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Tomographic Reconstruction of the Coronal Magnetic Field from Polarimetric Measurements of Magnetically Sensitive Ions

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Abstract

Magnetic field contains the dominant energy per unit volume in the solar corona and therefore plays important role in the most coronal phenomena. But untill now, there is no certain measurements of the magnetic field vector distribution over the corona. Methods which is applied for deriving magnetic field in photosphere are not suitable for the corona due to small magnetic field strength (~ 10 G) and high temperature ($\sim 10^6$ K). Some information about coronal magnetic field can be obtained using longitudinal Zeeman and Hanle effects. Polarimetric measurements of light scattered by Fe XIII and Fe XIV ions yelds information about magnetic field direction in the plane of the sky integrated over line-of-sight (LOS). The Zeeman effect provides the LOS integrated line-of-sight component of the magnetic field. We apply a tomographic technique based on these effects in order to reconstruct the configuration of the vector magnetic field in the whole solar corona. To make the problem more determined we introduced an additional regularization constraint $\nabla \cdot \mathbf{B} = 0$ for the inversion. This term together with solar surface magnetogram data allows us to more precisely reconstruct both strength and direction of the magnetic field.

Basics of Vector Tomography

Since corona is opticaly thin, data used in our tomography reconstruction are integrals over line-of-sight (LOS) of some function $S(\vec{B}(\vec{r}))$ depending on the magnetic field $\vec{B}$ at position $\vec{r}$ in the corona

$$I_{i,p} = \int_{\text{LOS of } i,p} S(\vec{r}, \vec{B}(\vec{r}), \hat{\vec{e}}_{\text{LOS}}) d\ell, \qquad (0.1)$$

where p is index of data point at some viewing direction with index of i, $\hat{\vec{e}}_{\text{LOS}}$ is the unit vector along the LOS. The function S here are either Stokes emission coefficients or some scalar function of it.

All volume of the corona is separated in the plane parallel layers which are parallel to the equliptic plane. And also we assumed that corona is stationary during half of it's rotation. These makes possible the observations from the Earth.

In the case of the longitudinal Zeeman and Faraday effects, the data can be described by the integral

$$D_{i,p} = \int_{\text{LOS of } i,p} K(\vec{r}) \cdot \vec{B} \cdot \hat{\vec{e}}_{\text{LOS}} d\ell, \qquad (0.2)$$

where $K(\vec{r})$ is scalar function.

The Helmholtz decomposition of the vector field $\vec{V} = K \cdot \vec{B}$ is

$$\vec{V}(\vec{r}) = \nabla \times \vec{\Psi}(\vec{r}) + \nabla \cdot \Phi(\vec{r}), \qquad (0.3)$$

where $\vec{\Psi}$ and Φ are the vector and scalar potentials, respectively. The Φ is strongly dependent on boundary conditions for $\vec{V}$ and can exactly be determined from them (MDI measurements) if the field $\vec{V}$ is incompressible (flows of incompressible fluids, for example). And with other hand in this case, the $\vec{\Psi}$ can be by tomography reconstruction [1].

We look for the solution by minimizing the function

$$F(\vec{B}) = \sum_{i,p} \left(D^{\text{obs}}_{i,p} - D^{\text{sim}}_{i,p}(\vec{B}) \right)^2 + \mu \cdot \int_{\text{corona}} \left| \nabla \cdot \vec{B} \right|^2 d\vec{r}, \qquad (0.4)$$

where $D^{\text{sim}}_{i,p}$ is simulated data obtained from the guessed field distribution $\vec{B}(\vec{r})$. The second term here is the regularization constraint based on the property of magnetic field $\nabla \cdot \vec{B} = 0$. The photospheric magnetic field must be included into the reg. term as a boundary condition.

Longitudinal Zeeman effect

The magnetograph formula:

$$\varepsilon_V(\omega, \hat{\vec{e}}_{\text{LOS}}) = g \frac{d\varepsilon_I(\omega, \hat{\vec{e}}_{\text{LOS}})}{d\omega} \omega_L \cos\theta, \qquad (0.5)$$

where ε_I and ε_V are the Stokes-I and -V emission coefficients, respectively, ω_L is the Larmour frequency, θ is the angle between $\vec{B}$ and the direction to the observer defining by the unit vector $\hat{\vec{e}}_{\text{LOS}}$, g is the effective Landé factor. The integrated Stokes-V signal is therefore of the form (0.2) and proportional

$$g \int_{\text{LOS}_{p,i}} \frac{d\varepsilon_I(\omega)}{d\omega} \omega_L \cos\theta d\ell = \int_{\text{LOS}_{p,i}} K(\vec{r}) \vec{B} \cdot \hat{\vec{e}}_{\text{LOS}} d\ell. \qquad (0.6)$$

The coefficient K depends here on the total emissivity $\varepsilon_I(\omega)$ and can therefore be found by scalar field tomography technique from spectroscopic measurements. For the simulations we chose $K(\vec{r})$ to be the same as electron density in [2].

