

Inversion of the RTE

Once solution of RTE is known:

- comparison between Stokes spectra of synthetic and observed spectrum
- trial-and-error changes of the initial parameters of the atmosphere („human inversions“)
- until observed and synthetic (fitted) profile matches

Inversions:

Nothing else but an optimization of the trial-and-error part

Problem:

Inversions always find a solution within the given model atmosphere. Solution is seldomly unique (might even be completely wrong).

Goal of this lecture:

Principles of genetic algorithms

Learn the usage of the HeLix⁺ inversion code, develop a feeling on the reliability of inversion results.

The merit function

- The quality of the model atmosphere must be evaluated
- Stokes profiles represent discrete sampled functions
- widely used: chisqr definition

$$\chi^2(\mathbf{x}) = \frac{1}{N_{free}} \sum_{s=0}^3 \sum_{i=1}^q \left[\mathbf{I}_s^{obs}(\lambda_i) - \mathbf{I}_s^{syn}(\lambda_i; \mathbf{x}) \right]^2 w_{s,i}^2$$

number of free parameters

sum over Stokes

sum over WL-pixels

weight (also WL-dep)

- RTE gives the Stokes spectrum \mathbf{I}_s^{syn}
- The unknowns of the system are the (height dependent) model parameters:

$$\mathbf{x} = (B, \Theta, \phi, v_{LOS}, \dots)$$

HeLix⁺ overview of features

- ⊙ includes Zeeman, Paschen-Back, Hanle effect (He 10830)
- ⊙ atomic polarization for He 10830 (He D3)
- ⊙ magneto-optical effects
- ⊙ fitting / removing telluric lines
- ⊙ fitting unknown parameters of spectral lines
- ⊙ various methods for continuum correction / fitting
- ⊙ convolution with instrument filter profiles
- ⊙ user-defined weighting scheme
- ⊙ direct read access to SOT/SP, VTT-TIP2, SST-CRISP, ...
- ⊙ flexible atomic data configuration
- ⊙ extensive IDL based display routines
- ⊙ MPI support (to invert maps)

Download from <http://www.mps.mpg.de/homes/lagg>

GBSO download-section → helix

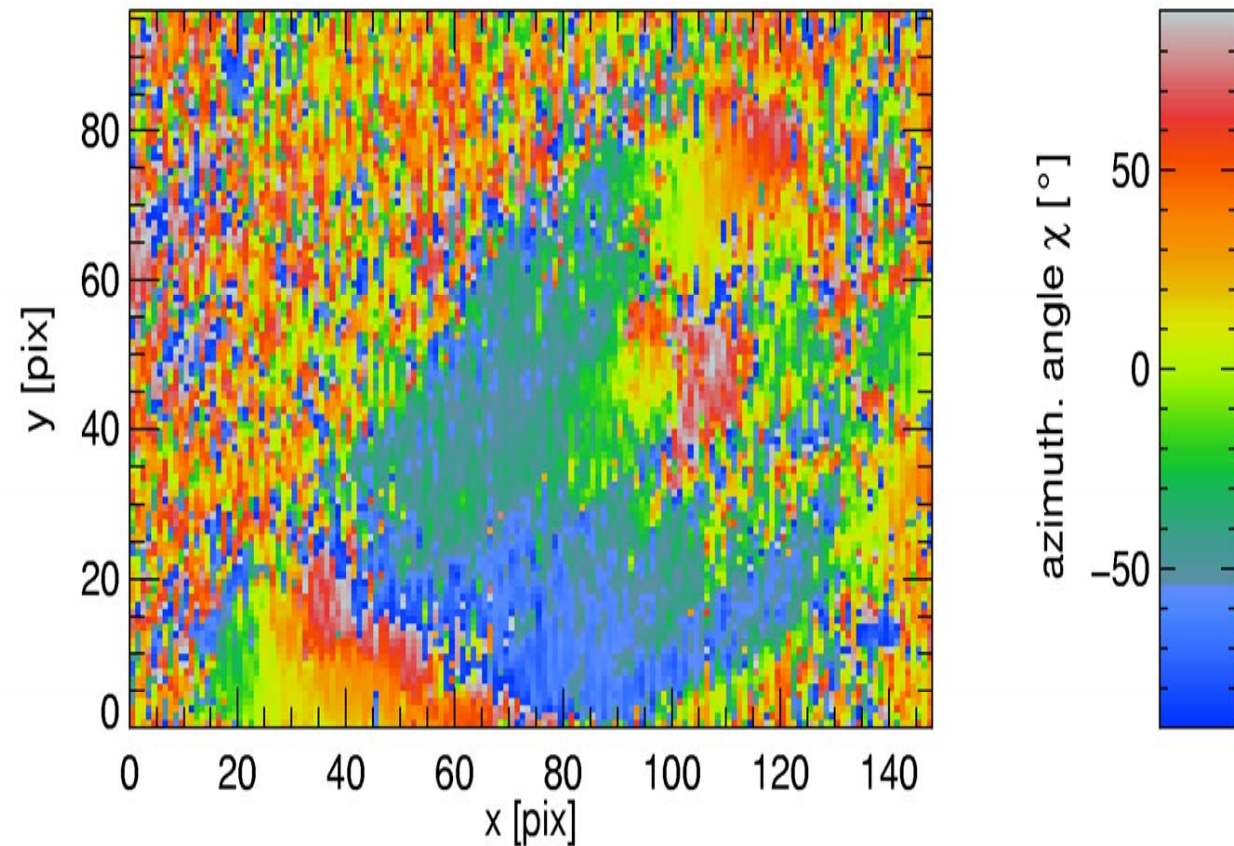
use *invert* and *IR\$soft*

The inversion technique: reliability

Two minimizations implemented:

