Numerical simulation of solar magneto-convection

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Abstract. Numerical simulations of the interaction of magnetic field and solar surface convection are reviewed. Emphasis is laid upon the relevant physical processes and on work with implications for the interpretation of observational results. We outline the development of the two main lines of research in the past 35 years, namely studies of idealized magneto-convection and ‘realistic’ solar simulations, and discuss representative results.

1. Introduction

The term ‘magneto-convection’ summarizes the physical processes resulting from the interaction between convectively driven flows and a magnetic field in an electrically conducting fluid. The large length scales of astrophysical systems lead to high (hydrodynamic and magnetic) Reynolds numbers, so that astrophysical magneto-convection typically involves nonlinear dynamics and interactions as well as the formation of structures and patterns. The Sun offers an ideal testbed for studies of astrophysical magneto-convection since the relevant physical processes can (in principle, at least) be studied observationally on their intrinsic spatial and temporal scales. Such processes involve the generation of magnetic flux by a self-excited dynamo mechanism, its spatial distribution by flux expulsion and the formation of intermittent structure, the dynamics originating from instabilities, wave excitation, and field-line reconnection, and the energetics due to non-thermal heating and the interference of the magnetic field with convective energy transport.

This contribution is intended to review the development of numerical simulations of solar magneto-convection in the past 35 years. Obvious limits of space (in the book), time (of the author), and patience (of the reader) force us to narrow the scope and to focus the discussion on work including both magnetic field and convection, describing processes at or near the solar photosphere with direct implications for our understanding of observational results. Thus we neither consider the dynamics of thin flux tubes nor magneto-convection in the lower convection zone, let alone dynamo theory. Although I try to sketch the main lines of development and include the key contributions, no attempt is made to give a fully comprehensive overview of all work done in the field. My apologies to all authors whose papers are not mentioned here: as this research field grows, so does my ignorance since it becomes more and more difficult to keep track of all developments.
Research in numerical magneto-convection can be roughly divided into two complementary approaches: the study of idealized magneto-convection pioneered by Nigel Weiss (Cambridge/UK) and ‘realistic’ solar simulations championed by Åke Nordlund (Copenhagen). We discuss both lines of research in the following sections and venture a brief outlook in the concluding section.

2. Idealized magneto-convection

Historically, research on numerical magneto-convection started with the pioneering paper by Weiss (1966) on the formation of magnetic field concentrations by flux expulsion (see Fig. 1). The severe computational limitations at that time imposed restrictive simplifications of the physics that could be included and strongly constrained the spatial resolution and simulation time that could be afforded. Therefore, the aim of the early simulations was not so much an approximation of the conditions prevailing in the Sun but the investigation of simplified model problems that could be studied systematically. After the advent of more comprehensive solar simulations in the 1980s, this approach of research remained to be a valuable and complementary line of research since it allows to study relevant processes in isolation, to use analytical tools like stability analysis and bifurcation theory, and to systematically examine the dependence of the results on parameter values. The results of from idealized models cannot be directly compared to observations simulations, but they contribute to a deeper understanding of the underlying physical processes. The development of this line of research is sketched here along the discussion of two such processes, namely, flux expulsion and convection in a strong magnetic field. More detailed reviews
of idealized magneto-convection have been given by Proctor & Weiss (1982), Proctor (1992), and Weiss (1997).

2.1. Magnetic flux expulsion

On the basis of analytical calculations, Parker (1963) proposed that the observed supergranular magnetic network can be understood in terms of the exclusion of magnetic flux from the regions of closed streamlines of a stationary, cellular flow. Weiss (1966) gave the first numerical demonstration of this process, showing that an initially homogeneous and vertical magnetic field is advected by the horizontal component of the flow velocity and gathers at the up- and downflow regions of the flow pattern, where a balance between advection and diffusion of magnetic flux is established (see Fig. 1). The width of the resulting flux concentrations therefore depends on the value of the magnetic Reynolds number, $R_m = UL/\eta$, where $U$ is the flow velocity, $L$ the size of the box, and $\eta$ the magnetic diffusivity (inversely proportional to the electrical conductivity of the fluid). Larger Reynolds numbers lead to thinner magnetic filaments with correspondingly larger field strength.

