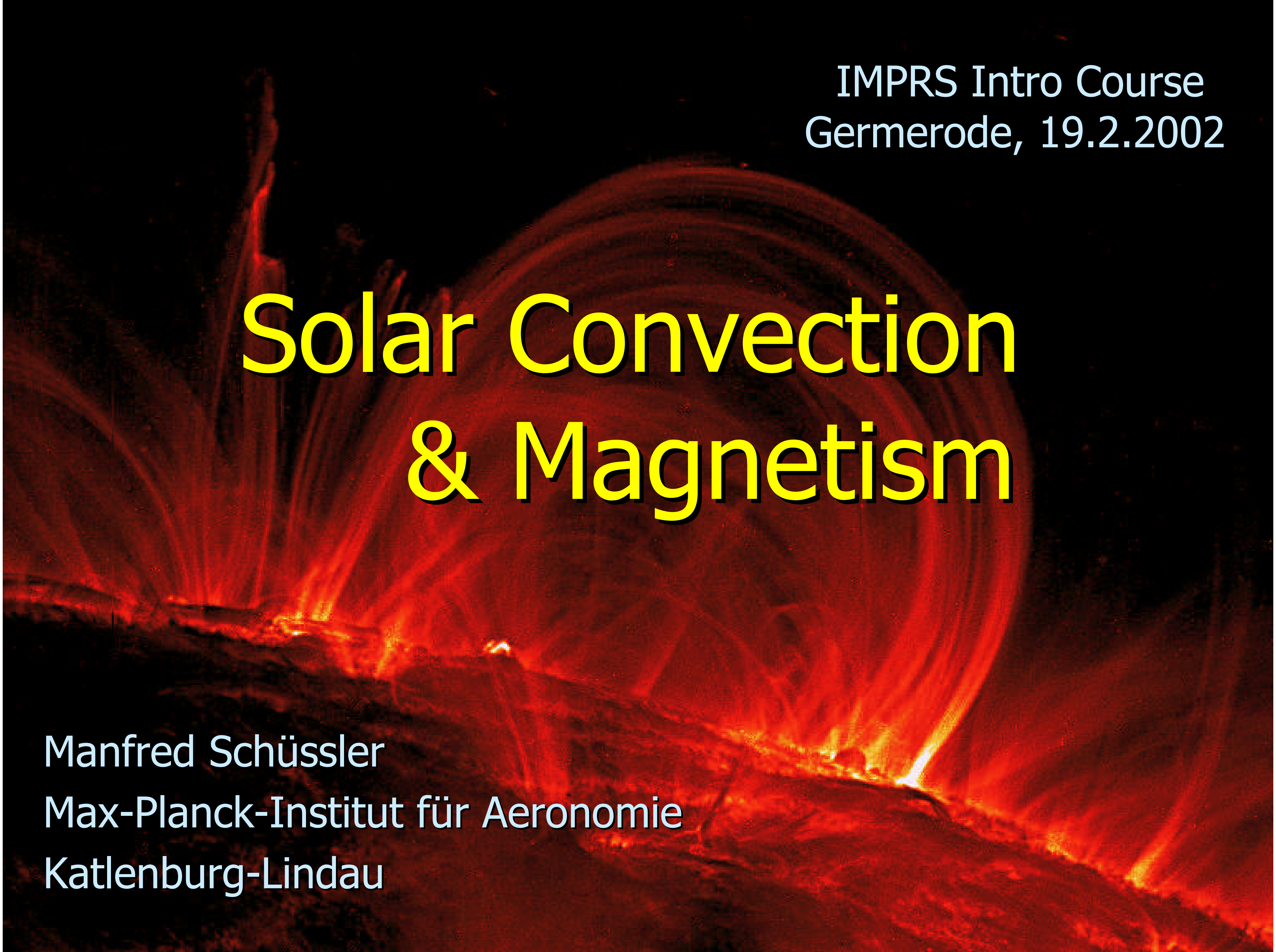
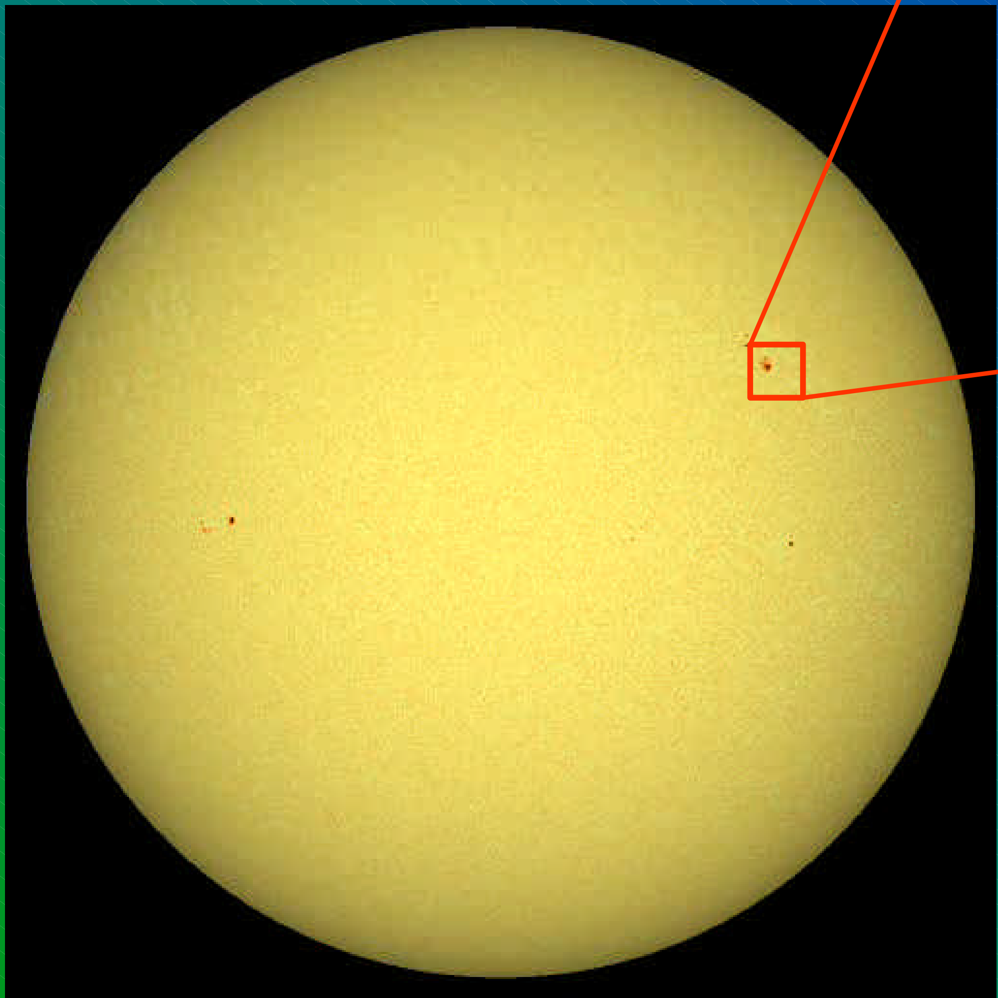


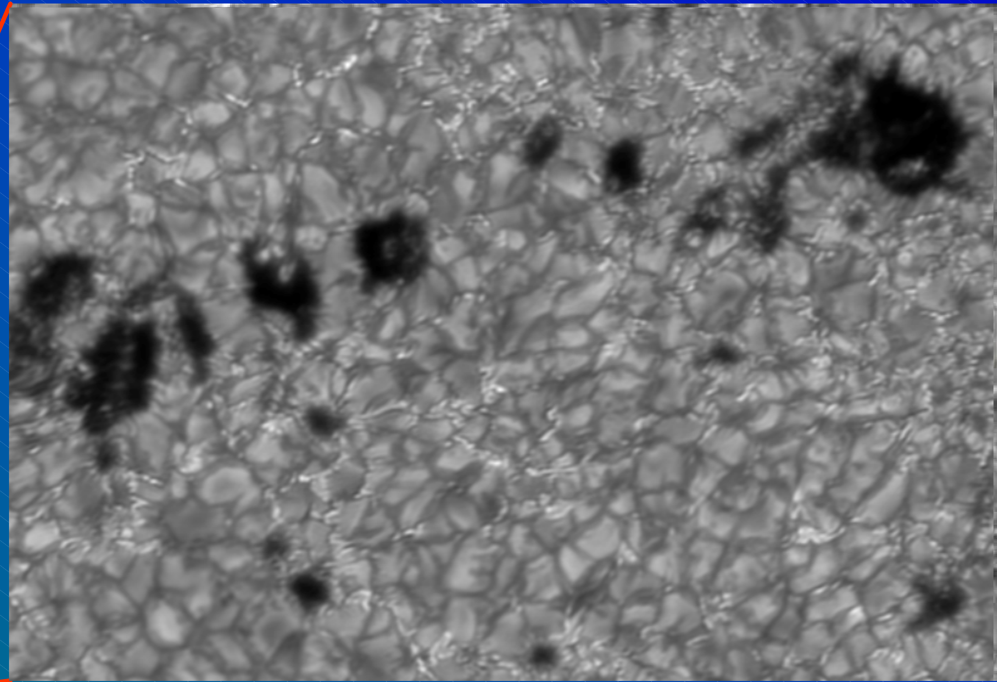
IMPRS Intro Course
Germerode, 19.2.2002

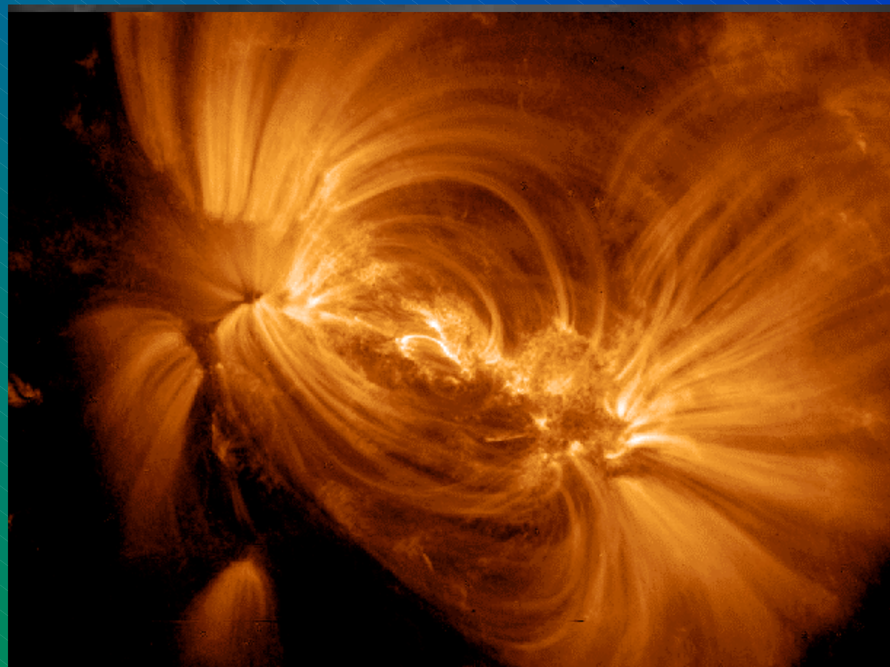
Solar Convection & Magnetism

Manfred Schüssler
Max-Planck-Institut für Aeronomie
Katlenburg-Lindau

Convection & magnetism: closely related

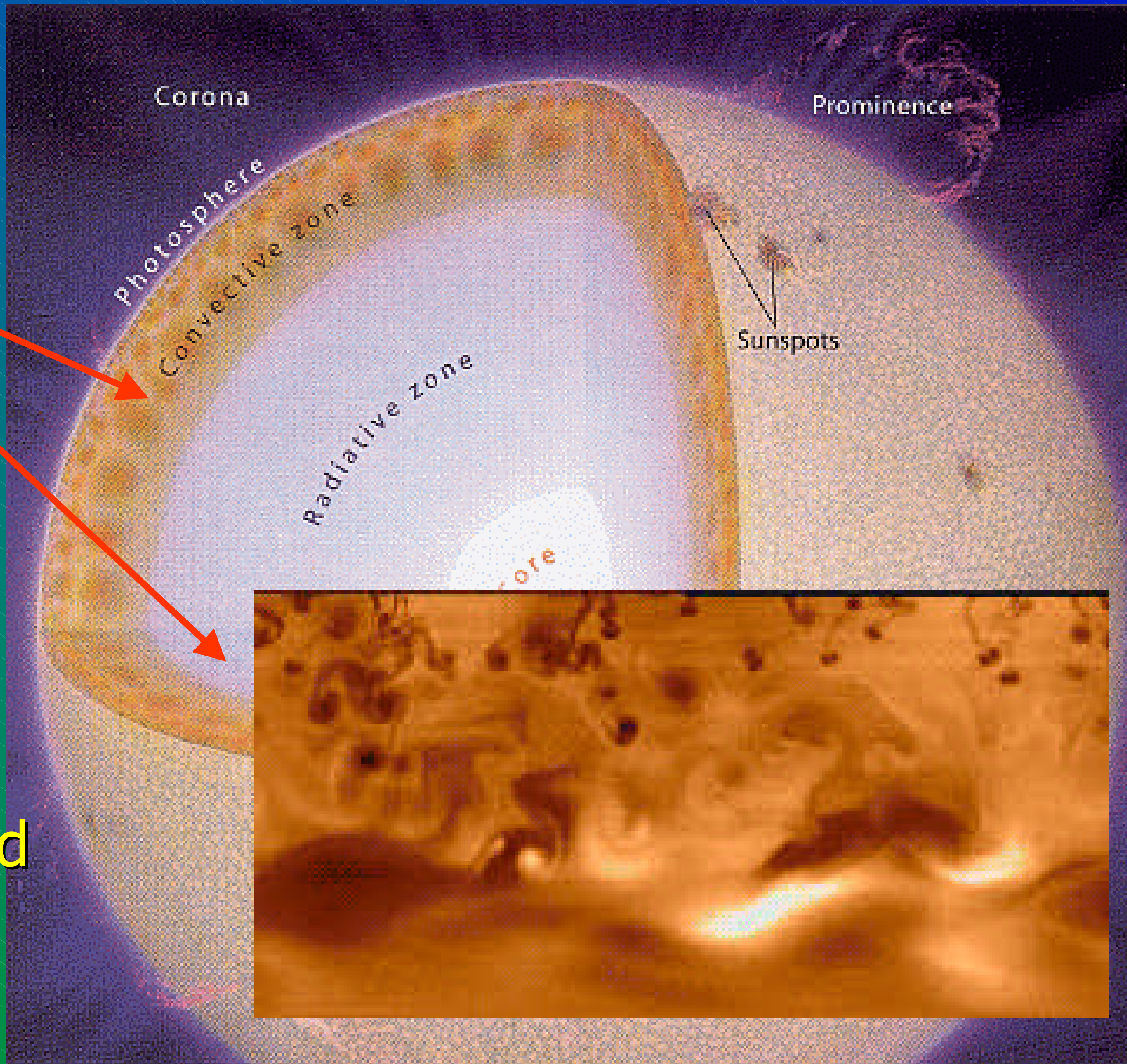




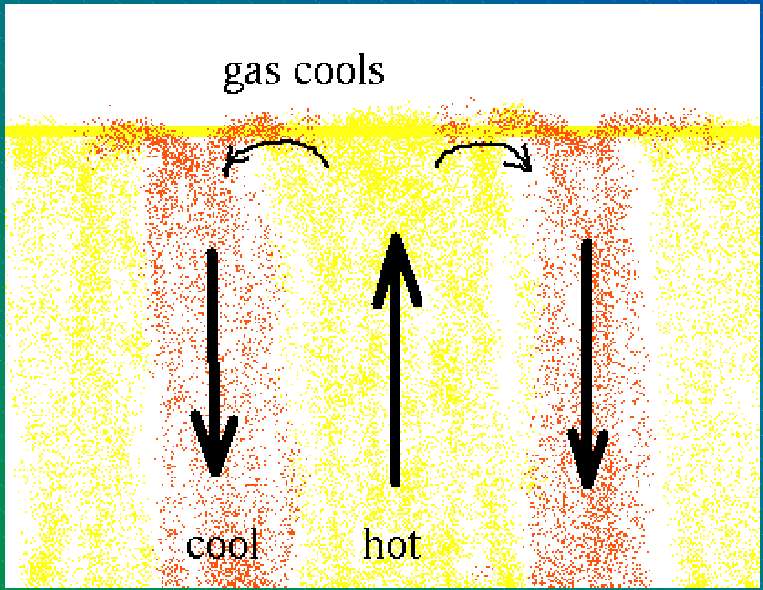


The solar convection zone

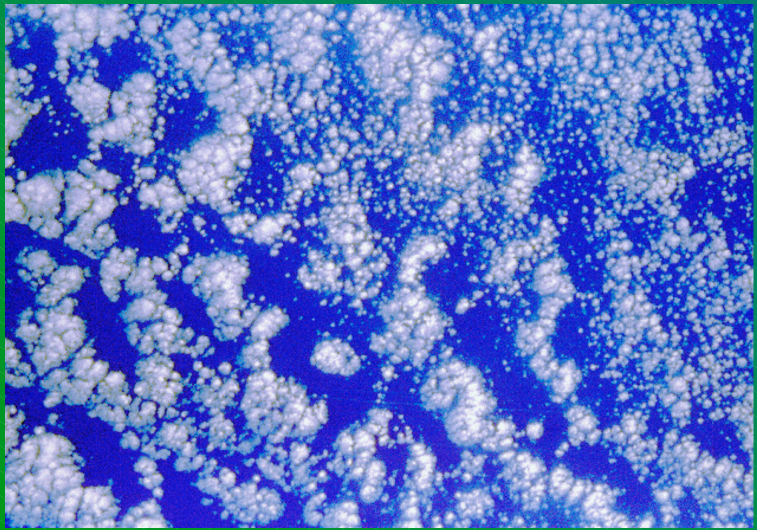
- 200 Mm thick layer in turbulent motion
- Velocities range from 100 m/s (bottom) to 10 km/s (top)
- Energy flux nearly completely transported by convective motion



What is convection?



- Flow driven by thermal buoyancy
- Convective instability



➔ Viewgraphs...

WHAT IS CONVECTION ?

- Transport of energy (internal, kinetic, latent, ...) by macroscopic motions (flows) driven by dynamical instability of a static equilibrium

- ... driven by an entropy gradient in thermodynamically open system

- Examples : air above a "radiator", earth's troposphere, stellar convection zones

- Convective motions normally are *overturning* ($\leftrightarrow$ oscillatory)

- treat solar atmospheric flow structures as "convective" in spite of stable stratification of observable layers ("overshoot")

- "Sun as a Lab": Concentrate on solar convection zone ($\rightarrow$ huge Reynolds number $Re = ul/\nu$, small Prandtl number $\sigma = \nu/\kappa$, strong stratification, pressure fluctuations important $\rightarrow$ laboratory convection results inapplicable)

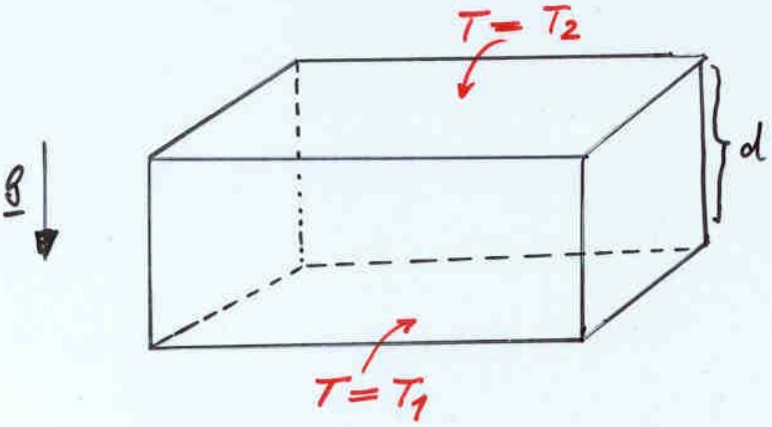
- *Not* discussed here, but important : Effects of/on rotation, oscillations, heating of upper atmosphere

CONVECTION IN ASTROPHYSICS

$\rightarrow$ *everybody's darling/problem*

- Basic *energy transport* mechanism, besides radiation
- Leads to strong *structuring* of late type star surface regions ($\rightarrow$ 1D-models inadequate) which affects the determination of atmospheric structure and abundances of elements
- Generates large-scale *magnetic fields* (dynamo) and small-scale magnetic structures (*flux tubes)*
- Provides mechanical energy for *heating* of chromospheres and coronae, drives stellar winds/mass loss
- Drives global *oscillations*
- Generates *differential rotation*
- Causes *mixing* and disturbs stellar evolution
- Is everywhere:
  $\Rightarrow$ envelopes of late-type stars
  $\Rightarrow$ cores of early-type stars
  $\Rightarrow$ accretion disks
  $\Rightarrow$ planetary interiors and atmospheres
- Solar atmosphere is a unique testing ground for understanding stellar convection : Processes can be observed on their natural time scales and length scales

LABORATORY CONVECTION
(RAYLEIGH- BÉNARD CONV.)



