Stability of magnetic flux tubes in close binary stars

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Photometric and Doppler imaging observations of active binaries indicate the existence of starspots at preferred longitudes (position angles with respect to the companion star). We investigate the stability of magnetic flux tubes in the convection zone of close, fast-rotating binary stars and explore whether the observed preferred longitudes could be caused by tidal forces and the deformation of the active star. We assume a synchronized binary system with spin axes perpendicular to the orbital plane and a rotation period of a few days. The tidal force and the deviation from spherical structure are considered in lowest-order perturbation theory. The magnetic field is in the form of toroidal magnetic flux rings, which are stored in mechanical equilibrium within the stably stratified overshoot region beneath the convection zone until the field has grown sufficiently strong for the undulatory instability to initiate the formation of rising loops. Frequencies and geometry of stable as well as growth rates of unstable eigenmodes are determined by linear stability analysis. Particular consideration is given to the question whether the effects of tidal forces and perturbations of the stellar structure can force a rising flux loop to enter the convection zone at specific longitudes.

Key words: binaries: close — stars: magnetic fields — stars: spots — stars: activity — stars: imaging

1. Introduction

Rapid rotation and deep convection zones are optimal ingredients for a dynamo mechanism generating magnetic fields in cool stars. The emergence of magnetic flux at the stellar surface leads to the various manifestations of stellar activity. Photometric and Doppler imaging observations reveal temperature inhomogeneities on the stellar surface, which are interpreted as cool spots caused by magnetic fields (Strassmeier 1999, and references therein). Such identification is supported by the results of Zeeman-Doppler imaging (e.g. Carter et al. 1996) by which method magnetic maps of the stellar surface are obtained. These observations and other tracers of activity show that magnetic activity of the kind shown by the sun (often with much larger strength) is common in cool stars with outer convection zones. Examples of very active stars are fast-rotating single stars and tidally coupled members of close binary systems (RS CVn- and BY Dra systems) as well as T Tauri stars. Contrary to the case of the sun, where spots emerge in an activity belt near the equator, fast-rotating stars typically exhibit spots at intermediate and high latitudes (Strassmeier 1994, Berdyugina et al. 1998).

There are indications for preferred longitudes in the spot distribution on magnetically active close binary stars with rotation periods of a few days (Zeilik et al. 1994, Rodonò et al. 1995, Lanza et al. 1998). The dependence on the system period suggests that this effect is caused by the proximity of the companion star (Heckert & Ordway 1995): systems with periods of about 1 day exhibit fixed active longitude belts near the quadrature points, whereas wider systems with rotation periods above about 10 days do not show spots at preferred longitudes. Some systems with intermediate rotation periods exhibit migrating active longitudes (Berdyugina et al. 1998) and may represent a transition from fixed active longitudes to no preferred longitudes at all.

We investigate the stability of magnetic flux tubes (in mechanical equilibrium) in the convection zone of an active component of a close binary system in order to examine whether the influence of the companion star is able to cause the emergence of magnetic flux at preferred longitudes. The quantitative elaboration of such a flux tube model is consistent with observed properties of solar active regions (Caligari et al. 1995), with the emergence of polar spots on rapidly rotating single stars (Schüssler et al. 1996, Granzer et al. 1999), and with the disappearance of strong surface activity in giants on the cool side of the 'coronal dividing line' (Schüssler et al. 1999).

2. Model assumptions

We assume a dynamo mechanism generating and amplifying magnetic flux in the lower part of the stellar convection zone. The magnetic field is stored in the form of magnetic flux tubes in the subadiabatically stratified overshoot region below the convection zone. Once a critical magnetic field strength is exceeded, the undulatory instability leads to the formation of growing flux loops, which rise through the convection zone. Upon emergence at the stellar surface, the flux loops form magnetic active regions and starspots (Schüssler et al. 1994).

