IMPRS SSP, March 2003
Dynamo Theory, Part 2

The Solar Dynamo

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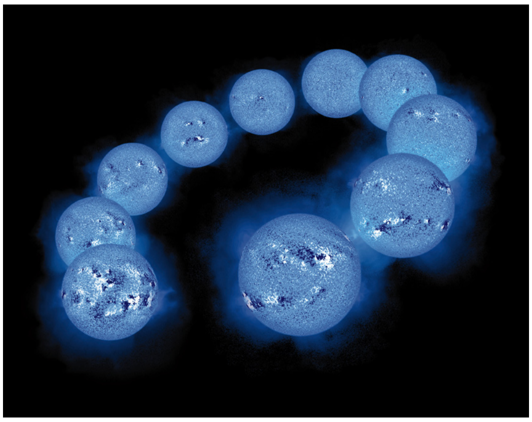
Mean-field dynamo models
The solar cycle
Long-term variability



1. Mean-field dynamo models

Mean-field vs. dynamical 3D models

1.1 Dynamo equation

Spherical coordinates (r, ϑ, φ)

axisymmetric mean fields $\boldsymbol{B}$, $\boldsymbol{v}$, $\partial/\partial\varphi = 0$, azimuthal averages

kinematic, i.e. $\boldsymbol{v}$ given

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B} + \alpha \boldsymbol{B} - \eta_T \nabla \times \boldsymbol{B}) \quad \text{mean induction equation}$$

$$\langle \boldsymbol{v}' \times \boldsymbol{B}' \rangle = \alpha \boldsymbol{B} - \beta \nabla \times \boldsymbol{B}, \quad \eta_T = \eta_m + \beta$$

$$\alpha \approx -\tau \langle \boldsymbol{v}' \cdot (\nabla \times \boldsymbol{v}') \rangle / 3, \quad \beta \approx \tau \langle \boldsymbol{v}'^2 \rangle / 3 \quad \text{for isotropic turbulence}$$

$$\boldsymbol{B} = \boldsymbol{B}_p + \boldsymbol{B}_t = \nabla \times (0, 0, A_\varphi) + (0, 0, B_\varphi), \quad B = B_\varphi, \quad A = A_\varphi$$

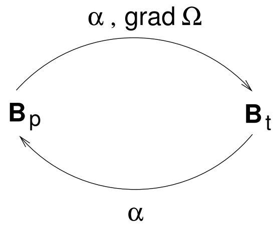
$$\boldsymbol{v} = \boldsymbol{v}_p + \boldsymbol{v}_t = \nabla \times (0, 0, \psi / r \sin\vartheta) + (0, 0, \Omega(r, \vartheta) r \sin\vartheta)$$

$$\frac{\partial \boldsymbol{B}_p}{\partial t} = \nabla \times (\boldsymbol{v}_p \times \boldsymbol{B}_p + \alpha \boldsymbol{B}_t - \eta_T \nabla \times \boldsymbol{B}_p)$$

$$\frac{\partial \boldsymbol{B}_t}{\partial t} = \nabla \times (\boldsymbol{v}_p \times \boldsymbol{B}_t + \boldsymbol{v}_t \times \boldsymbol{B}_p + \alpha \boldsymbol{B}_p - \eta_T \nabla \times \boldsymbol{B}_t)$$

$$\boldsymbol{v}_p = 0, \ \alpha = \eta_T = \text{const}, \ \nabla \times \nabla \times (F \boldsymbol{e}_\varphi) = -\Delta_1 F \boldsymbol{e}_\varphi, \ \Delta_1 = \Delta - \frac{1}{r^2 \sin^2 \vartheta}$$

$$\frac{\partial B}{\partial t} = r \sin\vartheta (\nabla \times \boldsymbol{A}) \cdot \nabla\Omega - \alpha \Delta_1 A + \eta_T \Delta_1 B$$
$$\frac{\partial A}{\partial t} = \alpha B + \eta_T \Delta_1 A$$



rigid rotation has no effect

no dynamo if $\alpha = 0$

$$\frac{\alpha\text{−term}}{\nabla\Omega\text{−term}} \approx \frac{\alpha_0}{|\nabla\Omega| L^2} \begin{cases} \gg 1 & \alpha^2\text{−dynamo} \\ \ll 1 & \alpha\Omega\text{−dynamo} \end{cases}$$

Sun: $|\nabla\Omega| L^2 \approx \Delta v \approx 400 \text{ ms}^{-1}$, $\quad \alpha \approx v'^2_{\text{rms}} \tau / L \approx 1 \text{ ms}^{-1}$