$\Delta T = T_2 - T_1 > 0$

„fluid in a box"

Conducted flux : $F_z \propto k \frac{dT}{dz} = \frac{k \Delta T}{d}$

Hydrostatic equil., fluid static for $\Delta T < \Delta T_c$

Instability for $\Delta T \geq \Delta T_c$

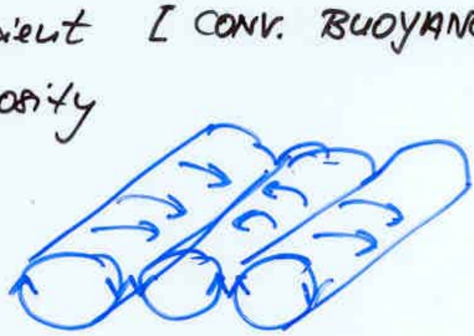
Rayleigh number : $\odot : \mathcal{R} \sim 10^{23}$

$$\mathcal{R} = \frac{\alpha g \Delta T d^3}{\nu \kappa} \geq \mathcal{R}_c \simeq 1700.$$

α : expansion coefficient [CONV. BUOYANCY-DRIVEN]

ν : (kinematic) viscosity

→ 2D rolls :



$\mathcal{R}\uparrow$: bifurcations → chaos

$$\frac{\partial T}{\partial t} + \underline{v} \cdot \underline{\nabla} T = \kappa \underline{\nabla}^2 T$$

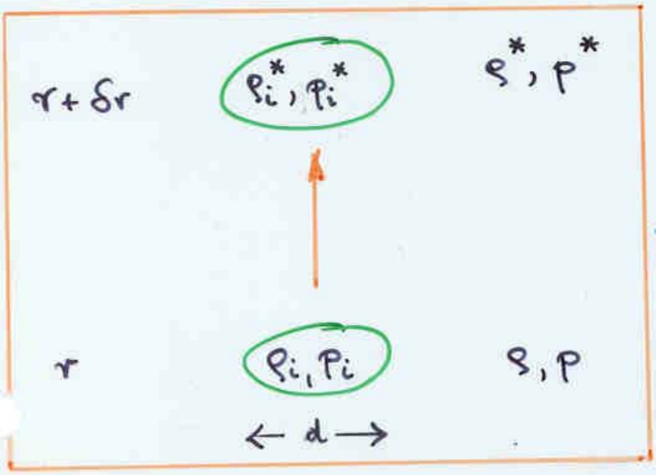
STARS : $\kappa = \kappa_{RAD}$

for $\mathcal{R} \geq \mathcal{R}_c$ is convection „more efficient":

Nusselt No. $Nu = \frac{F_{conv}}{F_{diff}(\underline{v} = \underline{0})} > 1$

3. CONVECTIVE INSTABILITY

HERE : „BLOB THEORY" → FLUCTUATIONS OF AN AVERAGE STATE



$d \ll H_P, \quad \delta\rho/\rho \ll 1$

BLOB TIMESCALE : $\tau_i = d/v$

THERMAL " : $\tau_{th} = d^2/\eta_R$

DYNAMICAL " : $\tau_{dyn} = d/c_s$

$\tau_i \ll \tau_{th} \quad \Rightarrow \quad S_i = S_i^*$ (ADIABATIC)

$\tau_i \gg \tau_{dyn} \quad \Rightarrow \quad p_i^* = p^*$ (NO PRESSURE FLUCTUATIONS)

INSTABILITY : $\rho_i^* < \rho^*$ (BUOYANCY)

$$\Rightarrow \quad \rho_i^* - \rho^* = \left[\left(\frac{d\rho}{dr}\right)_{ad} - \frac{d\rho}{dr} \right] \cdot \delta r < 0$$

EQ. OF STATE $\quad p = \mathcal{R}\rho T/\mu \quad \& \quad p_i^* = p_i$:

$$\Rightarrow \quad \frac{dT}{dr} < \left(\frac{dT}{dr}\right)_{ad} + \frac{T}{\mu}\left[\frac{d\mu}{dr} - \left(\frac{d\mu}{dr}\right)_{ad} \right]$$

$\frac{d\mu}{dr} \neq 0$
- → COMPOSITION CHANGES (STELLAR INTERIORS)
- → IONIZATION (STELLAR ENVELOPES)

→ CONSIDER SEPERATELY

SCHWARZSCHILD CRITERION

$$\frac{dT}{dr} < \left(\frac{dT}{dr}\right)_{ad}$$

$$\frac{1}{H_p} \equiv -\frac{1}{p}\frac{dp}{dr}$$ pressure scale height

REWRITE: $$\frac{dT}{dr} = \frac{T}{p}\underbrace{\frac{d\ln T}{d\ln p}}_{\nabla}\cdot\frac{dp}{dr} = -\frac{T}{H_p}\nabla$$

→ INSTABILITY FOR $\boxed{\nabla > \nabla_{ad}}$

ideal gas, no ionization : $\nabla_{ad} = \frac{\gamma-1}{\gamma} = 0.4 \ (\gamma=\frac{5}{3})$

RELATION TO ENTROPY GRADIENT

$$TdS = pdV + dE$$

ASSUME IDEAL GAS $dE = c_v dT$, $p = R\rho T$

+ USE $(dS)_{ad} = 0$ $R = c_p - c_v$

⇒ ... EXERCISE ...

$$\frac{dS}{dr} = \frac{c_p}{T}\left[\frac{dT}{dr} - \left(\frac{dT}{dr}\right)_{ad}\right] < 0 \text{ for INST.}$$

→ CONVECTIVE INSTABILITY

⇔ ENTROPY DECREASES OUTWARD

[ENTROPY SINK : STELLAR SURFACE]

5. OVERSHOOT

Convective motions extend into stable layers above and below the superadiabatically stratified region

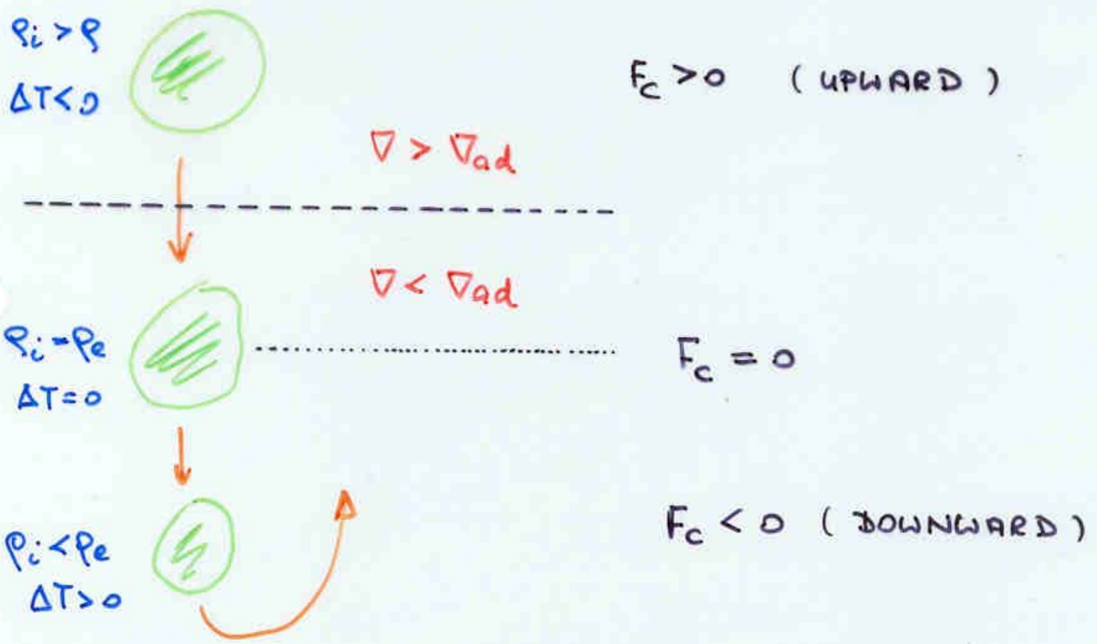
- Top (photosphere) : pressure gradients, radiative heating
- Bottom (radiative core) : penetration

⇒ Effective extension of the convection zone increases

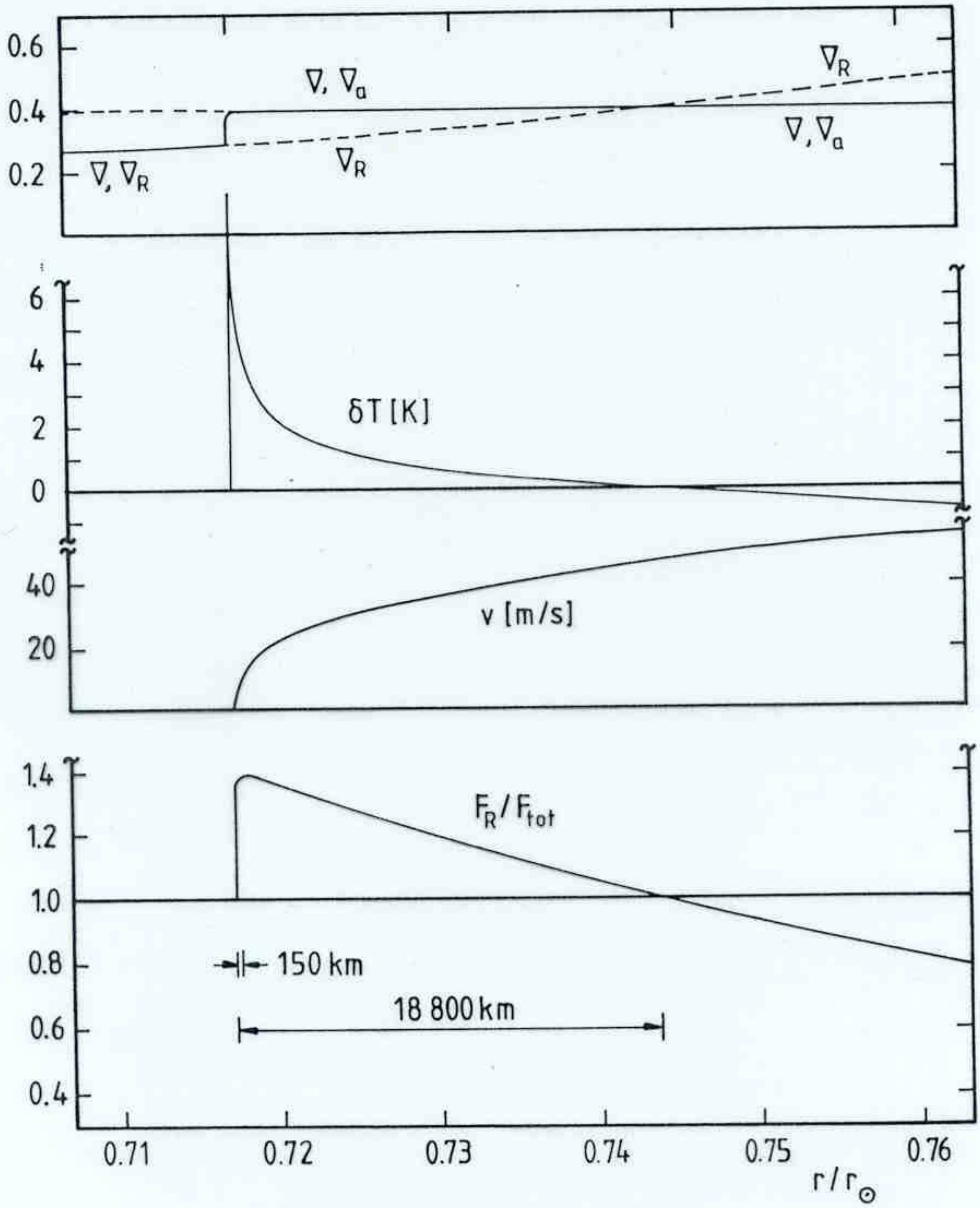
→ required by helioseismology

→ affects Li burning ($T > 2.5 \cdot 10^6$ K)

→ facilitates magnetic flux storage



NON-LOCAL TREATMENT NECESSARY



0.6
0.4
0.2
∇, ∇a
∇R
∇, ∇R
∇R
∇, ∇a
6
4
2
0
δT [K]
40
20
v [m/s]
1.4
1.2
1.0
0.8
0.6
0.4
FR / Ftot
150 km
18 800 km
0.71
0.72
0.73
0.74
0.75
0.76
r / r⊙

4. MIXING LENGTH DESCRIPTION

CONVECTION AS MIXTURE OF BLOBS WHICH MOVE VERTICALLY OVER A DISTANCE ℓ (THE MIXING LENGTH) AND DISSOLVE

[PRANDTL (1925), BIERMANN (1948), ŐPIK (1950), BÖHM-VITENSE (1953)]

TYPICALLY $\boxed{\ell = \alpha \cdot H_p}$, $\alpha = O(1)$

AIM: CALCULATE ENERGY FLUX AND MEAN QUANTITIES

(∇; S; ΔT, v of BLOBS, ...)

TEMPERATURE DIFFERENCE: [ignore factors of $O(1)$]

$$\Delta T = \left[\left(\frac{dT}{dr}\right)_i - \frac{dT}{dr}\right] \cdot \delta r = (\nabla - \nabla_i)\, T \frac{\delta r}{H_p} = (\nabla - \nabla_i)\, T \alpha$$

CONVECTIVE FLUX: [ERGS $\cdot$ CM^{-2} $\cdot$ S^{-1}]

$$F_c = \Delta T \cdot \rho c_p \cdot v = \rho c_p v T (\nabla - \nabla_i) \alpha$$

VELOCITY: [ACCELERATION BY BUOYANCY]

$$\ddot{\delta}_r = -g\, \Delta\rho/\rho = g \chi_p\, \Delta T / T = g \chi_p (\nabla - \nabla_i) \frac{\delta r}{H_p}$$

$\Rightarrow$ (BLOB HOMOGENEOUS) $$\dot{\delta}_r^2 = \frac{g \chi_p}{H_p} (\nabla - \nabla_i)\, \delta_r^2$$

$$\Rightarrow \quad \boxed{v = \left[\frac{g \chi_p}{H_p} (\nabla - \nabla_i)\right]^{1/2} \cdot \ell}$$

CONVECTIVE FLUX:

$$\boxed{F_c = \rho c_p T (g \chi_p H_p)^{1/2}\, \alpha^2 (\nabla - \nabla_i)^{3/2}}$$

ENERGY FLUX: $$F_{RAD} + F_c = L_\odot / 4\pi r^2$$

- $\nabla_i = \nabla_a \Rightarrow$ READY, $\nabla(r)$ DETERMINED
- $\nabla_i > \nabla_a$ DUE TO RADIATION → ... → CUBIC EQUATION
- TYPICAL VALUES IN DEEP CONVECTION ZONE:

  $\nabla - \nabla_a \approx 10^{-5} \ll 1$

  $\Delta T \approx 2\,K \ll T$

  $v \approx 100\,m/s \ll c_s$

  → CONVECTION IS VERY „EFFICIENT"
- ... AND NEAR THE SURFACE

  $\nabla - \nabla_a \approx 0.6$

  $\Delta T \approx 2000\,K$

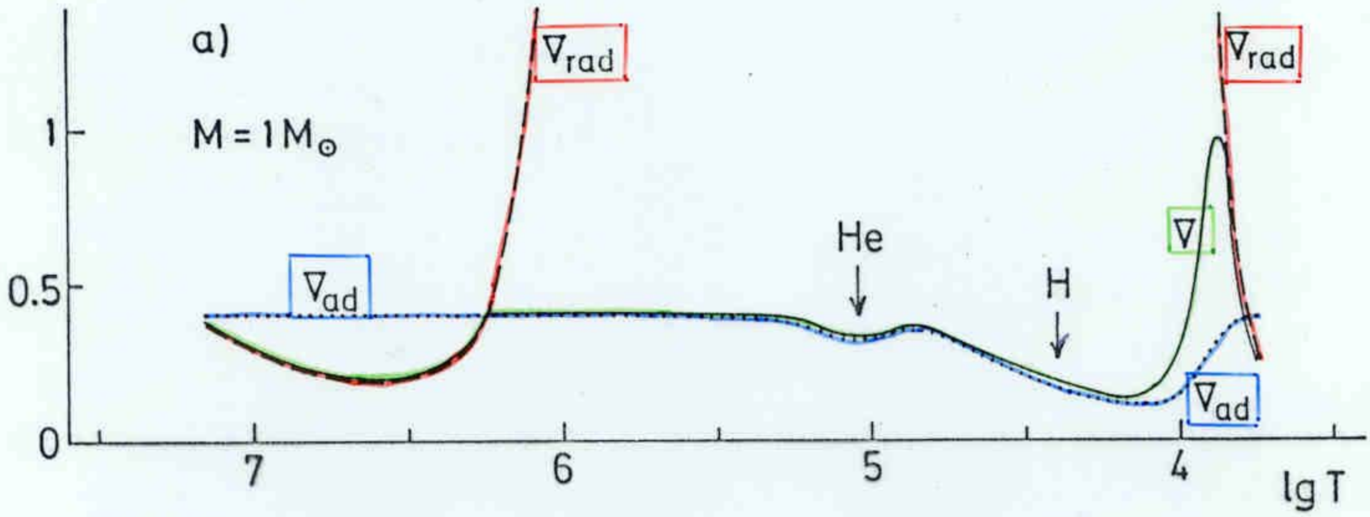
  $v \approx 2\,km/s$

  → ASSUMPTIONS BECOME INVALID
- MIXING LENGTH DESCRIPTION

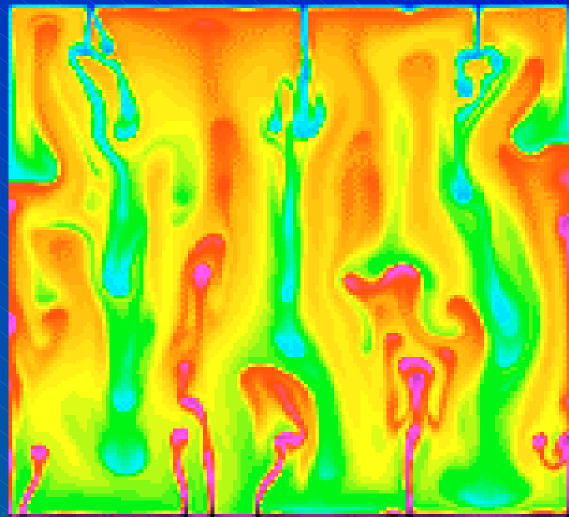
  ≈ TURBULENT DIFFUSION OF ENTROPY WITH $\eta \sim v \cdot \ell$
- IONIZATION (H, He) REDUCES ∇_{ad} (LATENT HEAT)

  → DESTABILIZING
- DRAWBACKS OF M.L.D.:
  - LOCAL (NO OVERSHOOT)
  - ADJUSTABLE PARAMETERS (NO PREDICTIONS)
  - EFFECTIVELY INCOMPRESSIBLE, NEGLECTS PRESSURE FLUCTUATIONS, STATIFICATION (BOUSSINESQ - APPROX.) (INAPPLICABLE TO OBSERVABLE SURFACE FLOWS)

