Helioseismology and Internal Rotation

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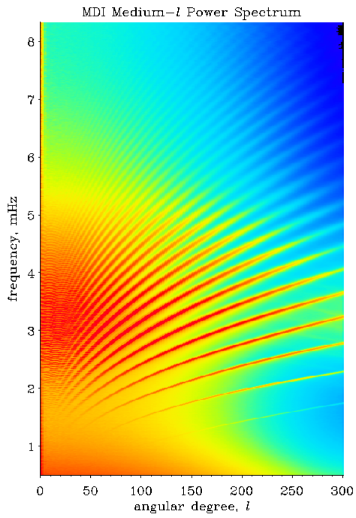
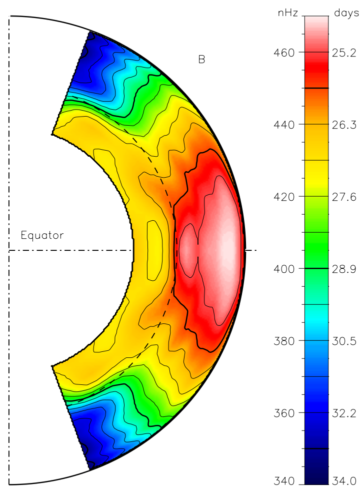
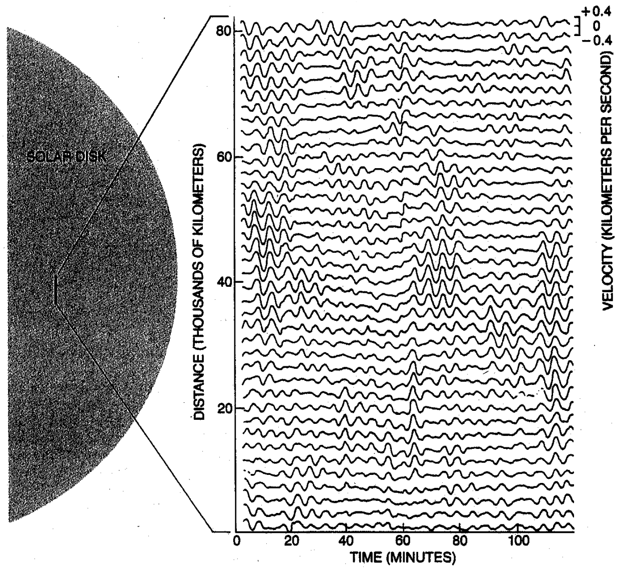


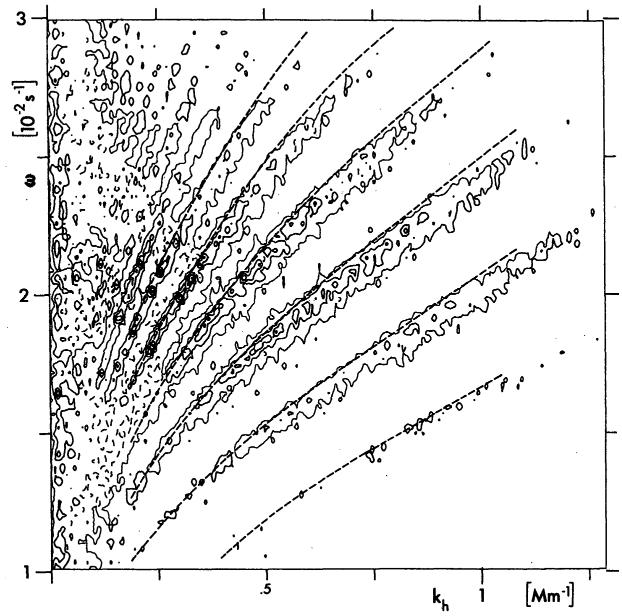


1. Introduction

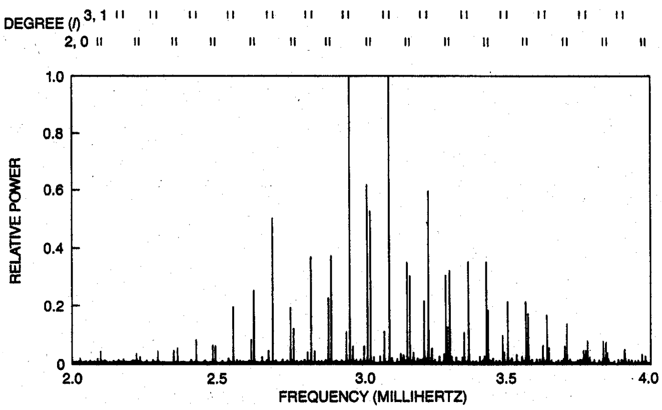
- Earth quakes ↷ compressional and shear waves ↷ propagation through interior ↷ information about Earth ↷ seismology
- free oscillations of Earth ↷ analogy to Sun
- Leighton, Noyes & Simon (1960): 5-min oscillation
  $P \approx 5\,\text{min}$, $v \approx 1/2\,\text{km}\,\text{s}^{-1}$
  spatial coherence ≈ 30 000 km, temporal coherence ≈ 1/2 h



- Ulrich (1970), Noyes & Simon (1971): superposition of many (~ 10 Mio) discrete global acoustic oscillations of Sun individual amplitudes < 20 cm s^{-1}
- Deubner (1975): confirmation by observation spatial and temporal Fourier analysis, power $k_h\varpi$-diagram, ridges



- Isaak et al.: discrete oscillations of spatially unresolved Sun ↷ basis for asteroseismology



- Each wave contains information about the solar interior, although largely smoothed appropriate combination of certain modes narrows contribution range ↷ spatial information ↷ helioseismology
- Physics of waves, approximate theory

