FREE MAGNETIC ENERGY AND FLARE PRODUCTIVITY OF ACTIVE REGIONS

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ABSTRACT

In this study, the photospheric vector magnetograms, obtained with the Spectro-Polarimeter of the Solar Optical Telescope on board *Hinode*, are used as the boundary conditions to extrapolate the three-dimensional nonlinear forcefree (NLFF) coronal magnetic fields. The observed non-force-free photospheric magnetic fields are preprocessed toward the nearly force-free chromospheric magnetic fields. The performance of the preprocessing procedure is evaluated by comparing with chromospheric magnetic fields obtained by the Vector SpectroMagnetograph instrument located on the Synoptic Optical Long-term Investigations of the Sun Tower. Then, the weighted optimization method is applied to the preprocessed boundary data to extrapolate the NLFF fields with which we are able to estimate the free magnetic energy stored in the active regions. The magnitude scaling correlation between the free magnetic energy and the soft X-ray flare index (FI) of active regions is then studied. The latter quantifies the impending flare production of active regions over the subsequent 1, 2, and 3 day time windows. Based on 75 samples, we find a positive correlation between the free energy and the FI. We also study the temporal variation of free magnetic energy for three active regions, of which two are flare-active and one is flare-quiet during the observation over a period of several days. While the magnitude of free magnetic energy unambiguously differentiates between the flare-active and the flare-quiet regions, the temporal variation of free magnetic energy does not exhibit a clear and consistent pre-flare pattern. This may indicate that the trigger mechanism of flares is as important as the energy storage in active regions.

Key words: magnetic fields - Sun: activity - Sun: corona - Sun: flares

1. INTRODUCTION

The solar magnetic field is the source of most (if not all) solar energetic events such as flares and coronal mass ejections (CMEs). Since the coronal magnetic fields cannot be precisely measured at present except in a few special cases (e.g., Gary & Hurford 1994; Lin et al. 2004), the efforts to identify the magnetic properties important for flare/CME production have been made almost exclusively with parameters derived from the photospheric magnetic fields. Generally speaking, these magnetic parameters quantify the size and/or the topological complexity of an active region. Several recently identified properties are: the total unsigned magnetic flux $\Phi = \int |B_{\rm rad}| dA$, where $B_{\rm rad}$ is the radial component of the magnetic field and the integral is performed over the field of view (FOV) A (Barnes & Leka 2008; Song et al. 2009); the amount of magnetic flux close to the strong-gradient magnetic polarity inversion line (PIL; Schrijver 2007); the length of the high-gradient and high-sheared PIL (Falconer et al. 2003); the total magnetic dissipation (Jing et al. 2006; Song et al. 2009); the effective connected magnetic field (Georgoulis & Rust 2007); and the photospheric excess energy (Leka & Barnes 2003; Barnes & Leka 2008). In particular, the photospheric excess energy E_{pe} measures the difference between the observed and the potential fields (i.e., the current-free fields, computed from the vertical component of magnetic fields) at the photospheric surface, i.e., $E_{\rm pe} = \int \frac{B_o^2}{8\pi} dA - \int \frac{B_p^2}{8\pi} dA$, where the superscripts *o* and *p* represent the observed field and the potential field, respectively. Note that E_{pe} is not truly magnetic energy stored in an active

region, since the integral is computed only at the photospheric surface and not throughout the coronal volume. Although all these photospheric magnetic parameters have been reported to bear a certain relation to flare occurrence, the limitations of using the state of photospheric magnetic fields to distinguish between flare-active and flare-quiet regions and to forecast flare occurrences have been addressed (Leka & Barnes 2007).

Compared with those photospheric magnetic parameters, free magnetic energy E_{free} derived from three-dimensional coronal magnetic configuration over an active region seems to be a more intrinsic physical parameter related to the flare/CME productivity of an active region. Efree quantifies the energy deviation of the coronal magnetic field from its potential state. Since the coronal magnetic field is commonly referred to as nonlinear force-free (NLFF) field, E_{free} can be estimated by

$$E_{\rm free} = E_N - E_p = \int \frac{B_N^2}{8\pi} dV - \int \frac{B_p^2}{8\pi} dV,$$
 (1)

where V is the volume of the computational domain from photosphere to corona, and the superscripts N and p represent the NLFF field and the potential field, respectively. E_{free} calculated in this way is regarded as the upper limit of the energy that is available to power the flares/CMEs. Knowledge of the amount of E_{free} and its temporal variation associated with flares/CMEs is important to our understanding of energy storage and release processes in active regions. Progress in this research area has been made recently by, e.g., Bleybel et al. (2002), Régnier et al. (2002), Régnier & Canfield (2006), Guo et al. (2008), Thalmann

& Wiegelmann (2008), Thalmann et al. (2008), and Jing et al. (2009).