∇_{RAD}, ∇_{AD}, ∇ IN A STANDARD MIXING LENGTH MODEL OF THE SOLAR CONVECTION ZONE



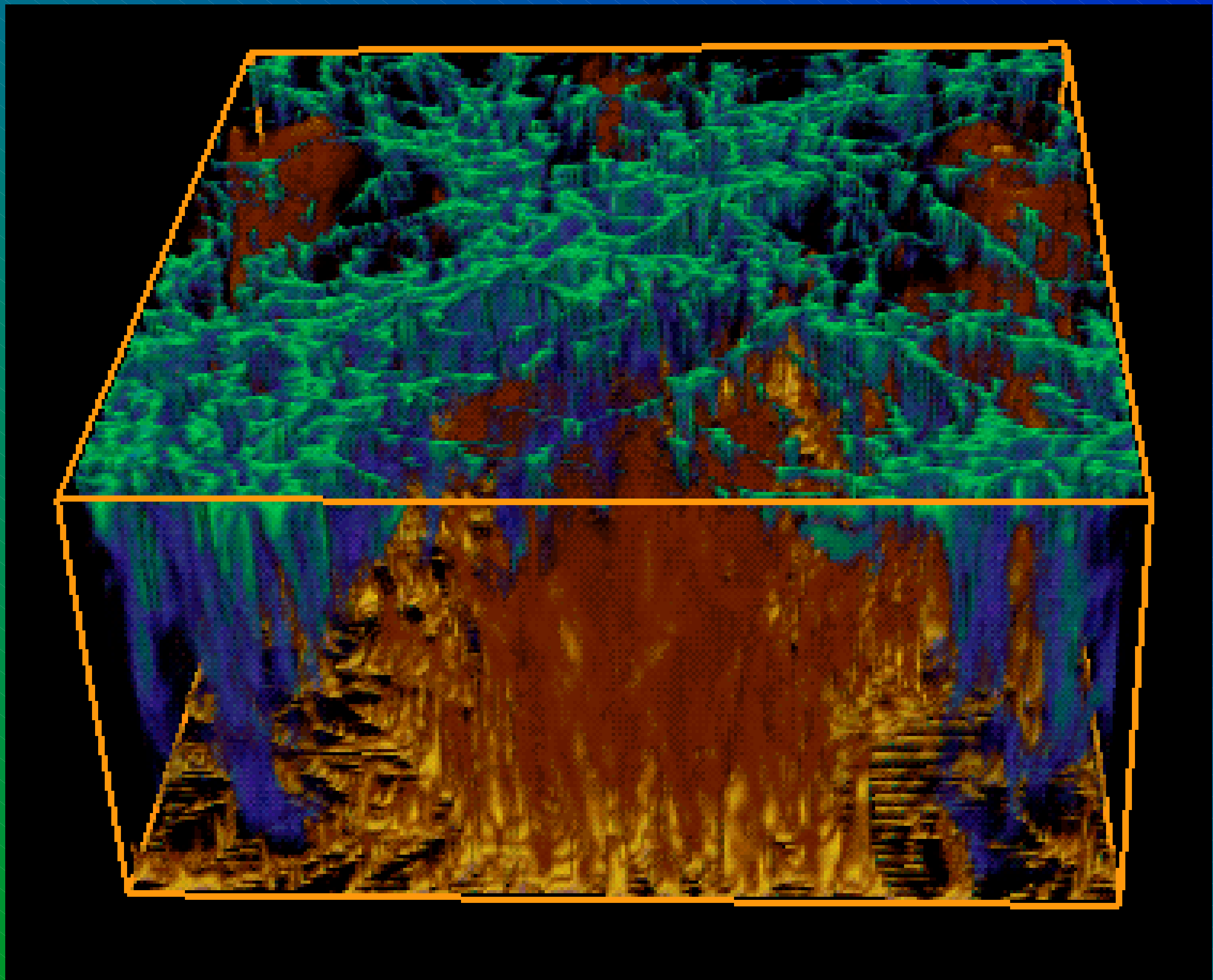
Laboratory results and numerical experiments for Rayleigh-Bénard convection with high Ra



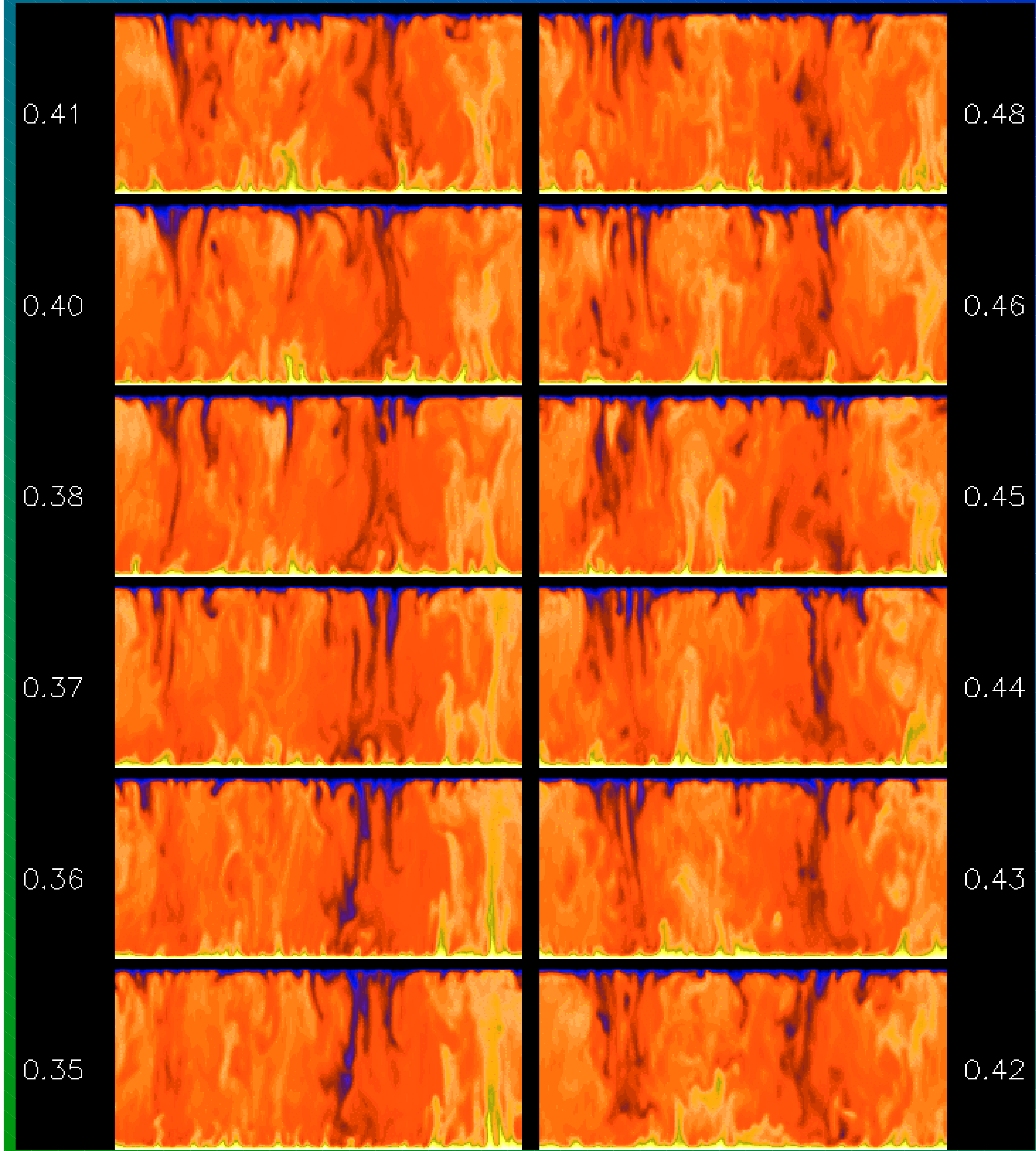
- lab: Ra up to 10^{17} (Helium gas at ~5 K)
- Ra > 2.5 10^{5} : turbulence, Nu ~ $Ra^{1/3}$
- Ra > 4 10^{7} : “hard turbulence”, Nu ~ $Ra^{2/7}$
- coherent threads/plumes/thermals of buoyant fluid connect top & bottom
- flows driven by threads, passive non-buoyant fluid between
- non-local energy transport, global circulation
- with rotation: vortex interaction of plumes, coalescence

- Solar convection strongly different from Rayleigh-Bénard: boundary structure, stratification, compressibility...

Filamentary flows in Rayleigh-Bénard convection



Kerr 1996



Time evolution of filamentary flows in Rayleigh-Bénard convection

Kerr 1996

SOLAR CONVECTION : OBSERVATIONAL APPROACHES

Continuum images, filtergrams

⇒ spatial structure : bright granules & interconnected network of dark intergranular lanes
⇒ temporal evolution : formation, dissolution, break-up of granules, "exploding granules"

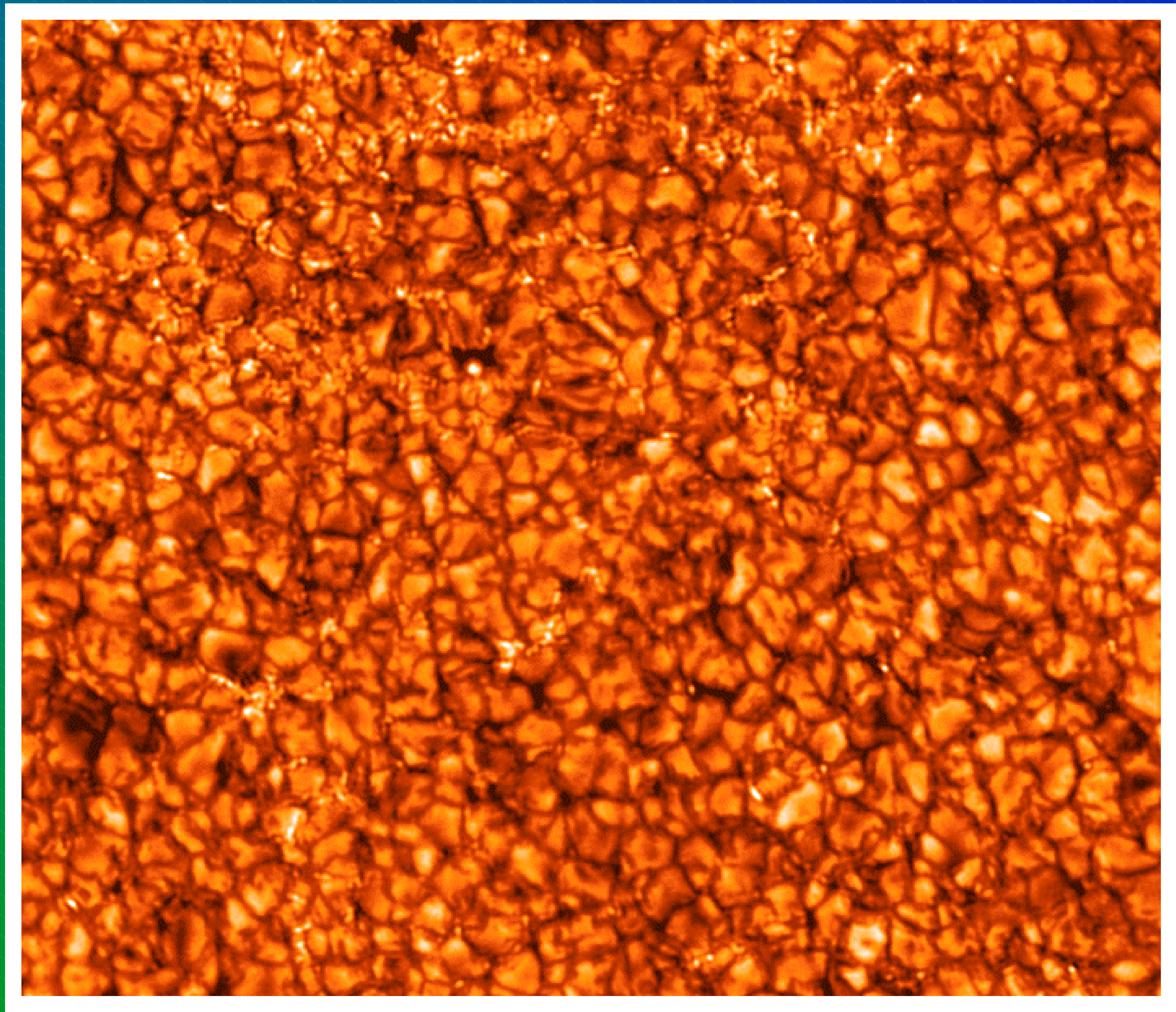
Spectrograms

⇒ line-of-sight velocities via Doppler effect ("line wiggles")
⇒ upflow in granules, downflow in intergranular lanes
⇒ velocity distributions & gradients (line asymmetries)
⇒ supergranulation (Dopplergrams)

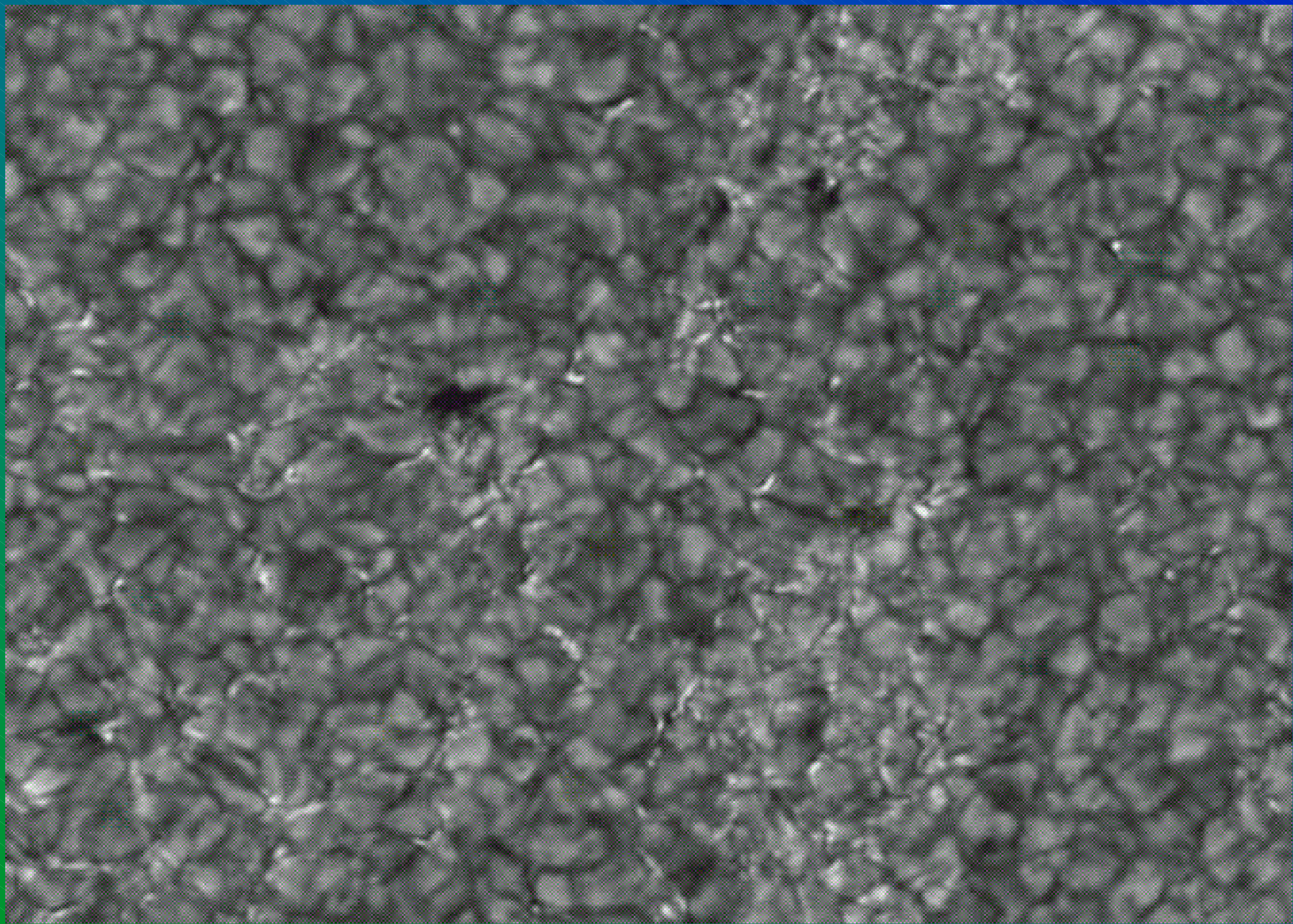
Proper motions of "tracers"

⇒ horizontal velocities
⇒ mesogranulation by "local correlation tracking"