2. Observational Constraints

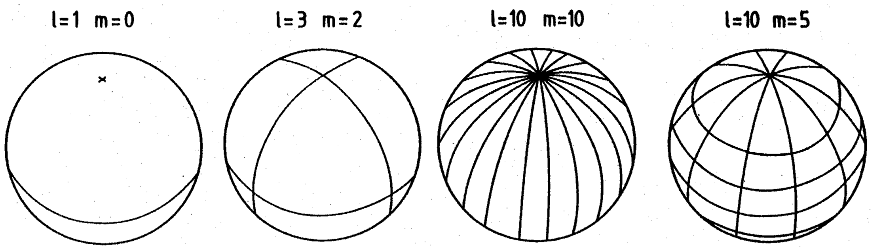
- Spectral lines, Doppler shift $\curvearrowright$ $v(r, t)$
- Intensity oscillations, solar irradiance variation
- Fourier transformation $f(\boldsymbol{k}, \varpi) = \int v(\boldsymbol{r}, t) \exp(-i\boldsymbol{k}\cdot\boldsymbol{r} - i\varpi t) d\boldsymbol{r} dt$
- power $p(k, \varpi) = ff^*$, $k = |\boldsymbol{k}|$
- Nyquist theorem, time resolution, ≥ 2 measurements per period, e.g. every 90 s
- Frequency resolution, $T \geq 2\pi/\Delta\varpi$
  $\Delta\varpi/\varpi \approx \Omega/\varpi \approx 10^{-4}$, $T \geq 30$ days
- Night gaps $\curvearrowright$ side peaks
  $\curvearrowright$ south pole, GONG, SOHO
- Spatial resolution, small wave length, large wave number
- Wave number resolution, whole Sun

3. Properties of p-Modes

- Sound and gravity waves or p- and g-modes
- Internal gravity waves $\neq$ gravitational waves
  gravitation / buoyancy restoring force
  levels inhomogenities on horizontal surfaces
  g-modes propagate in convectively stable layers
  i.e. in solar interior, evanescent in convection zone
  frequencies small $< N$, N buoyancy or Brunt-Väisälä frequency
  periods ≥ 30 min, not yet observed,
  thus not used in helioseismology
- Sound waves
  pressure gradient restoring force
  compression and expansion, longitudinal waves
  periods small 3 ... 12 min, maximum power at 5 min
  individual amplitudes small, linear treatment

- Infinite homogenous fluid
  - wave equation for pressure perturbation $\ddot{p}_1 - c^2 \Delta p_1 = 0$ with $c^2 = \gamma p_0/\rho_0 = \gamma R T_0$, c adiabatic sound speed
  - plane wave solution $p_1(\boldsymbol{r}, t) = \hat{p}_1 \exp i(\boldsymbol{k} \cdot \boldsymbol{r} - \omega t)$
  - angular frequency ω, wave vector $\boldsymbol{k}$
  - dispersion relation $\omega^2 = k^2 c^2$ with $k^2 = |\boldsymbol{k}|^2$
- Sun: stratification $c(r)$ and spherical geometry

  $p_1(r, \theta, \phi, t) = \hat{p}_1(r)\, Y_l^m(\theta, \phi) \exp(-i\omega t)$
- Spherical harmonics $Y_l^m(\theta, \phi) = P_l^m(\cos\theta) \exp(im\phi)$,
  Legendre functions P_l^m
  degree $l = 0, 1, 2, \ldots,$ order $m = -l, \ldots, 0, \ldots + l$
  l nodal lines, $l - |m|$ meridional nodal lines,
  $|m|$ azimuthal nodal lines



l, m determine horizontal wave numbers k_θ, k_ϕ
discrete numbers: wavelengths must fit on spherical surfaces
Y_l^m eigenfunctions of horizontal Laplace operator

$$\Delta_\theta^\phi Y_l^m = -\frac{1}{r^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right] Y_l^m = \frac{l(l+1)}{r^2} Y_l^m$$

$\curvearrowright$ eigenvalue $k_h^2 = k_\theta^2 + k_\phi^2 = \dfrac{l(l+1)}{r^2} \geq 0$ not dependent on m

$$k_\phi^2 = \frac{m^2}{r^2 \sin^2\theta} \geq 0\,, \quad k_\theta^2 = \frac{l(l+1) - m^2/\sin^2\theta}{r^2} \begin{cases} > 0 & \text{running} \\ < 0 & \text{evanescent} \end{cases}$$

running for $\sin^2\theta \geq \dfrac{m^2}{l(l+1)}$ latitudinal info for helioseismology

- No rotation $\curvearrowright$ no prescribed pole
  m does not occur in dispersion relation, degeneration
  $2l(l+1)$ eigenfunctions have same eigenfrequency ω

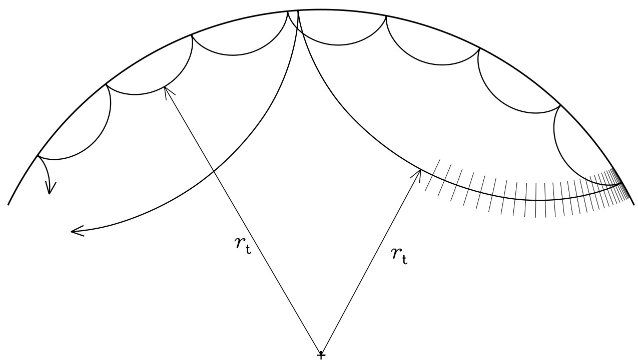
- Information on depth
  stratification, coefficients of wave equation depend on r
  local analysis, $\hat{p}_1(r) = \rho_0^{1/2} \exp(i k_r r)$
  approximate dispersion relation for $\varpi > N$:

  $\varpi^2 = \left(k_r^2 + k_h^2\right) c^2 + \varpi_{\rm ac}^2$

- Acoustic cut-off: $\varpi > \varpi_{\rm ac}$, $\varpi_{\rm ac} = c/2H$
  c, $\varpi_{\rm ac}$, k_h functions of r $\curvearrowright$ $k_r(r)$ local wavenumber

