Max-Planck-Research-School

"Physical Processes in the Solar System and Beyond"

18 – 22. February 2002

Numerical Simulation of Space Plasmas

M. Kotschmann, Braunschweig

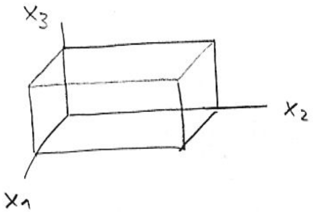
Numerical simulation of space plasmas

1. Introduction

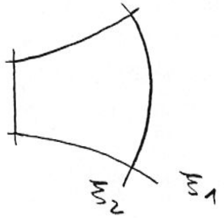
1.1. Systems and objects

System = domain in 3d space
= simulation box

- cartesian frame (x_1, x_2, x_3)



- curvilinear frame (ξ_1, ξ_2, ξ_3)

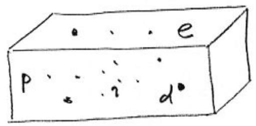


- domain is open (interaction with neighbourhood)

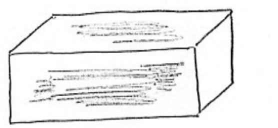
Objects = material + fields

- material : ionized gas
  neutrals
  dust particles

Microscopic view/ microscopic approach	Macroscopic view/ macroscopic approach
ensemble of small particles (points), physical parameters:	continuous gas or fluid, physical parameters:
mass	mass density
electric charge	charge density
position in space	pressure
velocity	streaming velocity

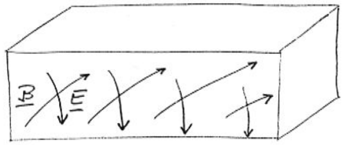


$\Rightarrow$ particle description

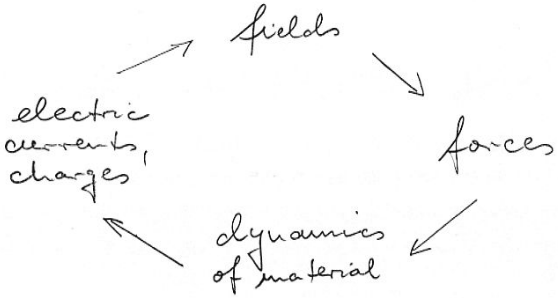


$\Rightarrow$ fluid description

- Fields: electric field $\underline{E}(\underline{x},t)$
  magnetic field $\underline{B}(\underline{x},t)$
  $\Rightarrow$ Lorentz force
  $$\underline{F}(\underline{x},t) = q(\underline{E} + \underline{v} \times \underline{B})$$



- Interaction of material and fields



- Computational plasma physics / numerical plasma simulation is the numerical handling of this cycle

1.2. Kinds of equations

- Field equations ($\underline{E}, \underline{B}$)

  Maxwells equations, up to 8 partial differential equations

- Material equations

(a) Fluid description

1 set of typically 5 equations for each fluid component (species),

2 ... 3 ... components (electrons, protons, heavy ions, charged dust particles, ...),

typ of equations: Navier-Stokes.

Particle description

(b) – Individual particles: $\sim 10^6$ O.DE

typ of equation: Newtons law

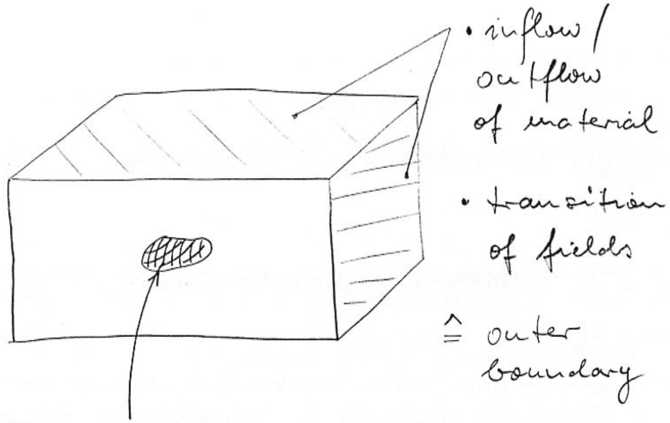
(c) – Particles are summarized by a distribution function:

typ of equation: Boltzmann, Vlasov

1.3. Boundary condition

- System is described by partial differential equations
  → boundary conditions are needfully

- Simulation box



Obstacle
source of material (evaporation)
≙ inner boundary

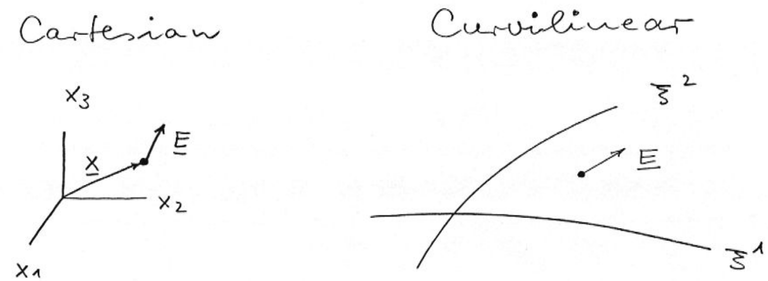
2. Basic equations

2.1. Field equations

<u>Fields:</u>

$\underline{E}(\underline{x},t)$	electric field
$\underline{B}(\underline{x},t)$	magnetic field
$\underline{j}(\underline{x},t)$	current density
$\rho(\underline{x},t)$	charge density