The binary model consists of a system with an active star of mass M_1 and a companion star with mass $M_2 = qM_1$. The two components are separated by a distance a and move on circular orbits with the orbital period T. The spin axes of the stars are assumed to be perpendicular to the orbital plane and the rotation rate is equal to the orbital rate of the system (synchronized rotation). For the active star under consideration, we use a perturbed single star model, whereas the companion star is taken as an idealized point mass.

In the following we will assume a binary system with two stars of $1\,\mathrm{M}_\odot$ each. For orbital periods in the range of $1\ldots10\,\mathrm{d}$, Kepler's law yields distances $a\sim5\ldots25\,\mathrm{R}_\odot$, which are large enough to apply lowest-order perturbation theory for the tidal forces and for the deviation from spherical symmetry. Using spherical coordinates (radius r, azimuth ϕ, latitude λ; $\phi=0$ is the direction towards the companion star), the expansion parameter of approximation, ϵ^3, is in the range

$$\epsilon^3 = \left(\frac{r}{a}\right)^3 \sim 10^{-2}\ldots10^{-5} \qquad \text{for} \quad r\sim 1\,\mathrm{R}_\odot \ . \tag{1}$$

The azimuthal component of the resulting effective gravitation, $\vec{g}_{\mathrm{eff}}$, can be written as

$$\vec{g}_{\mathrm{eff}}\cdot\vec{e}_\phi \approx -\epsilon^3\frac{3}{2}q\cos\lambda\sin2\phi \ . \tag{2}$$

For a weakly deformed star, we can assume that the value of any stellar quantity, f, (like pressure, density,...) is a function of the effective potential, Ψ_{eff}, only, and can be written as

$$f(r,\phi,\lambda) \approx f_0(r)\left(1+\epsilon^3\frac{3}{4}q\cos\lambda\frac{r}{H_f}\cos2\phi\right) \ , \tag{3}$$

where the corresponding scale height of this quantity, H_f, and the value f_0 are determined from an unperturbed stellar model. In Eq. (3) we have neglected the axisymmetric deformation caused by rotation since we are interested in the effect of the ϕ-dependence on the stability of the flux tubes. All calculations are carried out in the framework of the 'thin flux tube approximation' (cf. Spruit 1981)

3. Flux tube equilibrium and linear stability analysis

A stationary equilibrium of a magnetic flux tube with an internal toroidal flow along the magnetic field lines is determined under the assumption of an ideal plasma. For a non-buoyant toroidal flux tube (density inside the tube equal to that of its environment) lying in a plane parallel to the equator the outward directed Coriolis and inertial forces are compensated by the inward directed magnetic curvature force. Force equilibrium is obtained for

$$\frac{v}{\Omega R} = \frac{2M_\alpha^2}{1-M_\alpha^2} \ , \tag{4}$$

where v denotes the velocity of the toroidal flow, $M_\alpha = v/c_\mathrm{a}$ the Alfvénic Mach number, $c_\mathrm{a} = B/\sqrt{4\pi\rho}$ the Alfvén velocity, B the magnetic field strength, ρ the density, Ω the orbital rate ($\equiv$ angular velocity of rotation), and R the (constant) radius of curvature of the flux ring.

The deviation of the stellar structure from spherical symmetry leads to a periodic variation of the internal density along the toroidal tube [cf. Eq. (3)]. All other equilibrium quantities like magnetic field strength, gas flow, tube diameter etc. follow from the density and, therefore, inherit (in the lowest order of approximation) the same periodicity. Figure 1 gives a sketch of the circular equilibrium flux tube embedded in the tidally deformed star.

Due to the forces acting on the plasma within the flux tube (buoyancy, magnetic curvature, centrifugal, Coriolis, and tidal force), the ring responds to a small perturbation with oscillations consisting of a discrete spectrum of eigenfrequencies and eigenmodes. We use a linear stability analysis to determine these characteristic oscillations and to derive critical field strengths and growth rates of unstable perturbations. In particular, we are interested in the shape of the eigenmodes, since we want to examine whether unstable flux loops form at specific longitudes.