On the other hand, solar flares are classified as X, M, C, or B according to their peak soft X-ray (SXR) flux, as measured by the *Geostationary Operational Environmental Satellite (GOES)* and recorded in the NOAA Space Environment Center's solar event reports. The peak SXR flux of X-, M-, C-, and B-class flares is of 10^{-4} , 10^{-5} , 10^{-6} , and 10^{-7} W m⁻² magnitude order, respectively. As proposed by Abramenko (2005), the flare productivity of an active region can be measured by the SXR flare index (FI, hereafter) which counts the X-, M-, C-, and B-class flares by different weights. The weight of each class is 10 times stronger than the succeeding one, with X-class flares having a weight of 100 in units of 10^{-6} W m⁻², i.e.,

$$FI = \left(100 \times \sum_{\tau} I_X + 10 \times \sum_{\tau} I_M + 1 \times \sum_{\tau} I_C + 0.1 \times \sum_{\tau} I_B\right) / \tau, \qquad (2)$$

where τ is the length of the time window, usually measured in days, and I_X , I_M , I_C , and I_B are *GOES* peak SXR flux of X-, M-, C-, and B-class flares produced by the given active region within the time window τ . In other words, the FI measures an active region's daily average flare production within the time window. In this study, we use three different time windows ranging from the time of the analyzed magnetogram to 1, 2, and 3 subsequent days after that time, i.e., FI_{n-day}, where n = 1, 2, 3.

The goal of this paper is twofold. First, we examine the statistical correlation between free magnetic energy E_{free} derived from three-dimensional NLFF fields and FI measured within the 1, 2, and 3 time windows $FI_{n-\text{day}}$. To our knowledge, this correlation has not been explored before. Second, we study the temporal variation of E_{free} for both flare-active and flare-quiet regions. With the advances in the high-resolution vector-magnetographic capabilities of the *Hinode* spacecraft (Kosugi et al. 2007) and the computational capabilities of NLFF field extrapolation, we are presently in a good position to explore these issues. We anticipate that this study will help us disclose the energy storage and release mechanism of flares, and perhaps even provide a tool to forecast flares.

2. DATA PROCESSING AND NLFF FIELD EXTRAPOLATION

This study requires extrapolating the three-dimensional NLFF coronal fields from the photospheric boundary. The photospheric vector magnetograms, obtained by the Spectro-Polarimeter (SP) of the Solar Optical Telescope (SOT; Tsuneta et al. 2008) on board *Hinode*, are used as the boundary conditions. The SOT-SP obtains Stokes profiles of two magnetically sensitive Fe lines at 630.15 and 630.25 nm with a sampling of 21.6 mÅ. The polarization spectra are inverted to the photospheric vector magnetograms using an Unno–Rachkovsky inversion based on the assumption of the Milne–Eddington atmosphere (e.g., Lites & Skumanich 1990; Klimchuk et al. 1992). We use this data set solely, because it has the best polarization accuracy and seeing free condition to provide needed consistency in this statistical study.

The 180° azimuthal ambiguity in the transverse magnetograms is resolved using the "minimum energy" method that is the top-performing automated method among state-of-art algorithms in this area (Metcalf et al. 2006). This method uses the simulated annealing algorithm to minimize a function $|J_z| + |\nabla \cdot \mathbf{B}|$, where the former is the vertical electric current density and the latter is the field divergence (Metcalf 1994). The projection effects are removed by transforming the observed vector magnetograms to heliographic coordinates, i.e., the heliographic-Cartesian *x*-, *y*-, and *z*-components.