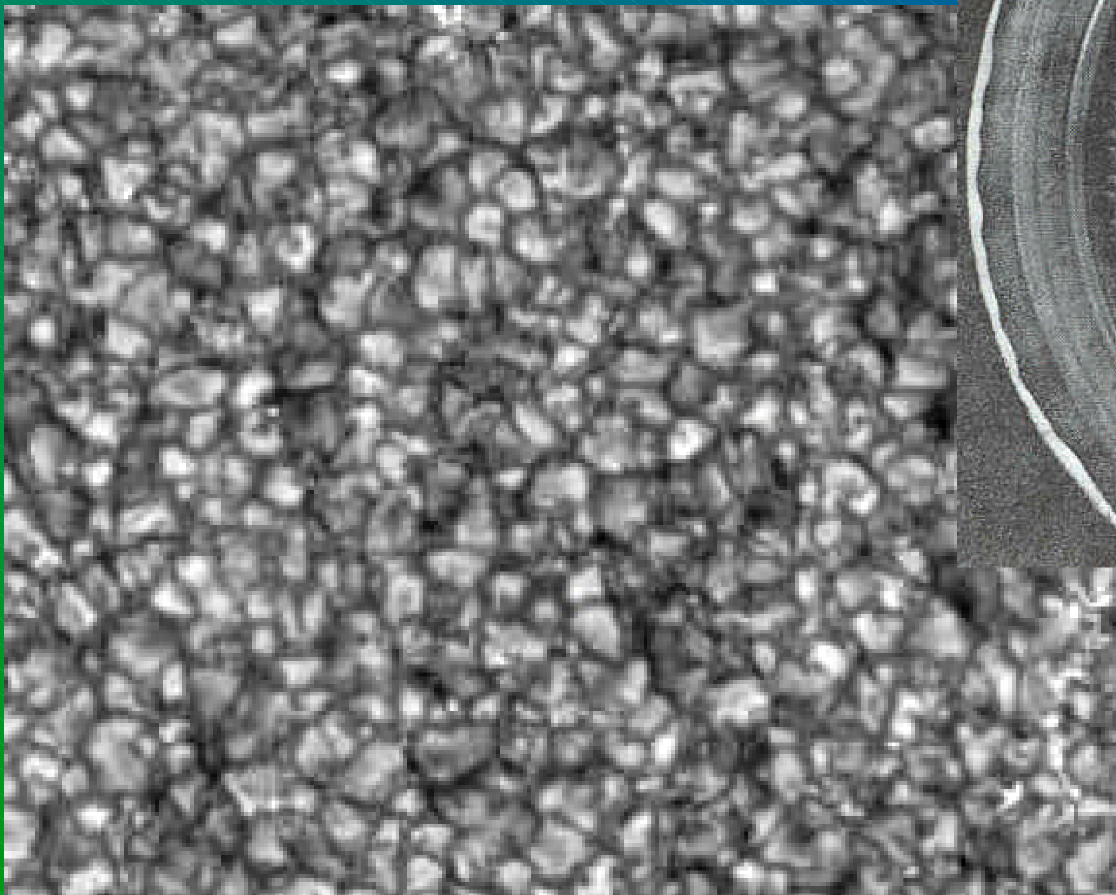
Granulation: Solar surface convection

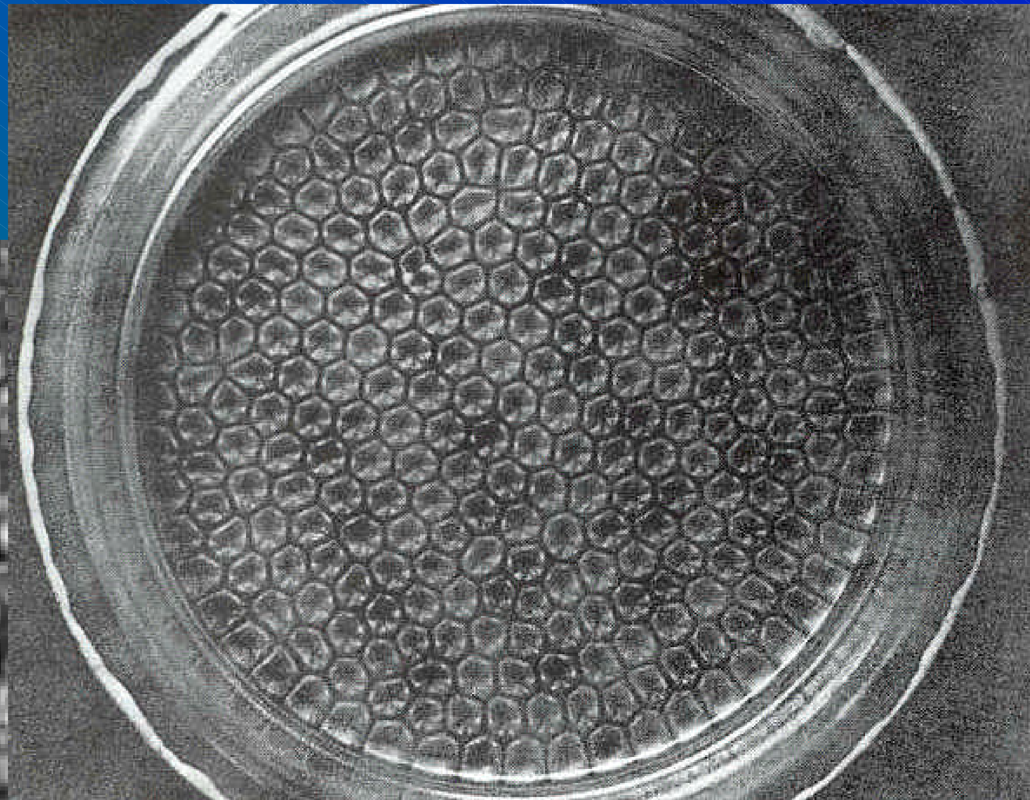


Solar granulation

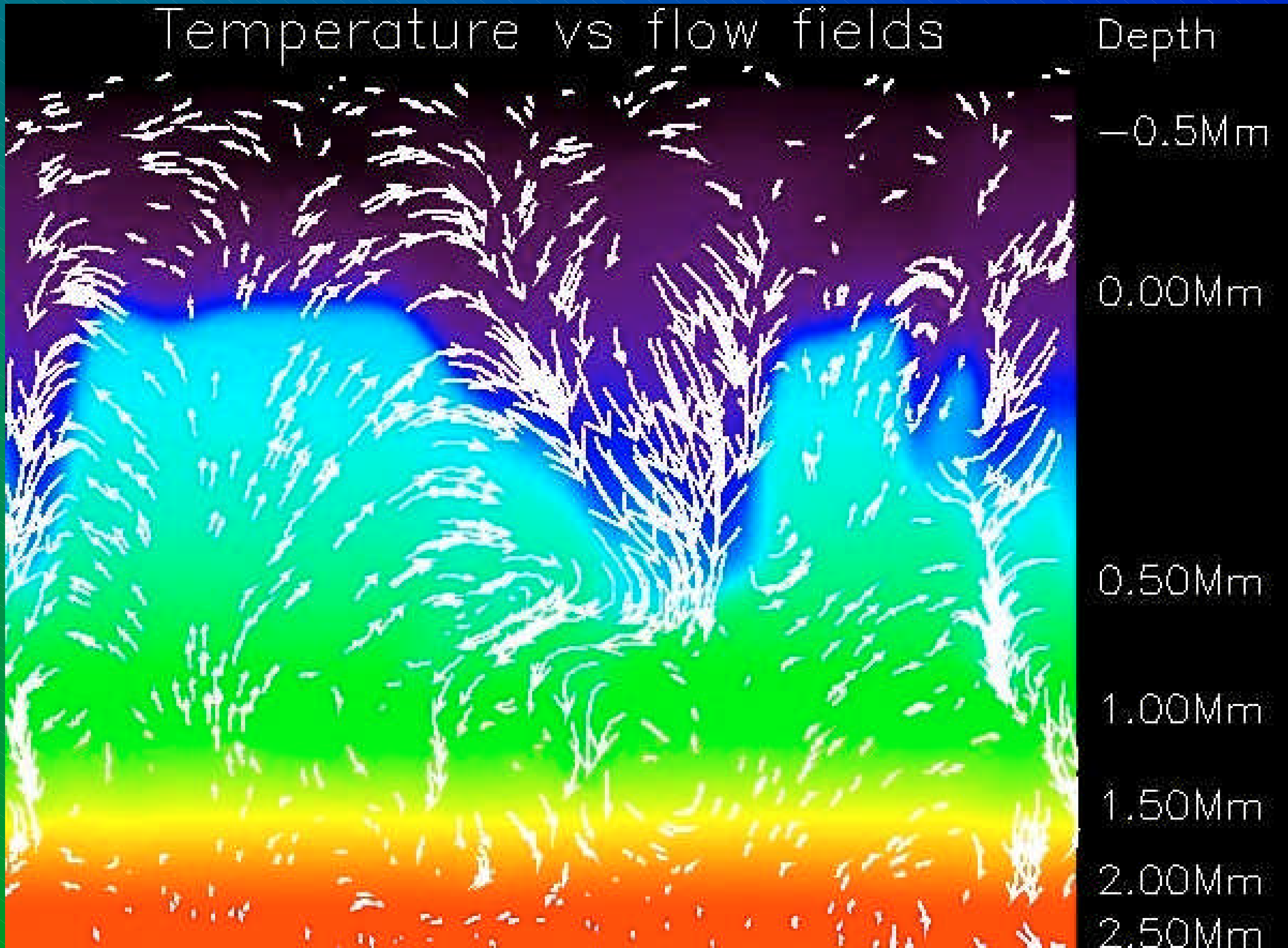


Granulation und laboratory convection

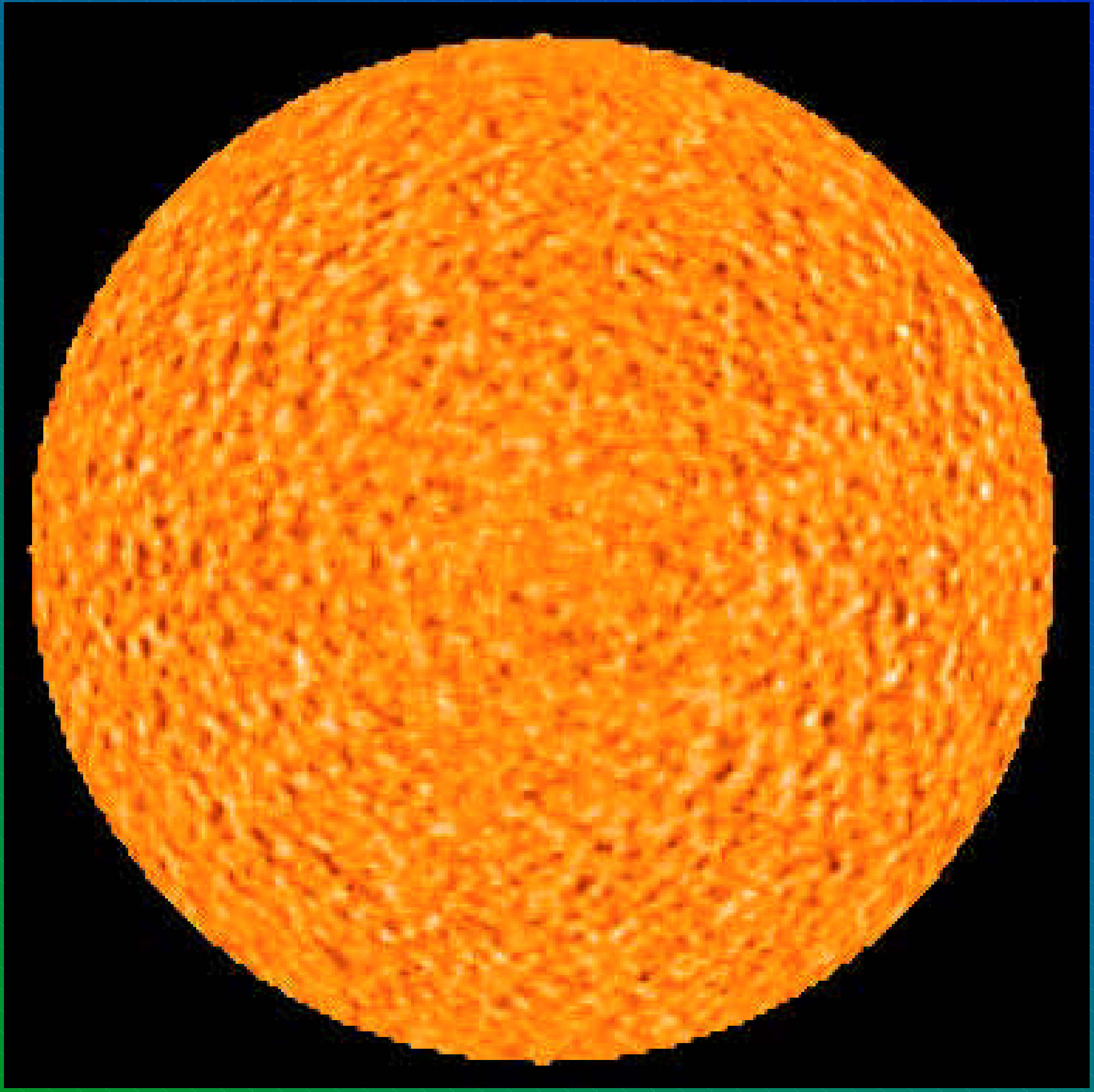




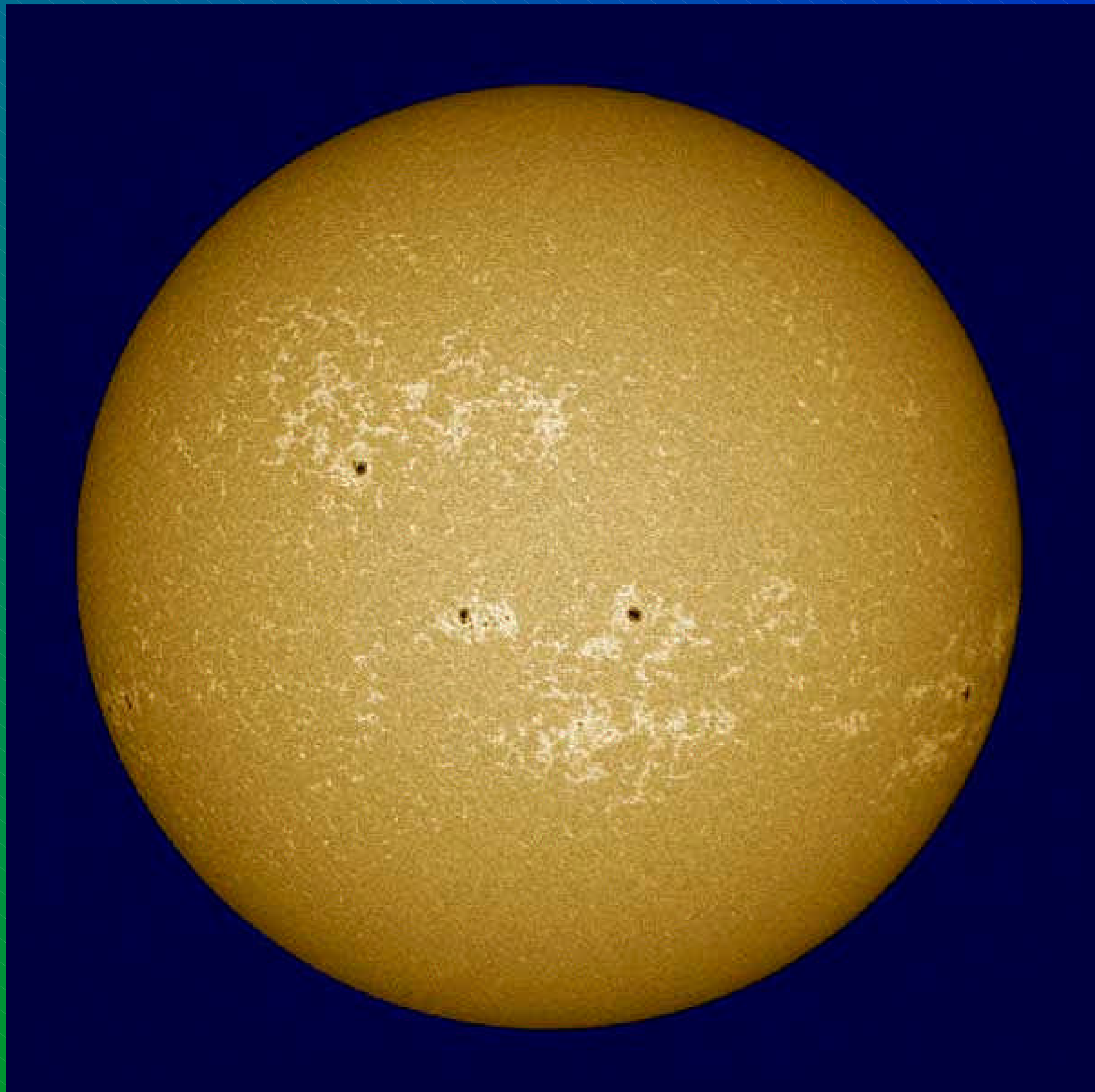
Granulation as a convective phenomenon



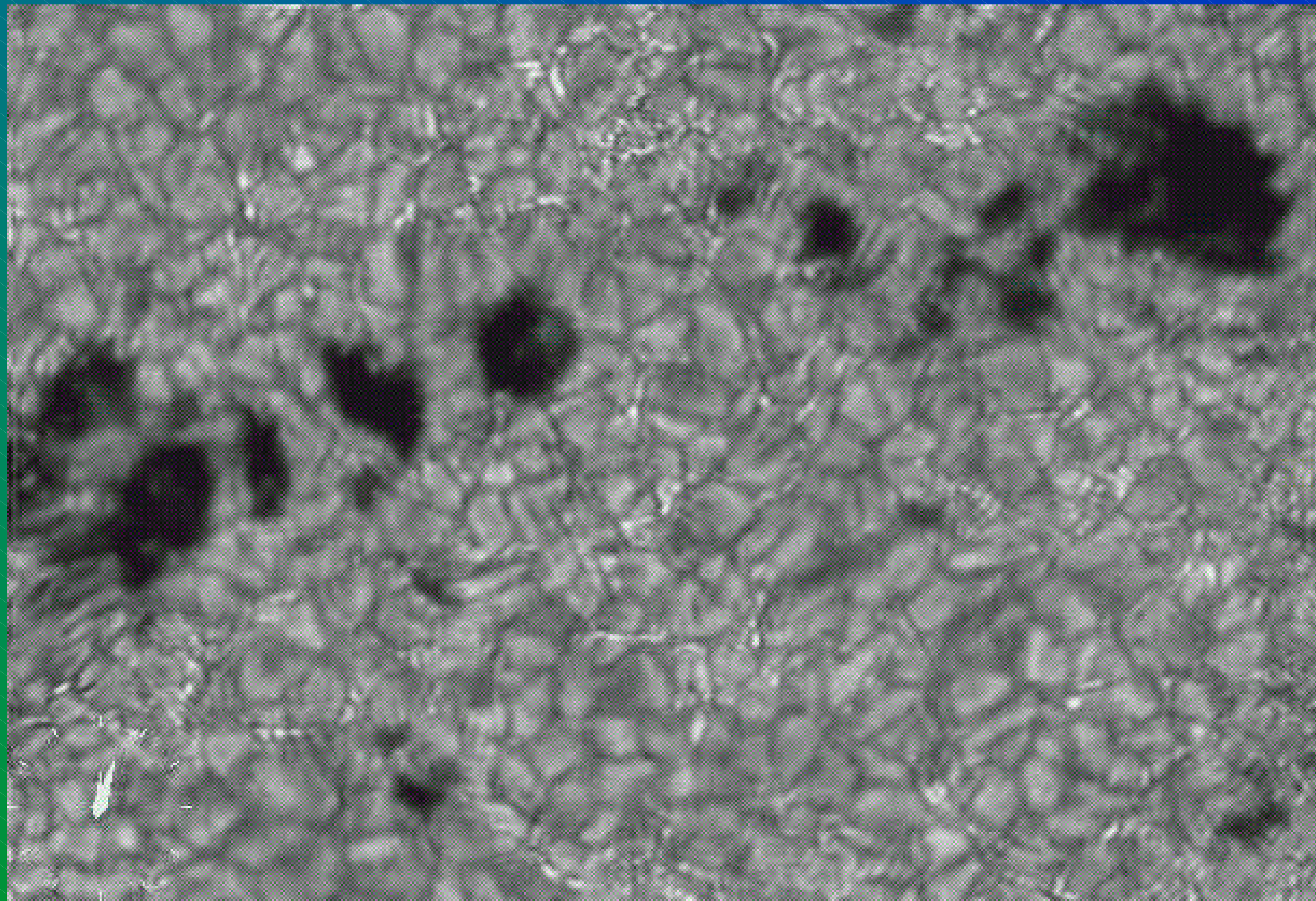
Supergranulation



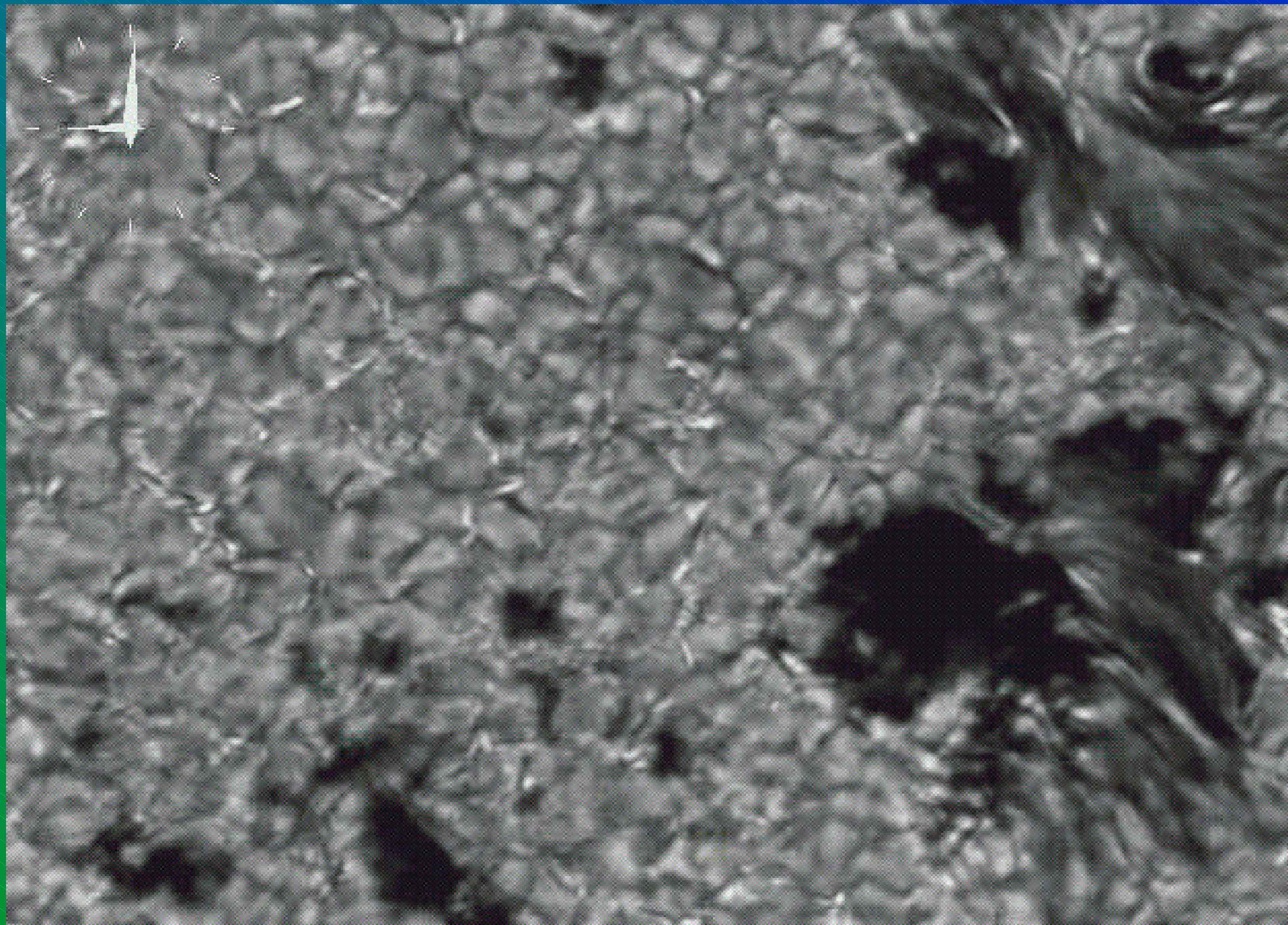
Supergranulation and magnetic field: the Ca^+ network



Granulation, sunspots, & small-scale magnetic field



"Mesogranulation" ?