$\curvearrowright$ $\alpha\Omega$-dynamo

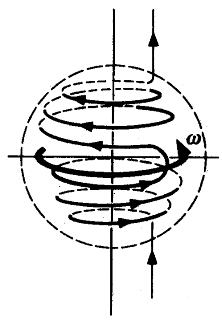
$$\frac{|\boldsymbol{B}_t|}{|\boldsymbol{B}_p|} \approx \left(\frac{|\nabla\Omega| L^2}{\alpha_0} \right)^{1/2} \approx 20 \quad \text{toroidal field dominates}$$

bipolar regions on surface erupted toroidal field

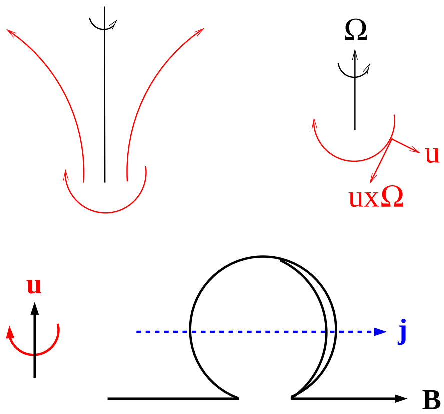
$\curvearrowright$ EW orientation

1.2 Dynamo effects

- Differential rotation

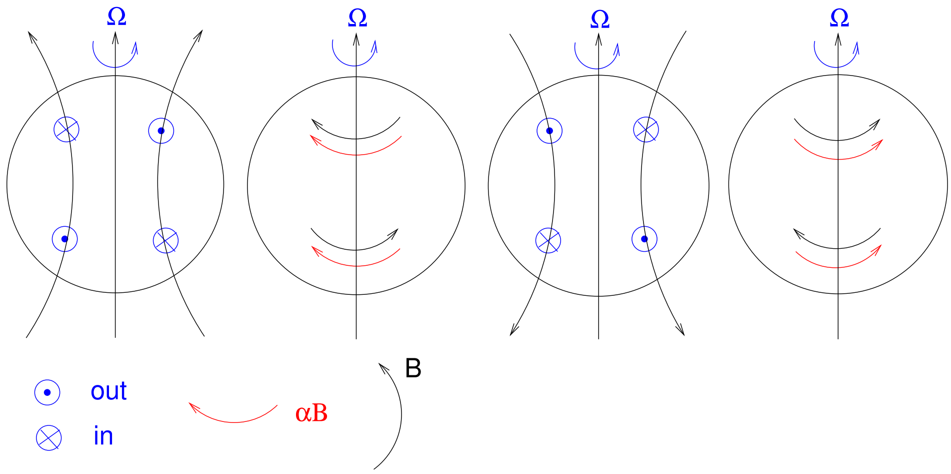


- Helical convection / α-effect



1.3 $\alpha\Omega$-dynamo

$$\frac{\partial \Omega}{\partial r} < 0, \quad \alpha \sim \cos\vartheta$$



poloidal field

toroidal field by differential rotation; electric currents by α-effect

poloidal field by α-effect

toroidal field by differential rotation; electric currents by α-effect

periodically alternating field, here antisymmetric with respect to equator

1.4 Dynamo waves

Consider $\alpha\Omega$-equations locally

Cartesian coordinates (x, y, z) corresponding to (ϑ, φ, r)

α = const, η_T = const, $\boldsymbol{v} = (0, \Omega z, 0)$ with Ω = const

$\boldsymbol{B}_t = (0, B(x,t), 0)$, $\boldsymbol{B}_p = (0, 0, \partial A(x,t)/\partial x)$

$\dot{B} = \Omega A' + \eta_T B''$, $\quad \dot{A} = \alpha B + \eta_T A''$, $\quad \dot{} = \partial/\partial t$, $\quad ' = \partial/\partial x$

ansatz $(B, A) = (B_0, A_0)\exp[i(\varpi t + kx)]$

dispersion relation $(i\varpi + \eta_T k^2)^2 = ik\Omega\alpha$

assume $\alpha\Omega < 0$, e.g. $\alpha > 0, \Omega < 0$ and take $k > 0$

$\varpi = i\eta_T k^2 - (1+i)|k\alpha\Omega/2|^{1/2}$ (Parker, 1955)

growth rate $-\varpi_I = -\eta_T k^2 + |k\alpha\Omega/2|^{1/2} \geq 0$ for

$|k\alpha\Omega/2|^{1/2} \geq \eta_T k^2$: inductive effects must exceed threshold