The NLFF field extrapolation endeavors have been plagued by the problem that the photospheric magnetic field, which has a plasma- β of the unity order, does not satisfy the force-free condition (Gary 2001). To find suitable boundary conditions for the NLFF field modeling, we have to preprocess the measured photospheric magnetograms by using a preprocessing scheme developed by Wiegelmann et al. (2006). This preprocessing scheme removes forces and torques from the boundary and approximates the photospheric magnetic field to the low plasma- β force-free chromosphere. In an effort to test the performance of the preprocessing procedure, we compare the unpreprocessed and preprocessed SOT-SP photospheric line of sight (LOS) magnetogram B_z of AR NOAA 10930 with the co-aligned LOS chromospheric magnetogram when the active region is close to the disk center. The chromospheric magnetogram was obtained by the Vector SpectroMagnetograph (VSM) instrument located on the Synoptic Optical Long-term Investigations of the Sun (SOLIS; Keller & NSO Staff 1998). SOLIS/VSM produces spectroheligram of HeI line at 1083.0 nm, photospheric LOS and vector magnetograms of Fe line at 630.2 nm, and chromospheric LOS magnetograms of Ca II line at 854.2 nm line. The latter are used here for comparison. The comparison is made on a pixel-by-pixel basis and the scatter plots are shown in Figure 1. As seen from this figure, the linear correlation coefficient (CC) increases from 0.78 to 0.93 in the unpreprocessed to preprocessed case, indicating that the preprocessed field is closer to the chromospheric field, and hence closer to the forcefree condition. Additionally, as confirmed by some model tests (Metcalf et al. 2008; Wiegelmann et al. 2008), the ability of the NLFF field extrapolation algorithms to reconstruct the coronal field morphology is substantially improved by using the preprocessed photospheric boundary.

Finally, the NLFF field and the potential field were extrapolated from the disambiguated and preprocessed magnetograms using the weighted optimization method (Wiegelmann 2004) and Green's function method (Aly 1989), respectively. The weighted optimization method is an implementation of the original work of Wheatland et al. (2000). It involves minimizing a joint measure (L) for the normalized Lorentz force and the divergence of the field throughout the computational domain V:

$$L = \frac{1}{V} \int_{V} [\omega_f(\mathbf{r}) B^{-2} | (\nabla \times \mathbf{B}) \times \mathbf{B} |^2 + \omega_d(\mathbf{r}) | \nabla \cdot \mathbf{B} |^2] dV, \quad (3)$$

where $B = |\mathbf{B}|$, ω_f and ω_d are weighting functions for the force and divergence terms, respectively. Both ω_f and ω_d are position dependent. They are chosen to be 1.0 in the center of the computational domain and drop to 0 monotonically in a buffer boundary region that consists of 16 grid points toward the side and top boundaries. More detailed descriptions of the method were given by Wiegelmann (2004) and Schrijver et al. (2006). Subsequently, we derived free magnetic energy E_{free} using the integration Equation (1) over the three-dimensional volume.

 Table 1

 Information of Active Regions

Index	NOAA	Number Of Frames	Maximum Flare Magnitude During Lifetime	Dimensions of V ^a (Mm ³)	$\frac{\mathrm{FI}_{\mathrm{overall}}^{\mathrm{b}}}{(10^{-6}~\mathrm{W}~\mathrm{m}^{-2})}$
1	10930	25	X3.4	$210 \times 115 \times 87$	112.7
2	10960	15	M1.0	$210 \times 115 \times 87$	18.1
3	10921	1	C3.7	$104 \times 104 \times 87$	0.4
4	10923	1	C3.3	$73 \times 87 \times 87$	1.1
5	10933	1	C1.7	$136 \times 115 \times 87$	0.6
6	10938	1	C4.2	$136 \times 115 \times 87$	0.9
7	10940	1	C3.4	$115 \times 115 \times 87$	1.0
8	10953	1	C8.5	$104 \times 104 \times 87$	1.1
9	10956	1	C2.9	$112 \times 112 \times 87$	0.7
10	10963	25	C8.2	$210 \times 112 \times 87$	3.2
11	10978	1	C4.5	$108 \times 108 \times 87$	1.6
12	10961	1	None	$115 \times 115 \times 87$	0.0
13	11005	1	None	$108\times108\times87$	0.0

Notes.

^a *V*: the volume of computational domain, see Equation (1).

^b FI_{overall} is calculated with Equation (2), the time window τ is the lifetime of the active regions.



Figure 1. Top: SOLIS chromospheric LOS magnetic field B_z vs. unpreprocessed *Hinode/SP* photospheric B_z ; bottom: SOLIS chromospheric B_z vs. preprocessed *Hinode/SP* photospheric B_z . The SOLIS chromospheric magnetogram was taken on 2006 December 11 at 18:15 UT in AR 10930, and the *Hinode/SP* photospheric magnetogram was taken at 17:00 UT on the same day and in the same active region. The solid line in each panel is the least-square best fit to the data points in a form of y = ax + b, where a and b are constants. The linear CCs between the quantities are shown in each panel.