While the first simulations of Weiss were carried out in two-dimensional Cartesian geometry, kinematic (i.e., the velocity field was prescribed and the effect of the Lorentz force was not considered), in subsequent work such limitation were eliminated step by step. Galloway, Proctor & Weiss (1977) considered axisymmetric thermal convection in the Boussinesq approximation (which takes the fluid to be essentially incompressible except for the buoyancy caused by thermal expansion) and included the back-reaction of the magnetic field on the flow. They found that the magnetic flux becomes concentrated into a flux tube forming within the cool convective downflow region near the axis of the cylindrical computational domain. A sufficiently strong magnetic field reacts back on the velocity field and expels the flow from the flux tube, resulting in a separation of the fluid into a strongly magnetic, non-convecting part (the flux tube) and its almost non-magnetic surroundings with vigorous convection.

Galloway & Proctor (1983) performed three-dimensional simulations of kinematic flux expulsion, prescribing a stationary velocity field with a hexagonal cell structure (cf. Clark & Johnson 1967). They again found that magnetic flux concentrations form in the regions of converging streamlines. Those are the cell corners (converging downflow vertices) in the upper layers of the computational domain and the cell center (converging upflow) near its bottom. As a result, a significant part of the magnetic flux always remains at the center of the cell, even near the top, where the flow is diverging from the center.

The first simulation of flux expulsion in a compressible medium was carried out by Hurlburt & Toomre (1988). They considered the dynamical problem in two-dimensional Cartesian geometry and simulated thermal convection in a closed box spanning a few pressure scale heights. Compressibility breaks the symmetry between up- and downflows and leads to concentrated, rapid downflows and more gentle, extended upflows. As a consequence, most of the magnetic flux becomes concentrated in the downflows. The growing vertical magnetic field suppresses the horizontal fluid motion but does not interfere with the vertical velocity component; therefore, the upper part of a forming flux sheet becomes partially evacuated and then compressed by the external fluid pressure, so that
the magnetic field becomes even stronger. The resulting quasi-stationary situation is characterized by a separation into an almost static strong magnetic flux sheet and surrounding convection with much weaker magnetic field.

More recently, Tao et al. (1998) considered the effect of a magnetic field on compressible thermal convection in a wide three-dimensional (aspect ratio = 8). They found flux separation with regions of strong field and small-scale convection with small plumes next to weakly magnetized domains with extended and vigorous convective plumes (see Fig. 2). Such a situation is reminiscent of the difference between the patterns of granulation in plage regions and in the adjacent ‘quiet’ Sun. Emonet & Cattaneo (this volume) revert to the Boussinesq approximation but employ a very high spatial resolution with a grid of $512 \times 512 \times 97$ cells and high Rayleigh number ($5 \cdot 10^5$), so that turbulent convection with a wide range of spatial scales develops. Depending on the amount of imposed vertical magnetic flux in the box, the simulations show flux expulsion in network patterns (reminiscent of granulation, ‘abnormal’ granulation, and mesogranulation on the Sun) as well as magnetically dominated small-scale convection patterns (relevant for sunspot umbrae). Without externally imposed magnetic field, the system exhibits dynamo action and generates a mixed-polarity (‘salt-and-pepper’) field from small and irregular initial seed field. Cattaneo (1999) suggests that such ‘local’ dynamo action could be the source of a substantial fraction of the observed small-scale field in the quiet solar photosphere.

2.2. Convection in a strong magnetic field

Flux expulsion leads to the formation of a network pattern of concentrated magnetic field if the average field strength (or, equivalently, the amount of magnetic flux) is not too large. Magneto-convection in a strong average magnetic field shows an interesting variety of nonlinear behavior, which is relevant for understanding the dynamics and the energy transport in pores and sunspots.
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Figure 3. Oscillatory magneto-convection at times $t = 0$ (top), $t = 0.2$ (middle), and $t = 0.4$ (bottom, all in units of the oscillation period). Drawn are streaklines (left), contours of the temperature fluctuation (center), and magnetic field lines (right). While the flow reverses in the upper half of the box, it keeps the same orientation in the lower half (adapted from Weiss et al. 1990).

