What drives a cyclic dynamo in a solar-like star with anti-solar differential rotation?

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(Received February 14, 2019; Revised; Accepted)

Submitted to ApJL

ABSTRACT

Global and semi-global convective dynamo simulations of solar-like stars are known to show a transition from an anti-solar (fast poles, slow equator) to solar-like (fast equator, slow poles) differential rotation for increasing rotation rate. The dynamo solutions in the latter regime can exhibit regular cyclic modes, whereas in the former regime such cyclic solutions have not been obtained so far. In this paper we present a semi-global dynamo simulation, which for the first time produces clear cyclic magnetic activity in the anti-solar differential rotation regime. We analyze the large-scale flow properties (differential rotation and meridional circulation) together with the turbulent transport coefficients obtained with the test-field method. We find that turbulent dynamo effects play an important role in the dynamics of the system as effective large-scale flows are significantly altered by turbulent pumping. Neither an $\alpha\Omega$ dynamo wave nor advection-dominated dynamo are able to explain the cycle period and the propagation direction of the mean magnetic field. Furthermore, we find that the $\alpha$ effect is comparable or even larger than the $\Omega$ effect in generating the toroidal magnetic field and therefore the dynamo seems to be $\alpha^2\Omega$ or $\alpha^2$ type.

Keywords: Magnetohydrodynamics — dynamo — rotation

1. INTRODUCTION

Recently, Brandenburg & Giampapa (2018) reported an abrupt increase of the magnetic activity level of solar-like stars with decreasing values of the Coriolis number\(^1\) in the vicinity of its solar value. Another observational study (Olspert et al. 2018) found that the degree of magnetic variability abruptly decreased, indicative of the disappearance of magnetic cycles, at solar chromospheric activity index towards smaller values. Brandenburg & Giampapa (2018) proposed a transition in differential rotation (DR) from solar-like (for younger stars) to anti-solar-like (at a later age) to be responsible for this phenomenon.

This transition (henceforth S-AS transition) has already been the subject of many numerical studies (see, e.g., Gastine et al. 2014; Käpylä et al. 2014; Mabuchi et al. 2015; Featherstone & Miesch 2015; Viviani et al. 2018) and they all pinpoint it around the solar rotation rate. However, only few of these studies considered dynamo solutions in the antisolar regime. Those that do, produce stationary dynamo solutions (e.g., Warnecke 2018) or irregular oscillations (Karak et al. 2015). The latter study shows an enhancement of magnetic field strength in accordance with Brandenburg & Giampapa (2018), which actually motivated their interpretation.

In a previous paper (Viviani et al. 2018), we reported on two convective dynamo simulations of solar-like stars showing anti-solar differential rotation. In contrast to previous studies, we find regular dynamo cycles. Both the meridional circulation and the differential rotation are stronger than in models with faster rotation, but we do not see significantly increased magnetic energy across the S-AS transition, in agreement with Warnecke (2018), but in contrast to the findings of Karak et al. (2015).

\(^1\) The Coriolis number describes the strength of the rotational influence on convection.
Solar-like DR is usually obtained only when rotation rates, somewhat elevated from the solar one, are used. In this regime, cyclic dynamo solutions with equatorward-dynamo waves are often obtained from global magneto-convection models (e.g. Käpylä et al. 2012; Augustson et al. 2015; Strugarek et al. 2017). Most of them can be explained in terms of Parker waves (see e.g., Warnecke et al. 2014, 2016, 2018; Käpylä et al. 2016, 2017; Warnecke 2018). The migration direction and cycle period of a Parker wave is determined by the product of the \( \alpha \) effect and the radial gradient of the local rotation rate \( \Omega \) (Parker 1955; Yoshimura 1975). For an equatorward-migrating field in the northern hemisphere (as observed on the Sun), one needs, for example, a negative radial gradient of \( \Omega \) and a positive \( \alpha \) effect. However, simplified dynamo models often invoke an advection-dominated concept (e.g. Choudhuri et al. 1995; Dikpati & Charbonneau 1999; Küker et al. 2001) to explain the migration and cyclic behavior of large-scale stellar magnetic fields. In this case, the meridional flow speed and direction at the location of the toroidal field generation determines the cycle period and latitudinal dynamo wave direction.