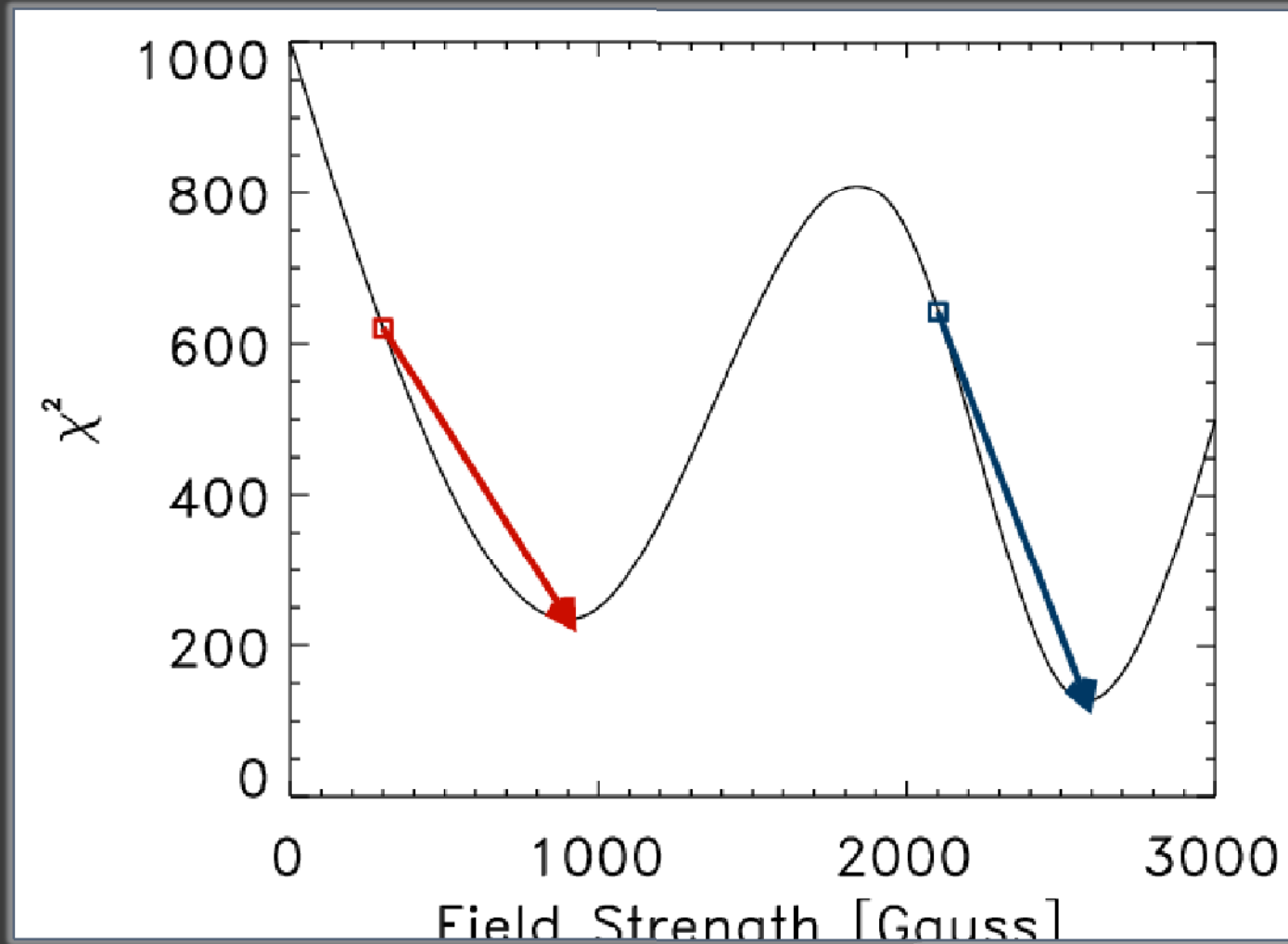
- Levenberg-Marquardt:
 - requires good initial guess
- PIKAIA (genetic algorithm, Charbonneau 1995):
 - no initial guess needed
- planned: DIRECT algorithm (good compromise between global min and speed)

Pikaia



Initial guess problem

Having a good initial guess for the iteration process improves both the speed and the convergence of the inversion.