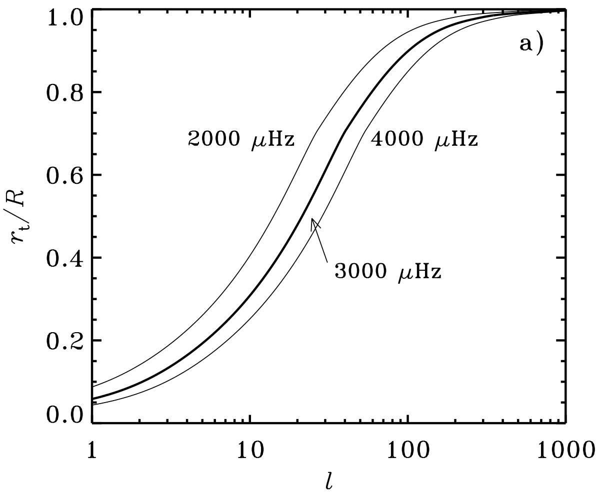
  $$k_r^2 = \frac{\varpi^2 - \varpi_{\rm ac}^2}{c^2} - \frac{l(l+1)}{r^2} \begin{cases} > 0 \text{ running} \\ < 0 \text{ evanescent} \end{cases}$$

  $r \searrow \quad c \nearrow \quad k_r \searrow$



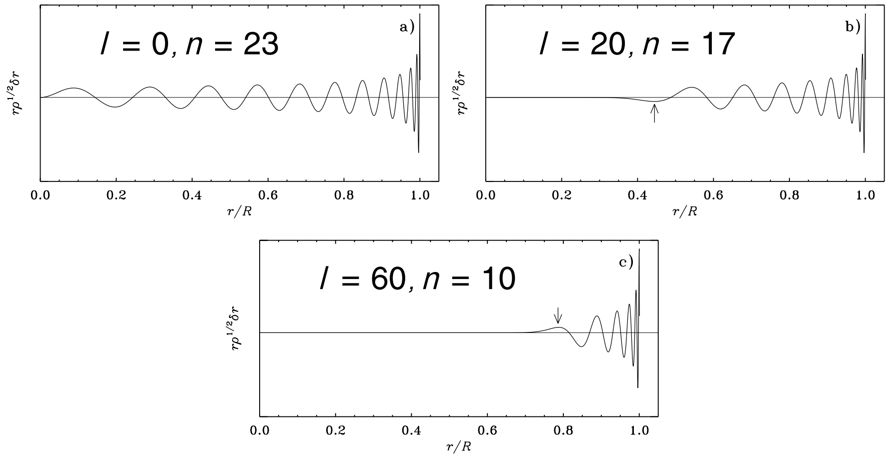
- Inner turning point r_t where $k_r = 0$, in solar interior $\varpi_{\rm ac} \ll \varpi$

  $$\frac{\varpi^2}{c^2(r_t)} = \frac{l(l+1)}{r_t^2} \quad \text{or} \quad r_t = \frac{\sqrt{l(l+1)}}{\varpi} c(r_t) \quad \text{i.e. function of } \frac{l}{\varpi}$$



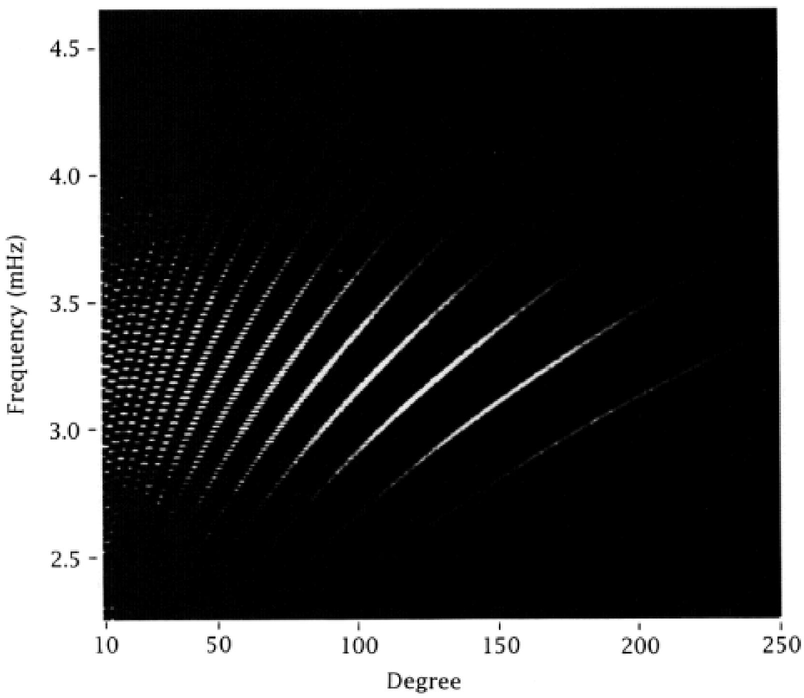
- Upper reflection point near surface, $\varpi_{\rm ac}$ increases (H decreases)
  R_t approx given by $\varpi = \varpi_{\rm ac}(R_t)$, $R_t \approx r_\odot$
- Cavity, constructive interference, discrete spectrum
  of standing sound waves

- Overtones, index n



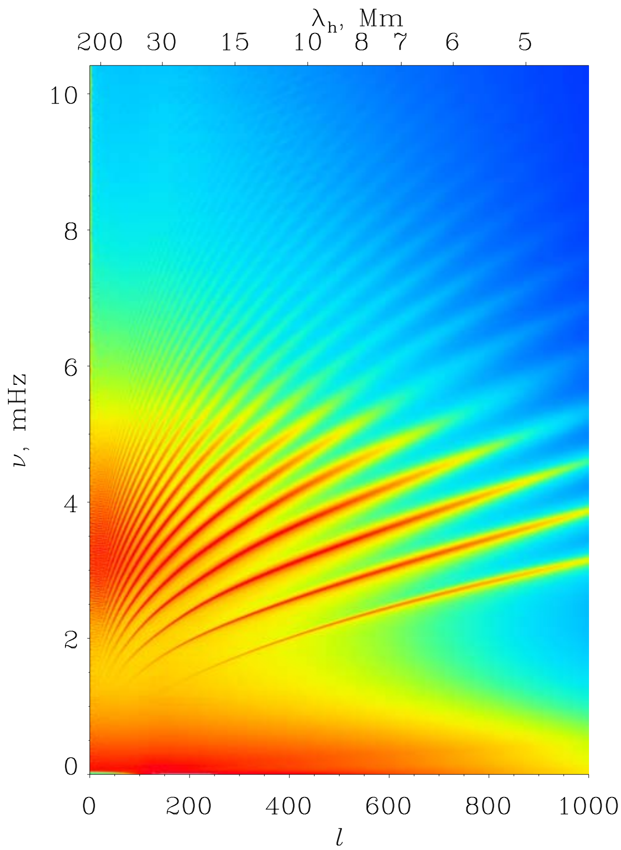
- Individual mode: three indices l, m, n