Another possible mechanism generating cyclic dynamo solutions is an \( \alpha^2 \) dynamo (Baryshnikova & Shukurov 1987; Rädler & Bräuer 1987). There, magnetic field generation is due to the \( \alpha \) effect alone and differential rotation is not needed. Such a dynamo was reproduced in forced turbulence simulations (Mitra et al. 2010), but global magneto-convection models have not yet yielded a similar solution.

In this work we will test these mechanisms and determine if one of them can explain the cyclic magnetic activity in a simulation with anti-solar DR. To achieve this goal we will use the test-field method (Schrinner et al. 2005, 2007) to extract the turbulent transport coefficients. This is possible due to the dominance of the axisymmetric magnetic field allowing us to try a description in terms of mean-field theory. The test-field method has been used successfully to explain cyclic dynamo solutions in previous studies of solar-type stars (Warnecke et al. 2018; Warnecke 2018) and their long-time variations (Gent et al. 2017) as well as of planetary dynamos (e.g. Schrinner 2011; Schrinner et al. 2011, 2012).

2. SETUP AND METHODS

We use the Pencil Code\(^2\) to solve the fully compressible MHD equations for the velocity \( \mathbf{U} \), the density \( \rho \), the specific entropy \( s \) and the magnetic vector potential \( \mathbf{A} \) with the magnetic field \( \mathbf{B} = \nabla \times \mathbf{A} \) in a semi-spherical shell, defined in spherical coordinates \( (r, \theta, \phi) \) by \( 0.7R \leq r \leq R \) for the radial extent, and \( \theta_0 \leq \theta \leq \pi - \theta_0 \) and \( 0 \leq \phi \leq 2\pi \) for the extents in colatitude and longitude, respectively, with \( \theta_0 = 15^\circ \).

A detailed description of the setup can be found from Käpylä et al. (2013) and Viviani et al. (2018).

The non-dimensional quantities are scaled to physical units using the solar radius \( R = 7 \cdot 10^8 \) m, solar rotation rate \( \Omega_\odot = 2.7 \cdot 10^{-6} \) s\(^{-1}\), the density at the bottom of the solar convection zone \( \rho(0.7R) = 200 \) kg/m\(^3\), and \( \mu_0 = 4\pi \cdot 10^{-7} \) Hm\(^{-1}\) (for further details see Appendix A of Käpylä et al. 2019). We indicate by \( \mathbf{B} \) and \( \mathbf{U} \) the mean, that is, longitude-averaged fields, and by \( b', \mathbf{u'} \) the corresponding fluctuating fields, so that, for example, \( \mathbf{B} = \overline{\mathbf{B}} + b' \).

The need to compute turbulent transport coefficients can be seen from the induction equation for the mean magnetic field, \( \overline{\mathbf{B}} \):

\[
\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times \left( \mathbf{U} \times \overline{\mathbf{B}} + \mathbf{u'} \times b' \right) - \nabla \times \eta \nabla \times \overline{\mathbf{B}}. \tag{1}
\]

The term \( \mathbf{\varepsilon} = \mathbf{u'} \times b' \) is the turbulent electromotive force; it can be expanded in terms of \( \overline{\mathbf{B}} \) and its derivatives. Further, the tensorial coefficients of the individual contributions can be divided into symmetric and anti-symmetric parts (see e.g. Krause & Rädler 1980):

\[
\mathbf{\varepsilon} = \alpha \overline{\mathbf{B}} + \gamma \overline{\mathbf{B}} \times \beta \nabla \times \overline{\mathbf{B}} - \delta \nabla \times \overline{\mathbf{B}} - \kappa \cdot (\nabla \overline{\mathbf{B}})^{(s)}, \tag{2}
\]

