GEOPHYSICAL RESEARCH LETTERS, VOL. 22, NO. 15, PAGES 2057-2060, AUGUST 1, 1995

Formation of thin current sheets in a quasistatic magnetotail model

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Abstract. Observations suggest that thin current sheets forming in the near-Earth magnetotail late in substorm growth phases may be a crucial part of substorm evolution. In a simple theoretical model the current density was shown to become singular for suitable external perturbations. Here, we address the same problem in a more realistic model based on the adiabatic MHD - theory developed by *Schindler and Birn* [1982]. We show that under suitable conditions the formation of a thin current sheet in the near-Earth tail is an intrinsic aspect of flux transfer to the magnetotail. The mechanism is based on the strong variation of flux tube volume with the magnetic flux function.

Introduction

Thin current sheets forming in space and astrophysical plasmas play an important role in solar [*Parker*, 1979, 1994] and geomagnetic [*Voge et. al.*, 1994] activity. Recent magnetospheric observations [*Sergeev et al.*, 1990; *Pulkkinen et al.*, 1991,1992], analytical theory [*Schindler and Birn*, 1993] and computer simulations [*Lee*, 1995; *M. Hesse*, private communication 1995; *J. Birn*, private communication 1995] have provided evidence for a thin current sheet with a thickness of the order of a few hundred kilometers forming late in the substorm growth phase, which might be an important feature of the onset mechanism. In their recent paper *Schindler and Birn* [1993] argued that the formation of thin current sheets might be understood as an inherent part of magnetospheric growth phase dynamics. Their model, however, was a considerably simplified approach using linearized perturbations. It is the aim of this paper to address the same question using a more realistic nonlinear model based on the adiabatic MHD model by *Schindler and Birn* [1982]. Our results confirm the formation of thin current sheets as a consequence of flux transfer to the magnetotail and shed new light on the physical mechanism of current sheet formation.

Numerical calculations by *Erickson* [1992] using a numerical MHD approach, however including a near-Earth dipol region, did not show strong evidence for current sheet formation. A possible explanation is given in the discussion section.

The model

To describe the growth phase evolution as a slow dissipation-free process, we use a model, which was developed by *Schindler and Birn* [1982]. This model has the following structure:

Paper number 95GL01980
0094-8534/95/95GL-01980$03.00

In the limit of small time-dependence, small ratio of characteristic lengths of variation in z- and x-directions, ignoring y-dependence, the equations of ideal MHD with an adiabatic pressure law can be put in the form

$$\frac{1}{2\mu_0}\left(\frac{\partial A}{\partial z}\right)^2 + p(A,t) = \frac{1}{2\mu_0}B_0^2(x,t) \tag{1}$$

$$\frac{d}{dt}\left(p(A,t)V(A,t)^\gamma\right) = 0 \tag{2}$$

Here, x, y, z are solar magnetospheric coordinates except that the positive x-axis points in the tailward direction and the origin is shifted along the x-axis to a location in the night-side magnetosphere, where the tail-like stretching of the magnetic fieldlines begins to play an important role. $A(x,z,t)$ is the magnetic flux function, assumed to be symmetric in z, and $V(A,t)dA$ is the volume of the flux tube $(A, A+dA)$ at time t. Within our approximation $V(A,t) = \int_A \frac{ds}{|\nabla A|}$ is given by

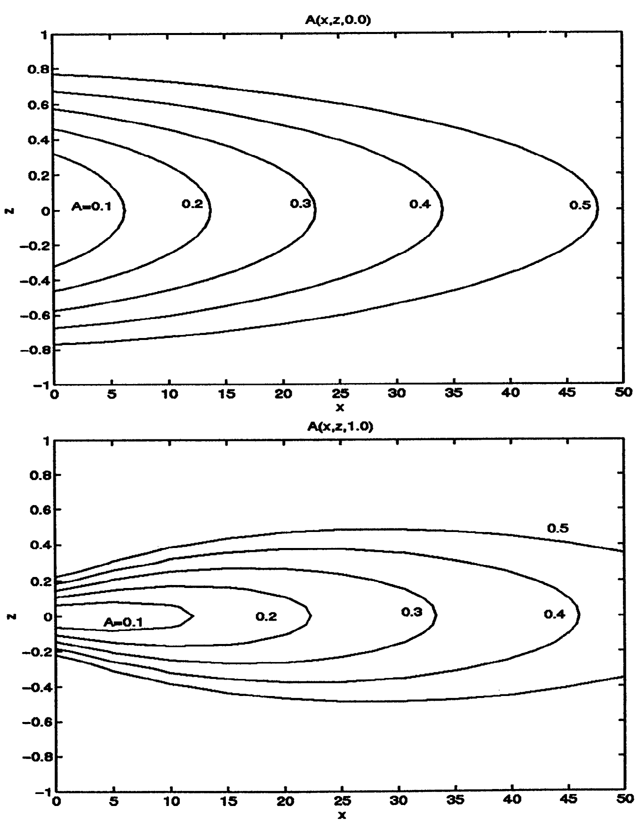

Fig. 1. Magnetic field lines for $t = 0$ and $t = 1$ (level curves of $A(x,z,t)$ with equal spacing of A)