Initial guess optimizations

Weak field initialization

$$\begin{aligned}B_L &= C_1 V \\B_T &= C_2 \sqrt{Q^2 + U^2} \\B &= \sqrt{B_L^2 + B_T^2} \\ \gamma &= \cos^{-1} \left(\frac{B_L}{B} \right) \\ \phi &= \tan^{-1} \left(\frac{Q}{U} \right)\end{aligned}$$

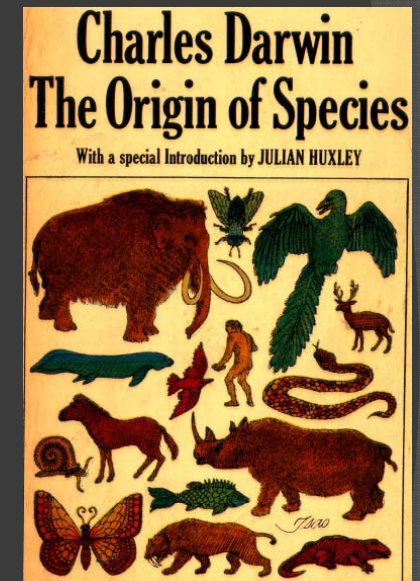
Auer77 initialization

$$\begin{aligned}R &= \frac{2V}{\sqrt{Q^2 + U^2}} \\ \gamma &= \cos^{-1} \left\{ \frac{1}{2} (\sqrt{R^4 + 4} - R) \right\} \\ \phi &= \frac{1}{4} \tan^{-1} \left\{ \frac{\sum \lambda Q U}{\sum \lambda (Q^2 - U^2)} \right\}\end{aligned}$$

Other methods:

- Artificial Neural Networks (ANN)
- MDI / magnetograph formulae
- use a minimization technique which does not rely on initial guess values

- Genetic algorithms (GA's) are a technique to solve problems which need optimization
- GA's are a subclass of Evolutionary Computing
- GA's are based on Darwin's theory of evolution
- History of GA's:
 - Evolutionary computing evolved in the 1960's.
 - GA's were created by John Holland in the mid-70's.



Advantages / drawbacks

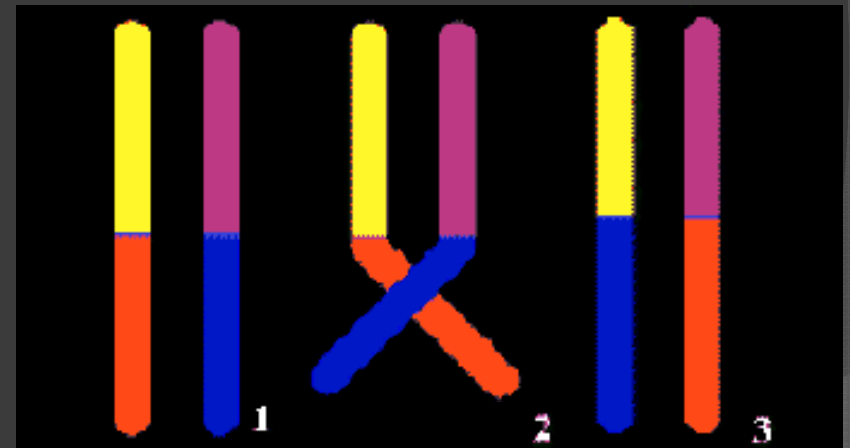
- No derivatives of the goodness of fit function with respect to model parameters need be computed; it matters little whether the relationship between the model and its parameters is linear or nonlinear.
- Nothing in the procedure outlined above depends critically on using a least-squares statistical estimator; any other robust estimator can be substituted, with little or no changes to the overall procedure.
- In most real applications, the model will need to be evaluated (i.e., given a parameter set, compute a synthetic dataset and its associated goodness of fit) a great many times; if this evaluation is computationally expensive, the forward modeling approach can become impractical.

Evolution in biology

- Each cell of a living thing contains chromosomes - strings of DNA
- Each chromosome contains a set of genes - blocks of DNA
- Each gene determines some aspect of the organism (like eye colour)
 - A collection of genes is sometimes called a genotype
 - A collection of aspects (like eye colour) is sometimes called a phenotype
- Reproduction involves recombination of genes from parents and then small amounts of mutation (errors) in copying
- The fitness of an organism is how much it can reproduce before it dies
- Evolution based on “survival of the fittest”

Biological reproduction

- During reproduction “errors” occur
- Due to these “errors” genetic variation exists
- Most important “errors” are:
 - Recombination (cross-over)
 - Mutation



Natural selection

- The origin of species: “**Preservation of favourable variations and rejection of unfavourable variations.**”
- There are more individuals born than can survive, so there is a **continuous struggle for life.**
- Individuals with an advantage have a greater chance for survive: **survival of the fittest.**
- Important aspects in natural selection are:
 - **adaptation to the environment**
 - isolation of populations in different groups which cannot mutually mate
- If small changes in the genotypes of individuals are expressed easily, especially in small populations, we speak of genetic drift
- “success in life”: mathematically expressed as **fitness**

- GA's often encode solutions as fixed length "bitstrings" (e.g. 101110, 111111, 000101)
- Each bit represents some aspect of the proposed solution to the problem
- For GA's to work, we need to be able to "test" any string and get a "score" indicating how "good" that solution is
- definition of "fitness function" required: convenient to use chisqr merit function

$$F(\mathbf{x}) = \frac{1}{\chi^2(\mathbf{x})}$$

GA's improve the fitness – maximization technique

- Imagine you had to drill for oil somewhere along a single 1km desert road
- Problem: choose the best place on the road that produces the most oil per day
- We could represent each solution as a position on the road
- Say, a whole number between [0..1000]

Solution1 = 300

Solution2 = 900



Road

0

500

1000

Encoding problem

- The set of all possible solutions [0..1000] is called the search space or state space
- In this case it's just one number but it could be many numbers or symbols
- Often GA's code numbers in binary producing a bitstring representing a solution
- In our example we choose 10 bits which is enough to represent 0..1000