$\varpi_R = -|k\alpha\Omega/2|^{1/2} < 0$: wave propagation in positive x-direction

identical result with $k < 0$

if $\alpha\Omega > 0$ wave propagation in negative x-direction

In general:

wave propagates along surfaces of constant rotation (Yoshimura, 1975)

direction of propagation depends on sign($\alpha\Omega$)

period geometric mean of $(k\alpha)^{-1}$ and Ω^{-1}

in the critical case $(\eta_T k^2)^{-1}$, decreasing with increasing excitation

1.5 Dynamo number

$$\Omega = \Omega_0\tilde{\Omega}, \quad \alpha = \alpha_0\tilde{\alpha}, \quad t = \frac{R^2}{\eta_T}\tilde{t}, \quad B = B_0\tilde{B}, \quad A = RB_0\tilde{A}$$

$$\tilde{\tilde{A}} = \frac{\Omega_0 R^2}{\eta_T}\tilde{A}$$

$$\frac{\partial B}{\partial t} = r\sin\vartheta(\nabla\times\boldsymbol{A})\cdot\nabla\Omega + \Delta_1 B$$

$$\frac{\partial A}{\partial t} = P\alpha B + \Delta_1 A$$

$$\boxed{P = R_\alpha R_\Omega = \frac{\alpha_0 R}{\eta_T}\cdot\frac{\Omega_0 R^2}{\eta_T} \quad \text{dynamo number}}, \quad B_t/B_p \approx (R_\Omega/R_\alpha)^{1/2}$$

1.6 $\alpha\Omega$ dynamo modes

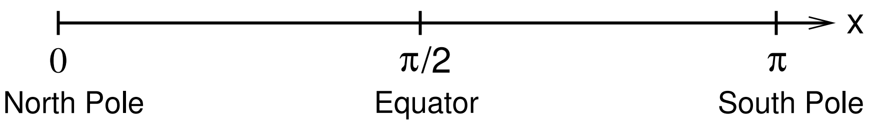
bounded $\alpha\Omega$ dynamo solutions, dimensionless

$\alpha = \alpha_0\cos x, \quad \partial u_y/\partial z = G_0\sin x$ dynamo effects

$\dot{A} = P\cos xB + A'', \quad \dot{B} = \sin xA' + B''$ dynamo equations

$P = R_\alpha R_\Omega = \frac{\alpha_0 L}{\eta_T}\cdot\frac{G_0 L^2}{\eta_T}$ dynamo number

boundary conditions, $L = \pi/2$



$x = 0 : A = B = 0$

$x = \pi : A = B = 0$

$x = \pi/2$: antisymmetric solution, dipolar : $A' = B = 0$

symmetric solution, quadrupolar : $A = B' = 0$

now antisymmetric solution

Free decay:

$\dot{A} = A'', \quad \dot{B} = B''$

$A_n = e^{\varpi_n t}\sin nx$ with $\varpi_n = -n^2, \quad n = 1, 3, 5, \ldots$

$B_n = e^{\varpi_n t}\sin nx$ with $\varpi_n = -n^2, \quad n = 2, 4, 6, \ldots$

Eigenvalue problem:

$$\dot{A} = P\cos x B + A'', \quad \dot{B} = \sin x A' + B''$$

expansion in decay modes (complete, orthogonal, satisfy b.c.)

$$A = e^{\varpi t} \sum_{n=1,3,5,\ldots} a_n \sin nx, \quad B = e^{\varpi t} \sum_{n=2,4,6,\ldots} b_n \sin nx$$