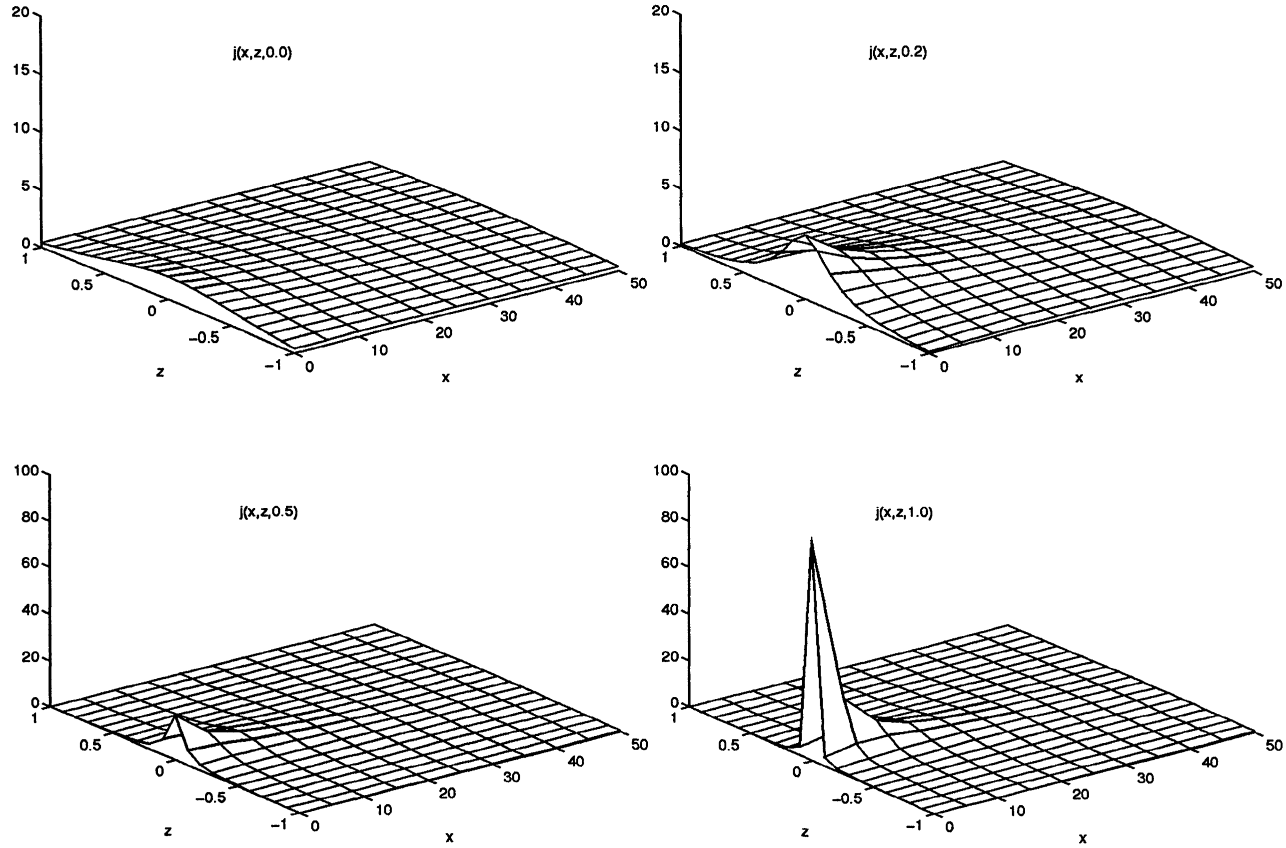

Fig. 2. Current density magnitude $j(x,z,t)$ at times $t = 0,\ 0.2,\ 0.5,\ 1.0$, demonstrating the formation of a thin current sheet in the near-Earth magnetotail. Note the difference in vertical scales in the upper and lower panels.

$$V(A,t) = \int_{p(A,t)}^{p_0(0,t)} \frac{dp_0}{-\frac{\partial p_0(x,t)}{\partial x}\sqrt{p_0 - p(A,t)}} \tag{3}$$

Here $p(A,t)$ is the time dependent pressure function and $p_0(x,t) = \frac{B_0^2(x,t)}{2}$ the pressure on the x-axis. $B_0(x,t)$ is the time-dependent lobe magnetic field as arising from flux transfer from the front side to the magnetotail, and γ is the adiabatic index. In the integrand $\frac{\partial p_0}{\partial x}$ is to be expressed by p_0 and t.