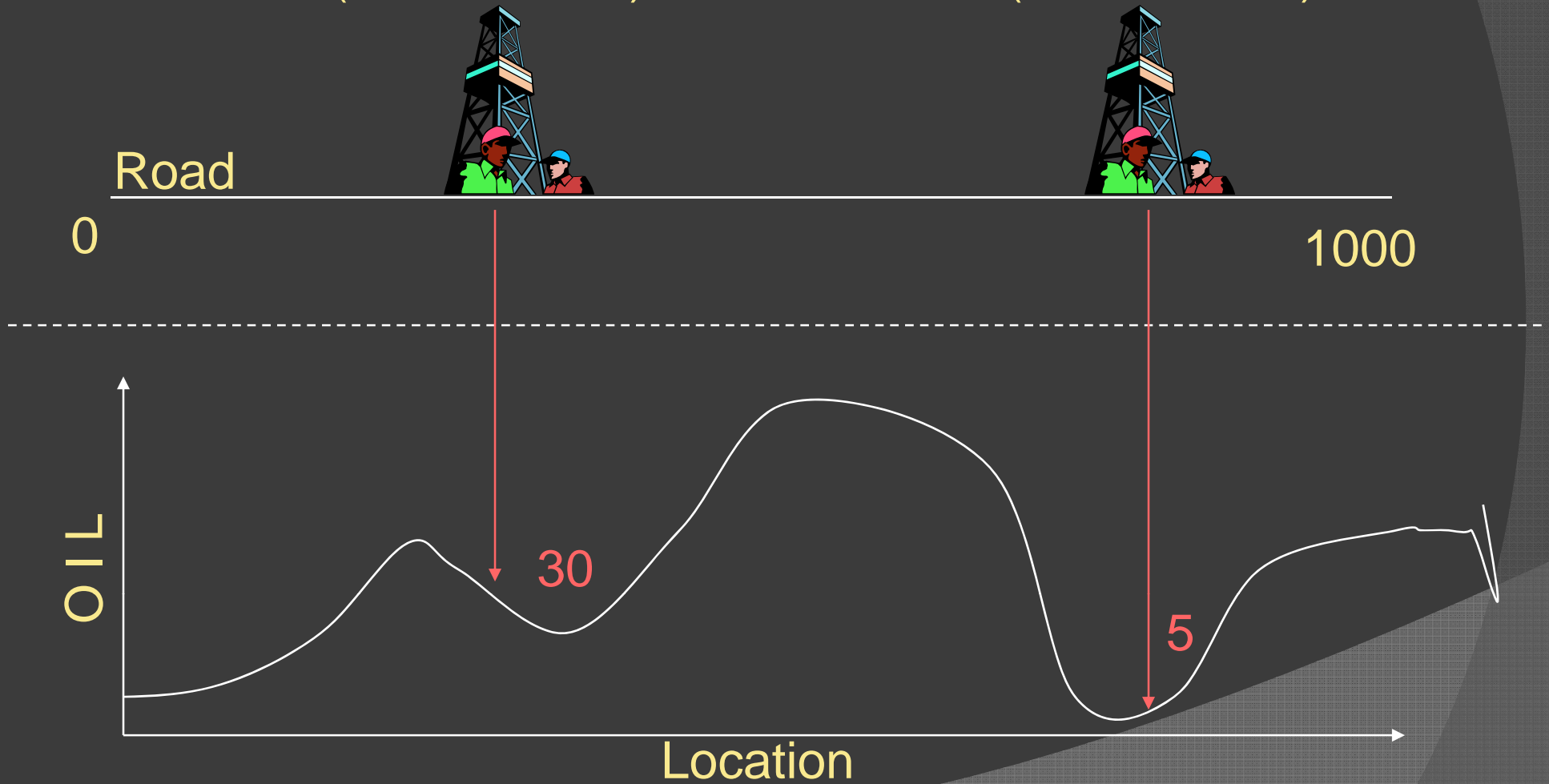
	512	256	128	64	32	16	8	4	2	1
900	1	1	1	0	0	0	0	1	0	0
300	0	1	0	0	1	0	1	1	0	0
1023	1	1	1	1	1	1	1	1	1	1

In GA's these encoded strings are sometimes called "genotypes" or "chromosomes" and the individual bits are sometimes called "genes"