- Interpretation of Deubner's observation, $\varpi(n, l) \leftrightarrow k_h(l)$
  ridges various n, $\varpi(l)$ not resolved

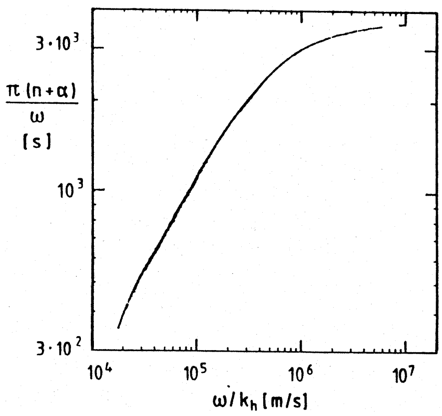


Duvall et al. (1988)
south pole, 50 h, resolution $\Delta l \approx 3 \ldots 5$
$\Delta l = 1$ needs information from around Sun ($l \approx k_h r_\odot$)

• SOHO data



• Duvall's law:



$$\frac{\pi(n+\alpha)}{\omega} \leftrightarrow \frac{\omega}{k_h} \quad \text{one curve}$$

phase difference $\Delta\Psi = \int_{r_t}^{r_\odot} k_r dr = \pi(n+\alpha)$

α because of evanescent boundaries,
string $\alpha = 0$, organ pipe $\alpha = 1/2$

$$r_t = \left(\frac{l(l+1)}{\omega^2}\right)^{1/2} c(r_t)\ , \quad k_r = \frac{\omega}{r}\left(\frac{r^2}{c^2} - \frac{l(l+1)}{\omega^2}\right)^{1/2}$$

$$\frac{\pi(n+\alpha)}{\omega} = \int_{r_t}^{r_\odot} \left(\frac{r^2}{c^2} - \frac{l(l+1)}{\omega^2}\right)^{1/2} \frac{dr}{r} = F\left(\frac{l(l+1)}{\omega^2}\right)^{1/2} = F\left(\frac{k_h}{\omega}\right)$$

- Ridges for large l: $F \sim (ld)^{-1}$, $\omega^2 \sim k_h$
- Frequencies for small l and large n

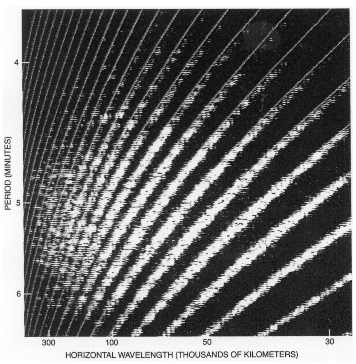
$$\int_{r_t}^{r_\odot} k_r dr \approx \frac{\omega}{\bar{c}} r_\odot - \left(l + \frac{1}{2}\right)\frac{\pi}{2} = \pi(n + \alpha)$$

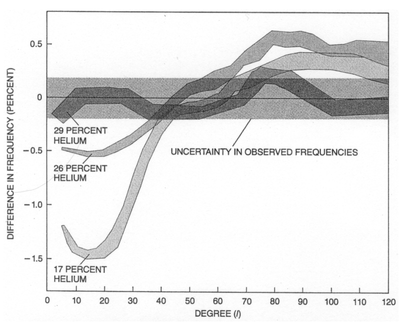
$$\curvearrowright \quad \omega \approx \left(n + \frac{l}{2} + \alpha + \frac{1}{4}\right)\frac{\pi \bar{c}}{r_\odot}$$

$$\curvearrowright \quad \omega_{nl} \approx \omega_{n-1,l+2} \quad \text{and} \quad \omega_{nl} \approx \frac{1}{2}(\omega_{n-1,l+1} + \omega_{n+1,l+1})$$

4. Direct Methods

- Theoretical model of Sun $\curvearrowright$ theoretical p-modes
  $\curvearrowright$ comparison with observation $\curvearrowright$ variation of solar model
- Good theory of p-modes necessary
- Often frequency differences $\delta\omega_{nl} = \omega_{nl} - \omega_{n-1,l+2}$ compared





- Determination:
  - equation of state: electrostatic correction to perfect gas law
  - $Z \curvearrowright$ opacity $\curvearrowright T(0) \leftrightarrow$ neutrino flux
  - Y
  - mixing length, depth of convection zone 200 000 km
- Confirmation of standard solar model

5. Inversion: Sound Speed

Duvall's law $\int_{r_t}^{r_\odot} \left(\frac{r^2}{c^2} - \frac{l(l+1)}{\omega^2} \right)^{1/2} \frac{dr}{r} = F\left(\frac{l(l+1)}{\omega^2} \right)^{1/2}$

$$u = \frac{l(l+1)}{\omega^2}, \ \xi = \left(\frac{r}{c}\right)^2, \ \xi_t = \left(\frac{r_t}{c(r_t)}\right)^2 = \frac{l(l+1)}{\omega^2} = u, \ \xi_\odot = \xi(r_\odot)$$