<u>Frame:</u>



<u>Constants</u> (system of units: SI)

ε_0 , μ_0 , $c = 1/\sqrt{\varepsilon_0 \mu_0}$

Maxwell's equations

Ampere's law:

$$\partial_{\underline{x}} \times \underline{B} = \frac{1}{c^2} \partial_t \underline{E} + \mu_0 \underline{j}$$

Faraday's law:

$$\partial_{\underline{x}} \times \underline{E} = - \partial_t \underline{B}$$

Gauss' law:

$$\partial_{\underline{x}} \underline{E} = \frac{1}{\varepsilon_0} \rho$$

No name law:

$$\partial_{\underline{x}} \underline{B} = 0$$

$\underline{j}$, ρ sources of $\underline{E}$, $\underline{B}$

2.2. Material equations

(a) Fluid description

- regard one species (e.g. protons)

$n(\underline{x},t)$	particle density
$\underline{u}(\underline{x},t)$	streaming velocity
$p(\underline{x},t)$	pressure

Continuity equation:

$$\partial_t n + \partial_{\underline{x}} (n \underline{u}) = 0$$

Momentum equation:

$$\partial_t (n \underline{u}) + \partial_{\underline{x}} (n \underline{u} \, \underline{u}) + \frac{1}{m} \partial_{\underline{x}} p = q n \left(\underline{E} + \underline{u} \times \underline{B} \right)$$

Pressure equation:

$$p = n k_B T$$

- analogous set for each species

- $$\rho(\underline{x},t) = \sum_{\text{species}} q \, n(\underline{x},t)$$

  $$\underline{j}(\underline{x},t) = \sum_{\text{species}} q \, n(\underline{x},t) \, \underline{u}(\underline{x},t)$$

(b) Particle description / Individual particles

- regard one species (e.g. protons)
- each particle gets a number: $p = 1, 2, .., 10^6, ..$

  $\underline{x}_p(t)$: position of particle p

  $\underline{v}_p(t)$: velocity of particle p

- Particle push

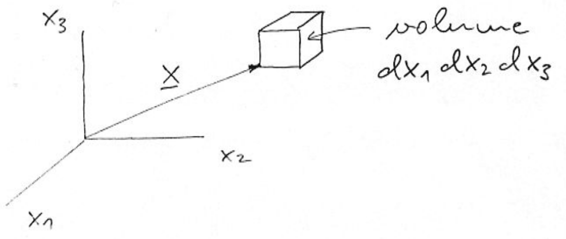
$$d_t \underline{x}_p = \underline{v}_p$$

$$d_t \underline{v}_p = \frac{q}{m}\left\{\underline{E}(\underline{x}_p, t) + \underline{v}_p \times \underline{B}(\underline{x}_p, t)\right\}$$

- analogous set of equations for each species (electrons, heavy ions, charged dust, ...)

- $$\rho(\underline{x}, t) = \sum_{\text{species}} \sum_{p \text{ in } dx_1 dx_2 dx_3} q$$

$$\underline{j}(\underline{x}, t) = \sum_{\text{species}} \sum_{p \text{ in } dx_1 dx_2 dx_3} q\, \underline{v}_p$$

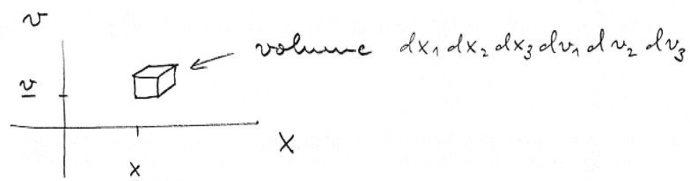


(c) Particle description / Particle distribution function

- regard one species
- introduction of distribution function

$$F(\underline{x}, \underline{v}, t) \sim \text{number of particles at position } \underline{x} \text{ with velocity } \underline{v}$$

- phase space (6 dim)



- Vlasov equation for each species

$$\partial_t F + \underline{v}\, \partial_{\underline{x}} F + \frac{q}{m}(\underline{E} + \underline{v} \times \underline{B})\, \partial_{\underline{v}} F = 0$$

- $$\rho(\underline{x}, t) = \sum_{\text{species}} \int q\, F(\underline{x}, \underline{v}, t)\, d^3v$$

$$\underline{j}(\underline{x}, t) = \sum_{\text{species}} \int q\, \underline{v}\, F\, d^3v$$