Fitness of oil function

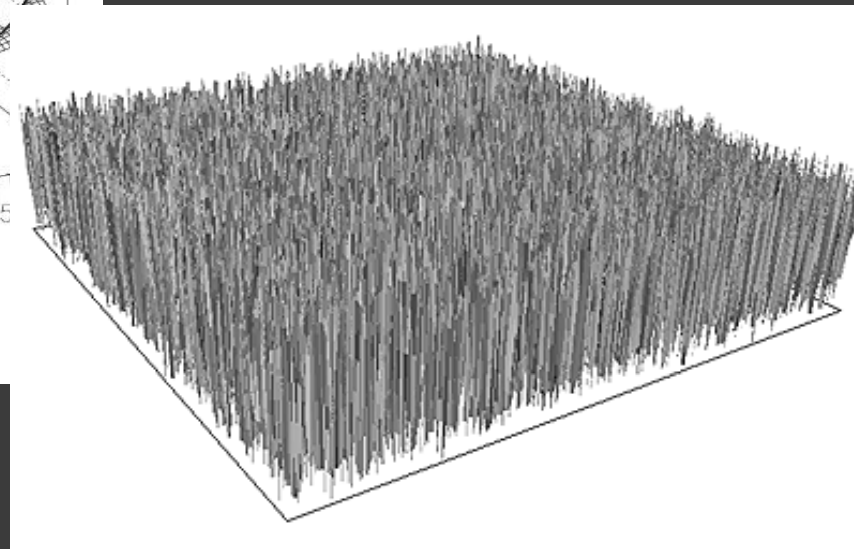
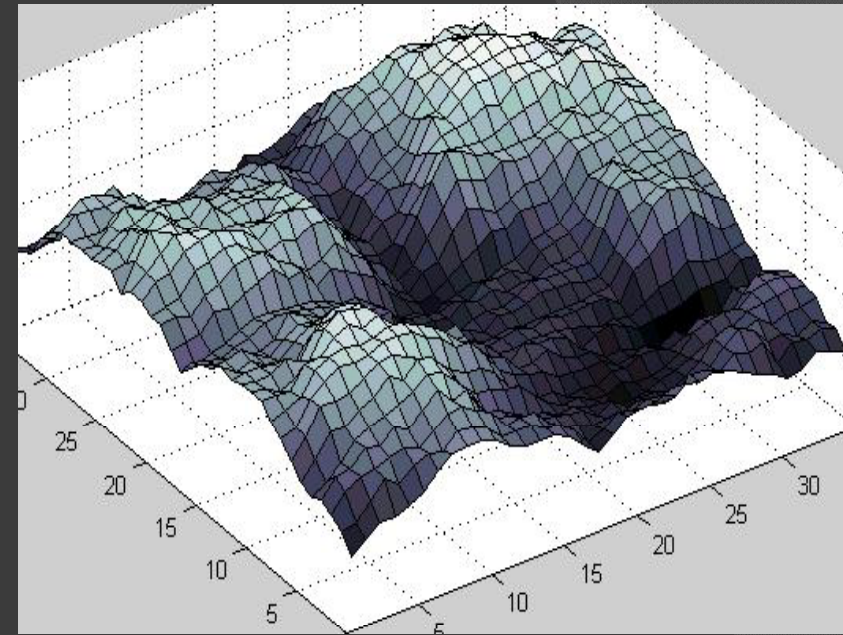
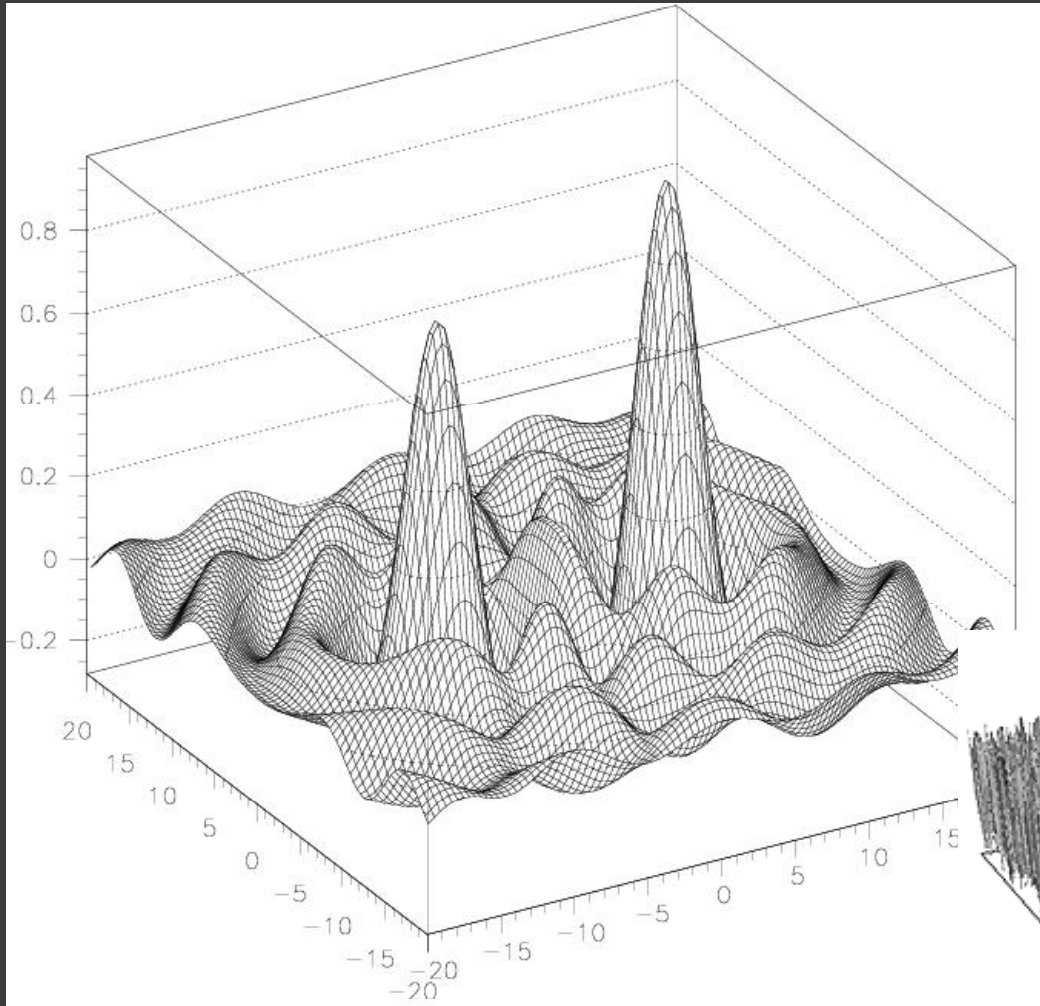
Solution1 = 300
(0100101100)