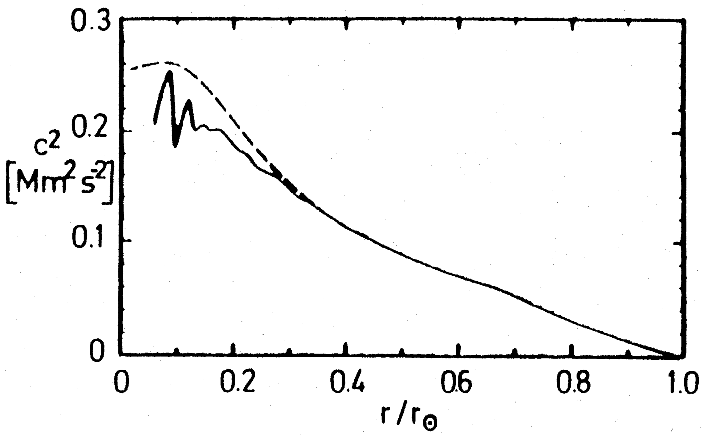
$$F(u) = \int_u^{\xi_\odot} (\xi - u)^{1/2} \underbrace{\frac{1}{r}\frac{dr}{d\xi}}_{d \ln r / d\xi} d\xi \quad \text{known from observation}$$

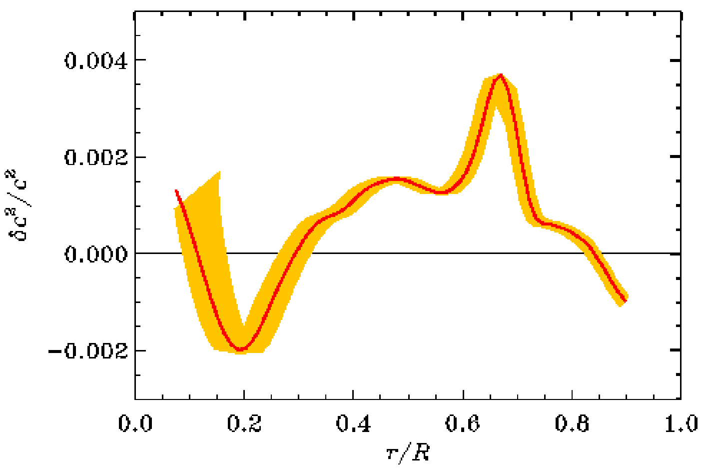
$$\frac{dF}{du} = -\frac{1}{2}\int_u^{\xi_\odot} \frac{d\ln r/d\xi}{(\xi - u)^{1/2}} d\xi \, , \quad \text{Abel's integral equation}$$

solution $\ln r(\xi) - \ln r(\xi_\odot) = -\frac{2}{\pi}\int_{\xi_\odot}^{\xi} \frac{dF/du}{(u-\xi)^{1/2}} du$

$$r = r_\odot \exp\left(-\frac{2}{\pi}\int_{\xi_\odot}^{\xi} \frac{dF/du}{(u-\xi)^{1/2}} du \right) = r(\xi)$$

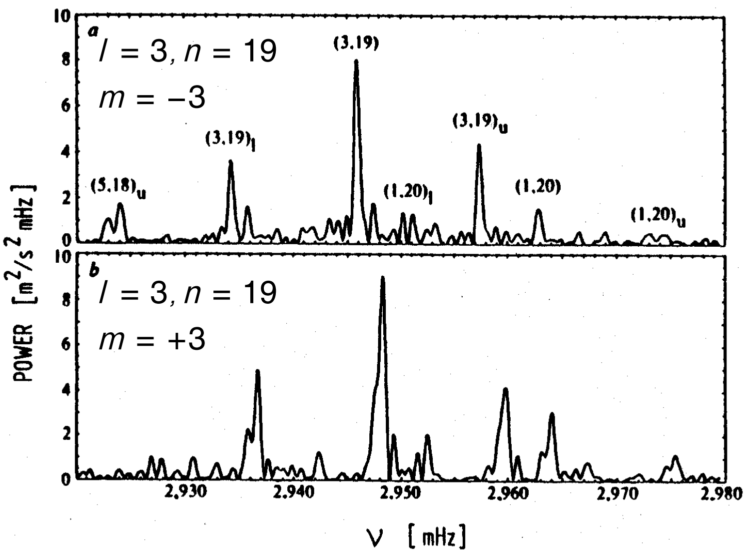
$r(\xi) \curvearrowright \xi(r) \curvearrowright c(r) \curvearrowright T(r)$





6. Internal Rotation

- No rotation: ω_{nlm} not dependent on azimuthal wavenumber m ↷ degeneration
- With rotation no degeneration
  - assumption for simplicity: rigid rotation
  - $\omega_{nlm} = \omega_{nl0} \pm m\Omega\,, \quad m = -l, \ldots, +l$
  - standing wave = superposition of two identical travelling waves in positive and negative azimuthal direction
  - rotation, Doppler effect, frequency shift and splitting
- $\omega_{nlm} - \omega_{nl,m-1} = \Omega\,, \quad \frac{\Omega}{\omega_{nl0}} \approx \frac{5\,\mathrm{min}}{30\,\mathrm{d}} \approx 10^{-4}$
- Observing time $T \geq 2\pi/\Omega \approx 30\,\mathrm{d}$ to resolve Ω with ≥ 2 measurements per period (Nyquist)
- Duvall and Harvey (1984)
  $l = 3$, $n = 19$, $m = +3, -3$



- Side peaks due to night gaps
  ↷ south pole, GONG, SOHO
- Frequency splitting weighted average of $\Omega(r, \theta)$

$$\Delta\omega_{nlm} = \int K_{nlm}(r,\theta)\Omega(r,\theta) r dr d\theta$$

- Kernel K_{nlm} given, i.e. $|\xi|^2$ of eigenfunction
- n of little influence, l determines contribution in depth, m in latitude

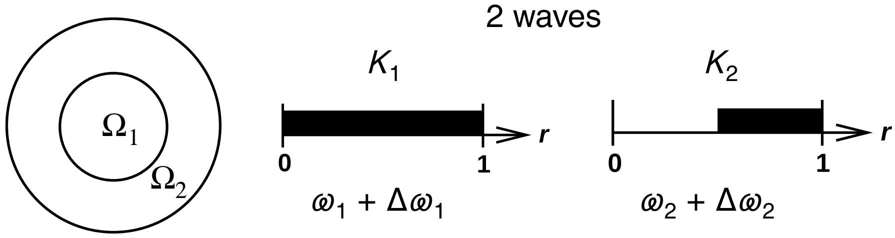
- Various methods to extract Ω, here optimal kernels (Backus & Gilbert, 1970):

$$\sum_i a_i(\boldsymbol{r}_0)K_i(\boldsymbol{r}) = \delta(\boldsymbol{r}-\boldsymbol{r}_0)\,,$$ combination of kernels to δ functions