4. Flux tubes in the equatorial plane

We consider the case of a toroidal magnetic flux tube in mechanical equilibrium located in the stellar equatorial plane. In this case, the binormal component of the perturbation (describing displacements perpendicular to the

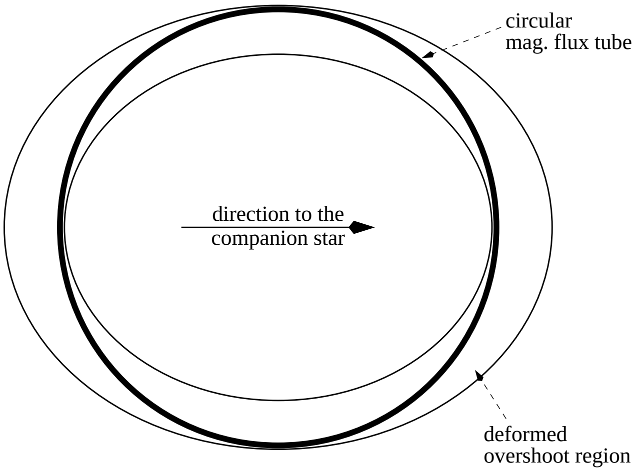

Fig. 1: A toroidal magnetic flux tube embedded in the tidally deformed overshoot region. The sections of the flux tube at the quadrature points are located at higher layers compared with those on the line connecting the stars.

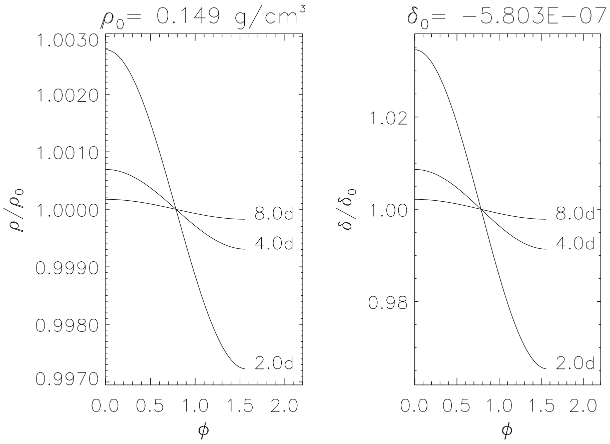

Fig. 2: Relative variation of the density, ρ, and the superadiabaticity, δ, between the direction towards the companion star ($\phi=0$) and a quadrature point ($\phi=\pi/2$) for different orbital periods in a system of two $1\,\mathrm{M}_\odot$ stars. Since in this example $H_\rho \approx 17 H_\delta$, the resulting longitudinal variation of δ is much larger.

equatorial plane) is decoupled from perturbations within the plane. In the following, we will consider only perturbations within the equatorial plane, since the binormal perturbations are found to be stable.
The corresponding linearized equation of motion can be written in the form

$$\xi'' + c_1\ddot{\xi} + c_2\dot{\xi}' + c_3 X\dot{\xi} + c_4 X\xi' + Y\xi = 0 \,, \tag{5}$$

where $\xi(\phi,t) = (\xi_\mathrm{t},\xi_\mathrm{n})^T(\phi,t)$ denotes the Lagrangian displacement vector of the mass elements of the tube, characterized by its local tangent, $\vec{t}$, and the normal, $\vec{n}$, unit vectors. Dots and primes indicate derivatives with respect to time, t, and azimuth, ϕ, respectively. The coefficients c_i are

$$c_1 = -\frac{1}{4}\frac{1}{\Omega^2}\left(\frac{1}{M_\alpha^2}-1\right) \quad , \quad c_2 = -\frac{1}{\Omega} \quad , \quad c_3 = \frac{1}{2}\frac{1}{\Omega}\left(\frac{1}{M_\alpha^2}+1\right) \quad , \quad c_4 = F g_\mathrm{n} - 1 \,. \tag{6}$$

