THREE-DIMENSIONAL NUMERICAL SIMULATION OF WAVE PROPAGATION THROUGH A MODEL SUNSPOT

Robert Cameron and Laurent Gizon

Max-Planck-Institut für Sonnensystemforschung, 37191 Katlenburg-Lindau, Germany

ABSTRACT

The interaction of waves with sunspots is being studied using numerical simulations. The code we have developed follows the linearized evolution of wave perturbations through an inhomogeneous solar atmosphere, including magnetic fields. The simulations are fully three-dimensional. As a first application of this code, we propagate surface-gravity waves through the model sunspot of Schüssler and Rempel. Several possible applications in the context of local helioseismology are discussed.

Key words: Solar oscillations; Sunspots; Sun: Magnetic field, Numerical simulations; Local helioseismology.

1. WAVE PROPAGATION

The solar atmosphere is, on the length-scales of interest to local helioseismology, far from horizontally uniform. Sunspots and granulation are two examples of large amplitude inhomogeneities. Theory and simulations have both been used to study how these inhomogeneities affect wave propagation (see e.g. [1]). We have developed a numerical code which treats the 3-D linearized initial value problem for a wide range of background states. The idea is to consider the evolution of small perturbations (here an f-mode wave packet) superimposed on a background state prescribed by its density, $\rho_0(\boldsymbol{r})$, pressure, $p_0(\boldsymbol{r})$, magnetic field $\boldsymbol{B}_0(\boldsymbol{r})$, and the first adiabatic exponent, $\Gamma_1(\boldsymbol{r})$. The background current, $\boldsymbol{J}_0$ and all other values, such as the sound speed $c_0 = (\Gamma_1 p_0/\rho_0)^{1/2}$, can be derived from the above quantities. Currently we assume that there is no background flow ($\boldsymbol{v}_0 = 0$).

The background, which will generally be very non-uniform, is assumed to be in equilibrium and the atmosphere is then perturbed by introducing a disturbance. The disturbance is assumed to be small and is treated in the linear regime. The perturbation is described by it displacement vector, $\boldsymbol{\xi}$, and velocity, $\partial\boldsymbol{\xi}/\partial t$.

For simplicity we currently assume that the waves are adiabatic. In addition, wave propagation is assumed to be ideal, i.e. radiative and diffusive processes are ignored.

The equation governing the displacement $\boldsymbol{\xi}$ is

$$\begin{aligned}\rho_0 \frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} &= -\boldsymbol{\nabla} p' + \frac{\rho'}{\rho_0}\boldsymbol{\nabla} p_0 + \boldsymbol{g}\rho' + \boldsymbol{J}' \times \boldsymbol{B}_0 \\ &\quad + \boldsymbol{J}_0 \times \boldsymbol{B}', \end{aligned} \tag{1}$$

where

$$\begin{aligned} \rho' &= -\boldsymbol{\nabla}\cdot(\rho_0 \boldsymbol{\xi}), && (2)\\ p' &= c_0^2(\rho' + \boldsymbol{\xi}\cdot\boldsymbol{\nabla}\rho_0) - \boldsymbol{\xi}\cdot\boldsymbol{\nabla} p_0, && (3)\\ \boldsymbol{B}' &= \boldsymbol{\nabla}\times(\boldsymbol{\xi}\times\boldsymbol{B}_0), && (4)\\ \boldsymbol{J}' &= \boldsymbol{\nabla}\times\boldsymbol{B}'. && (5)\end{aligned}$$

The primed quantities here are the Eulerian perturbations to the background state, and the gravitational acceleration is $\boldsymbol{g} = -g\hat{\boldsymbol{z}}$.

A complication necessarily arises because the background stratification is unstable (it is after all the convection zone) and perturbations grow exponentially. As we are not interested in these modes we have adjusted the stratification to make the background neutrally stable. Whilst this is not desirable it appears to be unavoidable and should not greatly affect the propagating modes we are interested in.

We only intend to simulate small regions of the solar surface and so have used a box-geometry with Cartesian coordinates $\boldsymbol{r} = (x, y, z)$. A spectral treatment in the horizontal directions and a finite difference scheme for the vertical direction are used. The boundary conditions are naturally important. The side boundaries are simple: the box is periodic. For the upper boundary we currently use the condition that the Lagrangian perturbation of the vertical component of the stress tensor vanishes [1]:

$$\Pi'_{iz} + (\boldsymbol{\xi}\cdot\boldsymbol{\nabla})\Pi_{iz} = 0, \tag{6}$$

where Π is the stress tensor

$$\Pi_{ij} = (p + B^2/2)\delta_{ij} - B_i B_j. \tag{7}$$

These are effectively boundary conditions on $\partial\boldsymbol{\xi}/\partial z$. In the simulation described here the boundary is quite close to the photosphere. In future efforts we intend to move the boundary higher where its effects will be reduced.