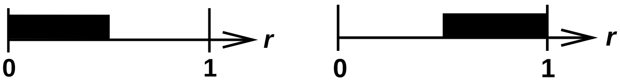
$$\sum_i a_i(\boldsymbol{r}_0)\Delta\varpi_i = \int \delta(\boldsymbol{r}-\boldsymbol{r}_0)\Omega(\boldsymbol{r})d^3\boldsymbol{r} = \Omega(\boldsymbol{r}_0)$$

- Example:

2 waves



optimal kernels: $\tilde{K}_1 = K_1 - K_2$ $\quad$ $\tilde{K}_2 = K_2$



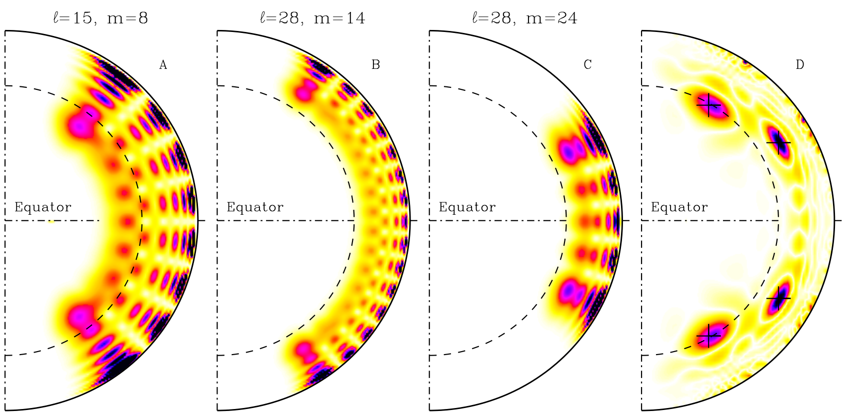
$a_1 = 1, a_2 = -1$ $\quad$ $a_1 = 0, a_2 = 1$

$$\Omega_1 = \Delta\varpi_1 - \Delta\varpi_2\,, \quad \Omega_2 = \Delta\varpi_2$$

$$\Delta\varpi_1 = \int K_1\Omega dr = K_1\left(\Omega_1\int_0^{1/2}dr + \Omega_2\int_{1/2}^1 dr\right) = \frac{K_1}{2}(\Omega_1+\Omega_2)$$

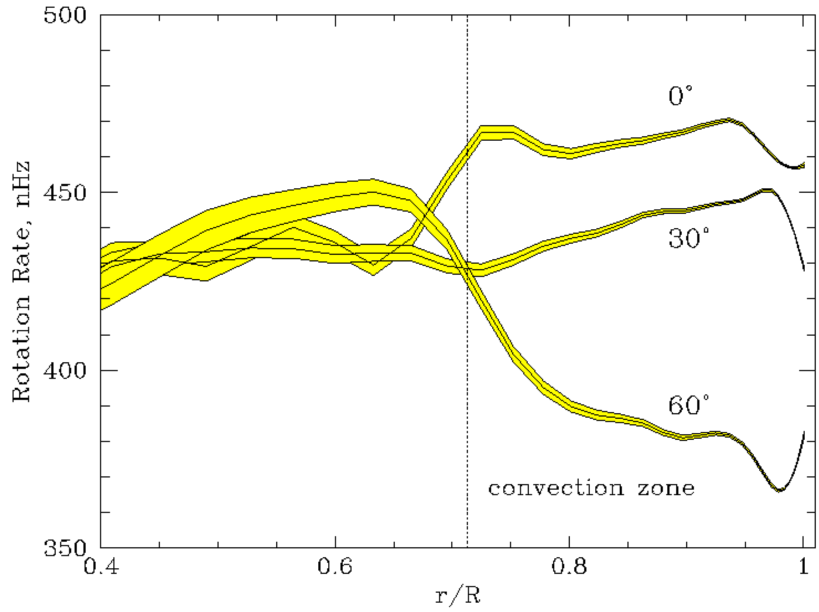
$$\Delta\varpi_2 = \int K_2\Omega dr = \frac{1}{2}K_2\Omega_2\,, \quad K_1 = K_2 = 2$$

