved measurements of the low degree ($\ell = 1, 2, 3$) rotational splitting frequencies. Assuming that the rotation law is known in the outer layers of the Sun, we investigate the implications of IRIS splittings for the central regions. Both a one-shell and a two-shell rotation model have been considered in the solar core. A core rotating slightly faster than the outer radiative envelope provides the best fit to the data. Some evidence for the reliability of the observations is shown by the visibility of differential rotation in the $\ell = 3$ multiplets.

Key words: Sun: rotation – Sun: oscillations – Sun: interior

1. Introduction

Since rotation lifts the azimuthal degeneracy of global modes of oscillation, helioseismology gives access to the Sun's internal angular velocity. Spatially resolved observations of moderate and high degree modes have already provided estimates of the angular velocity outwards from $R_\odot/3$, but the inference of the rotation in the core itself relies entirely on a knowledge of rotational splitting frequencies of p-modes with very low harmonic degree ℓ. Thanks to very stable full-disk seismometers, long observations with high duty-cycles can reveal the fine structure of the $\ell = 1, 2, 3$ multiplets. However, spectral line fitting is rendered very delicate by mode mixing, the presence of noise, and the χ^2 distribution of power spectra. A consequence is the

Send offprint requests to: L. Gizon

controversy between the results of the BiSON (Elsworth et al. 1995) and the IRIS (Lazrek et al. 1996) networks, the former implying a slower and the latter a faster than average rotation of the core. In order to improve the reliability of the measurements, several groups of observers tend to publish splittings averaged over several modes (Loudagh et al. 1993; Toutain & Kosovichev 1994; Lazrek et al. 1996). Here, we present an analysis of the most recent IRIS splittings, which take into account early data for the year 1989. This analysis follows the work of Gizon (1995). We note that Elsworth et al. (1995) used a similar approach to interpret their data.

2. Theoretical background

We work in an inertial frame, where we introduce the usual polar coordinates $\{r, \theta, \phi\}$. Let $\Omega(r, \theta)$ be the angular velocity, independant of longitude. If rotation is assumed to be symmetric about the equatorial plane, $\Omega(r, \theta)$ may be expanded in the form $\Omega_0(r) + \Omega_1(r)\cos^2\theta + \Omega_2(r)\cos^4\theta$. This truncated model should give a reasonable description of the rotation law over most of the Sun, especially in the radiative zone where little differential rotation is observed (Tomczyk et al. 1995). We define the rotational splitting by the scaled difference $S_{\ell,n}^m = (\nu_{\ell,n}^0 - \nu_{\ell,n}^m)/m$, where $\nu_{\ell,n}^m$ denotes the frequency of the mode of degree ℓ, radial order n and azimuthal order m. To the first order of approximation it is given by an expression of the form:

$$S_{\ell,n}^m = \int_0^{R_\odot} \frac{\Omega_0(r)}{2\pi} K_{\ell,n}^0(r)\, dr$$

$$+ \sum_{j=1}^{2} \int_{0}^{R_{\odot}} \frac{\Omega_j(r)}{2\pi} K_{\ell,n,|m|}^{j}(r)\, dr, \qquad (1)$$

where the kernel functions $K^j(r)$ depend on the unperturbed eigenfunctions and on the equilibrium solar model (e.g. Toutain & Kosovichev 1994). Note that differential rotation is responsible for the dependence of $S_{\ell,n}^m$ on the azimuthal order m through the second term in Eq. (1).

We want to employ full-disk observations to study rotation in the deep interior of the Sun, say within an arbitrary core limiting radius r_c. Assuming that the core does not rotate differentially, and splitting the integrals at $r = r_c$ into two parts yields

$$S_{n,\ell}^m = \int_{0}^{r_c} \frac{\Omega_0(r)}{2\pi} K_{\ell,n}^0(r)\, dr + \Sigma_{\ell,n}^m(r_c). \qquad (2)$$

The quantity $\Sigma_{\ell,n}^m(r_c)$ represents the contribution due to rotation in the region $r > r_c$; once an outer rotation model is specified, it is a known quantity.