6. Numerical Simulations

Solar convection, in particular near the surface, is unsufficiently described by concepts like mixing length, linear superposition of eigenmodes, Boussinesq approximation, etc. because of

→ strong density stratification,

→ nonlinearity of the flows,

→ non-stationarity,

→ compressibility effects, shocks.

⇒ Numerical simulations

Equations and problems

Continuity:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \, \mathbf{u})$$

⇒ compressibility, strong stratification

Equation of motion:

$$\rho \left(\frac{\partial \, \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \, \mathbf{u} \right) = -\nabla p + \rho \, \mathbf{g} + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} + \mathbf{f}_{visc}$$

⇒ $Re = u \cdot l / \nu \gg 1$ → small scales → "sub-grid" transport coefficients (viscosity, thermal & magnetic diffusion)

⇒ steep gradients, concentrated flows

Energy equation:

$$\frac{\partial T}{\partial t}+\mathbf{u}\cdot\nabla T = -\frac{T}{\chi_p}(\gamma-1)\nabla\cdot\mathbf{u}-\frac{\gamma}{\rho c_p}\nabla\cdot(\mathbf{F}_{rad}+\mathbf{F}_{visc}+\mathbf{F}_{mag}+\mathbf{F}_{turb})$$

$\Rightarrow$ partial ionization effects $\rightarrow$ thermodynamical quantities must be determined self-consistently

$\Rightarrow$ surface layers $\rightarrow$ radiation has to be treated accurately, radiative energy loss to space drives the whole flow $\rightarrow$ solve transfer equation, preferably non-grey (opacity distribution functions)

Magnetic induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times (\eta_m \nabla \times \mathbf{B})$$

$\Rightarrow$ steep gradients, concentrated fields & currents

$\Rightarrow$ "sub-grid" magnetic diffusivity

in addition...

$\rightarrow$ equation of state,
$\rightarrow$ equations for ionization equilibrium,
$\rightarrow$ equations for thermodynamic quantities (χ_p, γ, c_p, ...)
$\rightarrow$ determination of opacities,
$\rightarrow$ integration of radiative transfer equation along many lines of sight for many angles, calculation of mean intensity and radiative flux
$\rightarrow$ determination of diagnostic information (mean quantities, correlations, single and average spectral line profiles, continuum intensity maps...)
$\rightarrow$ visualization of flow properties and time evolution

- **Boundary problems**

Real boundaries:

→ top : $\tau = 1$, radiation, steep T-gradient

→ bottom : overshoot, thermal boundary layer

Artificial boundaries:

→ transmitting, non-reflecting, open

Initial conditions:

→ simulation time > thermal relaxation time

- **Approaches**

Numerical experiments

→ simplified physics, consider general properties without attempting to model the Sun → Graham (1975, 1977), Hurlburt et al, (1984, 1986), Chan & Sofia (1986, 1987), Hossain & Mullan (1990), Malagoli et al. (1990), Cattaneo et al. (1991)

Global convection

→ simulate bulk of solar convection zone, neglect surface regions, trade resolution for total size → Gilman & Glatzmaier (1981,...)

Surface convection

→ simulate observable solar convection on granular/mesogranular scale → Nordlund (1984,...), Gadun (1986), Stein & Nordlund (1989), Steffen (1989)

- **Results : Simulation of convection**

→ *Strong inhomogeneities, steep gradients (temperature, velocity)*

→ *Significant pressure fluctuations, "buoyancy braking"*

→ *Supersonic velocities, shocks*

→ *Asymmetry of up- and downflows, non-local dynamics*

⇒ strong, narrow downflows ↔ broad upflows
⇒ 3D, turbulent: Coherent pattern of downdrafts, disorganized upflows
⇒ topology changes with depth → "granulation" is a shallow phenomenon

→ *Downdrafts play a prominent role:*

⇒ sites of buoyancy driving

⇒ kinetic energy flux

⇒ helical motion due to "bathtub effect"

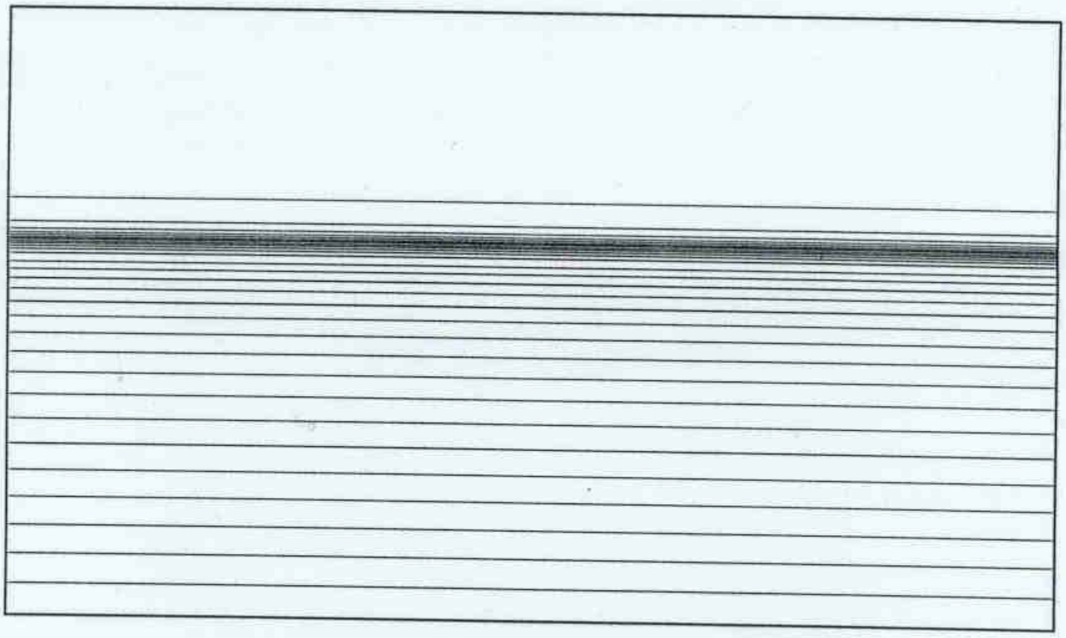
→ Other transport processes strongly non-local (magnetic fields, angular momentum...)

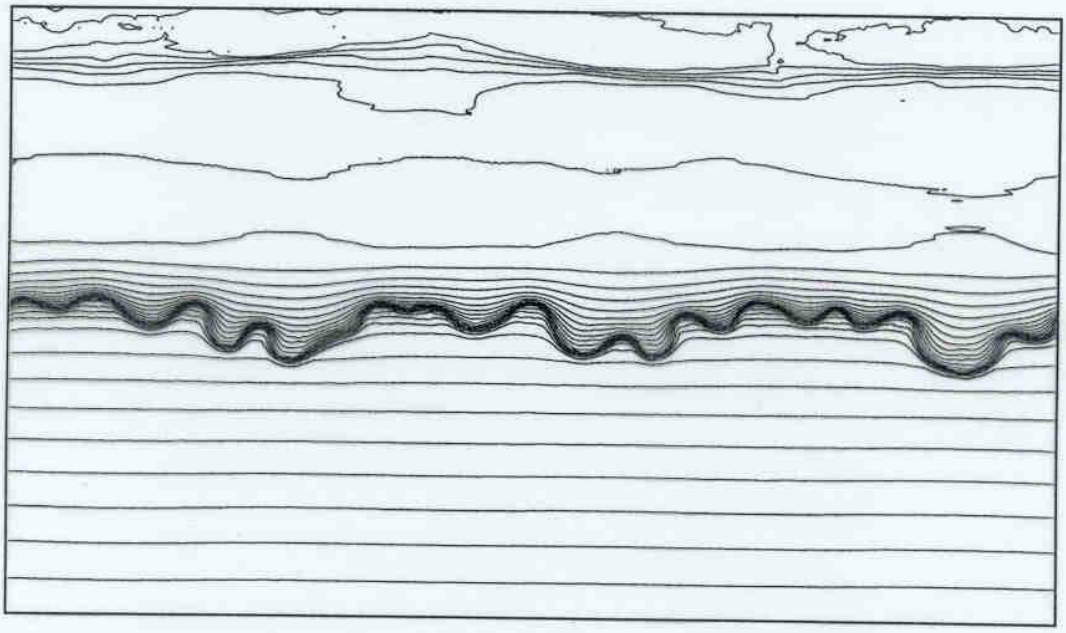
→ *Essential features of observed granulation are reproduced by simulations with radiative transport:*

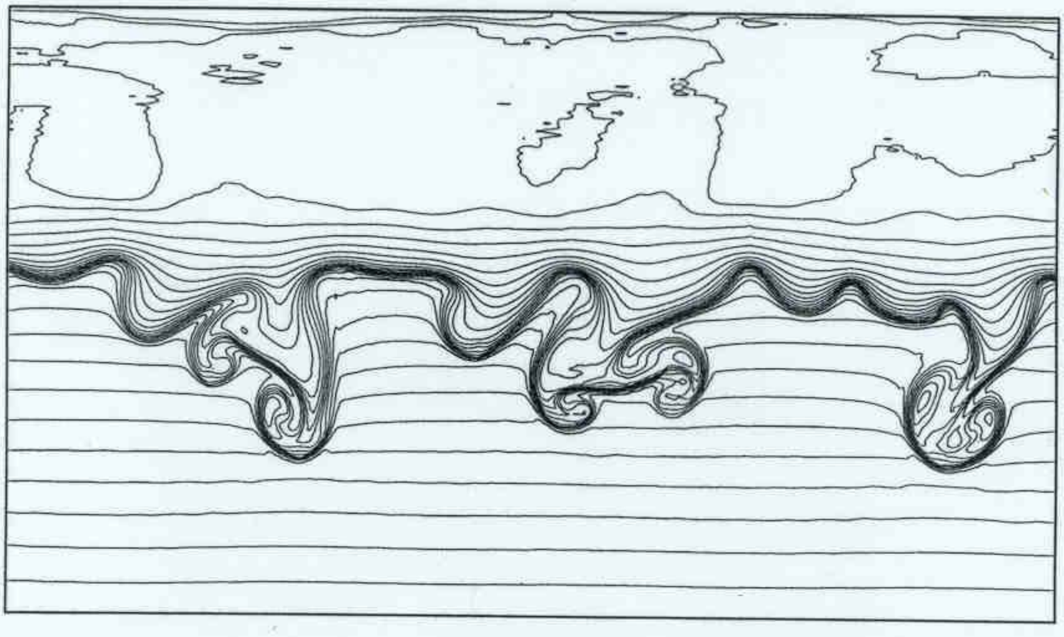
⇒ isolated, hot upflows – network of cool downflows

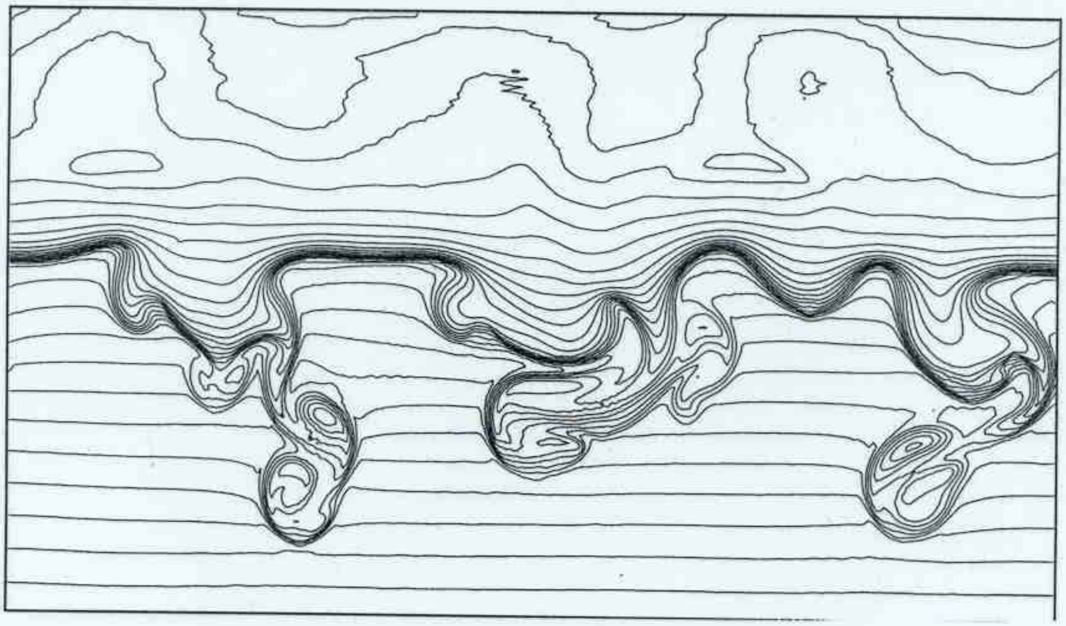
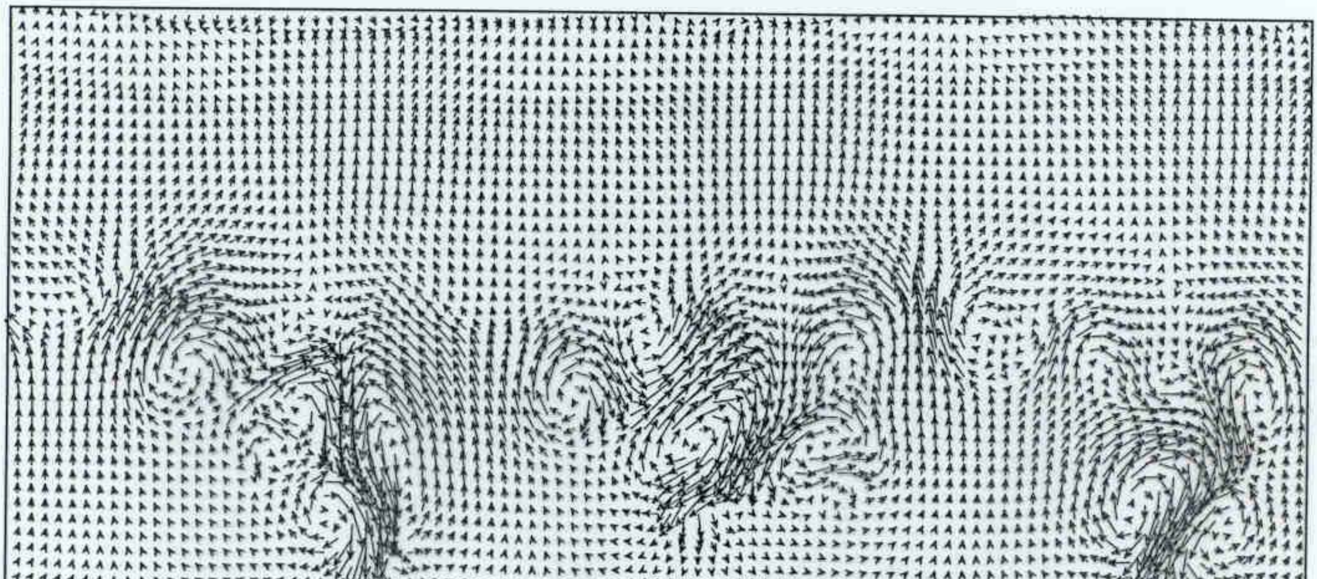
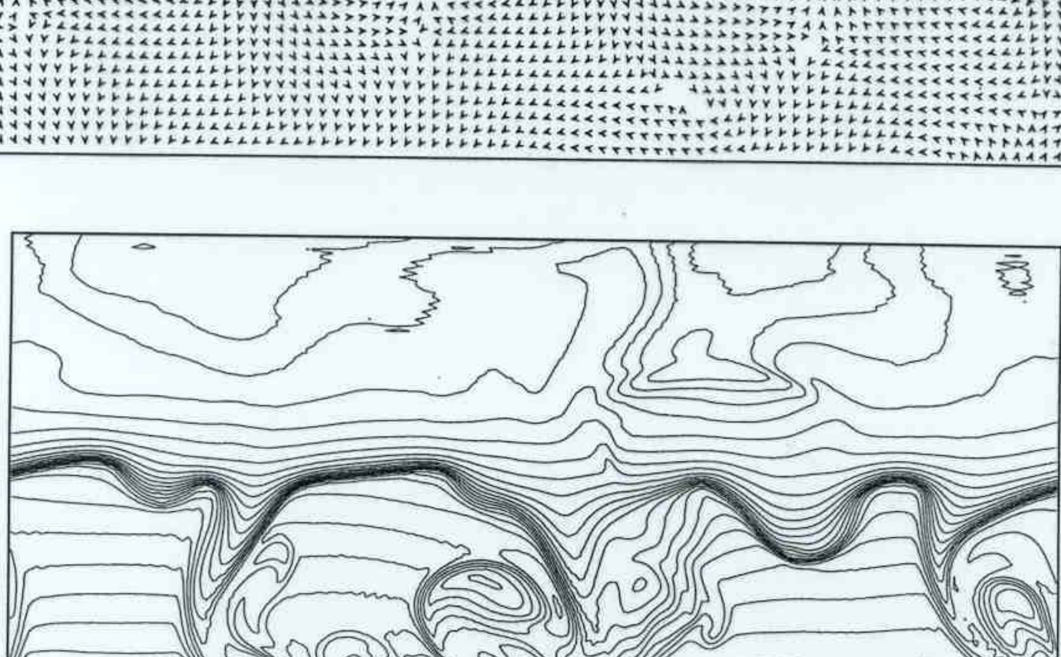
⇒ timescales, spatial scales, "exploding granules"

⇒ average line profiles







ASYMMETRY OF UP- AND DOWNFLOWS

HURLBURT ET AL. (1984) [2D SIMULATION]

$$\int \frac{dp}{\rho} + \frac{u^2}{2} \approx \text{const.}$$

ACC. — $\frac{P'}{\langle P \rangle} > 0$ ↓

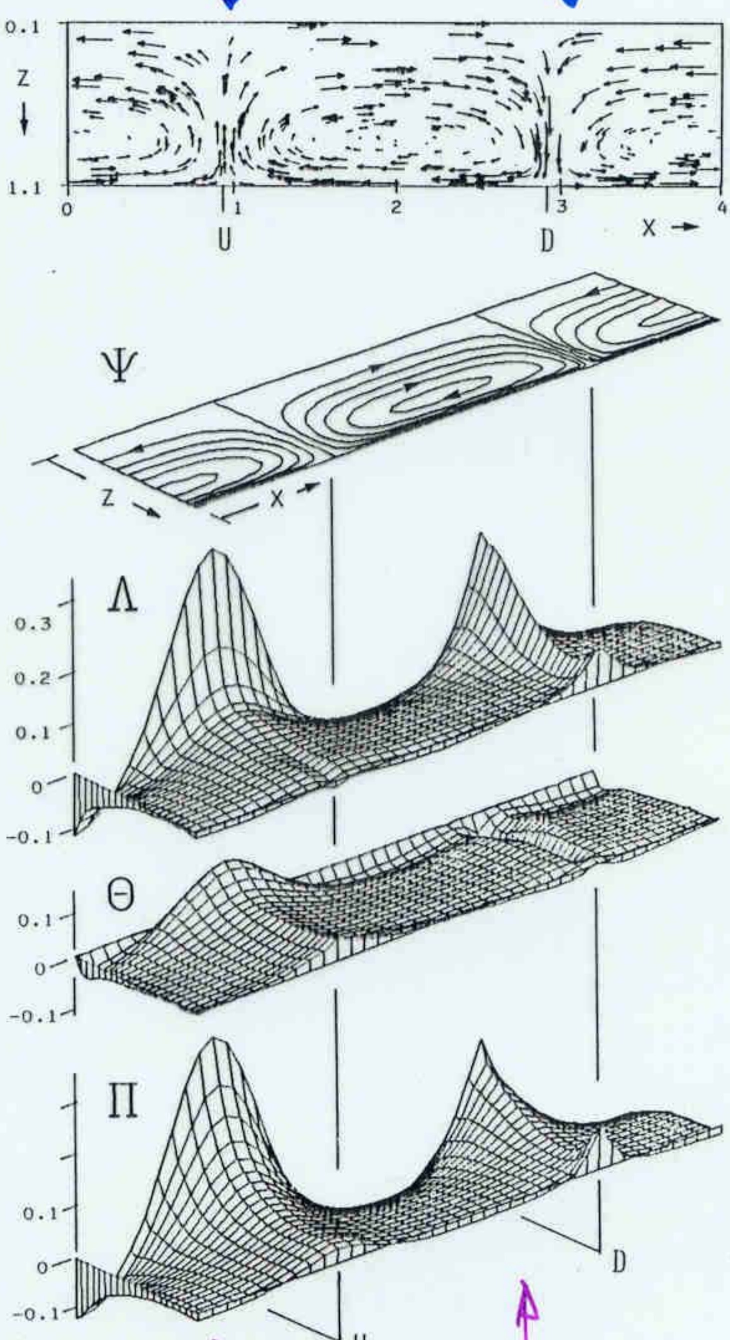
BRAKING — $\frac{P'}{\langle P \rangle} > 0$ ↓

$\frac{\rho_1}{\rho_2} = 11$

EQ. OF STATE ↓

$$\frac{\rho'}{\langle \rho \rangle} = -\frac{T'}{\langle T \rangle} + \frac{P'}{\langle P \rangle}$$

< 0 UP > 0 DOWN > 0



"BUOYANCY BRAKING" IN UPFLOW — $\frac{\rho'}{\langle \rho \rangle}$

↓

DOWNFLOW ENHANCED — $\frac{T'}{\langle T \rangle}$

↓

UP/DOWN ASYMMETRY (continuity !) — $\frac{P'}{\langle P \rangle}$

↓

KINETIC FLUX DOWNWARD

↓

CONVECTIVE FLOW MAINLY DRIVEN BY DOWNFLOWS!