3. DESCRIPTION OF ACTIVE REGIONS

Hinode was launched in 2006 September and the SOT-SP on board *Hinode* captured its first light on 2006 October. A total of 97 active regions were identified by NOAA from 2006 October 1 to 2008 December 31, i.e., NOAA 10913-11009. Of these active regions, during their lifetime, a considerable part (73 of 93) did not produce any flare activity above B-class (referred to as "flare-quiet"), 21 produced moderate flares (C-class), and three produced major flares (X- and/or M-class).

To check the statistical correlation between E_{free} and $\text{FI}_{n-\text{day}}$, it is important that the sample is comprised major flaring, moderate flaring, and flare-quiet regions. Therefore, we give priority to NOAA 10930 and 10960, as they are two of the very few active regions which produced major flares. Moreover, NOAA 10930 and 10960 are well covered by the SOT-SP observation over a period of several days. The data taken at both flare-active and flare-quiet phases not only expand the sample size but also diversify the values of FI_{n-dav} . Additionally, another 11 active regions are included in the sample to supplement the low end of FI_{n-day} . The data selection is mainly based on the availability of the SOT-SP data. A final tally of 75 vector magnetograms from 13 active regions are analyzed in this paper. All the vector magnetograms analyzed in this study are within $\pm 45^{\circ}$ in longitude and $\pm 30^{\circ}$ in latitude. The distribution of FI_{3-day} shows that 33 of 75 (44%) cases lie between 0 and 1 (i.e., flare quiet during the subsequent 3 day time window), 27 of 75 (36%) cases lie between 1 and 10 (equivalent to a daily average of a C-class flare during the subsequent 3 day time window), and the rest 15 (20%) cases are larger than 10 (equivalent to a daily average of a M/X-class flare during the subsequent 3 day time window). Table 1 lists the information on the 13 active regions.

As shown in the fifth column of the Table 1, the heights of the computational domain are chosen to be 87 Mm above the photosphere for all the cases, but the FOVs on the lower boundary vary from case to case according with the SOT-SP scan area. Since E_{free} is a volume-integrated parameter, the difference in the lower boundary FOVs may introduce an uncertainty in the statistical correlation and must be treated with caution.

Figure 2 presents an example in which we test how E_{free} of an active region changes with the varying FOV and height of the



Figure 2. Top: snapshot of LOS magnetograms of NOAA 10930. The whole FOV is 210 Mm × 115 Mm. The colored boxes mark the nine different FOVs. Bottom left: NLFF field energy E_N , potential field energy E_p , and free magnetic energy E_{free} as a function of the nine selected FOVs, given a fixed height of the upper boundary (87 Mm). The data dot with a certain color corresponds respectively to the FOV box with the same color. Bottom right: E_N , E_p , and E_{free} as a function of height of the upper boundary, given a fixed FOV of 210 Mm × 115 Mm.

computational domain V. The top panel in Figure 2 shows a LOS magnetogram of the active region NOAA 10930. The colored boxes mark the nine different FOVs, ranging from $22 \times 9 \text{ Mm}^2$ to 210×115 Mm². The smallest box only covers a small area around the flaring PIL, while the largest box covers not only the sunspots that comprise the major portion of this active region but also the weaker plage regions surrounding the sunspots. The bottom left panel shows the NLFF field energy E_N , the potential field energy E_p , and the free magnetic energy E_{free} as a function of the nine selected FOVs, given a fixed height (87 Mm) of the computational domain V. As shown in Equation (1), E_{free} is defined as the excess E_N from E_p . Each data point with a certain color corresponds respectively to the FOV box with the same color. We see that E_{free} first increases rapidly with an expanding FOV, then reaches its maximum when the major portion of this region is covered by the FOV, then changes little despite the continued growth of FOV. The bottom right panel shows E_N , E_p , and E_{free} as a function of height of the upper boundary of V, given a fixed FOV $(210 \times 115 \text{ Mm}^2)$ of the lower boundary. Evidently, E_{free} displays an impulsive increase from the lower boundary to a certain height (\sim 25 Mm in this case), then stays almost constant beyond this height. We run the E_{free} -FOV and $E_{\rm free}$ -height tests on other active regions and get similar results. It suggests that the difference in FOVs of our samples is not likely to significantly affect the statistical correlations that will be shown in Section 4, as long as the sampled active regions are well covered by the magnetograms. It also suggests that the magnetic fields approach potential beyond a certain height which is of a few tens of Mm magnitude order, consistent with our previous results (Jing et al. 2008). A height of 87 Mm of V adopted in this study seems to be enough to constrain the nonpotential magnetic fields.