Non-linear Boussinesq magneto-convection has been studied by Galloway & Moore (1979) for the axisymmetric case and by Weiss (1981a,b) in two-dimensional Cartesian geometry. The behavior of the system depends on the diffusivity ratio (also called magnetic Prandtl number), $\zeta = \eta/\kappa$, where $\kappa$ is the thermal diffusivity, and on the mean field strength in the box, measured by the (square root of the) dimensionless Chandrasekhar number, $Q$. While overturning convection is favored for $\zeta > 1$, oscillatory convection sets in for a sufficiently strong magnetic field if $\zeta < 1$. In the case $\zeta < 1$ and for increasing $Q$, the system first moves from kinematic flux expulsion to dynamically active flux concentrations with internally suppressed convection. At even higher values of $Q$, oscillatory convection sets in with strong nonlinear interaction between magnetic field and flow, including periodic reversals of the convective motion. Hurlburt et al. (1989) extended this work to the compressible case and simulated two-dimensional magneto-convection in a shallow polytropic layer. They found that, for a sufficiently strong imposed field, nonlinear oscillatory convection (standing waves) gives way to horizontally travelling waves, for which total pressure fluctuations (and, therefore, compressibility) are essential. Such horizontal waves in an inclined magnetic field could possibly be relevant for sunspot penumbras (Hurlburt, Matthews & Proctor 1996).

Weiss et al. (1990) extended the approach of Hurlburt et al. (1989) and considered a schematic model of a sunspot umbra with a depth-dependent diffusivity ratio. Taking $\zeta > 1$ in the lower part and $\zeta < 1$ in the upper part of the the (two-dimensional) computational box, they found mixed-mode periodic
solutions with flow reversals near the top (rising and sinking plumes) connected to a non-reversing flow pattern in the lower part (see Fig. 3). Rising plumes of hot gas could be related to the observed umbral dot phenomenon. Simulations of the corresponding three-dimensional problem have been carried out by Weiss et al. (1996) and Rucklidge et al. (2000). For decreasing strength of the imposed magnetic field they found a transition from steady convection with an ordered planform (possibly corresponding to the situation in a sunspot umbra) to a more irregular state with vigorous convection and intermittent bursts of larger convective plumes within a network of intense magnetic field (similar to a plage region). Both regimes of magneto-convection can coexist in sufficiently wide boxes.

A closer approximation of idealized models to the real solar situation has been attempted by Hurlburt & Rucklidge (2000). They consider an axisymmetric configuration, match the magnetic field to a potential field at the upper boundary and employ a radiative boundary condition corresponding to Stefan’s law. In a wide cylindrical box they find the magnetic flux to be confined by an inward ‘collar’ flow to the region around the axis. Further outside, the flow direction reverses and a ‘moat’ cell appears (see Fig. 4). The authors suggest that the collar flow holding sunspots together is hidden beneath their penumbrae, so that only the outflow in the moat cell directly becomes observable.

3. Realistic solar simulations

In contrast to the idealized models discussed above, ‘realistic’ simulations of solar magneto-convection aim at approximating the real Sun, so that the results can directly be compared to observations. Therefore, elaborate physics is included in such simulations: radiative transfer, partial ionization, open and transmitting boundary conditions, as well as spectral line and polarization diagnostics. Since the real solar values of the (magnetic and hydrodynamic) Reynolds numbers by far cannot be reproduced by the simulations due to obvious computational limitations, it is hoped that the parametrization of the effects of small-scale motion not explicitly included in the simulations (e.g., by sub-grid-scale diffusivities) does correctly describe their effect on the simulated larger scales. The results of
3.1. Convective intensification of magnetic field

Parker (1978), Spruit (1979), and Unno & Ando (1979) suggested that a 'convective collapse' of magnetic flux tubes, driven by the strongly superadiabatic stratification of the subphotosphere, is responsible for the observed concentration of magnetic field far beyond equipartition with the kinetic energy density of granulation. Nordlund (1983, 1986) performed three-dimensional simulations of solar granulation with an externally imposed magnetic field. He used the anelastic approximation (which filters out acoustic waves) and included the effects of radiative energy transport as well as partial ionization. Even with a rather modest horizontal spatial resolution of 250 km he clearly demonstrated the concentration of magnetic flux in the cool integranular downflow regions and the intensification of the field through downflows driven by radiative cooling. The simulations also show a continuous displacement and rearrangement of magnetic flux in the network of intergranular downflows.