where \( \alpha \) and \( \beta \) are symmetric tensors of rank two, \( \gamma \) and \( \delta \) are vectors, while \( \kappa \) is a tensor of rank three with \( (\nabla \overline{\mathbf{B}})^{(s)} \) being the symmetric part of the derivative tensor of \( \overline{\mathbf{B}} \). Each of these coefficients can be related to a physical effect: \( \alpha \) covers cyclonic generation (\( \alpha \) effect), \( \beta \) describes turbulent diffusion, \( \gamma \) represents turbulent pumping, \( \delta \) corresponds to the Rädler effect and the physical interpretation of \( \kappa \) is not yet established. The pumping enters the effective mean flow, \( \overline{\mathbf{U}}^{Eff} = \overline{\mathbf{U}} + \gamma \), (e.g Kichatinov 1991; Ossendrijver et al. 2002; Käpylä et al. 2006; Warnecke et al. 2018) and may thus be crucial in determining the nature of the dynamo.

To determine the turbulent transport coefficients, we continued a run showing anti-solar differential rotation and a cyclic dynamo solution (Run C1) from Viviani et al. (2018) with the test-field module of the Pencil Code activated (see also Schrinner et al. 2005, 2007).

3. RESULTS

The run considered has a Coriolis number of \( \text{Co} = 2.8 \) and is therefore in the anti-solar differential rotation regime close to the transition point. The Coriolis number is defined as \( \text{Co} = 2\Omega_\odot/\text{u}_{\text{rms}}k_u \), where

\( \text{u}_{\text{rms}} \) is the root mean square value
\[ u_{\text{rms}} = \sqrt{(3/2)(U_r^2 + U_\theta^2)} \text{ is the averaged rms velocity and } k_u = 2\pi/0.3R \approx 21/R \text{ is an estimate of the wavenumber of the largest eddies while } \Omega_0 \text{ is the overall rotation rate with } \Omega_0/\Omega_\odot = 1.8. \]

3.1. Mean magnetic field

The mean magnetic field is prevailingly symmetric about the equator (or quadrupolar) and shows regular cyclic behavior with poleward migrating \( B_\phi \), at only high latitudes near the surface, at all latitudes at larger depths (Figure 1b and c). It possesses also a strong steady contribution (Figure 1a). As our analysis is focussed on cyclic behavior, we subtract the steady constituent from the total field to obtain the residual field \( B_{\text{res}} = B - \langle B \rangle_t \), see Figure 1b and c for its toroidal component \( B_{\phi\text{res}} \) at two depths. Inspection reveals that polarity reversals occur at all latitudes. The steady mean magnetic field is typically 2-2.5 times stronger than the residual one. In Figure 1d, we also show the dependence of \( B_{\phi\text{res}} \) on radius and time at latitude +50\(^\circ\) where the residual field is strongest. The field topology is similar throughout the convection zone, and the poleward migration is present at all depths.

3.2. Mean flows

We start our analysis by investigating meridional circulation and differential rotation as shown in Figure 2a,b,c. The former has a dominant, large, anti-clockwise cell, producing a relatively strong (20 m s\(^{-1}\)) poleward flow near the surface at almost all latitudes. There is a slow equatorward return flow widely distributed in the bulk of the convection zone at mid to high latitudes. In the slowly rotating regime, the anti-solar DR is often accompanied by a single cell anti-clockwise meridional circulation. In contrast, in the fast rotating regime, the solar-like DR drives multi-cellular meridional circulation aligned with the rotation axis. The cell pattern in this run represents a transitional state between these two regimes (e.g., Käpylä et al. 2014; Karak et al. 2015; Featherstone & Miesch 2015).

The differential rotation profile shows a decelerated equator and accelerated poles hence an anti-solar profile. The pole-equator difference at the surface is comparable to runs with similar rotational influence (e.g. Karak et al. 2015; Warnecke 2018). However, the energy in the differential rotation compared to the total kinetic energy, neglecting the rigid rotation, is smaller than in runs with slightly slower and faster rotation with solar-like DR (Viviani et al. 2018). This is most likely due to our run being very close to the actual S-AS transition. In the differential rotation profile, we find two distinct features: At mid latitudes there is a local minimum of \( \Omega \) which has also been found in simulations with about three times faster rotation. In these, the resulting shear drives a Parker dynamo wave (e.g. Warnecke et al. 2014). Furthermore, we find a strong negative shear layer near the surface at low latitudes.