To study the temporal variation of E_{free} , we select three active regions, NOAA 10930, 10960, and 10963. As mentioned, the former two exhibited one major flare and several moderate flares during the SOT-SP observations over a period of days. The latter produced a few moderate flares at some point in its lifetime, but did not flare during the observation period. For each region, a sequence of the vector magnetograms at a general cadence of a few hours is used as the boundary conditions to extrapolate the three-dimensional NLFF field. These magnetograms are deprojected, co-aligned, and rescaled so that they have the same target location, the same FOV, and the same dimensions. In Figure 3, the snapshots of the vector magnetogram of the three regions and the corresponding NLFF fields are shown in the left and right columns, respectively.

4. RESULTS

In Figure 4, the left-to-right top panels show the scatter plots of FI_{3-day} versus E_{free} , FI_{2-day} versus E_{free} , and FI_{1-day} versus E_{free} , respectively. The solid line superposed in each plot indicates the least-square best fit to the data points. The cross CCs between the quantities are also given in the panels. Note that FI_{n-day} is plotted in a logarithmic scale. The FI_{n-day} with 0 value is set to 0.01 to avoid arithmetic error and is shown as gray points. These gray points are excluded from the fitting and CC calculation. While the points are widely scattered, the result still reveals a positive correlation between the quantities (0.55 < CCs < 0.76), suggesting that major flares generally come from active regions with high-energy content.

Furthermore, to test the ability of E_{free} to distinguish between flare-quiet and flare-active populations, we divide the data points into four groups (denoted by 1-4, seen in the middle panel of Figure 4) with the horizontal and vertical dashed lines in each panel. The horizontal dashed line shows the observed $FI_{n-day} = 1$, which is equivalent to a daily average of a C1.0 flare within the time window and is taken as a threshold for flare-active regions. The vertical dashed line is drawn according with the derived maximum E_{free} of the flare-quiet regions. We wish to test the hypothesis that the population on the right side of the vertical line are flare active. In this sense, group 1 refers to incorrectly rejected flare-active cases (Type I error) and group 2 refers to failing to reject the flare-quiet cases (Type II error). Groups 3 and 4 (the shaded areas) mean that flare-quiet and flare-active populations are well separated by the vertical line. The frequencies of the groups 1-4 are also given in Figure 4. Inspection of Figure 4 immediately reveals that most of data points fall into groups 3 and 4. The Type I and Type II error rates are $\sim 8\%$ -12% in total. This means that, based on the sample presented here, flare-quiet and flare-active populations can be separated by E_{free} with a ~88%–92% success rate.

For comparison, the middle and bottom panels of Figure 4 show similar diagrams in which the photospheric excess energy E_{pe} and the total unsigned photospheric magnetic flux Φ are plotted against the FI_{n-day}. The definitions of E_{pe} and Φ are given in Section 1. The former is regarded as a proxy for E_{free} , while the latter is a simple measure of an active region's size (Barnes & Leka 2008). The positive correlations are still evident in each panel. We also note the following properties.

First, when two populations (flare quiet and flare active) are concerned, Φ and E_{free} perform better than E_{pe} in separating two populations. Taking the 3 day time window, for example, the success rates of Φ , E_{free} , and E_{pe} are 95%, 90%, and 77%, respectively.



Figure 3. Left panels: snapshots of the *Hinode*/SP vector magnetograms. From top to bottom, they are NOAA 10930 taken on 2006 December 11 at 11:10 UT, NOAA 10960 taken on 2007 June 6 at 12:30 UT, and NOAA 10963 taken on 2007 July 13 at 19:07 UT. The background images are the LOS magnetograms. Green arrows indicate the transverse fields. The FOVs of three magnetograms are 210 Mm \times 115 Mm, 210 Mm \times 115 Mm, and 210 Mm \times 112 Mm, respectively. Right panels: extrapolated NLFF fields of NOAA 10930, 10960, and 10963. The boundary images are the vector magnetograms shown in the left panels.

Second, when only considering the flare-active population, all three parameters are moderately to strongly correlated with FI_{n-day} . In addition, despite the fact that E_{free} is one of the most direct measures for the available energy in a three-dimensional magnetic field, its correlations with FI_{n-day} are found to be quite similar to the correlations of the photospheric magnetic parameters, E_{pe} , and Φ . In particular, E_{pe} performs best in relating to FI_{3-day} with a CC as high as 0.82 and performs worst in relating to FI_{1-day} with a CC of 0.45. Basically, the difference among these three parameters is not significant. The similar magnitudes of the CCs imply that E_{free} , E_{pe} , and Φ have approximately equal predictability for flares.