Solution2 = 900
(1110000100)



- Oil example: search space is one dimensional (and stupid: how to define a fitness function?).
- RTE: encoding several values into the chromosome many dimensions can be searched
- Search space can be visualised as a surface or fitness landscape in which fitness dictates height (fitness / chi-sqr hypersurface)
- Each possible genotype is a point in the space
- A GA tries to move the points to better places (higher fitness) in the space

Fitness landscapes (2-D)



- Obviously, the nature of the search space dictates how a GA will perform
- A completely random space would be bad for a GA
- Also GA's can, in practice, get stuck in local maxima if search spaces contain lots of these
- Generally, spaces in which small improvements get closer to the global optimum are good

The algorithm

- Generate a **set of random solutions**
- Repeat
 - **Test each solution** in the set (rank them)
 - **Remove some bad solutions** from set
 - **Duplicate some good solutions**
 - **make small changes** to some of them
- Until best solution is good enough

How to duplicate good solutions?

Adding Sex

- Two high scoring “parent” bit strings (chromosomes) are selected and with some probability (crossover rate) combined
- Producing two new offsprings (bit strings)
- Each offspring may then be changed randomly (mutation)

} sex
} result of sex
} parents are seldom happy with the result

- Selecting parents: many schemes possible, example:

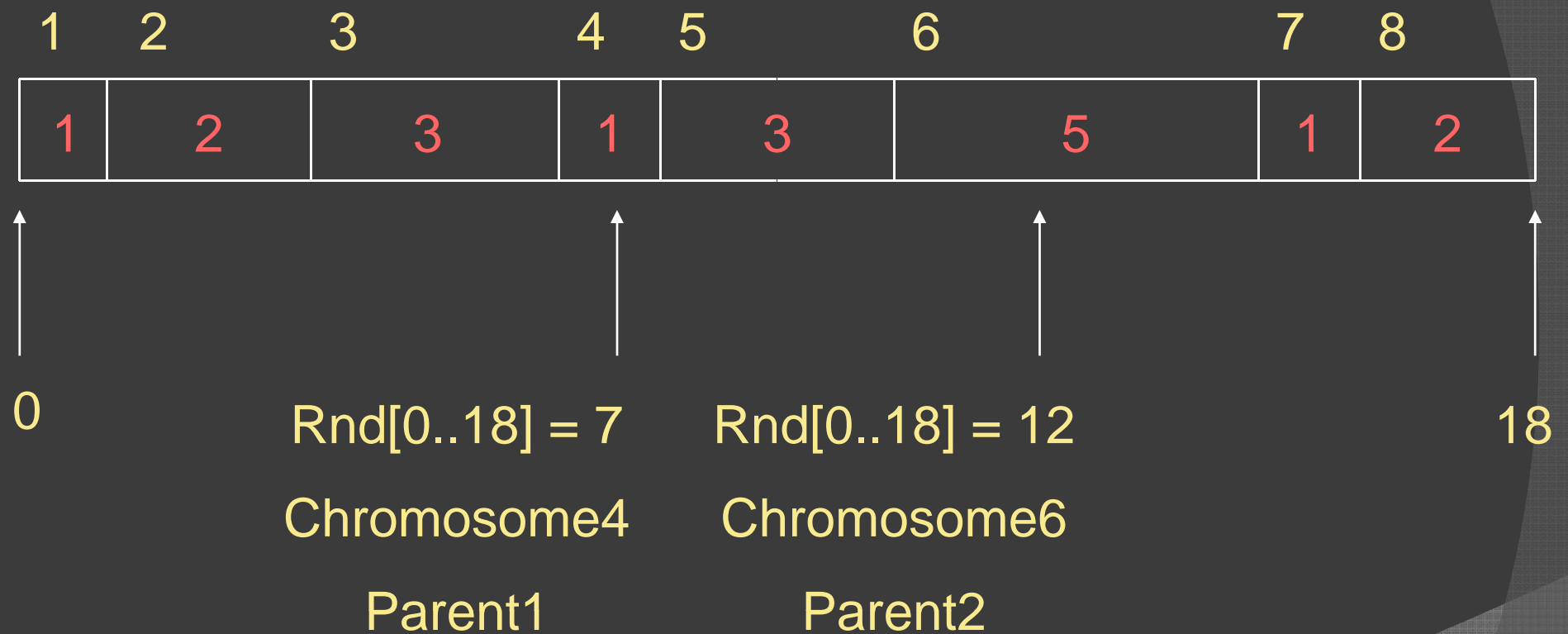
Roulette Wheel

- Add up the fitness's of all chromosomes
- Generate a random number R in that range
- Select the first chromosome in the population that - when all previous fitness's are added - gives you at least the value R