The problem of inferring the angular velocity within the core from low-ℓ splitting data is compounded by the fact that the eigenfunctions for these modes have their maximum amplitudes near the surface. Therefore, we need to have a reliable model for the angular velocity in the outer regions. Here, we make use of a simplified version of the two-shell rotation model produced by Goode & Dziembowski (1991) using Big Bear data. The convection zone, $r > 0.7R_{\odot}$, is such that $\Omega_0/2\pi = 462.4$ nHz, $\Omega_1/2\pi = -60.5$ nHz and $\Omega_2/2\pi = -81.7$ nHz. The radiative interior, $r < 0.7R_{\odot}$, is assumed to rotate rigidly at the rate $\Omega_{\text{RAD}}/2\pi = 438.7$ nHz. Although no reliability is claimed below $0.4R_{\odot}$, it is assumed that this outer model can be extended downward to r_c, so that $\Sigma_{\ell,n}^m(r_c)$ can effectively be computed.

3. New IRIS splitting frequencies

Lazrek et al. (1996) analysed three different IRIS time series obtained in 1990, 1991 and 1992. They showed how reliability could be achieved by measuring averages of the rotational splittings of the $\ell = 1$, $\ell = 2$ and $\ell = 3$ multiplets over selected ranges of n values.

Here we report improved data, using a fourth time series (6 months) for the year 1989. As in Lazrek et al. (1996) we use four different collective methods. First, peaks with a given degree are added together after renormalisation, and the fit is performed on this synthetic spectrum. The second method consists of fitting iteratively and simultaneously all the peaks of a given degree. In the third method we calculate the cross-correlation of two peaks of a given radial order from two power spectra for different years; then we fit the mean of all cross-correlation spectra of a given degree. For the last method, we compute the auto-correlation of each peak of a given degree, and we obtain the mean auto-correlation spectrum to be fitted. All sidereal splittings are obtained by adding 31 nHz.

The sidereal splittings $\mathscr{S}_{\ell}$ for $\ell = 1, 2, 3$ are given in Table 1, with their statistical uncertainties σ_{ℓ}. In the case $\ell = 3$, only

Table 1. IRIS sidereal rotational frequency splittings

ℓ	n	$\mathscr{S}_{\ell}$	σ_{ℓ}
1	9–23	456 nHz	10 nHz
2	11–22	434 nHz	15 nHz
3	12–21	466 nHz	18 nHz

the first two methods have been used in order to make the interpretation of the splitting easier as explained in the next section.

4. Statement of the problem

In order to make use of the measurements $\mathscr{S}_{\ell}$ we have to understand what theoretical averages they refer to. So as to make things clear, $\bar{S}_{\ell}$ will denote these theoretical averaged splittings.

It is important to remember that, without any spatial resolution, the IRIS observational technique does not give access to the modes for which $\ell + m$ is odd. In the case $\ell = 1$, the observation $\mathscr{S}_1$ refers to the quantity $\bar{S}_1$ obviously defined as the arithmetic mean of $S_{1,n}^1$ over n. Similarly, for $\ell = 2$, $\bar{S}_2$ is the arithmetic mean of $S_{2,n}^2$ over n.

The case $\ell = 3$ deserves special attention. In effect, $S_{3,n}^1$ and $S_{3,n}^3$ are theoretically slightly different because of differential rotation in the convection zone. The measured splitting $\mathscr{S}_3$ is not only an average over n, but also over m. When simultaneously fitting four equally spaced Lorentz profiles to the $\ell = 3$ synthetic spectrum, one has to be aware of the fact that the fitting procedure does not attribute equal weights to $S_{3,n}^1$ and $S_{3,n}^3$. If ρ denotes the relative power in peaks $m = \pm 3$ with respect to peaks $m = \pm 1$, then a simple least squares analysis shows that the azimuthal average is given by $\frac{1}{1+9\rho} S_{3,n}^1 + \frac{9\rho}{1+9\rho} S_{3,n}^3$. The fit provides the value $\rho \sim 1.2$. Finally, $\bar{S}_3$ is obtained by taking the arithmetic mean over n. Thus, $\mathscr{S}_3$ is an estimate of this well defined quantity.