The matrix X is given by

$$X = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{7}$$

and the elements of the ϕ-dependent matrix Y are

$$Y_{1,1} = F g_\mathrm{t}' \quad , \quad Y_{1,2} = \left[-\left\{(1-\nabla)\,M_\alpha^2 + \nabla\left(1-M_\alpha^2\right)\right\}\gamma F g_\mathrm{n} + \frac{\gamma^2}{2}\Delta\left(1-M_\alpha^2\right)F g_\mathrm{n} - 1\right] F g_\mathrm{t} \,, \tag{8}$$

$$Y_{2,1} = \left[\frac{\gamma^2}{2}\Delta\left(1-M_\alpha^2\right)F g_\mathrm{n} + 1\right] F g_\mathrm{t} \quad , \quad Y_{2,2} = \left[\frac{\gamma^2}{2}\Delta\left(1-M_\alpha^2\right)F g_\mathrm{n} + 2\right] F g_\mathrm{n} \,, \tag{9}$$

with

$$g_\mathrm{t} = (\vec{g}_\mathrm{eff}\cdot\vec{t}) \quad , \quad g_\mathrm{n} = (\vec{g}_\mathrm{eff}\cdot\vec{n}) \quad , \quad F = \frac{R}{c_\mathrm{s,e}^2\left(1-M_\alpha^2\right)} \quad , \quad \Delta = \beta\delta - \frac{2}{\gamma}\left(\frac{1}{\gamma}-\frac{1}{2}\right) \,. \tag{10}$$

Furthermore, we have $\gamma = c_\mathrm{p}/c_\mathrm{v}$, $\beta = 8\pi p/B^2$, the superadiabaticity $\delta = \nabla - \nabla_\mathrm{ad}$, and the sound velocity $c_\mathrm{s,e}$ outside the tube.
All equilibrium quantities exhibit a harmonic 2ϕ-dependence [cf. Eqs. (2) and (3)], so that the coefficients of Eq. (5) contain components $\propto \cos 2\phi$ and $\propto \sin 2\phi$. Therefore, Eq. (5) represents a Hill-type system of differential equations, which we solve numerically in order to determine unstable eigenmodes. The corresponding single-star problem already showed that the stability of flux tubes depends sensitively on the value of the superadiabaticity δ. Since the corresponding scale height, H_δ, is rather small (cf. Fig. 2) we expect that the influence of this quantity dominates over all other ϕ-dependent effects.