- Solar p-mode kernels

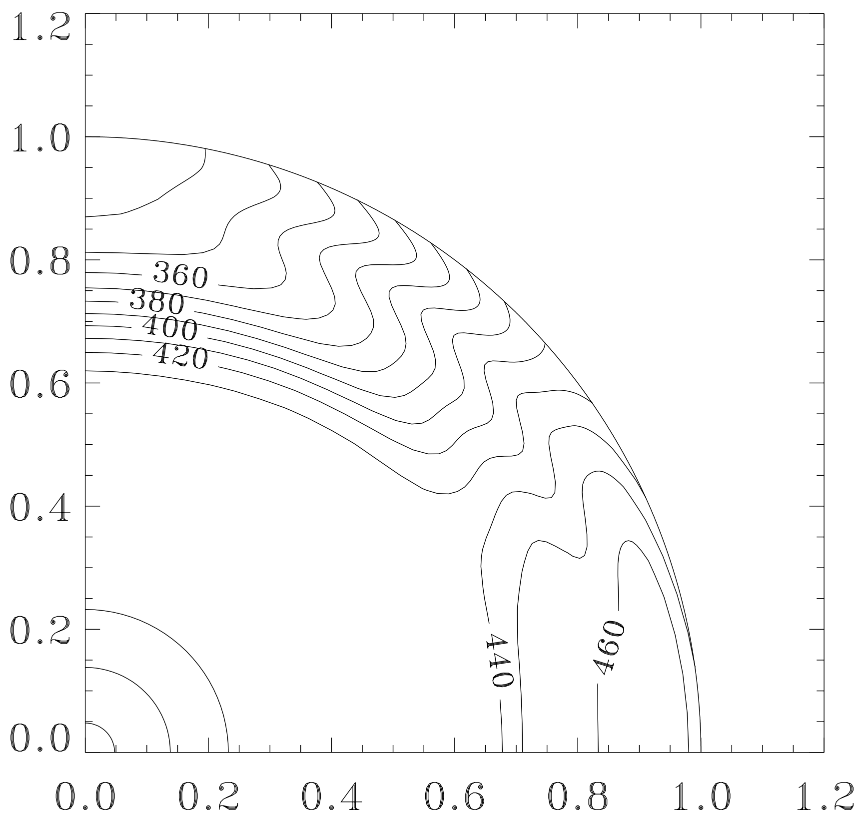


- Such each location in Sun addressable $\curvearrowright$ $\Omega(r,\theta)$

• Result

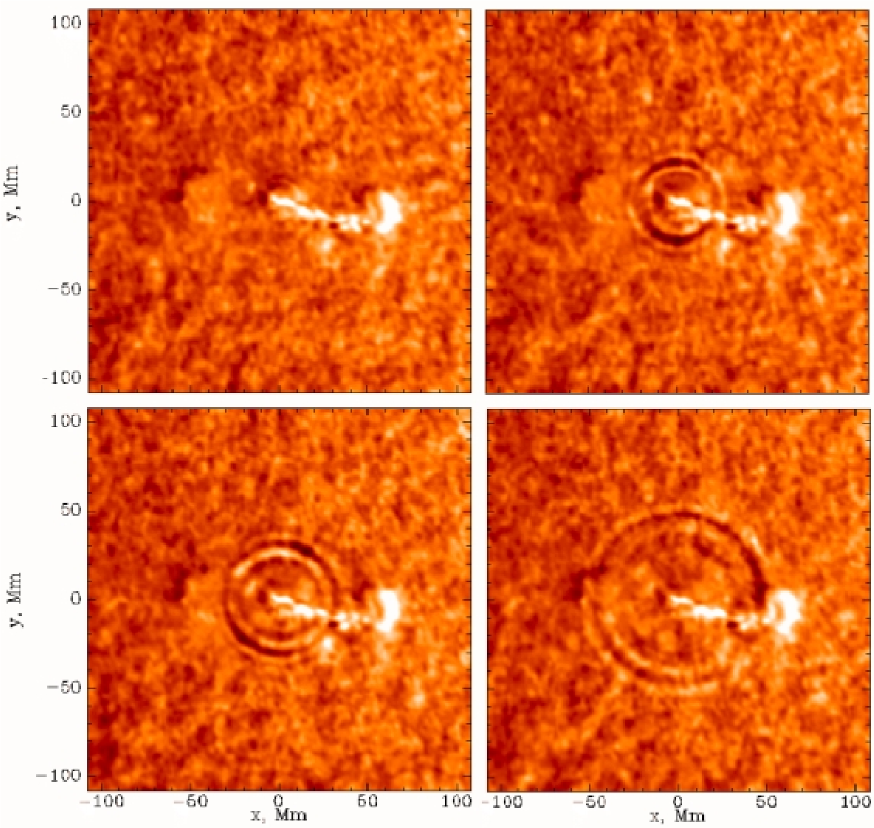


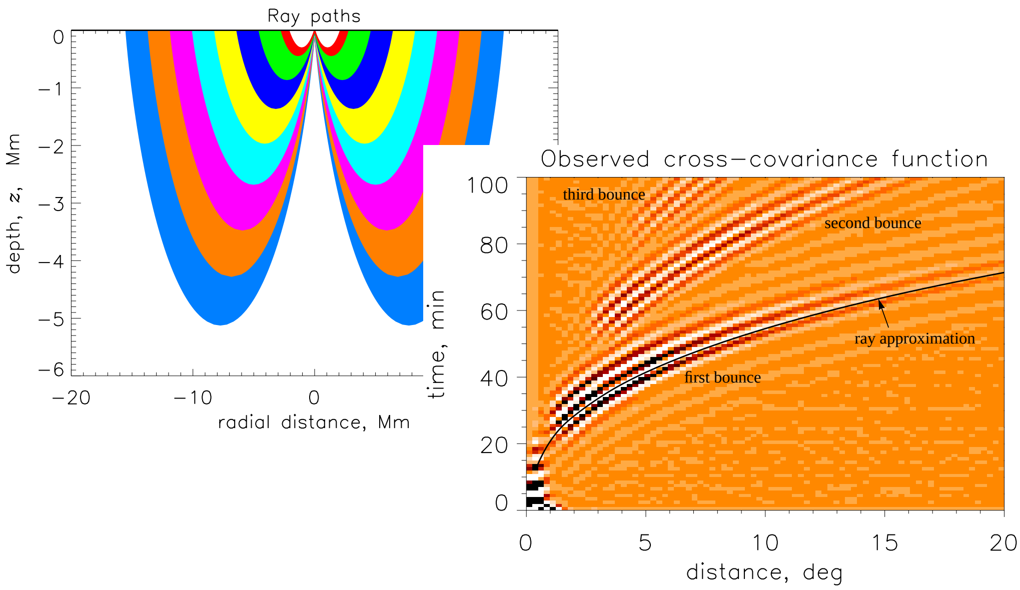
Rotation Rate, nHz
r/R
0°
30°
60°
convection zone