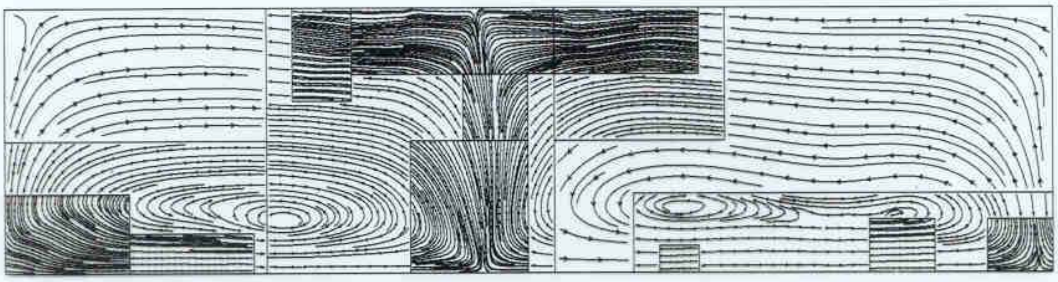
UPFLOW

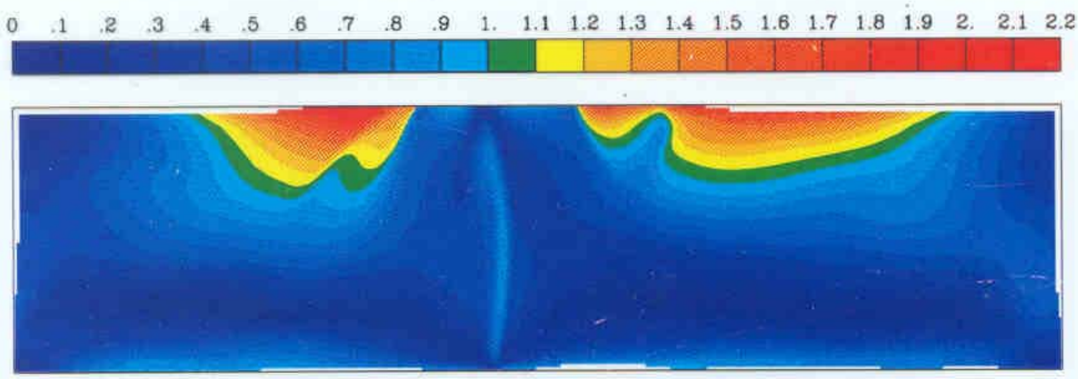
DOWNFLOW

(MUCH ENHANCED BY RADIATIVE COOLING)

SIMULATION OF SUPERSONIC CONVECTION
WITH ADAPTIVE GRID REFINEMENT

(STEINER, 1992)

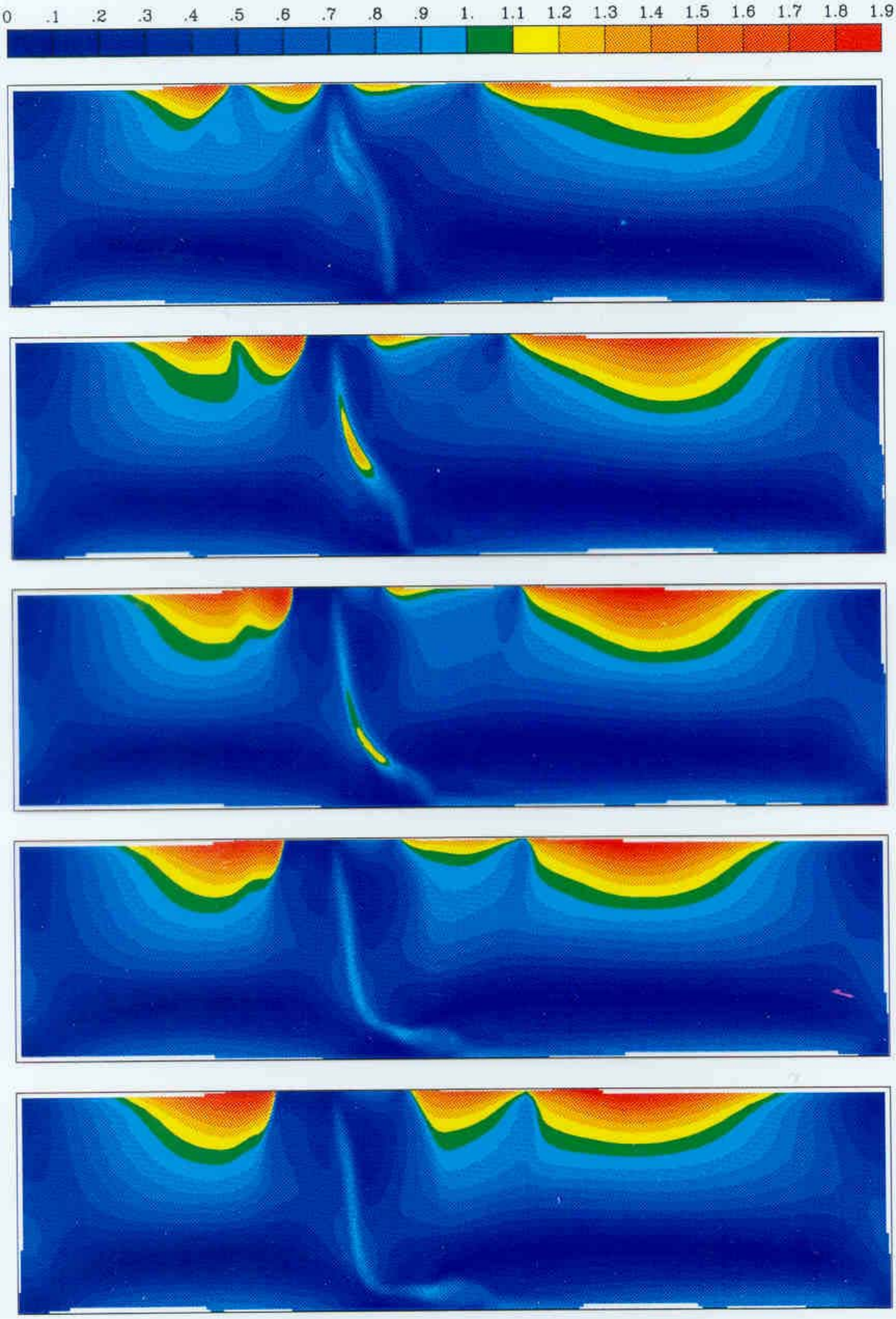




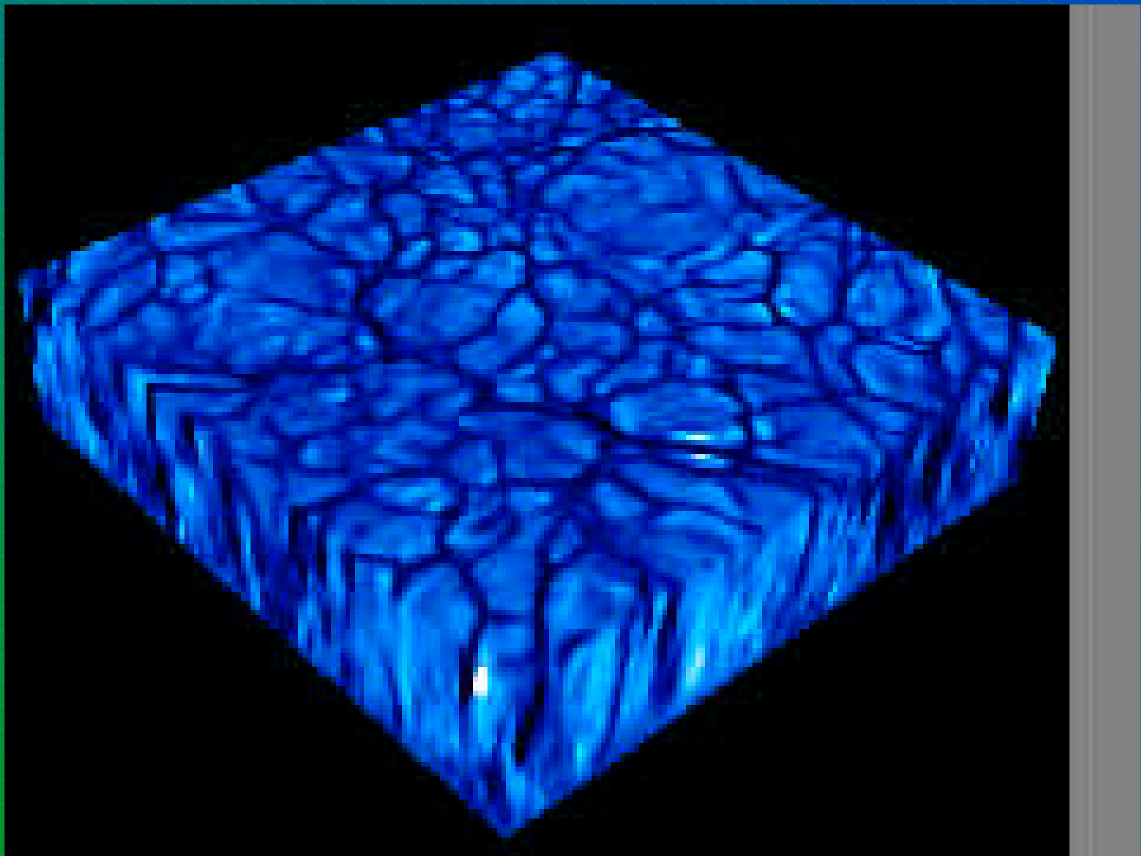
$\chi = 11$; $r = 4$; $\sigma = 0.3$; $m = 1$; $\gamma = 1.4$
$R = 10^6 \approx 10^4 R_{crit}$

base grid 80 × 20 ; refinement ratio = 2
ℓ_1 : 1 grid ; ℓ_2 : 4 grids ; ℓ_3 : 9 grids

SIMULATION SUPERSONISCHER KONVEKTION
MIT ADAPTIVER GITTERVERFEINERUNG (STEINER, 1992)



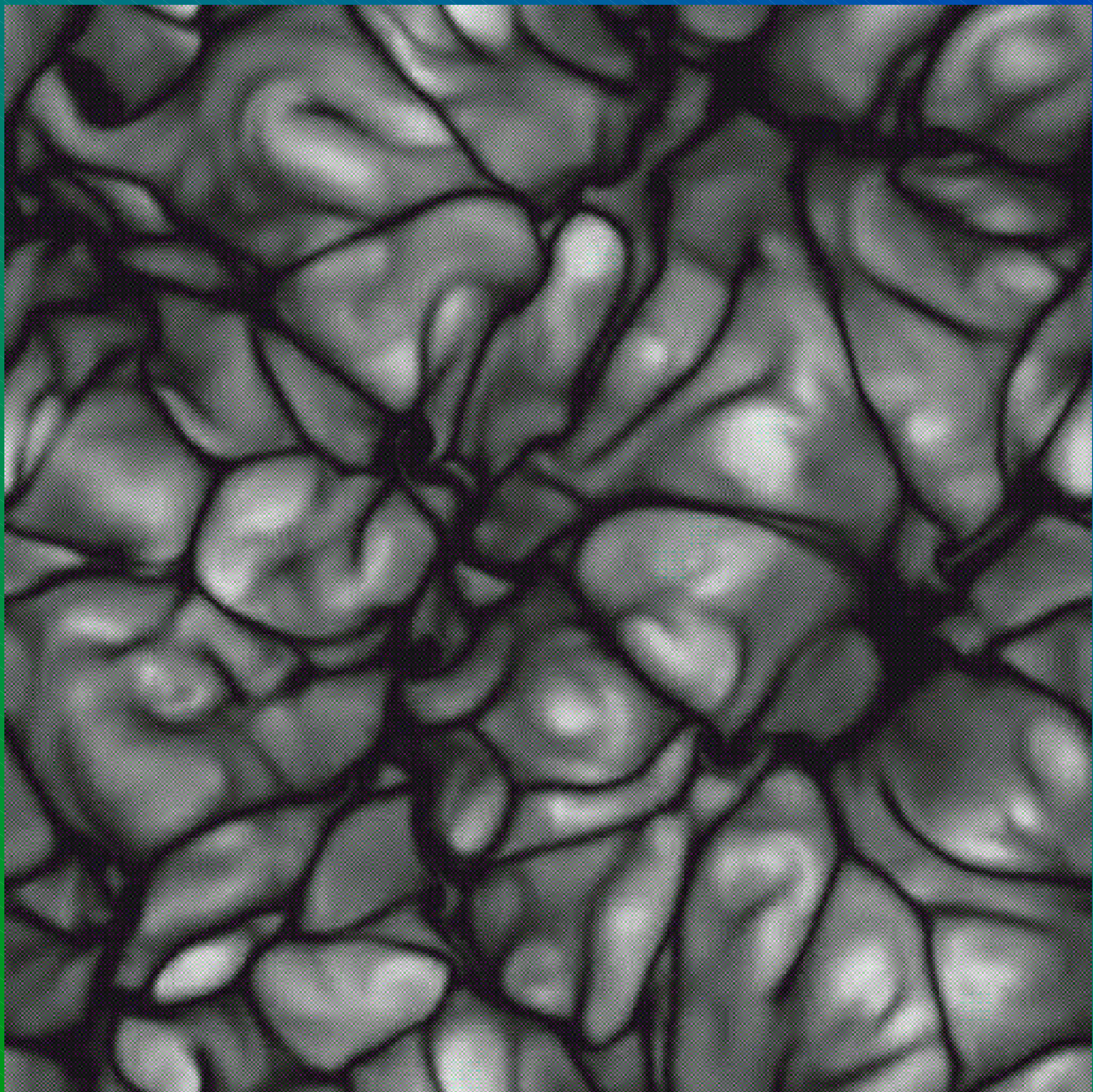
Computer-simulated convection



Cattaneo & Emonet (2001)

- Boussinesq model
- Rayleigh number: $5\ 10^5$
- 3D, $512\times512\times97$ mesh
- wide box, aspect ratio: 10
- "(meso)granulation"

Computer-simulated convection

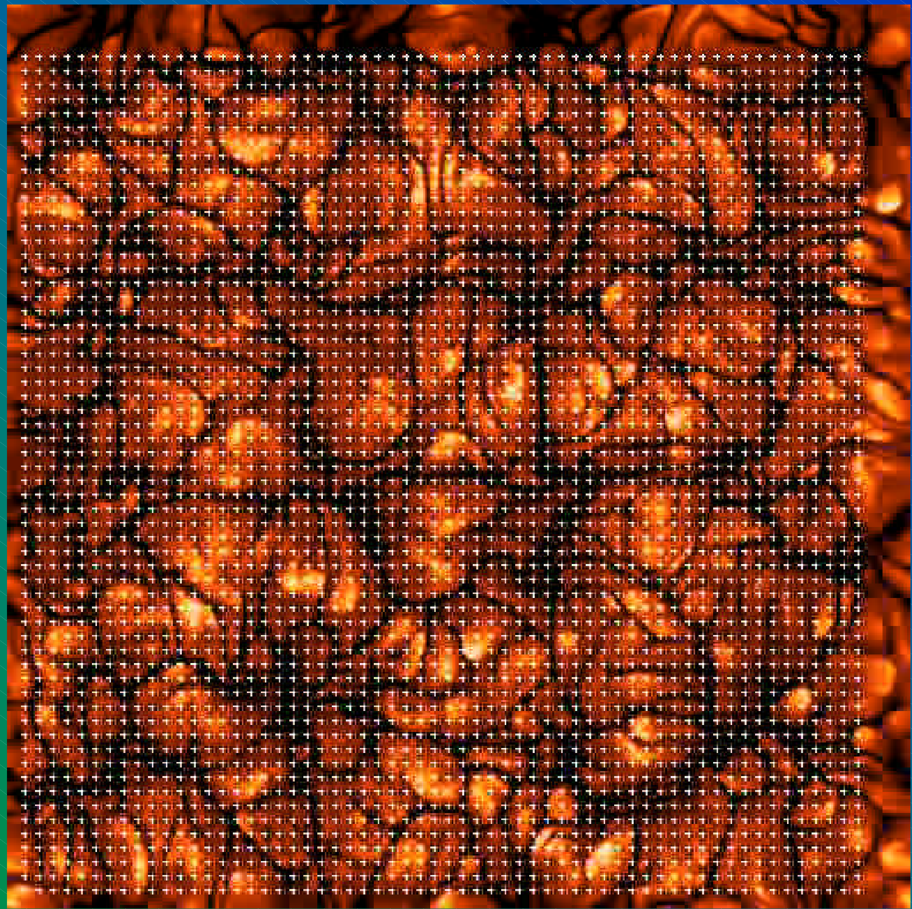


Temperature fluctuations

- Boussinesq model
- Rayleigh number: 5 10^5
- 3D, 512×512×97 mesh
- wide box, aspect ratio: 10
- "(meso)granulation"

Cattaneo & Emonet (2001)

Simulated long-lived convective downflows



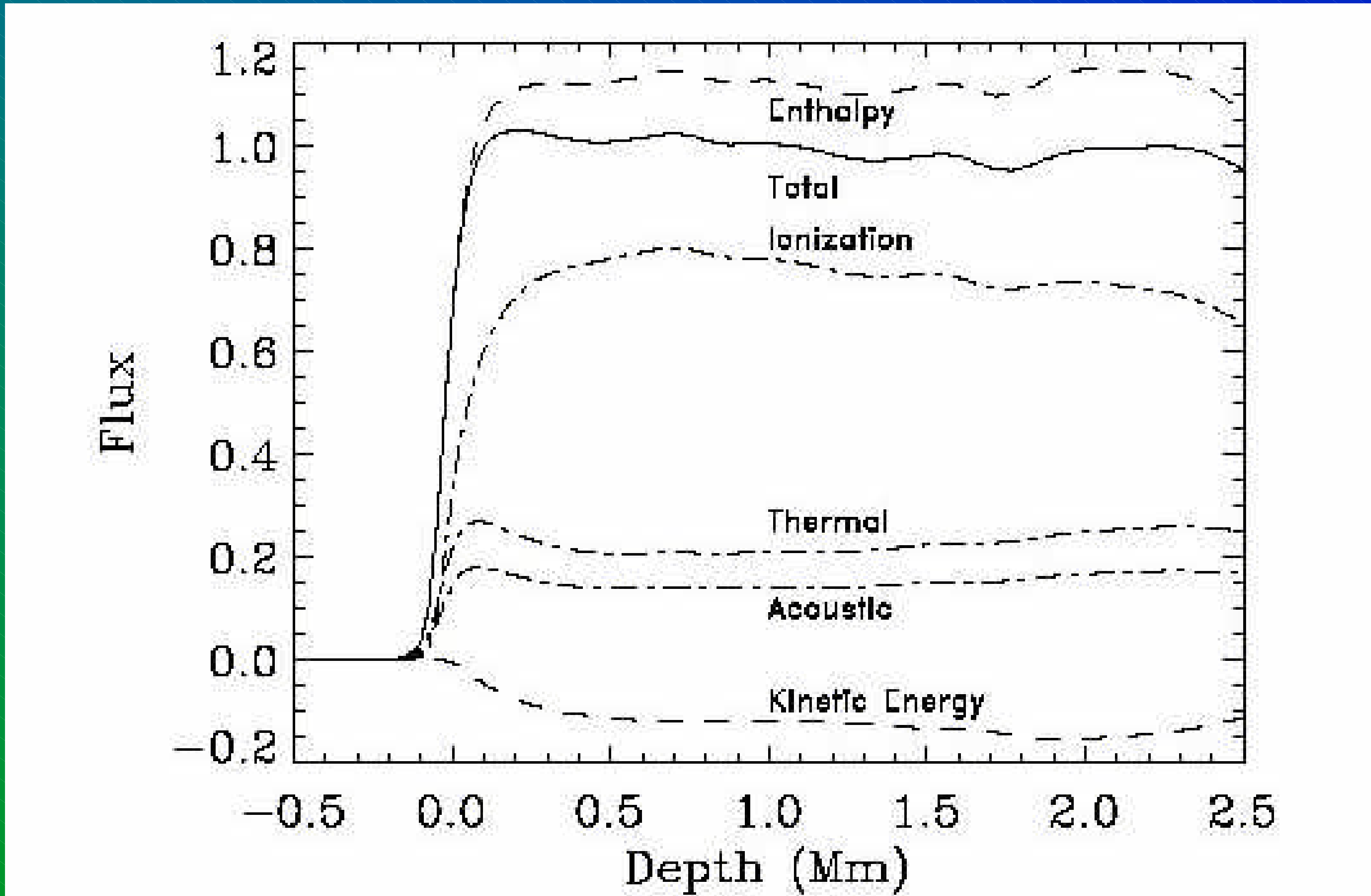
Virtual "corks" are carried by the horizontal flow.
They accumulate in downflow regions.