3.3. Turbulent transport coefficients

Next, we look at the turbulent transport coefficients, where we focus on \( \alpha \) and \( \gamma \), and compare them with their counterparts of the faster rotating dynamo run with solar-like DR of Warnecke et al. (2018). The profiles of the other coefficients (\( \beta, \delta, \kappa \)) are similar to those in Warnecke et al. (2018), but with higher magnitudes. Regarding \( \alpha \) (see first two rows of Figure 3), the \( \alpha_{rr} \) and \( \alpha_{\phi\phi} \) components have very similar magnitudes and profiles while \( \alpha_{\theta\theta} \) is somewhat larger and shows an opposite sign near the surface close to the equator. \( \alpha_{r\theta} \) and \( \alpha_{\phi\theta} \) are now stronger, while \( \alpha_{r\phi} \) and \( \alpha_{\theta\phi} \) show opposite signs at the equator near the surface. We associate these

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\(^3\) Angle brackets indicate averaging over the coordinate(s) in the subscript.
Figure 2. Time-averaged radial (a) and latitudinal (b) components of the meridional circulation ($U_r, U_\theta$), (c) angular velocity $\Omega = \frac{U_\phi}{r \sin \theta} + \Omega_0$ and (d) temporal rms value of the residual azimuthal mean magnetic field $B_{\text{res}}^\phi$. Panels e)-h): same as a)-c), but using the effective mean velocity. The flow lines in panels b), f) represent meridional and effective meridional circulation, respectively. Arrows in panels d), h) represent the direction of the PY dynamo wave with normalized arrows.

differences to Warnecke et al. (2018) with the slower rotation in this run.

Concerning the turbulent pumping (see last row of Figure 3), $\gamma_r$ is upward everywhere else except in the bulk of the convection zone at mid and high latitudes. $\gamma_\theta$ is equatorward (poleward) in the upper (lower) half of the convection zone. $\gamma_\phi$ is prograde near the surface and at mid-latitudes near the base, and negative everywhere else. The magnitudes of all three components are around $0.3 u_{\text{rms}}'$, where $u_{\text{rms}}'(r, \theta) = \langle u'^2 \rangle^{1/2}$ is the local turbulent rms velocity in the meridional plane. Compared to Warnecke et al. (2018), the profiles remain mostly unchanged, but $\gamma_r$ and $\gamma_\phi$ are now around twice as large, while $\gamma_\theta$ is 20% weaker. The resulting effective mean velocity is shown as its time average in Figure 2e,f,g. Its radial component $U_{r,\text{eff}}$ is completely dominated by $\gamma_r$ leaving nearly no trace of the actual flow. $\gamma_\theta$ changes the sign of $U_\theta$ only slightly below the surface and reduces its magnitude by around 30%, however, the meridional flows cells are completely destroyed, as shown by the flow lines in Figure 2f. $\gamma_\phi$ is accelerating the equator and decelerating the polar region. The larger change in $\Omega_{\text{eff}}$ compared to Warnecke et al. (2018) is because $\gamma_\phi$ increases with decreasing $\Omega_0$.

3.4. Dynamo cycles and migration

As a first step in determining the possible dynamo mechanism we compare the period of the magnetic field cycle with theoretical expectations. We compute the magnetic cycle period by Fourier transforming $B_\phi$ at $r = 0.98 R$ and then averaging the spectra over latitude. As a result we get $P_{\text{cyc}} = (3.2 \pm 0.3)$ yrs, where the error is obtained from the width at half maximum. The two main dynamo scenarios both make predictions for the dynamo cycle length $P_{\text{cyc}}$. The Parker-Yoshimura wave period is locally defined as (Parker 1955; Yoshimura 1975)

$$P_{\text{PY}} = 2\pi \left| \frac{\alpha_{\phi\phi} k_\theta}{4} r \cos \theta \partial_r \Omega \right|^{-1/2}$$