We shall cite the above averages as the IRIS-averages, $\bar{f}_{\ell}$, applicable to any function of the form $f = f_{\ell,n}^m(r)$. Taking IRIS-averages of Eq. (2) yields

$$\bar{S}_{\ell} = \int_{0}^{r_c} \frac{\Omega_0(r)}{2\pi} \bar{K}_{\ell}^0(r)\, dr + \bar{\Sigma}_{\ell}(r_c). \qquad (3)$$

We see that each rotational splitting is related to an IRIS-averaged kernel $\bar{K}_{\ell}^0(r)$ which can easily be computed. For a given ℓ, the constraint applied to $\Omega_0(r)$ is given by $\{ \bar{S}_{\ell} \in [\mathscr{S}_{\ell} - \sigma_{\ell}, \mathscr{S}_{\ell} + \sigma_{\ell}] \}$ at a one sigma confidence level.

In the next section, we shall search for classes of functions $\Omega_0(r)$, such that, for a given core radius r_c, all constraints have been met. Of course, we can extend this notion to averages which would include modes with various harmonic degrees ℓ as well. However, we have to keep in mind that different modes do not probe exactly the same part of the Sun: there is no reason why all low-ℓ modes should have exactly the same splittings.
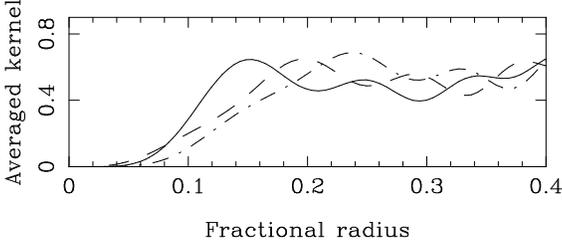
Fig. 1. IRIS-averaged kernels $\bar{K}_\ell^0(r)$ as a function of fractional radius $r/R_\odot$, for $\ell = 1$ (solid line), $\ell = 2$ (dashed line) and $\ell = 3$ (dot-dashed line).
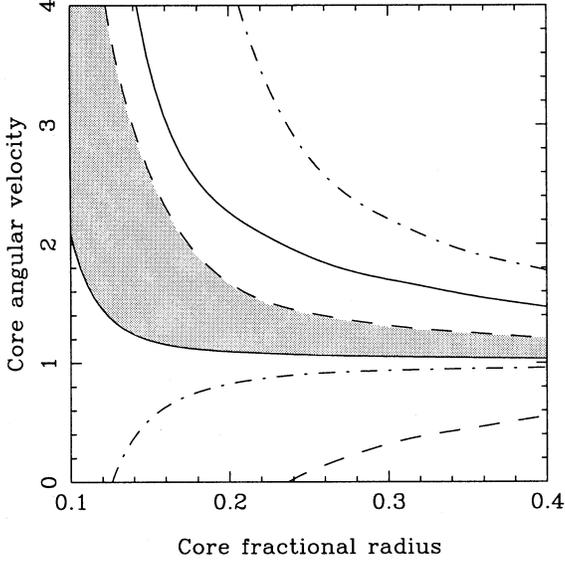
Fig. 2. Regions in which the angular velocity Ω_c of a core of radius r_c agrees with IRIS splittings $\mathcal{S}_\ell$ within 1.5-σ_ℓ. The regions defined by $\ell = 1$, $\ell = 2$ and $\ell = 3$ are enclosed by the solid, dashed and dot-dashed lines respectively. The vertical axis is graduated in units of Ω_{RAD}.

5. Interpretation of the data

The IRIS-averaged kernels are plotted in Fig. 1 for $r < 0.4R_\odot$. We first notice that they are negligible for $r < 0.1R_\odot$, so that the angular velocity in this region cannot be determined by these observations. The first maxima of $\bar{K}_1^0(r)$, $\bar{K}_2^0(r)$ and $\bar{K}_3^0(r)$ are respectively found at $\sim 0.15R_\odot$, $\sim 0.20R_\odot$ and $\sim 0.24R_\odot$. Because of the averaging procedure, the kernels are not markedly different from each other. Hence, it will be difficult to resolve the radial variations of $\Omega_0(r)$, and we choose to investigate a rigid-body rotation law for the core.