Hanle-effect Application

In the case of Hanle-effect for 10747 Å emission line of Fe XIII, function $S(\vec{B}(\vec{r}))$ is represented by components of Stokes-vector integrated over line profile [3]:

$$\begin{pmatrix} \varepsilon_I \\ \varepsilon_Q \\ \varepsilon_U \end{pmatrix} \sim \begin{pmatrix} 2\Sigma + \Delta V(\theta) V(\Theta) \\ 3\Delta V(\Theta) \sin^2\theta \cos 2\alpha \\ 3\Delta V(\Theta) \sin^2\theta \sin 2\alpha \end{pmatrix}, \qquad (0.7)$$

where Θ is the angle between $\vec{B}$ and radius vector (Fig. 1), θ is the angle between $\vec{B}$ and LOS, angle α describes the orientation of the linear polarization plane relative to the projected on the plane of the sky (POS) local radius vector, Σ and Δ are quantities proportional to the Zeeman sublevels populations and ion density, and $V(\Theta) = 3\cos^2\Theta$-1 is the van Vleck factor which introduce van Vleck ambiguity [4]. There is also $180°$-ambiguity introduced through the angle α.

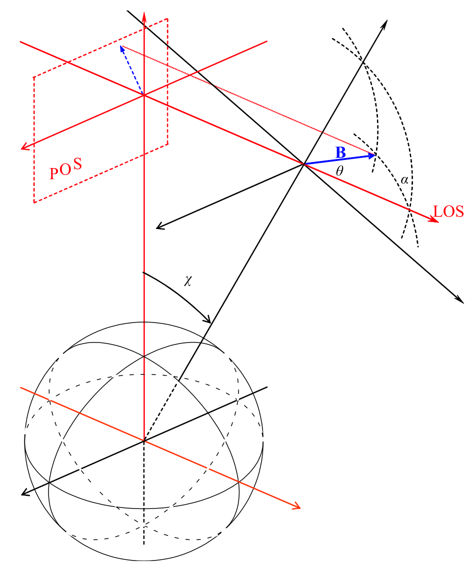

Fig. 1: Scattering geometry: Hanle effect

We have from the Hanle measurements no information about the magnetic field strength. During the inversion, the information about the strength of the magnetic field comes entirely from the photospheric measurements via the regularization term in (0.4). This and ambiguities properties make the inversion problem more ill-posed.

Test simulations

Model I. The dipole inclined for $10°$ with perturbation introduced by the circular current situated in the plane $y = 0$.

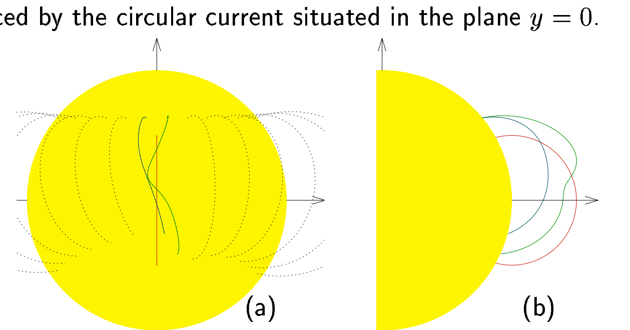

Fig. 2: Test model field I. The view from the $x = +\infty$ direction (a), and the view from the $y = -\infty$ direction (b). The perturbing current loop is shown by the red lines.