where $k_\theta = 2\pi/(r \Delta \theta)$ is the latitudinal wavenumber describing the path length that the dynamo wave has to travel during the cycle, with $\Delta \theta = \pi/2 - \theta_0$. The justification of using only $\alpha_{\phi\phi}$ in Eq. (3) is that the other contributions to the poloidal field generation are sub-dominant in the regions where the wave will turn out to be poleward.
The cycle period of an advection dominated dynamo (ADD) is related to the travel time of the meridional circulation, \( \tau_{MC} \), such that
\[
P_{MC} \approx 2 \tau_{MC} (\text{Küker et al. 2001, 2019}).
\] Hence, in our notations, the expected cycle period can be written as
\[
P_{MC} = 2 r \Delta \theta U_{MC}(r),
\] (4)
where \( U_{MC} \) is the amplitude of the mean meridional flow at the location of the dynamo wave. Traditionally, ADD models assume meridional flow and the resulting migration to be significant near the bottom of the convection zone, which would correspond to setting \( r = 0.7 R \), but in the case of our simulation it is not so straightforward to determine the location of the dynamo wave.

We start using \( \alpha_{\phi\phi} \) and the radial differential rotation in Eq. (3), and the mean meridional flows in Eq. (4), we obtain for the averages over the convection zone
\[
\langle P_{PY} \rangle_r = 3.1 \text{ yr} \quad \text{and} \quad \langle P_{MC} \rangle_r = 8.2 \text{ yr}.
\]
Using the prediction for the Parker-Yoshimura wave propagation direction given by (Yoshimura 1975)
\[
\xi(r, \theta) = -\alpha_{\phi\phi} \hat{e}_\phi \times \nabla \Omega,
\] (5)
we obtain the migration directions for the shear from \( \Omega \) (Figure 2d) and \( \Omega^{\text{eff}} \) (Figure 2h), respectively. Near the bottom of the convection zone, where also the residual field is strongest, the migration direction is poleward at almost all latitudes, which would agree with the actual field propagation. In the bulk of the convection zone, however, the predicted direction is equatorward, failing to explain the actual migration. Hence, neither the Parker-Yoshimura dynamo wave or the ADD alone can be responsible for the oscillating poleward-migrating magnetic field throughout the convection zone.

3.5. Dynamo drivers

To understand the failure of the simple dynamo scenarios in explaining cycles and migration of the field, we finally turn to computing the terms, contributing to the magnetic field generation, in detail. We present the contributions of the \( \Omega \) and \( \alpha \) effects, that is, of \( \mathbf{B} \cdot \nabla \Omega \) and \( \nabla \times (\alpha \cdot \mathbf{B}) \), in Figure 4 for the total rms magnetic field (upper row), and show the corresponding rms magnetic fields in the lower row. The two leftmost (rightmost) columns show the generators for the poloidal (toroidal) meridional circulation in the lower quarter of the convection zone we obtain, instead, \( P_{MC,\text{bot}} = 63.8 \text{ yr} \). Considering the relevant role of the turbulent pumping, especially in \( U_r \), we also calculated the periods using the effective velocity, obtaining \( \langle P_{PY}^{\text{eff}} \rangle_r = 2.8 \text{ yr} \), \( \langle P_{MC}^{\text{eff}} \rangle_r = 5.6 \text{ yr} \) and \( P_{MC,\text{bot}}^{\text{eff}} = 22.0 \text{ yr} \). The Parker-Yoshimura period is less affected than the ones for meridional circulation, as the \( \gamma \) contribution is more significant for the meridional circulation than for the differential rotation. In conclusion, the Parker-Yoshimura periods are very well consistent with the measured magnetic cycle, while advection by meridional flow cannot well explain it.

If the mean magnetic field were advected by the meridional flow or its effective counterpart, one would not be able to explain poleward migration virtually everywhere within the convection zone. This becomes evident from Figure 2b,f, where equatorward flows are present. Whether the meridional circulation is able to overcome diffusion, can be assessed by help of the corresponding dynamo number
\[
C_U = 0.3 R U_{MC}^{\text{rms}} / \tau_r \{ \beta \},
\]
where \( \tau_r \{ \cdot \} \) indicates the trace. The averaged value for the mean and the effective flow is 0.3 and 0.8, respectively. Despite the increase due to \( \gamma \), values below unity imply that the (effective) flow cannot overcome the diffusion, therefore the ADD scenario is not applicable here.