We normalize B_0 by $\hat{B} = B_0(0,0)$, coordinates by the initial plasma sheet width L at $x = 0$, t by a typical growth phase duration, A by $\hat{B}L$ and pressures by $\frac{\hat{B}^2}{\mu_0}$. To solve equations (1) and (2) we need appropriate boundary and intial contitions. We choose for the initial state

$$p(A,0) = \exp(-2A) \tag{4}$$

We model the flux transfer to the tail by prescribing the lobe magnetic field as

$$B_0(x,t) = \sqrt{2}(1+t)\left(1 + \Lambda(1+t)^2 x\right)^{-\frac{n}{2}} \tag{5}$$

which takes the role of a boundary condition. The choice (5) with $\Lambda = 0.036$ and $n = 1.1$ is motivated by observations [*Schindler and Birn*, 1982], (4) corresponds to the standard local Harris sheet profile for B_x. Note that (4) and (5) imply $A(0,0,t) = 0$, using $p(A(x,0,t),t) = p_0(x,t)$ which follows from (1).

After integrating (2) with respect to time and using (3) we get

$$p(A,t)^{\frac{1}{\gamma}} \int_{p(A,t)}^{p_0(0,t)} \frac{dp_0}{-\frac{\partial p_0(x,t)}{\partial x}\sqrt{p_0 - p(A,t)}} = M(A) \tag{6}$$

Defining the inverse function of $p(A,t)$ as $A(p,t)$ we can write M as a function $\hat{M}(p,t)$

$$M(A,t) = \hat{M}(p,t) \tag{7}$$

and get

$$p^{\frac{1}{\gamma}} \int_{p}^{p_0(0,t)} \frac{dp_0}{-\frac{\partial p_0(x,t)}{\partial x}\sqrt{p_0 - p}} = \hat{M}(p,t) \tag{8}$$

With (7) we can calculate $p(A,t)$ with the initial condition (4) and the boundary condition (5), evaluating the integrals numerically. We use the time-dependent pressure function $p(A,t)$ and again the boundary condition (5) to solve (1) and calculate $A(x,z,t)$. Note that in our model a pressure increase beyond the lobe pressure [*Erickson and Wolf*, 1980] is avoided by the (self consistent) time-dependent evolution.

The current density is given by

$$j(A,t) = \frac{\partial p(A,t)}{\partial A} \tag{9}$$

which with the help of (2) (after integration with respect to time), assumes the form

$$j(A,t) = j(A,0)G(A,t)^{\gamma} + p(A,0)\frac{\partial}{\partial A}G(A,t)^{\gamma} \tag{10}$$

with $G(A,t) = \frac{V(A,0)}{V(A,t)} = \left(\frac{p(A,t)}{p(A,0)}\right)^{\frac{1}{\gamma}}$. Note that large values of $|j|$ can develop only from the factor $\frac{\partial G^\gamma}{\partial A}$ in (10). The function G^γ itself is bounded and assumes its maximum at $A = 0$. In particular one finds $G(0,t)^\gamma = \left(\frac{p_0(0,t)}{p_0(0,0)}\right) = (1+t)^2$. If $\frac{\partial G^\gamma}{\partial A}$ becomes large locally in some region of A, (10) would imply that a thin region of large current density magnitude is embedded in a broader region with a smooth current distribution.

Results

We solve equations (1) and (2) with (5) and (4) in the range $0 \leq t \leq 1$, $0 \leq x \leq 50$ and $0 \leq |z| \leq 1$. Note that the lobe field increases by a factor of 2 and smoothly varies with x. Doubling of the lobe field (at $t = 1$) is at the upper end of the realistic domain of magnetic field increase during growth phases. Therefore, we also discuss 20% and 50% lobe field increases (at $t = 0.2$ and $t = 0.5$) as more representative values. There is nothing in the initial or boundary conditions that would directly enforce the formation of a thin current sheet. Nevertheless, as we will show, the system organizes itself such that a thin current sheet forms with maximum current densities increasing with time.