Finally, as a general trend, the magnitude of the CCs decreases as the time window of FI becomes narrower from 3 days to 1 day. It reveals that the magnetic parameters have relatively less predictability for flares within a 1 day time window than a 3 day time window. It is understandable, because flares as a result of electromagnetic instabilities may occur only under certain circumstances and after a substantial waiting time (Schrijver et al. 2005). Forecasting imminent flares certainly faces more uncertainties than forecasting long-term flares.

Figure 5 illustrates how three magnetic parameters, $E_{\rm free}$, $E_{\rm pe}$, and Φ , correlate with each other. The colored data points from dark to light refer to increasing levels of FI_{3-day} from flare quiet to major flaring. Apparently, the major flaring samples generally exhibit higher values of $E_{\rm free}$, $E_{\rm pe}$, and Φ than the flare-quiet ones. It is interesting to note that, the data points of $E_{\rm free}-\Phi$ in

the top panel can be fitted by two lines with different slopes. The lower line mainly contains the data from flare-quiet samples, while the higher line is comprised data from two major flaring active regions (NOAA 10930 and 10960) that were observed on a timescale of several days. E_{pe} does not exhibit such a pattern with Φ . The result suggests that Φ is fairly constant for major flaring regions, but E_{free} may change significantly over days. It is not very surprising, because E_{free} , as a difference between the NLFF field and the potential field, should be expected to be intrinsically more variable than Φ . In addition, the correlation between E_{free} and Φ is weak (CC=0.3) for flare-quiet samples, but very strong (CC=0.86) for flare-active ones.

Theoretically speaking, the statistical relation between E_{free} and $\text{FI}_{n-\text{day}}$ and the temporal variation of E_{free} derived from individual active regions can provide clues to distinguish between flare-active and flare-quiet regions. We compare the long-term variation of E_{free} for three active regions, NOAA 10930, 10960, and 10963. Figure 6 shows the time profiles of E_{free} (gray histogram), E_{pe} (green diamonds), Φ (blue pluses), and the *GOES* SXR 1–8 Å light curves (red curves). The flares that originated from the active regions are indicated by the black arrows. As shown in Figure 6, NOAA 10930 dominated the solar activity during the observation from 2006 December 9–14 and produced a X3.4 flare on 2006 December 13. E_{free} in NOAA 10930 is remarkably built up during the 2 day prior to the X3.4 flare and even continues to increase after the flare. NOAA 10960 is also flare active. It produced a M1.0 flare on 2007 June 9, at the very





Figure 4. Top panels: scatter plots of FI_{n-day} vs. E_{free} ; middle panels: scatter plots of FI_{n-day} vs. E_{pe} ; bottom panels: scatter plots of FI_{n-day} vs. Φ , where n = 3, 2, 1 from left to right. The FI_{n-day} swith 0 value are set to 0.01 to avoid arithmetic error and shown as gray points. The solid lines indicate the least-square best fits to the data points, and CCs are CCs, with gray points excluded. In each panel, the data points are divided into four groups, denoted by (1)–(4) in the middle panel, with the horizontal and vertical dashed lines. The horizontal dashed line shows $FI_{n-day} = 1$, while the vertical dashed line is drawn according with the maximum E_{free} of the flare-quiet regions. The percentages refer to the frequencies of each group.

end of our observation and four C-class flares during the observation. $E_{\rm free}$ in NOAA 10960 fluctuates and does not have any obvious association with the flare occurrence. NOAA 10963 did not flare during the observation from 2007 July 12 to 16 and contains a clearly lower amount of $E_{\rm free}$ in comparison with the other two regions. Since the long-term trend of $E_{\rm free}$ varies from case to case, we find no particular pre-flare signatures useful in predicting flares.

5. QUALITY CONTROL AND UNCERTAINTY ANALYSIS

The quality of the extrapolated NLFF fields is influenced by a number of inadequacies of the boundary data and uncertainties in the data reduction and NLFF field modeling process. These inadequacies and uncertainties are caused by, for instance, the non-force-free nature of the photospheric magnetic field, the instrumental effects, the FOV issue and the noise level of vector magnetograms, and the physical/mathematical problems relating to the NLFF field extrapolation techniques. In Section 2, we show that the preprocessing method employed in this study improves the non-force-free photospheric magnetogram toward the nearly force-free chromospheric magnetogram. In Section 3, we show how the FOV and the height selections of the computational domain affect the estimation of E_{free} . Here, we would like to further examine the quality of and the uncertainties in the NLFF fields.