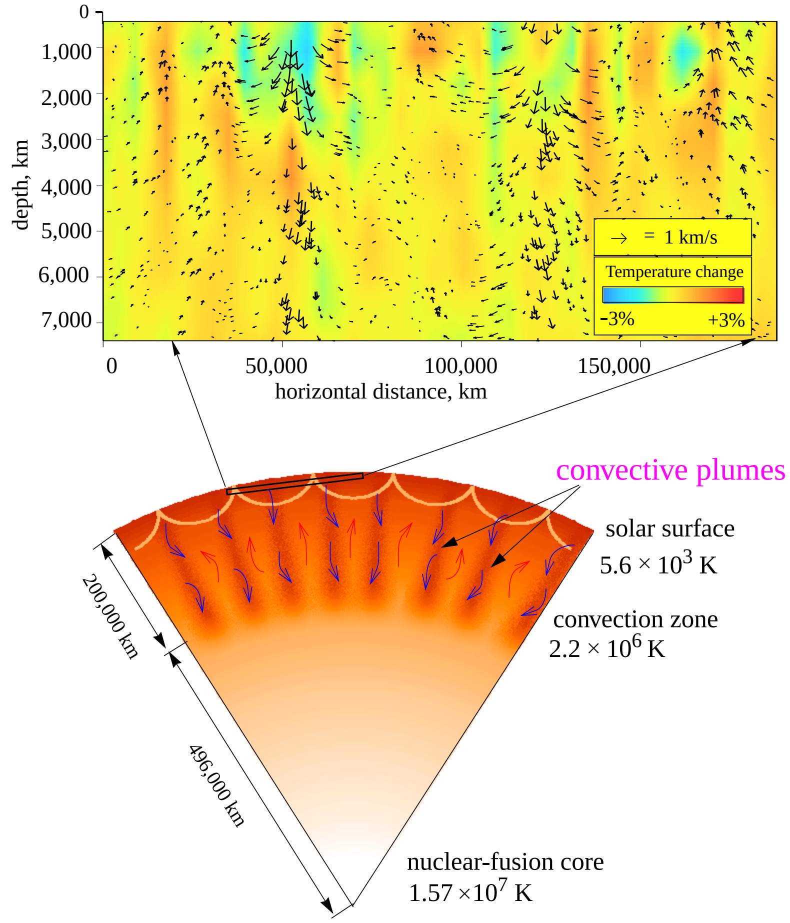
360
380
400
420
440
460

7. Time-Distance Helioseismology

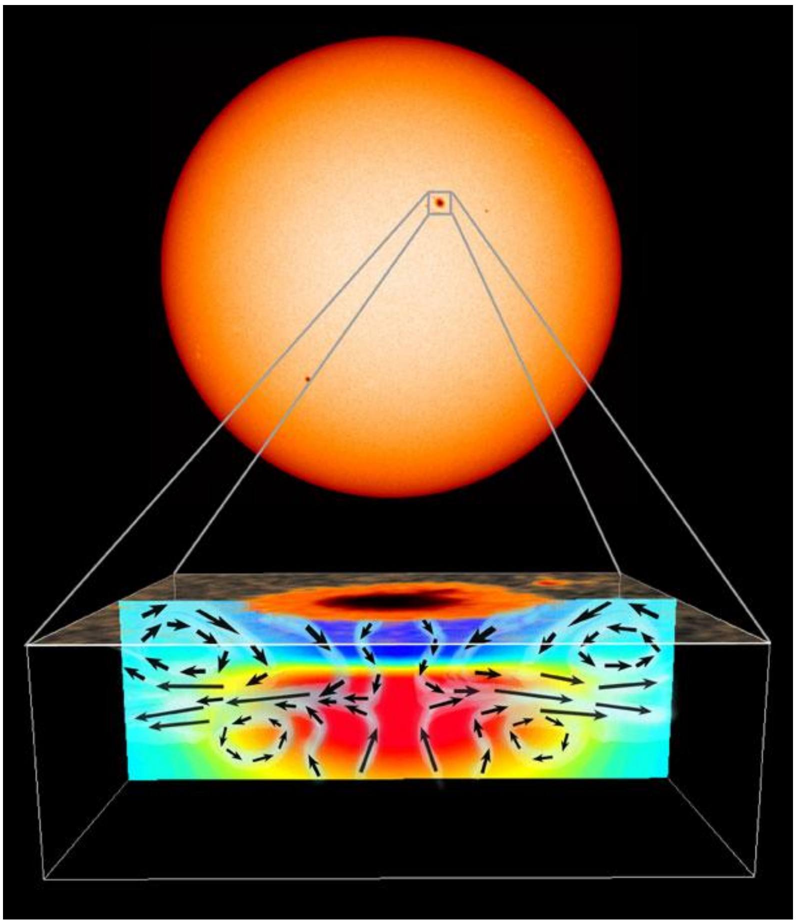




- Ray paths: horizontal distance ≈ $\pi \cdot$ depth, length ≈ $4 \cdot$ depth
- Cross-correlation of time series at two separate points as function of travel time
- Speed up in hotter regions
- Speed up for wave in flow direction, slow down in reciprocal direction
- Magnetic field $\curvearrowright$ anisotropy in travel time

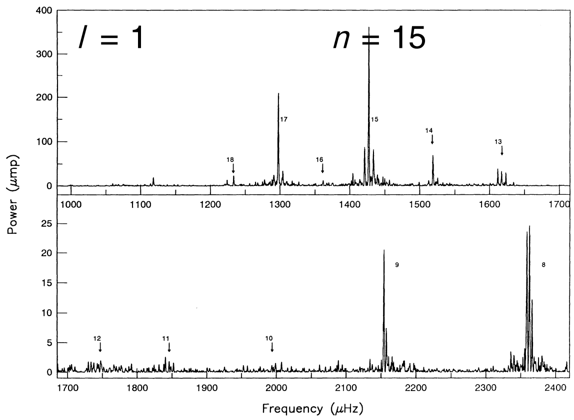


depth, km
0
1,000
2,000
3,000
4,000
5,000
6,000
7,000
horizontal distance, km
0
50,000
100,000
150,000
→ = 1 km/s
Temperature change
-3%
+3%
convective plumes
solar surface
5.6×10^3 K
convection zone
2.2×10^6 K
nuclear-fusion core
1.57×10^7 K
200,000 km
496,000 km



8. Asteroseismology

- Whole solar disk, small l
  spectral line shift – Doppler effect
- Oscillations in intensity
  → asteroseismoloy by photometry
  good photometers $\Delta m \sim 0.01$
  long time series (weeks): Whole Earth Telescope (WET)
- Spectrum of white dwarf GD 358



→ g-modes
→ determination of mass, rotation, magnetic field, ...

Appendix