Since the flux function A ist constant on magnetic field lines the field lines can be illustrated by a contour plot of $A(x,z,t)$. The initial magnetic field is determined by (4) and by (5) specified for $t = 0$. Although the initial flux function is available analytically, we evaluated it numerically as we did for $t \neq 0$. The agreement is satisfactory. Figure 1 shows the magnetic field lines of the plasma sheet region for $t = 0$ and $t = 1$. At $t = 1$ the innermost field lines are strongly stretched which indicates large current density. The current densities are shown in figure 2 for four times.

A pronounced peak exists for later times. The maximum current density magnitude at $x = 0$ and $t = 0.2$ is by a factor of ≈ 2.8 higher than the maximum of the initial current density magnitude. The corresponding factors for $t = 0.5$ and $t = 1.0$ are ≈ 9.7 and ≈ 48, respectively.

The mechanism of current sheet formation is evident from (10) together with figure 3, where $G(A,t)$ is displayed for several values of time t. Clearly, a large gradient of G develops without G becoming large itself. The gradient is restricted to a small region in A near $A = 0$. The maximum of the current density magnitude in that region grows unlimited as time procedes. The problem of unaccceptably large $\vec{j} \times \vec{B}$ forces, does not arise because inside the current sheet B becomes very small where $|j|$ is large.

Thus, the current sheet formation can be regarded as a consequence of the strong tail stretching and the associated large gradients in flux tube volume.

We would like to point out that our results show that current sheets do not form under all circumstances. To show this we have also analyzed the following form of $B_0(x,t)$, which is more general than (5)

$$B_0(x,t) = \sqrt{2}(1+t)^{\frac{k_1}{2}} \left(1 + \Lambda(1+t)^{k_2} x\right)^{-\frac{n}{2}} \quad (11)$$

where k_1 and k_2 are positive constants. We find that $k_2 > k_1 \left(\frac{1}{\gamma} - \frac{1}{2}\right)$ is a necessary condition for the formation of current sheets. This condition means that for large values of t the second term in (10) (at $A = 0$) growth faster with time than the first term.

The limiting choice $k_2 = k_1 \left(\frac{1}{\gamma} - \frac{1}{2}\right)$ corresponds to the similarity solutions discussed by *Schindler and Birn* [1982]

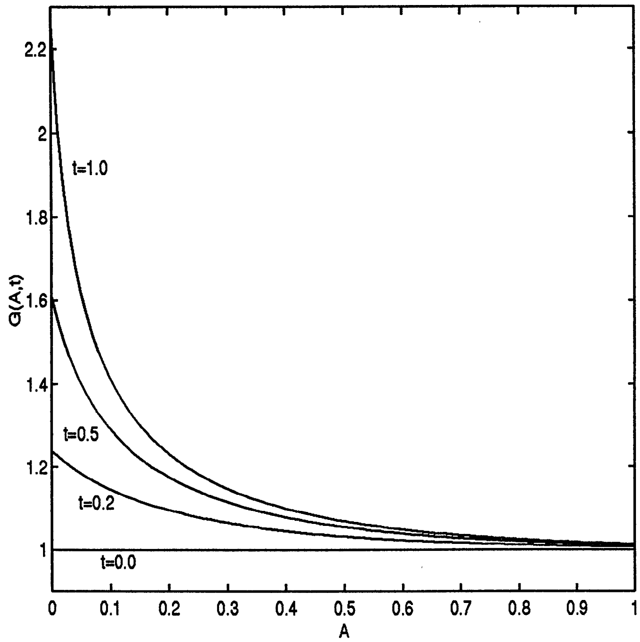

Fig. 3. The development of the function $G(A,t) = \frac{V(A,0)}{V(A,t)}$. The strong gradient forming near $A = 0$ for later times is directly related to the current sheet formation (see Eqn. (10))

with $p(A,t) = p_1(t)p_2(A)$ and $j(A,t) = j_1(t)j_2(A)$, which excludes current sheet formation.

If one uses a function $B_0(x,t)$ that satisfies the above condition only marginally, one would expect that current sheets form which are less pronounced than the ones that we find using (5). This might provide an explanation of the fact that the computation of *Erickson* [1992] did not exhibit evidence for strong current sheets.

A major limitation of our model is the absence of a near-Earth dipolar field region. Although we try to model the effect of rather rigid dipolar field lines by our boundary condition at $x = 0$, it is quite possible that a more realistic model would arrive at results that quantitatively differ from ours. Nevertheless, we expect that the formation of thin current sheets as a qualitative feature is not affected by our boundary conditions.

In summary, our results provide further support to the view that under suitable conditions the formation of thin current sheets may be considered as an inherent part of the evolution of the magnetotail during substorm growth phases. The occurrence of current sheets and their structure depend on the detailed shape of the lobe field $B_0(x,t)$.

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(Received: March 15, 1995 Revised: May 21, 1995
Accepted: June 6, 1995)