‘Realistic’ solar simulations

- elaborate physics: partial ionization, radiation, compressible, open box, transmitting boundaries, spectral line diagnostics (Stokes profiles)
- + : approximation to solar conditions
- + : direct comparison with observations
- – : computational restrictions (box size, resolution)
- – : Reynolds numbers much below solar values

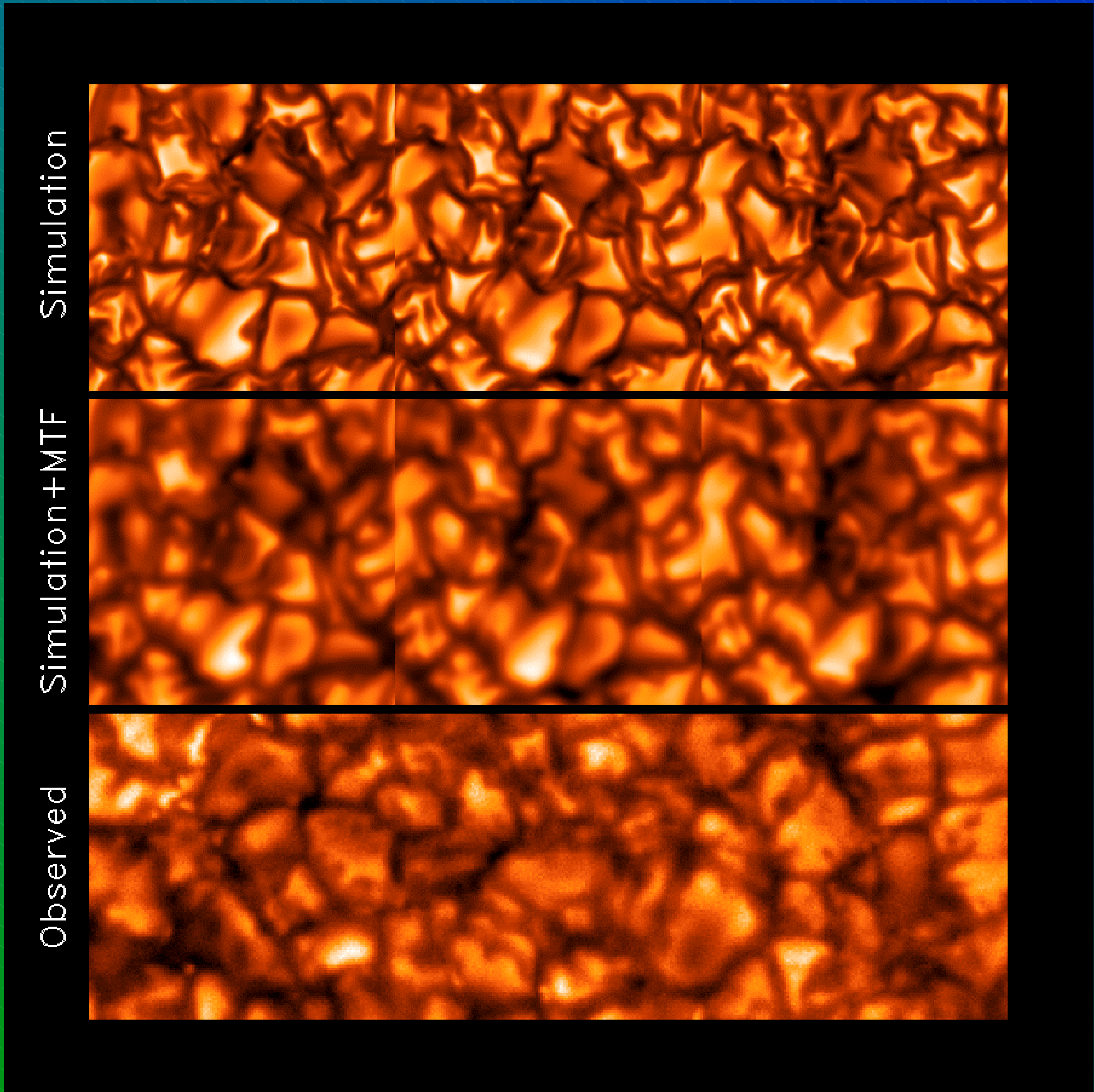
➔ viewgraphs

Averaged energy fluxes in a simulation of solar convection



Stein & Nordlund, 2000

Simulation and observation

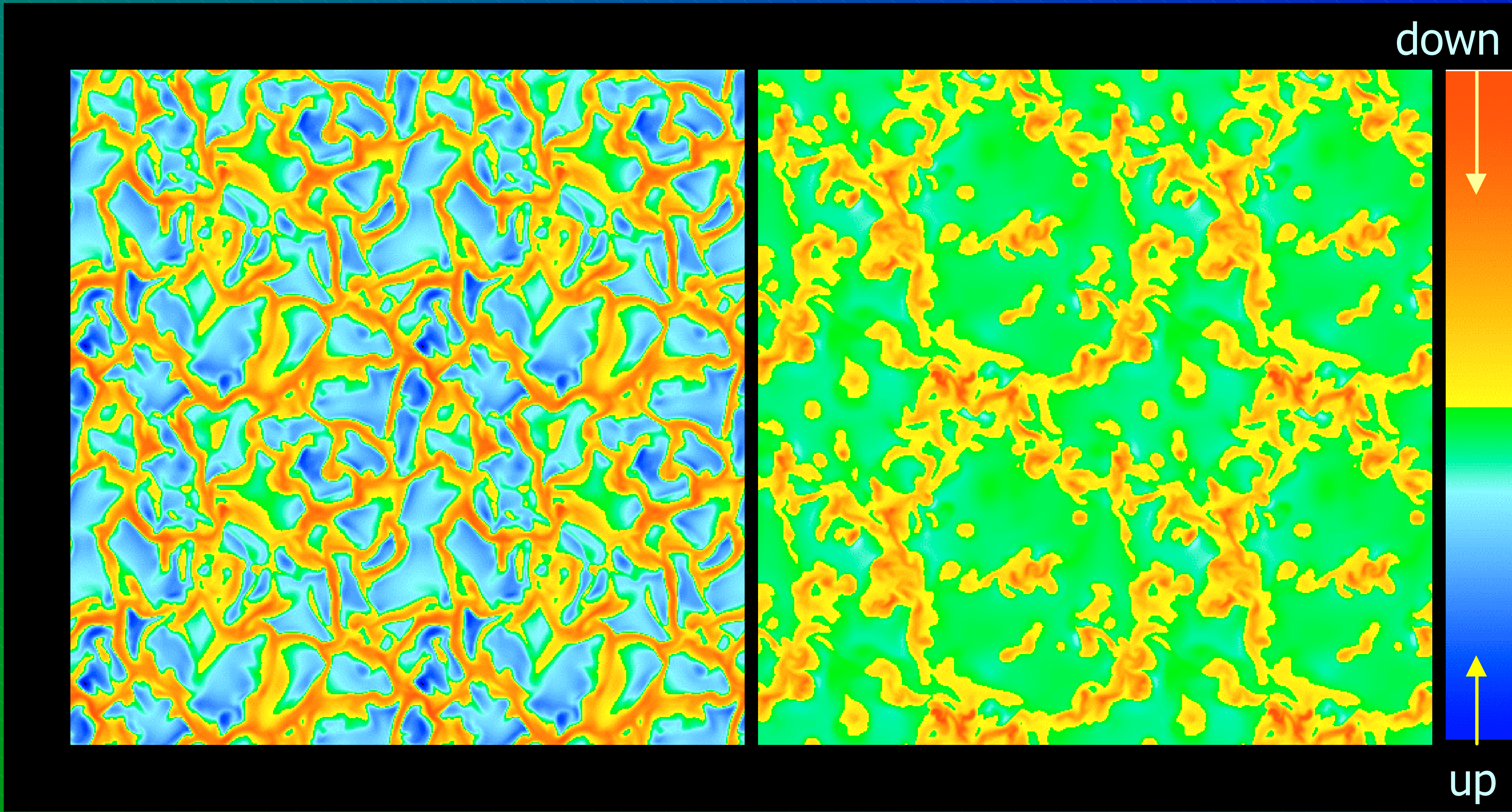


Simulation (original)

Simulation (smoothed)