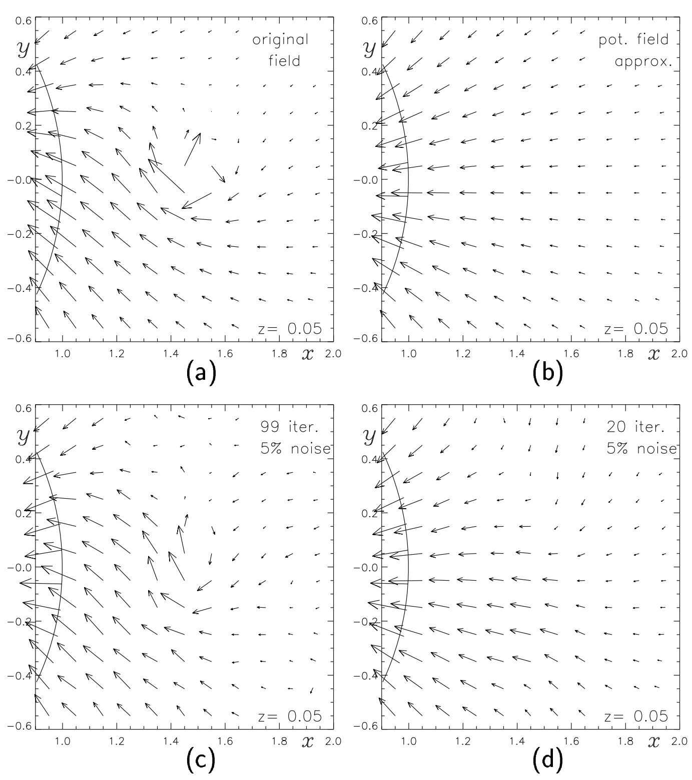

Fig. 3: Cross section by $z = 0.05$ plane: the original model field (a), potential field approximation (b), the reconstruction based on the Zeeman effect (c), and Hanle effect (d).

Model II. The dipole inclined for $10°$ with perturbation introduced by the circular current situated in the plane $z = 0$.

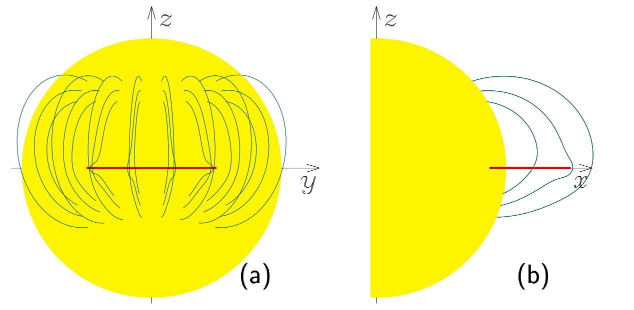

Fig. 4: Test model field II. The view from the $x = +\infty$ direction (a), and the view from the $y = -\infty$ direction (b). The perturbing current loop is shown by the red lines.

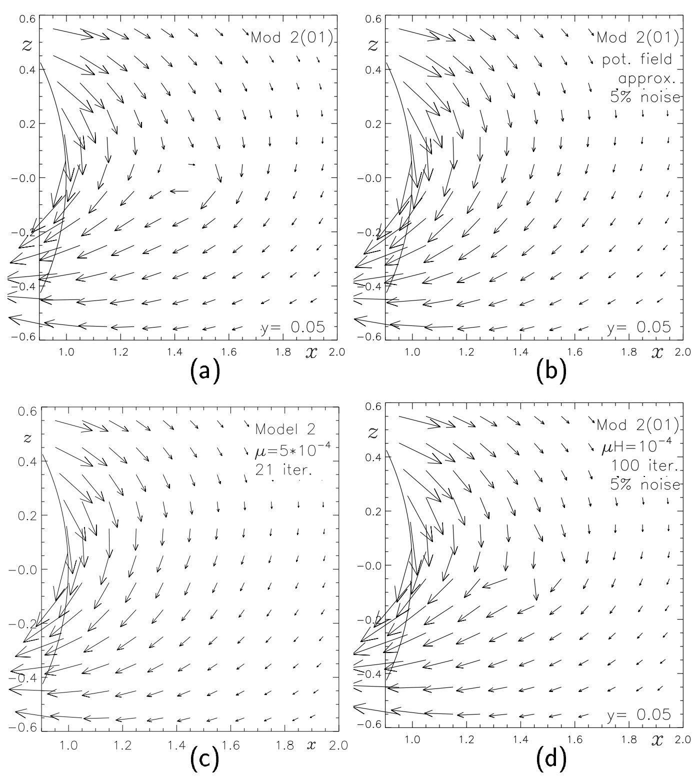

Fig. 5: Cross section by $y = 0.05$ plane: the original model field (a), potential field approximation (b), the reconstruction based on the Zeeman effect (c), and Hanle effect (d).

Summary and Outlook

- Using tomography technique based on the data obtained from longitudinal Zeeman-effect measurements, it is possible to reconstruct non-potential field having vortex-like configuration (Model I) when the vortex plane is perpendicular to the rotation axis. The Hanle-effect observations do not seem to be suitable for the reconstruction of such a field geometry.
- On other hand, in contrary with Model I, the Hanle-effect observations is helpful for reconstruction a field configuration presented by Model II, but Zeeman-effect data does not allow to reconstruct this field.

Questions still remain to be answered:

- How much noise is tolerable to achieve a certain precision of the solution.
- The influence of data gaps on the inversion result.
- The inversion technique presented here could be looked at as a first step towards a systematic LOS inversion of all Stokes components.

References

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