We first visually compare the extrapolated NLFF fields with the observed coronal images. An example, NOAA 10960, is shown in Figure 7. The coronal images were taken by the *Transition Region and Coronal Explorer (TRACE)* at 171 Å (top



Figure 5. Top panels: Scatter plots of E_{free} vs. Φ ; Middle panels: Scatter plots of E_{pe} vs. Φ ; Bottom panels: Scatter plots of E_{free} vs. E_{pe} . The solid lines are the least-square best fits to the data points, and CCs are CCs. The different colors of the data points refer to the different levels of the flare activity within the subsequent 3 days.

panel) and by the X-Ray Telescope (XRT) on board *Hinode* (bottom panel) on 2007 June 7 when the active region is near the disk center. The overlaid field lines are extrapolated from the *Hinode*/SOT-SP vector magnetogram that was taken at almost the same time. We see a reasonable correspondence between projections of the three-dimensional NLFF magnetic field lines and the coronal loops in some regions (e.g., the central closed loops with a footpoint near [-225, -120]). The footpoints of these structures are well apart from the lateral boundary of the vector magnetogram. Unfortunately, the NLFF field lines deviate from the observed coronal loops when one footpoint of the field line is close to the lateral boundary of the FOV, which is the case for many open field lines (e.g., in the top right part of the active region). Such deviations are commonly seen in other active regions and have been reported in other

studies (Wiegelmann et al. 2005; Schrijver et al. 2008; DeRosa et al. 2009). It suggests that the NLFF field model can only reproduce structures well apart from the lateral boundaries of the vector magnetogram and is not able to accurately reproduce the coronal field and the magnetic energy contained in the entire active region. On the other hand, it is still unclear if coronal loops are an accurate reflection on the complete coronal structure, as only loops with certain temperature and emission measures are visible.

Second, we check how well the force-free and divergencefree conditions are satisfied over the computational domain by computing two metrics, the current-weighted sine metric, and $\langle | f_i | \rangle$ metric (Wheatland et al. 2000), for each extrapolated field. Two metrics are defined as

$$CWsin = \frac{\sum_{i} |\mathbf{J}_{i}|\sigma_{i}}{\sum_{i} |\mathbf{J}_{i}|},$$
(4)

where

$$\sigma_i = \frac{|\mathbf{J}_i \times \mathbf{B}_i|}{|\mathbf{J}_i||\mathbf{B}_i|} = |\sin\theta_i|, \tag{5}$$

and

$$f_i| = \frac{|(\nabla \cdot \mathbf{B})_i|}{6|\mathbf{B}|_i / \Delta x},\tag{6}$$

where Δx is the grid spacing. Clearly, CWsin and $\langle |f_i| \rangle$ respectively reach a optimal value of zero for a perfect forcefree and divergence-free field, with a smaller value indicating a more force-free and divergence-free state. Thereby, they have been used in many papers to evaluate the performance of NLFF fields (e.g., Metcalf et al. 2008; Schrijver et al. 2008; DeRosa et al. 2009). Figure 8 shows the distribution of CWsin and $\langle |f_i| \rangle$ for our 75 samples. We note that the final state of the NLFF fields does contain residual forces and divergences.

Third, we estimate the uncertainties in E_{free} caused by the image noise by employing a Monte Carlo method that was described by Guo et al. (2008), i.e., we add artificial noise to the vector magnetograms. The noise is in a normal distribution with the standard deviation of 5 G for B_z and 50 G for B_x and B_y , which is consistent with the sensitivity of *Hinode*/SOT-SP measurements. Then, we redo the 180° ambiguity resolution, preprocessing, and NLFF field extrapolation as described in Section 2. We repeat the same process 30 times, and find that the addition of noise leads to a standard deviation of E_{free} by ~6%.

Additionally, it is worth mentioning that Wiegelmann et al. (2010) systematically test how strongly the instrumental effects (e.g., limited spatial resolution and spectral sampling) could influence the quality of NLFF fields. Since they used the same data set (*Hinode*/SOT-SP) and the data reduction methods as we do in this study, their result can be seen as a good object lesson for our reference. According to Wiegelmann et al. (2008, 2010), the combination of preprocessing and NLFF field extrapolation allows an estimation of free magnetic energy (in the low- β corona) from forced photospheric data with an error of a few percent.

Although the limitations of the NLFF field models in fully recovering the real coronal fields may undermine the value of this work, it is worthwhile experimenting with the correlation between free energy and flare productivity. Nevertheless, researchers should be aware of the lack of operational readiness of the NLFF field models and not over-interpret the results.