Using the prediction for the Parker-Yoshimura wave propagation direction given by (Yoshimura 1975)
magnetic field, respectively. From the magnitudes of the toroidal generators, it is evident that the \( \alpha \) effect is equally important, or even dominant compared to the \( \Omega \) effect. The poloidal field generation is as efficient as the toroidal field generation by the \( \Omega \) effect; hence the most likely dynamo mechanism is an \( \alpha^2 \Omega \) or even an \( \alpha^2 \) dynamo.

The \( \Omega \) effect generates toroidal field efficiently at mid-latitudes both near the surface, and in the bulk of the convection zone, co-inciding with the location of the local minimum of \( \Omega \). The \( \alpha \) effect is strongest near the surface, but shows also toroidal field generation at the location of the local minimum of \( \Omega \). The rms toroidal field, however, does not co-incide strongly with its generators, but clearly its profile is offset deeper into the convection zone. One reason might be the radial field boundary condition, which would supress a toroidal field near the surface. The \( \alpha \) effect generates poloidal field mostly at high latitudes at all depths of the convection zone, although there are also regions of strong field generation close to the surface near the equator. The high-latitude field generator profiles match qualitatively better to the rms field distribution than for the toroidal field, but still the match is not very strong. The mismatch in between the generators and the actual field distribution indicates that our conclusion of the mechanism being a simple \( \alpha^2 \Omega \) or \( \alpha^2 \) dynamo is not a very solid one, but that other dynamo effects than considered here might be at play.

The dynamo solutions obtained in narrow wedges by Warnecke (2018) turn into oscillatory ones in the anti-solar DR regime when the longitudinal wedge assumption is relaxed (here and Viviani et al. (2018)), indicating that there are indeed various dynamo modes excited, which possess very similar critical dynamo numbers. In terms of dynamo theory, the coexistence of a steady and an oscillating field constituent can be understood as follows: sufficiently overcritical flows enable in general the growth of more than one dynamo mode. Under the assumption of steady mean flows and statistically stationary turbulence, the time dependence of the mean-field eigenmodes is exponential with an in general, complex increment. It is well conceivable that a non-oscillating and an oscillating mode are both excited and even co-
time to coexist in their nonlinear stage, although their kinematic growth rates were different.

4. CONCLUSIONS

We presented and analyzed a spherical convective dynamo simulation showing for the first time regular magnetic cycles with anti-solar differential rotation. We tried to explain the oscillating magnetic field as a Parker dynamo wave or within the advection dominated framework. Neither of the two approaches alone can explain the results in terms of cycle period and migration direction, even if we take the turbulent contributions to the effective mean flow into account. One reason might be that in this situation the \( \alpha \) effect plays a more dominant role than in a simple \( \alpha \Omega \) dynamo. Our claim is validated by the analysis of the field generators shown in Figure 4: the mean field is generated by cyclonic convection and differential rotation together, suggestive of \( \alpha^2 \Omega \) or \( \alpha^2 \) dynamos. However, the spatial distributions of the generators do not match very well with those of the actual mean fields. This likely indicates that other dynamo effects also play important roles. However, mean-field models that take into account all turbulent effects are needed to address this issue.

We acknowledge the HPC-EUROPA3 project (INFRAIA-2016-1-730897), supported by the EC Research Innovation Action under the H2020 Programme. MV, MJK, PJK, and MR acknowledge the support of the Academy of Finland ReSoLVE Centre of Excellence (grant number 307411). MV acknowledges being enrolled in the International Max Planck Research School for Solar System Science at the University of Göttingen (IMPRS). JW acknowledges funding by the Max-Planck/Princeton Center for Plasma Physics. PJK acknowledges support from DFG Heisenberg grant (No. KA 4825/1-1). We acknowledge also support from the supercomputers at GWDG, at RZG in Garching, and in the facilities hosted by the CSC—IT Center for Science in Espoo, Finland, which are financed by the Finnish ministry of education.

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