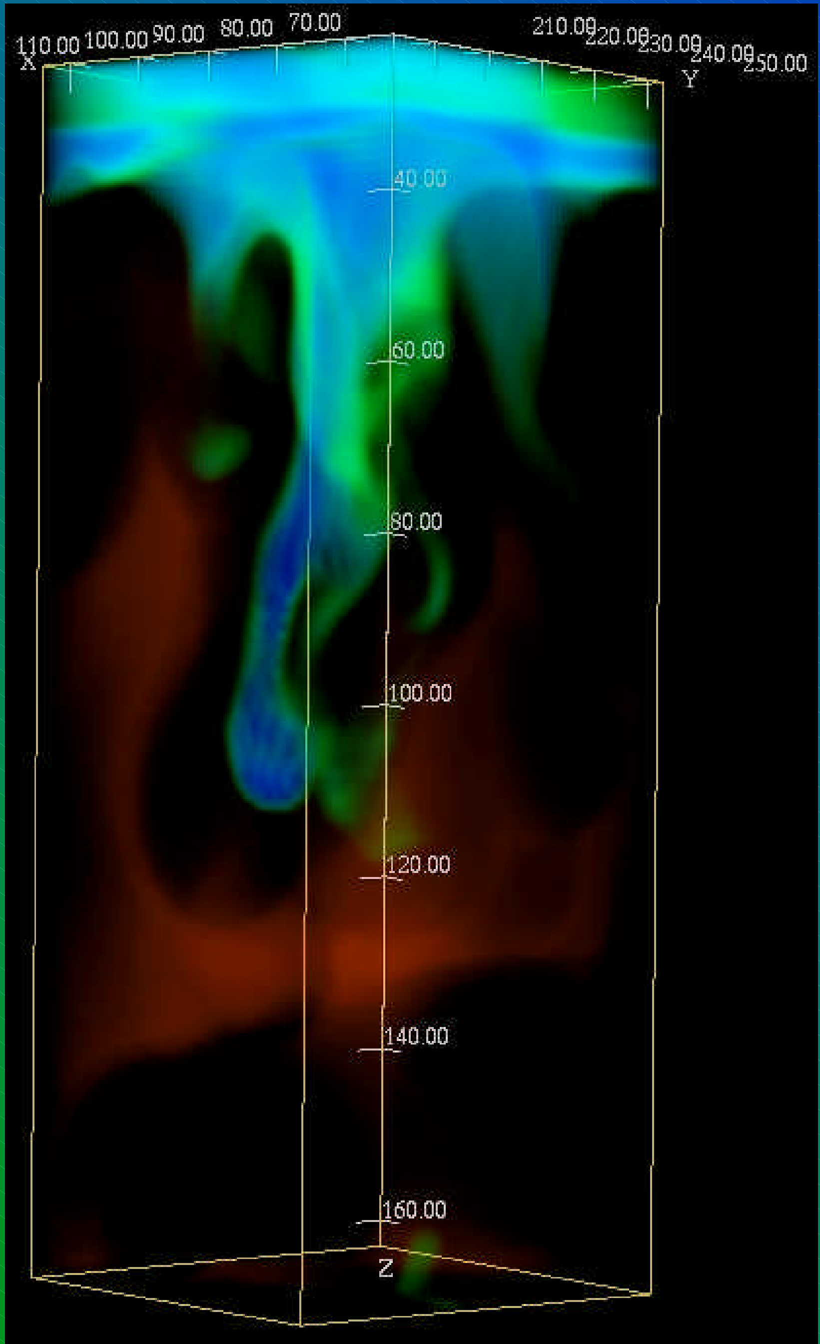
Observation

Change of downflow topology

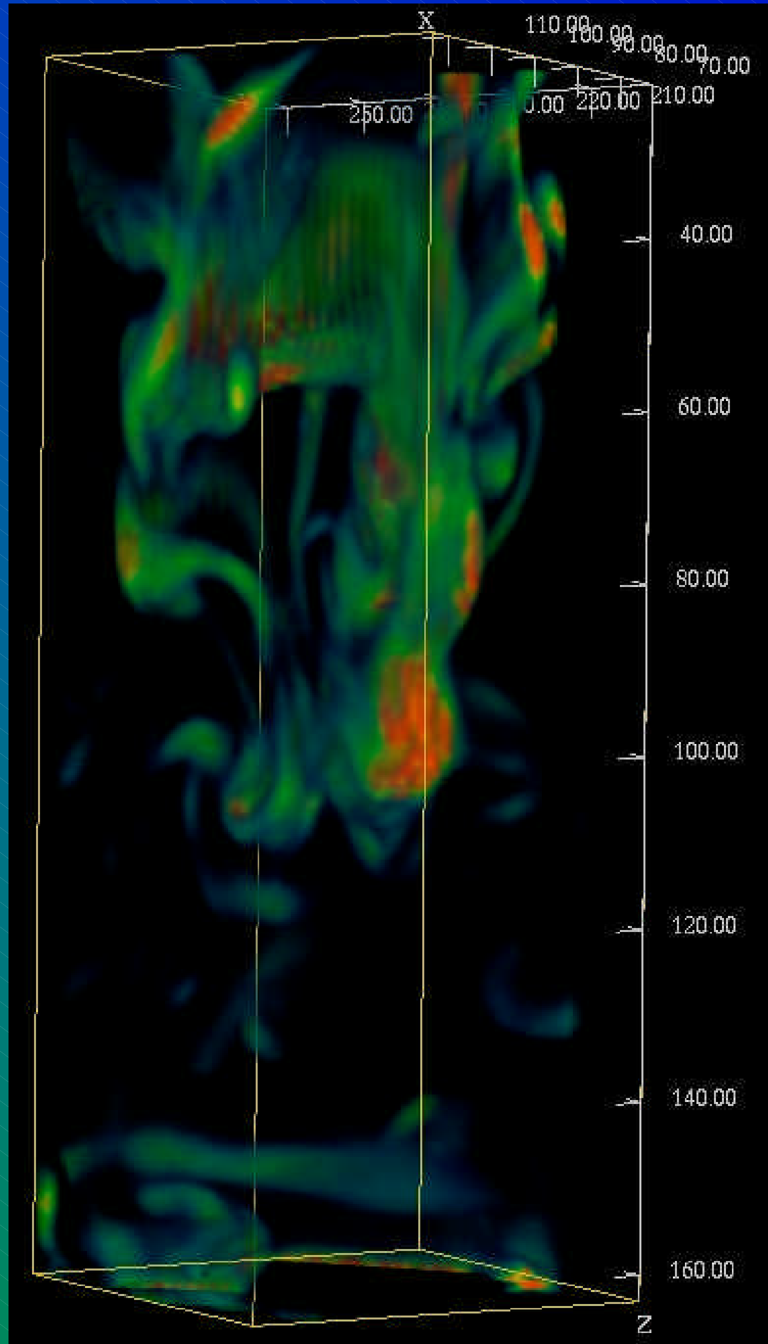


Stein & Nordlund, 1998

Downflow structure: plumes/thermals

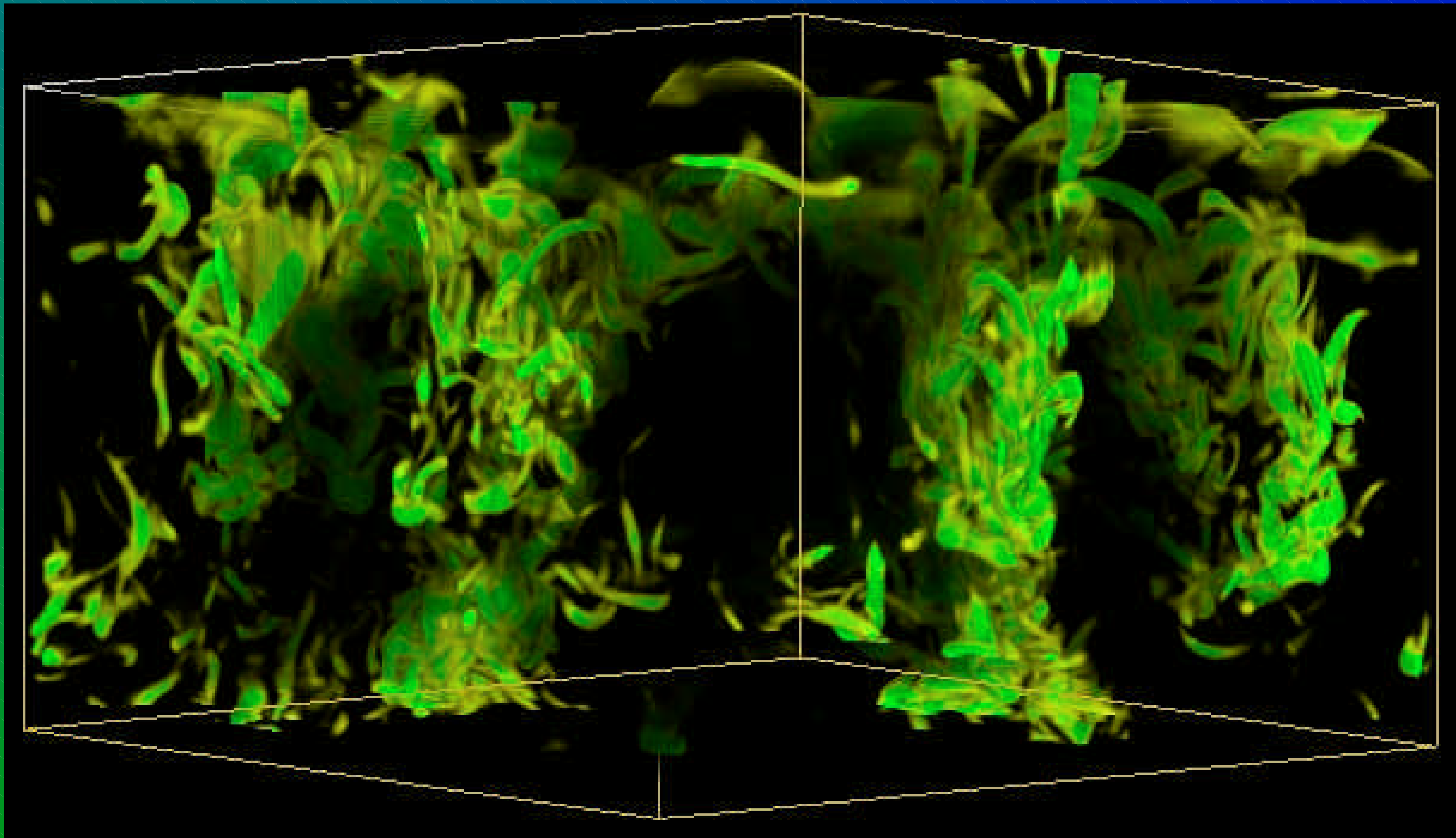


Entropy



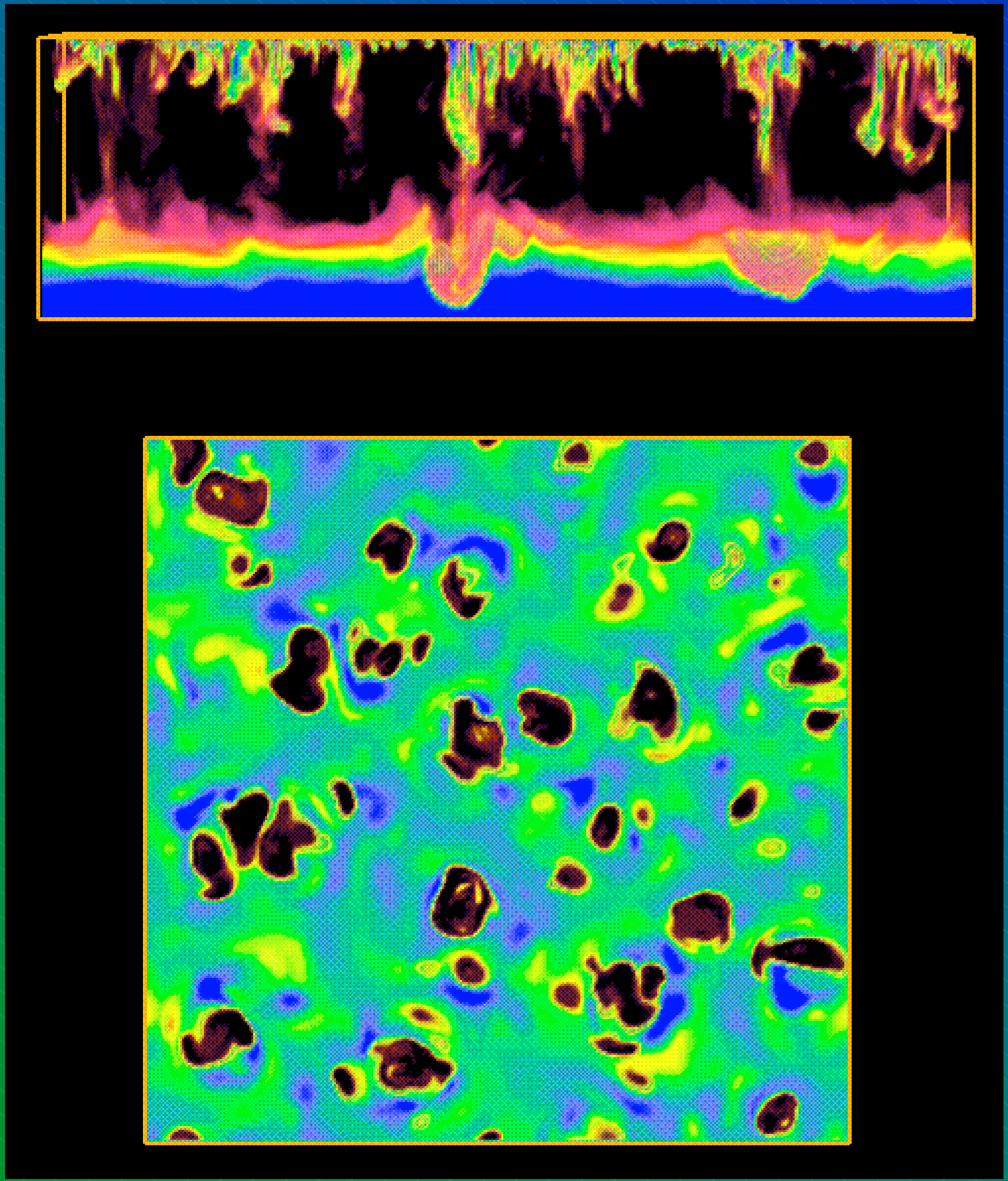
Vorticity

Distribution of vorticity



Stein & Nordlund, 1998

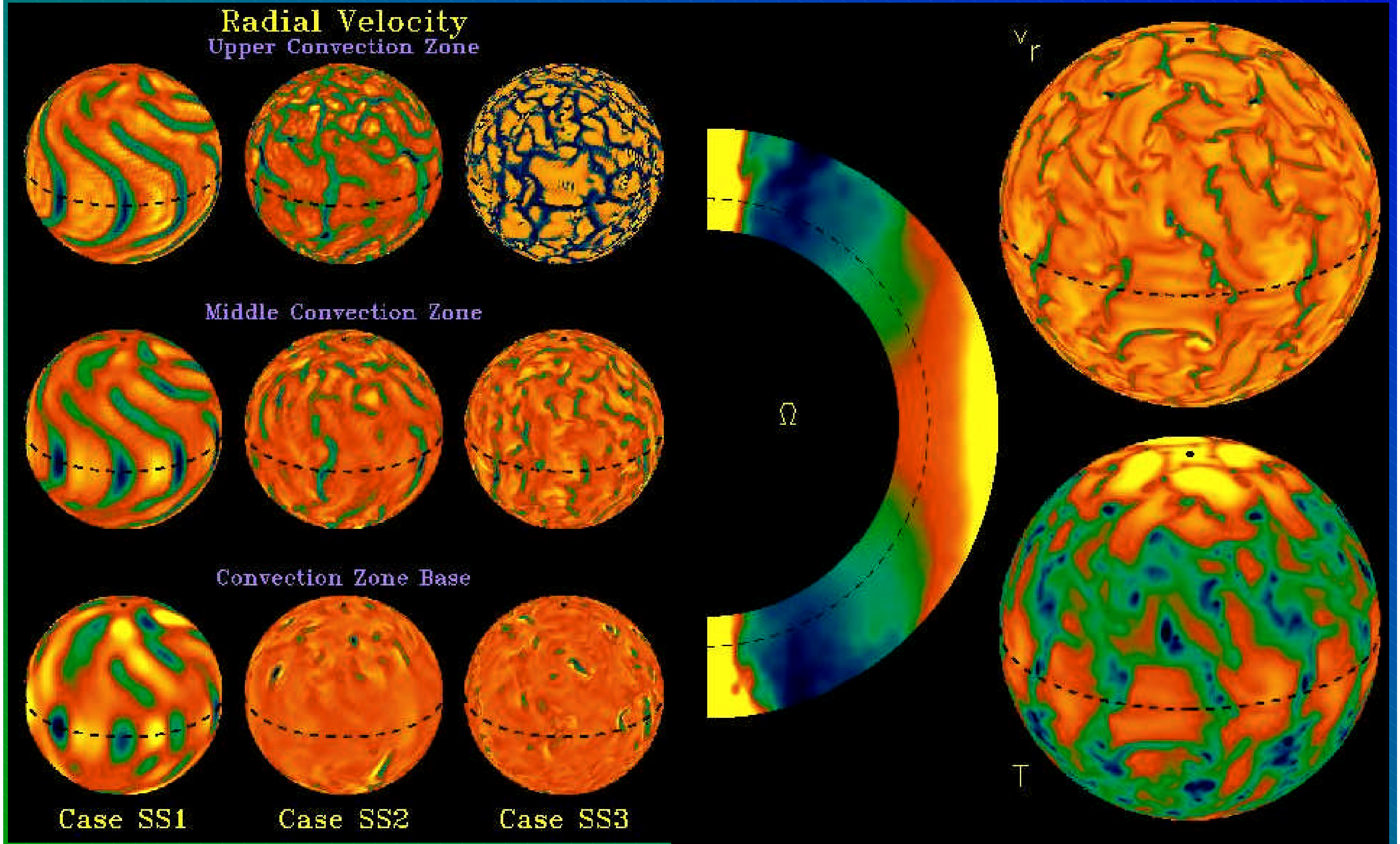
Penetrating convection/overshoot



Shown is the temperature field for convection penetrating down into a stable fluid layer. The top image shows a side-view through the layer. The bottom image shows a view from above, within the stable region. The dark "holes" are plumes punching down into the stable fluid.

Simulated convection in a solar-like spherical shell

Miesch 1998



A “new paradigm” for solar/stellar convection ?

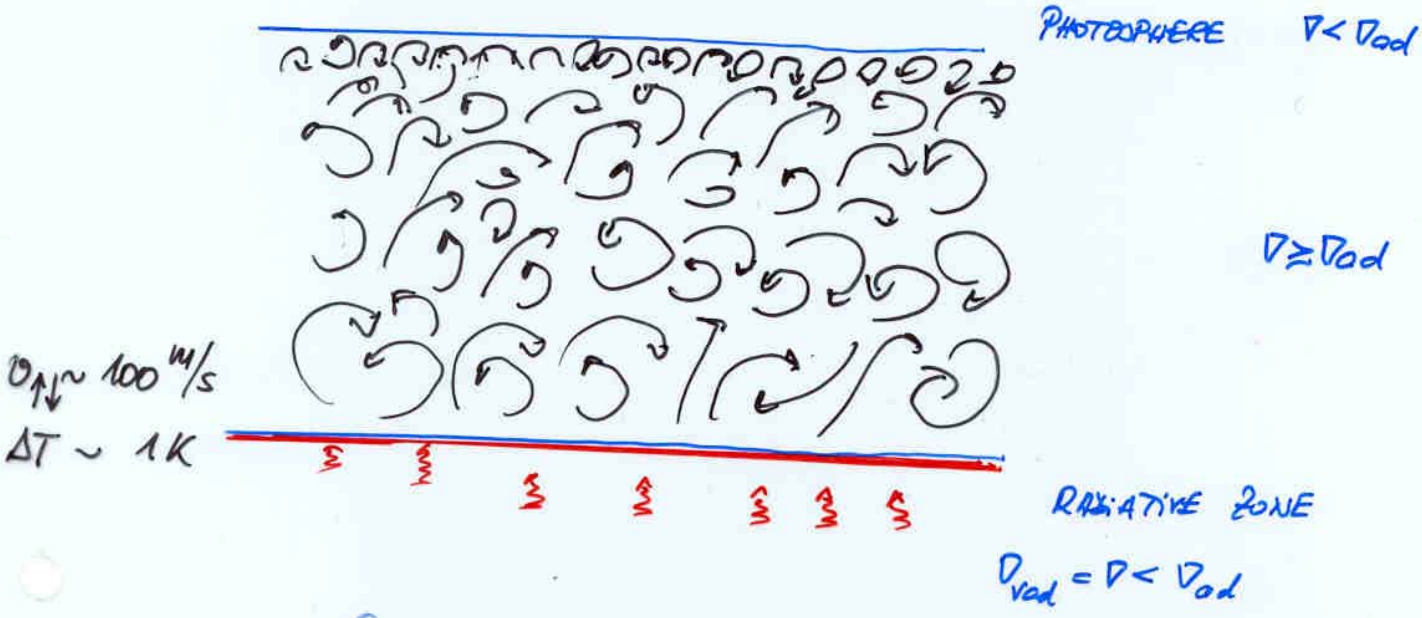
(Spruit, 1997)

- **Old paradigm (mixing-length model):**
- fully developed turbulence with a hierarchy of “eddies”
- quasi-local, diffusion-like transport
- flows driven by local entropy gradient

- **New paradigm (lab & numerical experiments):**
- turbulent downdrafts, laminar isentropic upflows
- flows driven by surface entropy sink (radiative cooling)
- non-diffusive transport
- larger scales (meso/supergranulation) driven by compressing and merging downdrafts

➔ viewgraph

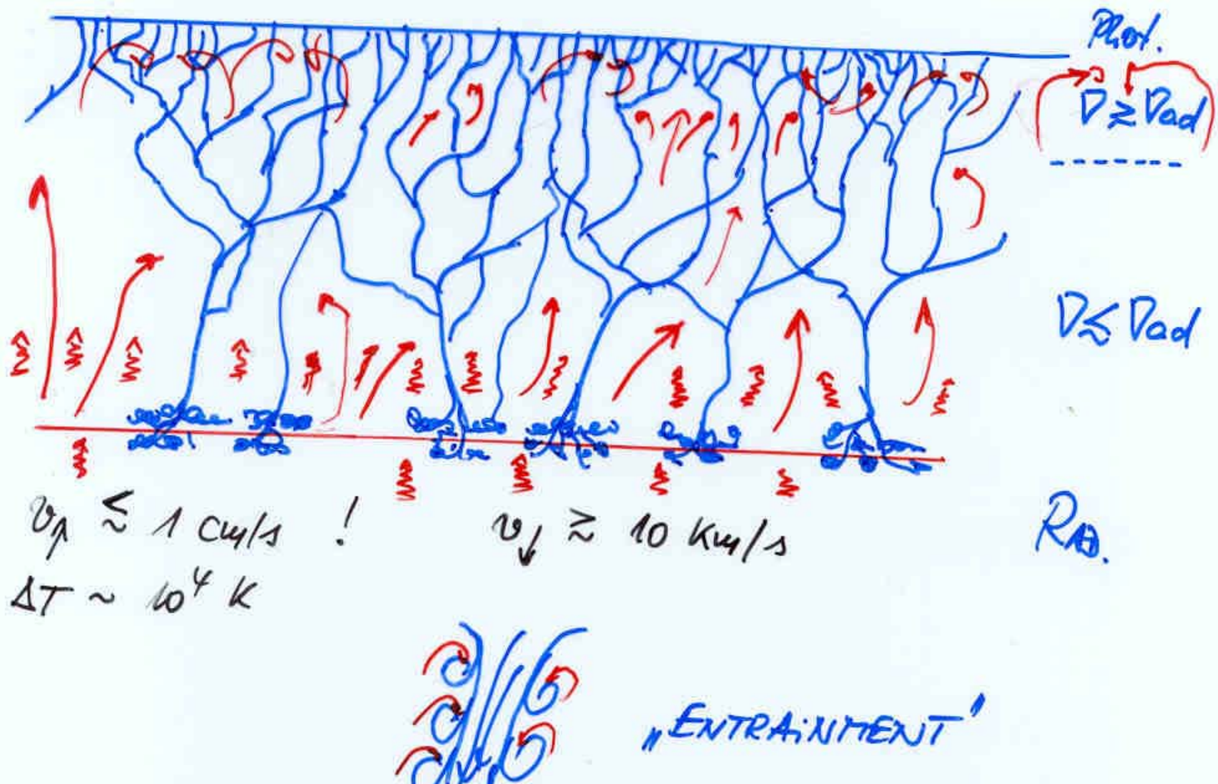
OLD PARADIGM



SURFACE LAYERS LIDICAR / SAME

DEEP LAYERS VERY DIFFERENT

NEW PARADIGM

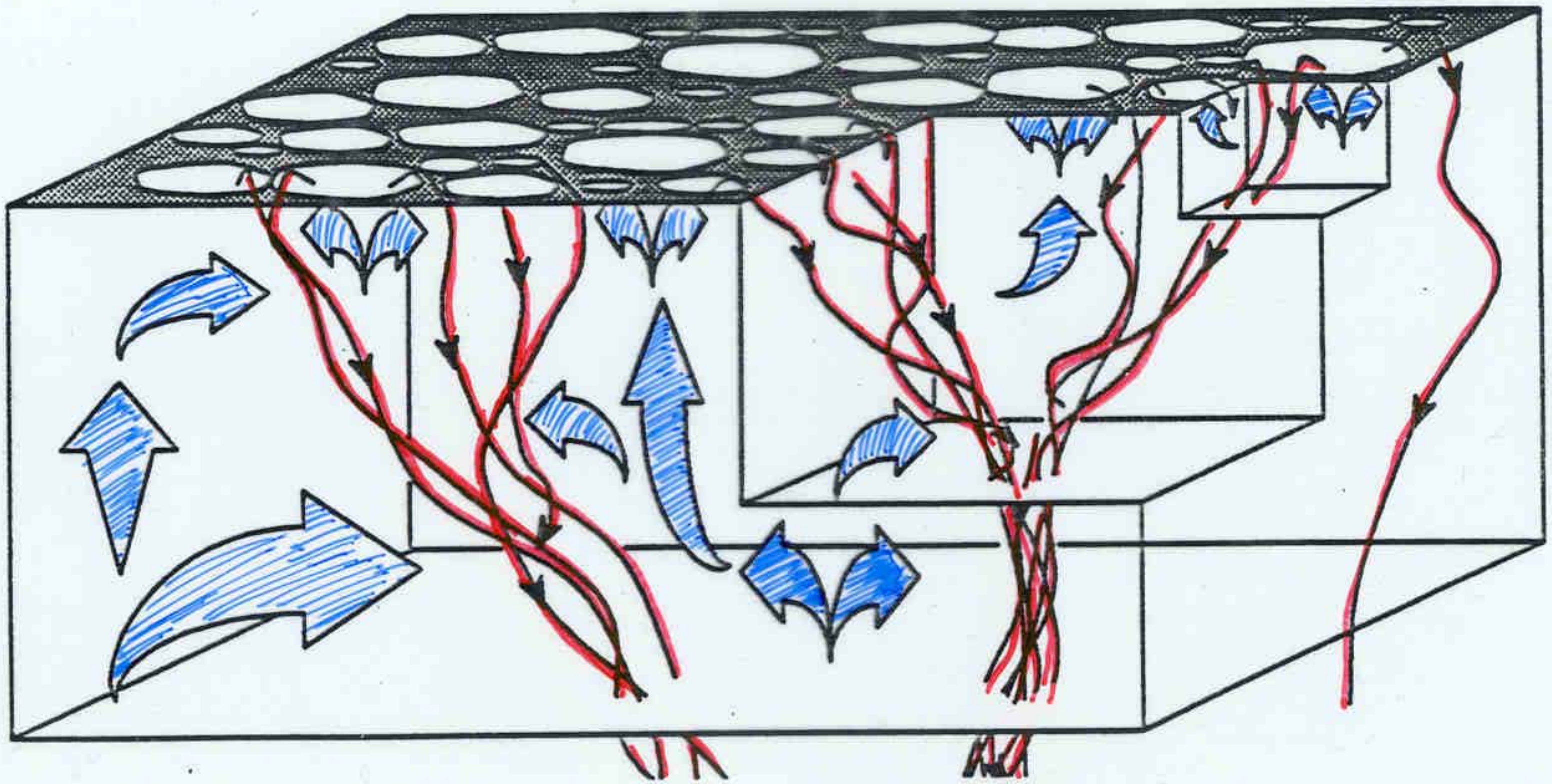


„ENTRAINMENT"

→ ENTROPY MIXING

(SPECULATIVE) PICTURE OF SOLAR CONVECTION : INVERSE CASCADE

(SPRUIT ET AL., 1990)



FILAMENTARY DOWNDRAFTS MERGE (DUE TO HORIZ. FLOWS ON LARGER SCALES,) DRIVEN BY THE ENTROPY DEFICIT OF THE DOWNDRAFTS THEMSELVES)