Assuming a constant angular velocity $\Omega_0(r) = \Omega_c$ for $r \leq r_c$, Eq. (3) contains two free parameters r_c and Ω_c. In the plane (r_c, Ω_c), we find no overlap between the solution regions defined by $\ell = 1$, 2 and 3, at a 1-σ confidence level. In Fig. 2, we show this overlap for 1.5-σ error bars. The core does not rotate much faster than the radiative zone: a $0.3R_\odot$ solar core would be most likely rotating in the range $1.1 < \Omega_c/\Omega_{\mathrm{RAD}} < 1.3$. However,
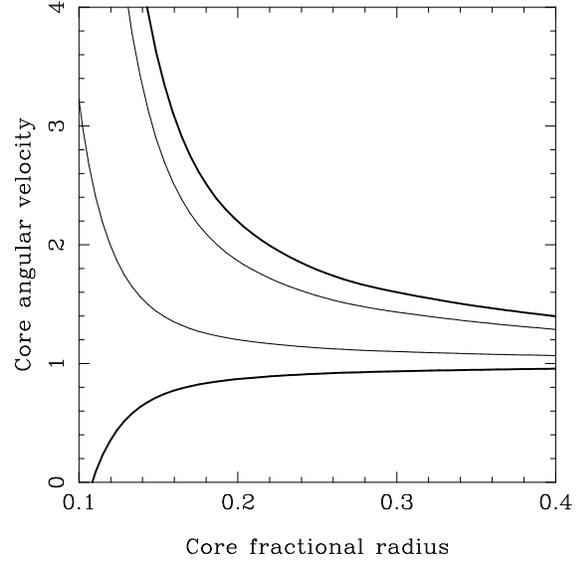
Fig. 3. Axes are the same as in Fig. 2. The angular velocity Ω_c is constrained by the mean frequency splitting $\mathcal{S}_{\mathrm{TOT}}$. The thin and thick boundaries respectively refer to the 1-σ and 2-σ confidence levels.

note that faster (or slower) rotation in the very centre cannot be excluded.

Alternatively, if we assume that the three splittings are measuring a similar quantity, we can work out a mean splitting value by weighting each $\mathcal{S}_\ell$ by $1/\sigma_\ell^2$. This global average $\mathcal{S}_{\mathrm{TOT}} = 452 \pm 8$ nHz leads to the solution shown in Fig. 3. Here again, the core is found to be rotating at a rate relatively close to Ω_{RAD}. For instance, for $r_c = 0.3R_\odot$ we have $0.9 < \Omega_c/\Omega_{\mathrm{RAD}} < 1.6$ with a probability of 95%.

6. Attempt for better localised kernels

We shall now investigate the possibility of detecting some variations of the core angular velocity as a function of radius. For this purpose, we divide the core into two shells bounded by radii r_{IN} and $r_{\mathrm{OUT}} = r_c$, assumed to be rotating rigidly at unknown rates Ω_{IN} and Ω_{OUT}. Figure 1 suggests the choices $r_{\mathrm{IN}} = 0.2R_\odot$ and $r_{\mathrm{OUT}} = 0.3R_\odot$, so as to be able to probe both shells equally well.

As in Sect. 5, we could perform an analysis in the plane $(\Omega_{\mathrm{IN}}, \Omega_{\mathrm{OUT}})$ by drawing the domains in which $\bar{S}_\ell$ and $\mathcal{S}_\ell$ are in agreement. It turns out that such a way of proceeding does not provide any clear nor convincing answer. As a consequence, we recommend measuring two specific averages (over ℓ and n) of the splittings, in order to probe separately the inner and the outer parts of the core. We select two sets of modes as follows.