Figure 6. Temporal variation of free magnetic energy E_{free} (gray histograms), photospheric excess energy E_{pe} (green diamonds), the photospheric unsigned magnetic flux Φ (blue pluses), and the *GOES* SXR 1–8 Å light curves (red curves) of NOAA 10930, 10960, and 10963. The flares that originated from the active regions are indicated by the black arrows.

6. SUMMARY AND DISCUSSION

In this paper, we examine the magnitude scaling correlation between FI_{n-day} and E_{free} based on the 75 samples, and also study the temporal variation of E_{free} with respect to the *GOES* SXR light curves for three active regions. The most important results are summarized as follows.

- 1. E_{free} is moderately to strongly correlated with FI_{*n*-day}. The correlation confirms the physical link between magnetic energy and flare productivity of active regions. However, compared with two photospheric magnetic parameters E_{pe} and Φ , E_{free} shows little improvement for flare predictability.
- 2. While the magnitude of E_{free} unambiguously differentiates between the flare-active and the flare-quiet regions, the

temporal variation of E_{free} does not exhibit a clear and consistent pre-flare pattern.

One likely cause of the lack of satisfactory results is that the storage of E_{free} in magnetic fields is a necessary but not sufficient condition for the onset of solar flares. The trigger mechanism of flares is the determining factor of whether and when an active region will flare. Recently, there has been growing evidence relating emerging flux regions (EFRs) and magnetic helicity to the flare trigger mechanism. Magnetic helicity is a measure of magnetic topological complexity such as twists, kinks, and linkages of magnetic field lines (Berger & Field 1984). As suggested by a numerical simulation and supported by many observations, flares preferentially occur in the presence of a



Figure 7. Top: *TRACE* 171 Å image of NOAA 10960, with over-plotted NLFF field lines. Bottom: *Hinode*/XRT SXR image, with over-plotted NLFF field lines. The NLFF field model is extrapolated from the *Hinode*/SOT-SP vector magnetogram that was taken on 2007 June 7 at 03:16 UT. *TRACE* 171 Å image was taken on the same day at 03:10 UT. *Hinode*/XRT SXR image was taken on the same day at 03:10 UT. *Hinode*/XRT SXR image was taken on the same day at 03:10 UT.



Figure 8. Histograms of CWsin (top) and $\langle |f_i| \rangle$ (bottom) metrics for the 75 samples.

particular magnetic topology, which is prone to the annihilation of magnetic helicities with opposite signs between the newly EFR and the preexisting region (Kusano et al. 2003a, 2003b; Yokoyama et al. 2003; Wang et al. 2004; Jing et al. 2004). In addition, the monotonically increasing helicity over days prior to major flares has been found, which can be used as a warning sign of the flare onset (LaBonte et al. 2007; Park et al. 2008). We expect that a combination of E_{free} and magnetic helicity in future studies would carry extra weight in predicting flares.

Moreover, the energy release process involves a variety of dynamic phenomena such as flare heating, particle acceleration, and CME dynamics (Gibson et al. 2009). Consequently, the released E_{free} is converted and partitioned into the forms of thermal and non-thermal emissions and kinetic energy of CMEs. Considering that the SXR FI only quantifies a fraction of the released energy, i.e., the thermal part, we probably should not expect a very strong correlation between E_{free} and $FI_{n-\text{dav}}$.

Besides the concerns on flare trigger and energy release mechanisms, the NLFF field modeling from the photospheric boundary is subject to both observational limitations and intrinsic physical and/or mathematical problems. The observational limitations include the large uncertainties in transverse field measurements (Klimchuk & Canfield 1994), and 180° azimuthal ambiguity (Metcalf et al. 2006), etc. The physical/mathematical problems are related to the non-force-free nature of the photospheric boundary (Metcalf et al. 2008) and the difficulties of guaranteeing the existence and uniqueness of the NLFF field solutions (Rudenko & Myshyakov 2009). Although the use of high-quality magnetogram data and the advanced preprocessing and modeling algorithm improves the situation to some extent, the ability of the NLFF field model to reproduce the real coronal field, and E_{free} is seriously compromised (DeRosa et al. 2009). Moreover, the NLFF field is only a good approximation in the force-free domain (i.e., chromosphere and lower corona), not in the high- β photosphere and upper corona. Future improvements on magnetic field modeling may come from developing a self-consistent magnetohydrostatic (MHS) modeling (Wiegelmann & Neukirch 2006) and incorporating information on the coronal field topology as seen, for example, by TRACE and/or the twin Solar Terrestrial Relations Observatory (Wiegelmann et al. 2009).

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