Example population

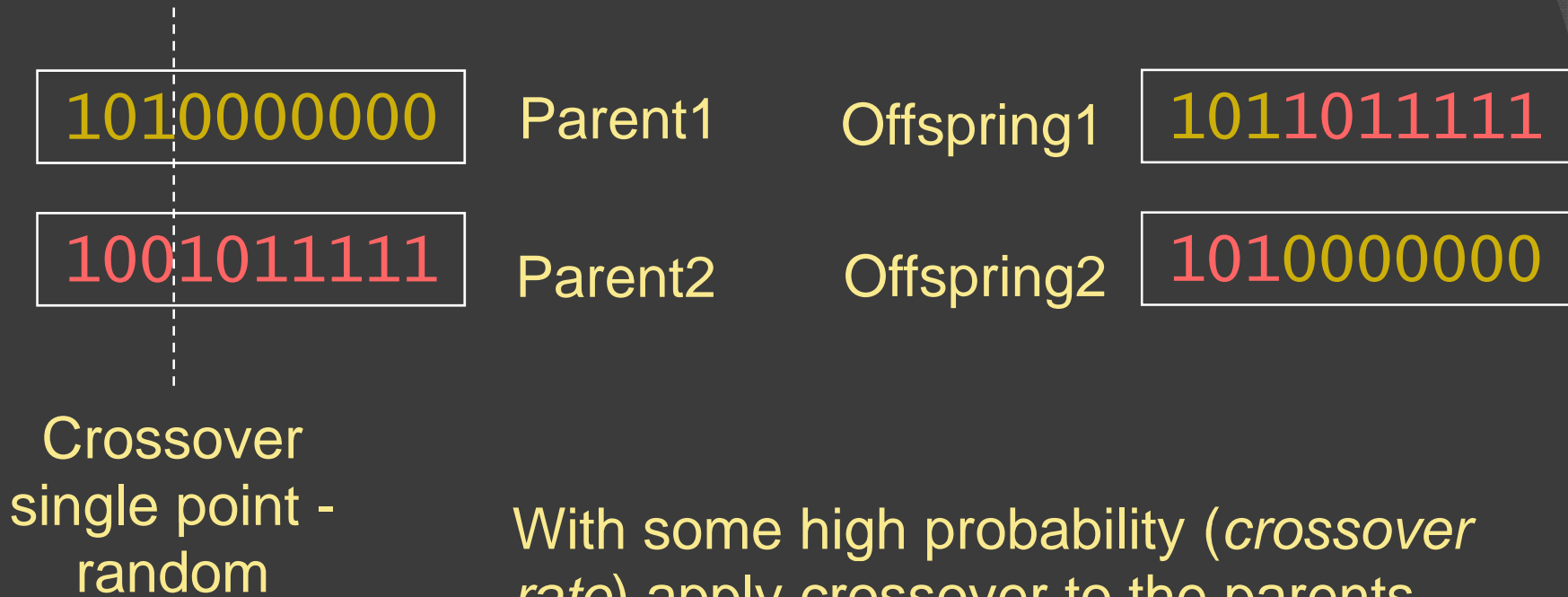
No.	Chromosome	Fitness
1	1010011010	1
2	1111100001	2
3	1011001100	3
4	1010000000	1
5	0000010000	3
6	1001011111	5
7	0101010101	1
8	1011100111	2
		sum: 18

Roulette Wheel Selection



Higher chance of picking a fit chromosome!

Crossover - Recombination



With some high probability (*crossover rate*) apply crossover to the parents. (*typical values are 0.8 to 0.95*)

Mutation

Offspring1 **1011011111**
Offspring2 **1010000000**

Original offspring

mutate

Offspring1 **1011001111**
Offspring2 **1000000000**

Mutated offspring

With some small probability (the *mutation rate*) flip each bit in the offspring (*typical values between 0.1 and 0.001*)