The first set contains all the available modes such that

$$\int_0^{0.2R_\odot} K_{\ell,n}^0(r)\,dr > \int_{0.2R_\odot}^{0.3R_\odot} K_{\ell,n}^0(r)\,dr, \tag{4}$$

i.e. the modes $\{\ell = 1, 12 \leq n \leq 23\}$ and $\{\ell = 2, 17 \leq n \leq 23\}$. The $\ell = 1$ subset gives a splitting of 453 nHz, and the $\ell = 2$ subset

LETTER
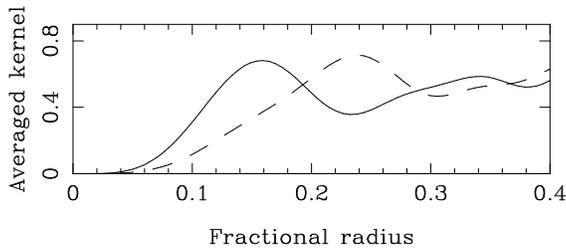
Fig. 4. The kernel functions $\bar{K}_{\mathrm{IN}}^0(r)$ (solid line) and $\bar{K}_{\mathrm{OUT}}^0(r)$ (dashed line) as a function of fractional radius $r/R_\odot$.

gives 457 nHz. The weighted average is equal to $\mathscr{S}_{\mathrm{IN}}$ = 454 nHz with an uncertainty σ_{IN} = 9 nHz.

Conversely, the second set contains all the modes such that inequality (4) is false: $\{\ell = 1, 10{\leq}n{\leq}11\}$, $\{\ell = 2, 10{\leq}n{\leq}16\}$ and $\{\ell = 3, 11{\leq}n{\leq}21\}$. These three subsets correspond respectively to the splitting values 453 nHz, 446 nHz and 465 nHz. The averaged splitting is found to be $\mathscr{S}_{\mathrm{OUT}}$ = 457 nHz with σ_{OUT} = 12 nHz.

As expected, the corresponding IRIS-averaged kernels $\bar{K}_{\mathrm{IN}}^0(r)$ and $\bar{K}_{\mathrm{OUT}}^0(r)$ have maxima in the inner and outer core respectively (Fig. 4). By construction, the kernels are as orthogonal as possible in the two shells. Figure 5 shows the plane $(\Omega_{\mathrm{IN}}, \Omega_{\mathrm{OUT}})$ where the two-shell rotation law has been constrained by the measurements $\mathscr{S}_{\mathrm{IN}}$ and $\mathscr{S}_{\mathrm{OUT}}$. It is found that the solution with the highest probability is $\Omega_{\mathrm{IN}} \sim 1.1\Omega_{\mathrm{RAD}}$ and $\Omega_{\mathrm{OUT}} \sim 1.5\Omega_{\mathrm{RAD}}$. Error bars are still too large to give any definite answer, but IRIS data point to a core rotating somewhat faster than the envelope, and this increase can already be noted at a radius $r \sim 0.25 R_\odot$.

7. Conclusion

In Sect. 4 we noted that the peaks in a $\ell = 3$ quadruplet are not equally spaced. The model we took for differential rotation in the convection zone predicts a difference $S_{3,n}^3 - S_{3,n}^1 \sim 28$ nHz. This is actually confirmed by measuring this difference on the $\ell = 3$ averaged power spectrum; we find 34 ± 20 nHz. This consistency for such a difficult measurement makes us confident about the general validity of all our error bars, and thus about the main conclusion: a core rotating slightly faster than the outer radiative zone.

Fig. 5. Regions in the plane $(\Omega_{\mathrm{IN}}, \Omega_{\mathrm{OUT}})$ as constrained by the observed splittings which specifically probe the inner and outer parts of the core. The region enclosed by the solid (dashed) lines matches $\mathscr{S}_{\mathrm{IN}}$ ($\mathscr{S}_{\mathrm{OUT}}$) within 1-$\sigma_{\mathrm{IN}}$ (1-σ_{OUT}). Angular velocities are in units of Ω_{RAD}.

References

Elsworth Y., Howe R., Isaak G.R., et al., 1995, Nat 376, 669
Gizon L., 1995, The rotation of the solar core. In: Vincent R.A. et al. (eds.) Proc. Conf. STEP 1995 (24-29 Sept.). University of Adelaide, Adelaide, p. 217
Goode P.R., Dziembowski W.A., 1991, Nat 349, 223
Lazrek M., Pantel A., Fossat E., et al., 1996, Sol. Phys. 166, 1
Loudagh S., Provost J., Berthomieu G., et al., 1993, A&A 275, L25
Tomczyk S., Schou J., Thompson M.J., 1995, ApJ 448, L57
Toutain T., Kosovichev A.G., 1994, A&A 284, 265