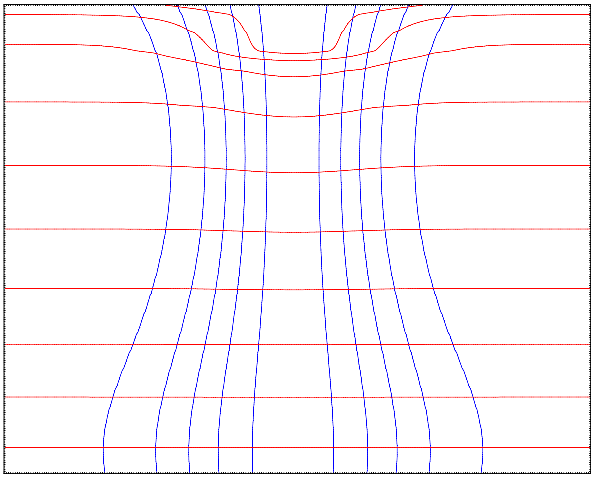

Figure 1. A vertical cut through a model sunspot. The top of this box is near the photosphere and extends (shown with a linear scale) to a depth of 6 Mm. *The horizontal extent of this figure is* 20 Mm. *The blue lines are magnetic field lines of the sunspot model, the red lines are temperature isolines. The field strength near the surface is approximately* 2 kG.

2. THE SUNSPOT MODEL

We have applied the wave code to the case of a model sunspot. Static sunspot models have existed since the work of Deinzer [2]. The main assumption of this type of model is that the radial dependence at all heights is similar, i.e. there exists functions f and b_0 such that $B_{0z}(r,z) = b_0(z) f[r\sqrt{b_0(z)}]$, where r is the horizontal distance from spot center. In addition, the temperature profile is fixed. The magnetohydrostatic equations are then solved to determine the field lines. The essential feature of monolithic models is that the flux tube becomes increasingly narrow with depth.

Recently Schüssler & Rempel [3] have relaxed the assumption of a fixed thermal stratification and have considered evolutionary behavior. They begin with a monolithic sunspot. Photospheric cooling produces a cooling front inside the sunspot. This cooling front propagates into the spot, weakening the field until the sunspot "disconnects" from its roots. The evolved field line structure is shown in blue in Fig. 1.

3. PRELIMINARY RESULTS

We study the interaction of an f-mode wave packet with the model sunspot of Fig. 1. We choose the following

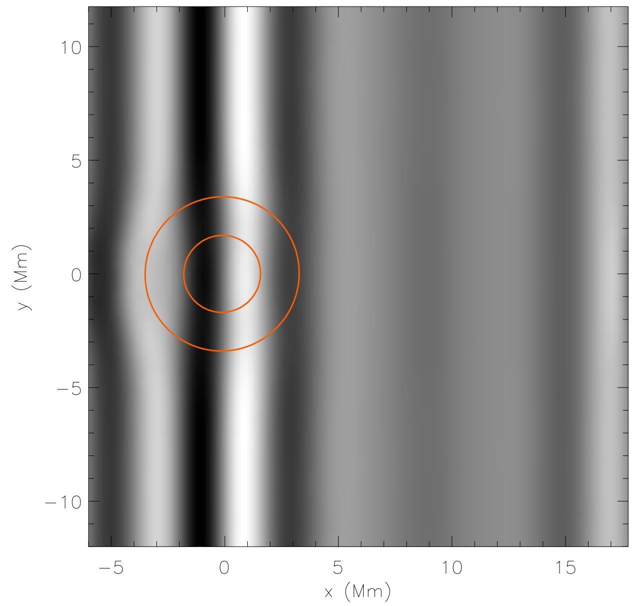

Figure 2. Preliminary example of an f-mode wave packet passing through a sunspot (from left to right). Shown is a horizontal slice from a 3D simulation, with the position of the sunspot indicated in red. The grey-scale shows the vertical component of the velocity at a constant geometrical height near the solar surface. In this snapshot, the wave packet has maximum amplitude at $x = 1$ Mm *and has passed through the center of the sunspot.*

initial conditions at $t = 0$:

$$\boldsymbol{\xi} = (-\mathrm{i}\hat{\boldsymbol{x}} + \hat{\boldsymbol{z}}) \int_0^\infty A(k) e^{\mathrm{i}k(x-x_0)} e^{kz} \mathrm{d}k + \text{c.c.}, \quad (8)$$

$$\frac{\partial \boldsymbol{\xi}}{\partial t} = (\hat{\boldsymbol{x}} - \mathrm{i}\hat{\boldsymbol{z}}) \int_0^\infty A(k) \omega_k e^{\mathrm{i}k(x-x_0)} e^{kz} \mathrm{d}k + \text{c.c.}, \quad (9)$$

where $\omega_k^2 = gk$ and $A(k)$ is a Gaussian function centered around wavenumber 1.83 rad Mm^{-1} (typical of solar f modes with frequencies near 3.5 mHz). The wave packet starts at $t = 0$ from position $x_0 = -6$ Mm (away from the sunspot) and propagates in the $+\hat{\boldsymbol{x}}$ direction.

We have begun performing preliminary test runs and Fig. 2 shows a snapshot taken from such a run. This figure currently only shows the vertical velocity at a fixed geometrical height; we intend, however, to improve this in the near future. A first observation is that the f-mode wave packet speeds up as it traverses the sunspot. This is consistent with earlier measurements of travel-time perturbations (time-distance helioseismology) and scattering phase shifts (Fourier-Hankel analysis) using real solar data.