Nordlund & Stein (1990) extended these calculations to the fully compressible case and improved spatial resolution (50 km horizontal grid spacing). They found that a strong vertical magnetic field (500 G average field strength) sta-

realistic simulations are often quite complex, while powerful analytical tools for their analysis (like in the case of the idealized models) are not available.
Figure 6. Flux expulsion and convective intensification in a two-dimensional simulation with radiative transfer and ionization. Shown are magnetic field lines at a time 4 minutes after imposing a homogeneous vertical field of 100 G strength on a developed granulation pattern: a strong magnetic flux concentration has formed in a convective downflow region around $x = 2000$ km (adapted from Grossmann-Doerth, Schüssler & Steiner 1998).

Figure 6

bilizes the granulation pattern in depth, so that the network of integranular downflows continues down to the bottom of the computational box at 1 Mm depth and does not fragment into isolated downdrafts like in the non-magnetic case (see Fig. 5). Bercik et al. (1998) used the same code for a systematic study, varying the strength and orientation of the imposed magnetic field (50–1000 G vertical field, 500–2000 G horizontal field). The results show the transition from undisturbed granulation to ‘abnormal granulation’ and pore-like structures.

Grossmann-Doerth et al. (1998) studied the process of convective intensification in a two-dimensional simulation with high spatial resolution (10 km horizontal grid spacing). They included ionization, radiative transfer, and spectral/polarization diagnostics. Depending on the amount of vertical magnetic flux initially put into the computational box, smaller or larger flux concentration form by the flux expulsion process (see Fig. 6). The intensification of the field proceeds according to the ‘convective collapse’ scenario: suppression of the horizontal convective flow by the growing magnetic field, radiative cooling of the surface layers, and the superadiabaticity of the stratification drive a strong downflow that evacuates the upper part of the flux concentration and leads to kilogauss field strength. For the larger flux concentrations, the downflow rebounds at the high-density layers below the photosphere and the resulting upflow develops into a strong shock, which could possibly be related to the spicule phenomenon. Synthetic Stokes profiles of spectral lines provide criteria through which the intensification process can be identified in observational data.
3.2. Structure and dynamics of magnetic elements

Another line of research addresses the interaction of already formed magnetic flux concentrations with the convective environment. This includes the thermal state, which is strongly affected by radiative energy transport, the excitation of waves and shocks by granular ‘buffetting’, and detailed spectroscopic/polarimetric diagnostics in order to compare with observational data.

Deinzer et al. (1984a,b) studied magnetic flux sheets in two-dimensional Cartesian geometry. They considered a fully compressible plasma, ionization, radiation (in the diffusion approximation) and an adaptive mesh (moving finite elements) in order to adequately resolve sharp interfaces between the flux sheets and their non-magnetic environment (cf. Schüssler 1986). The simulations showed the maintainance of convective downflows adjacent to the flux sheets and the internal heating caused by the ‘hot wall’ effect, so that the flux sheet appears bright in comparison to the average photosphere. Knölker, Schüssler & Weisshaar (1987) varied the width of the transition layer between a flux sheet and its non-magnetic exterior; by comparison with observations, they found that models with a thin transition layer are favored. Knölker & Schüssler (1988) studied larger flux concentrations with diameters up to 1000 km. They showed that flux sheets with diameters in excess of 500 km appear darker than the average photosphere in continuum radiation if observed near the center of the solar disk (vertical incidence of the line of sight), but become brighter near the limb (cf. Spruit 1976).