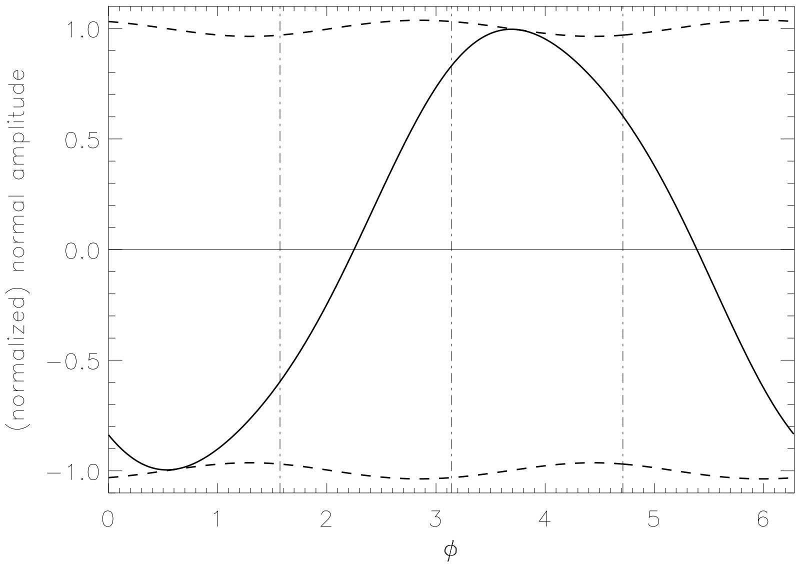

Fig. 3: Snapshot of an unstable $m=1$-mode in a binary system consisting of two $1\,\mathrm{M}_\odot$ stars with an orbital period of 2 days. The equilibrium flux tube with field strength $B=1.0\cdot10^5$ G is located in a depth $r=5.08\cdot10^{10}$ cm. The solution corresponds to a wave propagating in ϕ-direction with exponentially growing amplitude (solid curve). Superposed upon this growth is a ϕ-dependent amplitude modulation. The *relative* modulation of the normal amplitude is given by the dashed lines. Vertical dashed-dotted lines indicate the longitudes of the quadrature points ($\phi=\pi/2$ and $\phi=3\pi/2$) and the direction opposite to the companion star ($\phi=\pi$). The asymmetry of the wave is caused by the influence of the Coriolis force on the varying flow velocity of the gas inside the tube.

5. Results

The time dependence of the solution ξ of Eq. (5) can be separated with an exponential ansatz, $\xi\sim\exp(\mathrm{i}\omega t)$, which leads to a system of ordinary differential equations containing the complex eigenvalue ω. Using a shooting method, this eigenvalue is varied until the (periodic) boundary conditions for the integrated solution, $\xi(2\pi)\equiv\xi(0)$ and $\xi'(2\pi)\equiv\xi'(0)$, are fulfilled. A solution of Eq. (5) can formally be written as

$$\xi(\phi,t)=\mathrm{e}^{-\Im(\omega)t}\begin{pmatrix}|\xi_t|\mathrm{e}^{\mathrm{i}\alpha_t}\\|\xi_n|\mathrm{e}^{\mathrm{i}\alpha_n}\end{pmatrix}\mathrm{e}^{\mathrm{i}(m\phi+\Re(\omega)t)}\ . \tag{11}$$

The real part of the eigenvalue, $\Re(\omega)$, describes the oscillatory component of the solution, i.e. the propagation of a wave with azimuthal wave number m around the flux ring, whereas the imaginary part, $\Im(\omega)$, determines its stability: for $\Im(\omega)<0$ the amplitude increases with time, for $\Im(\omega)>0$ it decreases.
In contrast to the axially symmetric single-star problem, the moduli $|\xi_t|$ and $|\xi_n|$ as well as the relative phase differences α_t and α_n are functions of the azimuth ϕ. The ϕ-dependence of α_t and α_n causes varying phase velocities for the tangential and normal components of the propagating perturbation wave, whereas the moduli $|\xi_t|$ and $|\xi_n|$ cause a modulation of the wave's amplitude, respectively.

Because growing flux loops preferentially form at locations where the normal displacement of the tube is largest, we are particularly interested in the modulus $|\xi_n(\phi)|$, since we want to examine whether unstable flux loops form at specific longitudes. Figure 3 shows a snapshot of a wave with azimuthal wave number $m=1$ and $\Im(\omega)<0$, i.e. an unstable wave with a growing amplitude. Note that the maxima of the modulation of the normal amplitude, $|\xi_n(\phi)|$, are shifted with respect to the quadrature points. We find that such a phase shift relative to the π-periodic variation of the stellar structure (dominated by the variation of the superadiabaticity δ) is common to all unstable eigenmodes, whereas stable modes have their extrema in most cases at the quadrature points.

Figures 4 and 5 show the dependence of the (normalized) modulation of the normal amplitude, $|\xi_n|/\min(|\xi_n|)$, on the magnetic field strength for the wave numbers $m=1$ and $m=2$, respectively. The dependence of the phase shift of the modulation on field strength, azimuthal wave number, and growth rate, indicate a complex interaction of the eigenmode with the periodically varying environment, which cannot be described in simple terms. The decrease of the fractional variation of the modulation with increasing field strength, B, can be explained by the sensitive dependence of the problem on the term Δ [cf. Eq. (10)]: for growing B the quantity β decreases and therefore the influence of $\beta\delta$ becomes smaller.