The perturbations due to the sunspot are more easily seen in Fig. 3, which shows the scattered wave field, i.e. the

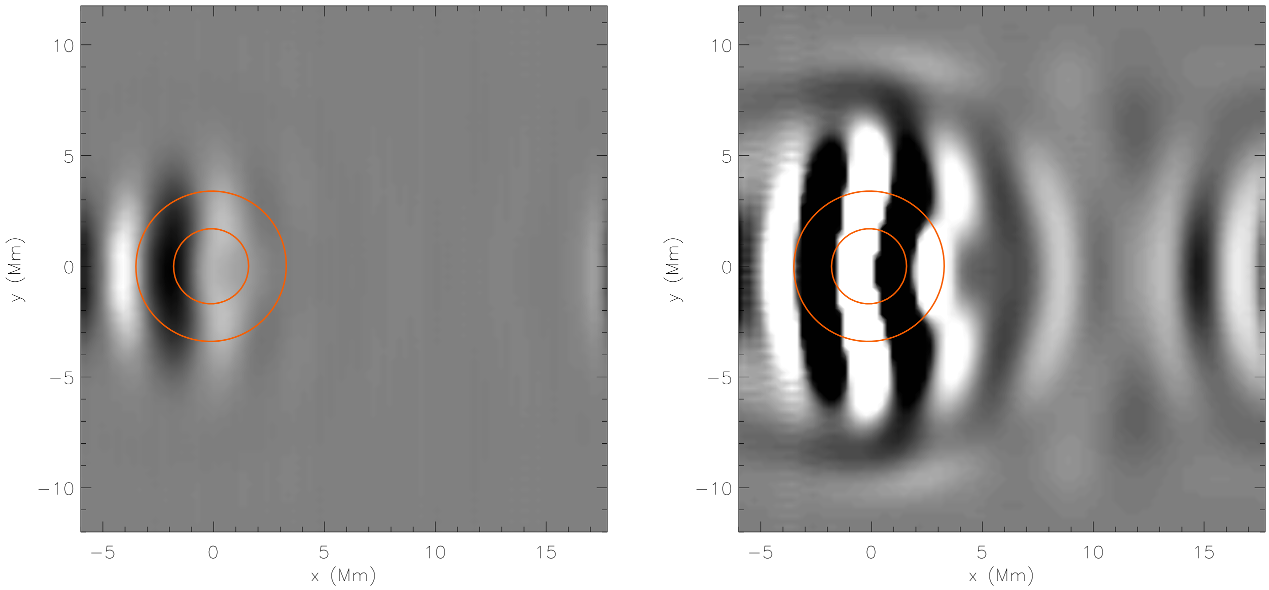

Figure 3. Scattered wave field (full wave field minus unperturbed wave field) shown for the same snapshot as in Fig. 2. The vertical component of velocity is shown a constant geometrical height. The greyscale of the right panel is saturated to bring out the details of the scattered waves away from the sunspot itself. The saturation is set at 1% of the maximum value of Fig. 2 (and the circular wavefrounts coming out from the sunspot are at some reasonably small fraction of this). The contribution of the scattered waves is such that the f-mode wave packet accelerates as it traverses the sunspot. Note that the perturbations on the right side of the box are an artifact due to the periodic nature of the solution: they would not be present if the box was big enough.

difference between the full wave field and the wave field in the absence of the sunspot. A movie of the scattered wave field reveals a fast propagating, small amplitude, nearly circular wavefront away from the sunspot (see Fig. 3, right panel).

4. DISCUSSION

We have presented a three-dimensional numerical simulation of wave propagation through a model sunspot.

At present we have performed only preliminary runs, and only looked at the perturbations at a constant geometrical height. We have also excluded looking at background flow fields. These are significant limitations which we expect to lift shortly. Looking ahead, we expect that we will soon be able to model the propagation of waves through different sizes and types of sunspots in order to see which sunspot features have significant helioseismological signatures. In particular, it will be of great interest to separate the contributions to the scattered wave field due to temperature or density inhomogeneities and to the direct effect of the magnetic field through the Lorentz force.

In principle, our code can be used to study wave propagation through completely general solar models, including time-varying models, and could have many possible applications in local helioseismology (see e.g. [4]). One application, which we are currently investigating, is the propagation of waves through existing numerical models of magneto-convection. We expect to learn a lot, in particular, about the propagation of wave packets in random media. The advantage of our approach is that it does not require very large computer resources compared to fully nonlinear calculations. Its limitation is that solar oscillations are not naturally excited by convection, but prescribed by initial conditions.

ACKNOWLEDGMENTS

The code used to generate the sunspot was kindly provided by Manfred Schüssler and Matthias Rempel.

REFERENCES

[1] Cally P., Bogdan T.J., 1997, ApJ 486, L67

[2] Deinzer W., 1965, ApJ 141, 548

[3] Schüssler M., Rempel M., 2005, A&A 441, 337

[4] Werne J., Birch A.C., Julien K., 2004, in Proc. SOHO 14, ESA SP-559, 172