Improved algorithm

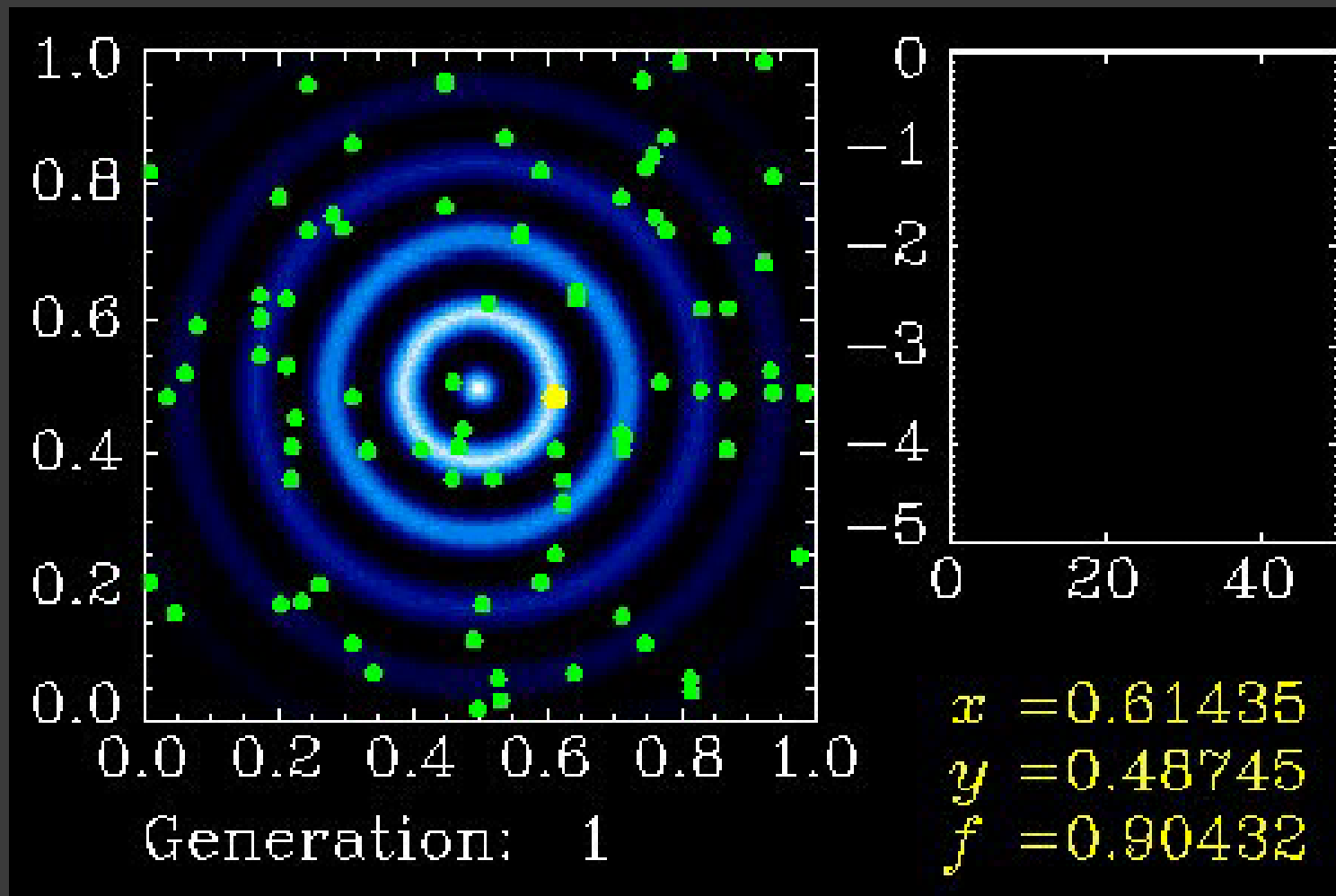
- Generate a *population* of random chromosomes
- Repeat (each generation)
 - Calculate fitness of each chromosome
 - Repeat
 - Use roulette selection to select pairs of parents
 - Generate offspring with crossover and mutation
 - Until a new population has been produced
- Until best solution is good enough

Many Variants of GA

- Different kinds of selection (not roulette):
Tournament, **Elitism**, etc.
- Different recombination:
one-point crossover, multi-point crossover, 3 way crossover etc.
- Different kinds of encoding other than bitstring
Integer values, Ordered set of symbols
- Different kinds of mutation
variable mutation rate
- Different reduction plans
controls how newly bred offsprings are inserted into the population

PIKAIA (Charbonneau, 1995)

How PIKAIA works...



List of ME Codes (incomplete)

- **HeLix⁺**
A. Lagg, most flexible code (multi-comp, multi line), He 10830 Hanle slab model implemented. Genetic algorithm Pikaia. Fully parallel.
- **VFISV**
J.M.Borrero, for SDO HMI. Fastest ME code available. F90, fully parallel. Levenberg-Marquardt with some optimizations.
- **MERLIN**
Written by Jose Garcia at HAO in C, C++ and some other routines in Fortran. (Lites et al. 2007 in Il Nuovo Cimento)
- **MELANIE**
Hector Socas at HAO. In F90, not parallel. Numerical derivatives.
- **HAZEL**
Artoro Lopez Ariste et al. (2008). Optimized for He 10830, He D3, Hanle-slab model.
- **MILOS**
Orozco Suarez et al. (2007), IDL, some papers published with it

Installation & Usage of HeLIX⁺

Follow instructions on user's manual:

He-Line Information Extractor⁺ HELIX⁺



Andreas Lagg · Max-Planck-Institut für Sonnensystemforschung · Katlenburg-Lindau, Germany

Basic usage:

- 1-component model, create & invert synthetic spectrum
- discuss problems:
 - parameter crosstalk
 - uniqueness of solution
 - stability & reliability
 - influence of noise

Download from <http://www.mps.mpg.de/homes/lagg>
GBSO download-section → helix
use *invert* and *IR\$soft*

Exercise II:

HeLix⁺ installation and basic usage



- install and run IDL interface of HeLix⁺
- the first input file: synthesis of Fe I 6302.5
 - change atmospheric parameters (B, INC, ...)
 - change line parameters (quantum numbers, g_{eff})
 - display Zeeman pattern
- add noise
- 1st inversion
- play with noise level / initial values / parameter range
- weighting scheme

Synthesis

- add complexity to atmospheric model (stray-light, multi-component)
- add 2nd spectral line (Fe 6301.5)

blind tests:

- take synthetic profile from someone else and invert it
- Which parameters are robust?
- How can robustness be improved?

Download first input file: `abisko_1c.ipt`
<http://www.mps.mpg.de/homes/lagg/>