6. Discussion

Our results show that the breaking of axial symmetry introduced by the companion star via tidal forces and the corresponding disturbance of the stellar structure induces asymmetries in the shape of the eigenmodes of perturbed flux tubes. Since the probability for the formation of an unstable flux loop depends on the modulus of the perturbation's normal amplitude, it can be seen that the location of formation depends on the longitude relative to the companion star.

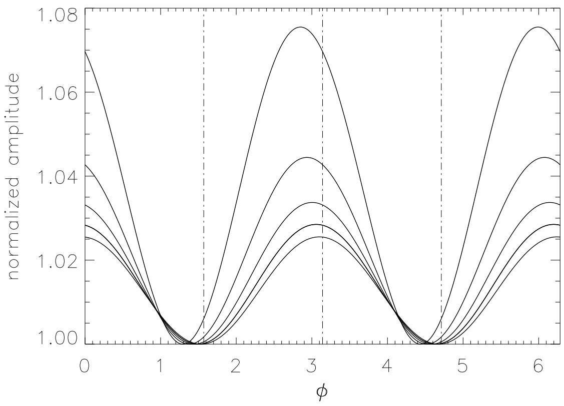

Fig. 4: Modulation of the normal amplitude of the eigenmode $m=1$. The system consists of two $1\,\mathrm{M}_\odot$ stars with an orbital period of 2 days. The flux tubes are located in a depth $r = 5.08 \cdot 10^{10}\,\mathrm{cm}$ and have field strengths $B = (1.0, 1.2, 1.4, 1.6, 1.8) \times 10^5\,\mathrm{G}$ (largest amplitude corresponds to smallest field strength).

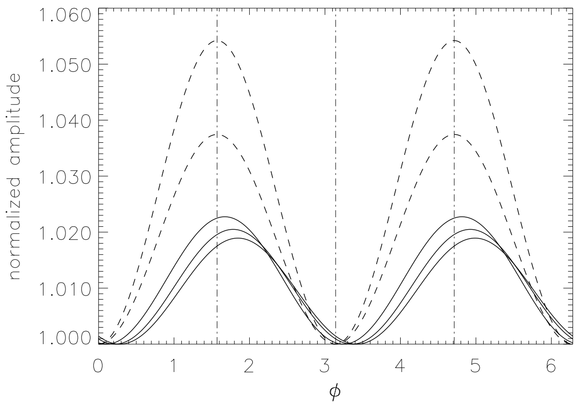

Fig. 5: Same as Fig. 4 for the eigenmode $m = 2$. For $B = 1.0 \times 10^5\,\mathrm{G}$ and $B = 1.2 \times 10^5\,\mathrm{G}$, respectively, the perturbations are stable (dashed lines). The maxima of their modulation are located at the quadrature points. The maxima of the modulation of unstable perturbations are shifted with respect to the quadrature points (solid lines).

The decreasing fractional variation of the perturbations amplitude with increasing magnetic field strength indicates that the introduced asymmetry is more important for flux tubes with low field strengths. Furthermore, the preferred longitudes of the eigenfunctions show a complicated dependence on the equilibrium properties and on the wave number of the perturbation. For the given examples, the maxima of the mode $m=1$ are located near the connecting line to the companion star, whereas the maxima of the mode $m=2$ are situated near the quadrature points.

On the other hand, the ϕ-dependent modulation of the eigenfunction is only one of the factors that affect the emergence longitude of a rising flux loop. The longitude at which the growing loop first enters the superadiabatically stratified part of the convection zone (from which point on it rises rapidly towards the surface) depends also on the distance between the equilibrium position of the flux tube and the upper boundary of the overshoot region, which is minimal at the quadrature points (cf. Fig. 1). Moreover, while a loop rises through the convection zone, it continues to propagate in ϕ-direction, so that the actual longitude of emergence can only be determined by following the evolution of the instability into the nonlinear regime. This requires numerical simulations, which will be performed as the next step of this investigation.

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