Knölker et al. (1991) and Grossmann-Doerth et al. (1994) used a revised and extended version of the code of Deinzer et al. They introduced an implicit integration method with a strongly reduced numerical viscosity and added a full, grey radiative transfer (Feautrier method) as well as elaborate intensity and Stokes diagnostics. The simulations carried out with this code showed the development of strong downflow jets with velocities up to 6 km·s$^{-1}$ besides the magnetic flux sheets. Such downflows lead to asymmetries of the Stokes $V$-profiles by way of the ‘canopy effect’ (Grossmann-Doerth, Schüssler & Solanki 1988). The upper layers of the flux sheet atmosphere become hotter than the environment through radiative illumination by the hot bottom and walls of the flux concentrations. By detailed comparison with observational data, the simulation results suggest that the typical size of magnetic structures in the network is about 200 km whereas somewhat larger values of 300–350 km are indicated in plages.

Steiner, Knölker & Schüssler (1994) developed a new MHD code with adaptive mesh refinement that allowed higher spatial resolution and smaller numerical viscosity. Steiner et al. (1995,1996,1998) used this code to simulate the interaction of magnetic flux sheets with granulation in a two-dimensional box with a grid resolution of 10 km. They found that the external motions can significantly bend a magnetic element, leading to a swaying motion with strong horizontal flows; moreover, they drive field-aligned upward flows that develop into shocks. The propagation of these shocks causes a distinctive signature in the corresponding time series of synthetic Stokes $V$-profiles (see Fig. 7).

Another two-dimensional MHD code with radiative transfer and ionization has been developed by the recently deceased A. Gadun (Kiew Observatory, Ukraine). Ploner et al. (two contributions in these proceedings) have used this
Figure 7.  

Left: Velocity vectors for a snapshot from a simulation of magnetic flux sheet dynamics (Steiner et al. 1998). Two shocks propagate upward side by side within the flux sheet, which is located around $x = 1200$ km. 

Right: Time sequence (from bottom to top) of Stokes $V$-profiles (Fe I 525.02 nm), separated by 10 s each, showing the diagnostic of a shock transition within the flux sheet. The profiles show a transition from a redshifted pre-shock profile to a strongly blueshifted post-shock profile.

code to simulate the formation and dynamics of magnetic flux sheets over extended periods of time. They find ‘recycling’ of magnetic flux as an alternative to local dynamo action to interpret the large flux emergence rates at small scales in the quiet Sun, which are inferred from observations. Moreover, the long simulation runs provide a rich data base to study the formation of Stokes profiles, in particular the ‘abnormal’ $V$-profiles, and so evaluate their diagnostic content.

4. Outlook

The past decades have seen impressive development and progress in the study of magneto-convection with numerical simulations. Three-dimensional simulations of non-magnetic solar surface convection have already reached a stage where they reproduce all essential observed properties (i.e., morphology and evolution, detailed profiles of many iron lines) on the scale of granulation (Stein & Nordlund 1998, Asplund et al. 2000) with a grid resolution of 10–25 km. Apparently, small-scale turbulence does not affect the flows at the observed scales otherwise than through effectively enhanced diffusivities, which are parametrized in the simulations by means of the sub-grid-scale recipe. Three-dimensional simulations with a similar spatial resolution including a magnetic field are within the reach of present-day computing facilities. Do we expect a similar situation as for granulation, i.e., will this resolution prove sufficient to capture the essential physics, at least at spatial scales that can be observed with existing telescopes?
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Probably not. The magnetic field has a topology that in most non-academic cases cannot be adequately described by using simple ‘turbulent’ magnetic diffusivities. Flows with chaotic streamlines lead to an amazingly rich structure of the magnetic field (e.g., Brummell, Cattaneo & Tobias 1999) and the pictures from TRACE reveal the breathtaking complexity of the coronal magnetic field. Current sheets, field line reconnection, dynamically dominant field structure at small scales are genuine to magnetic fields, so that really realistic simulations of solar MHD probably require, at least locally, a much better spatial resolution (of the order of 1 km ?) than purely hydrodynamic simulations. Codes with adaptive, locally refined grids may be able to reach this resolution level in the foreseeable future, but a lot of work still needs to be done.

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