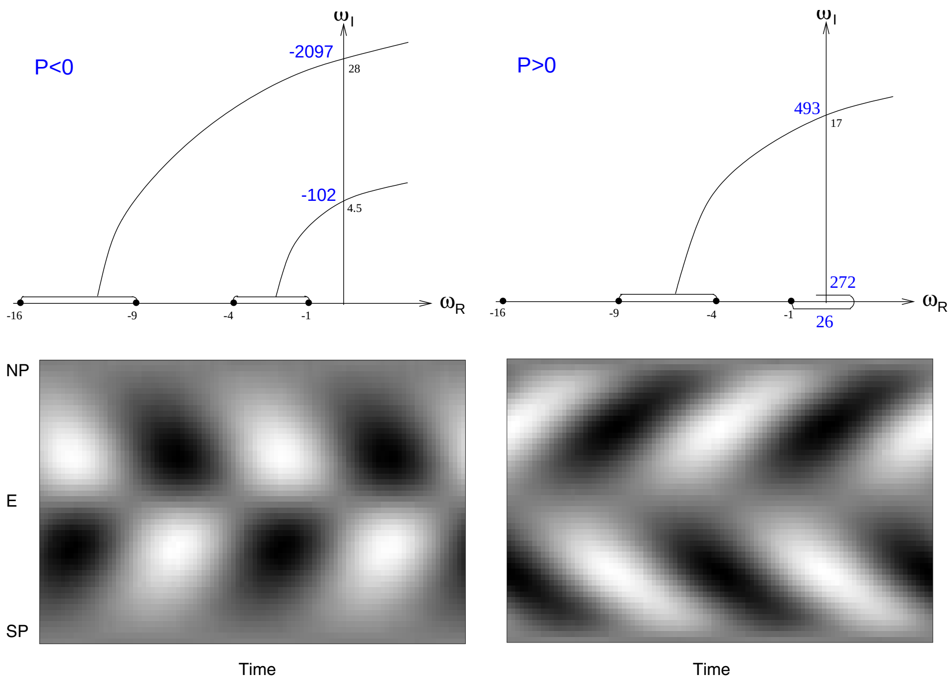
$$\sin x \cos nx = 1/2\,[\sin(n+1)x - \sin(n-1)x]$$
$$\cos x \sin nx = 1/2\,[\sin(n+1)x + \sin(n-1)x]$$
$$\int_0^{\pi/2} \sin nx \sin mx \, dx = \pi/4\,\delta_{nm}$$

$$\varpi a_m = P/2(b_{m-1} + b_{m+1}) - m^2 a_m, \qquad m \text{ odd}$$
$$\varpi b_m = 1/2((m-1)a_{m-1} - (m+1)a_{m+1}) - m^2 b_m, \qquad m \text{ even}$$

$$\varpi \begin{pmatrix} a_1 \\ b_2 \\ a_3 \\ b_4 \\ \vdots \end{pmatrix} = \begin{pmatrix} -1 & P/2 & & & \\ 1/2 & -4 & -3/2 & & \\ & P/2 & -9 & P/2 & \\ & & 3/2 & -16 & -5/2 \\ & & & & \ddots \end{pmatrix} \begin{pmatrix} a_1 \\ b_2 \\ a_3 \\ b_4 \\ \vdots \end{pmatrix}$$

vary P until $\varpi_R = 0$: P_{crit}

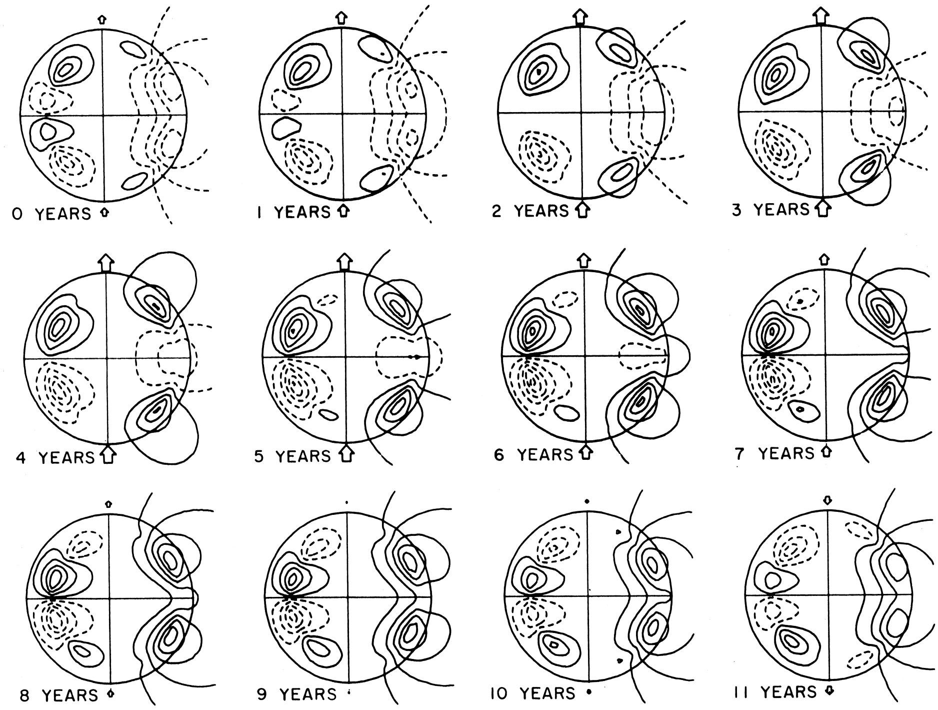
Dipole: antisymmetric with respect to equator



Pcpublic/schmitt/dynamo/dynewp.f and dynew.f

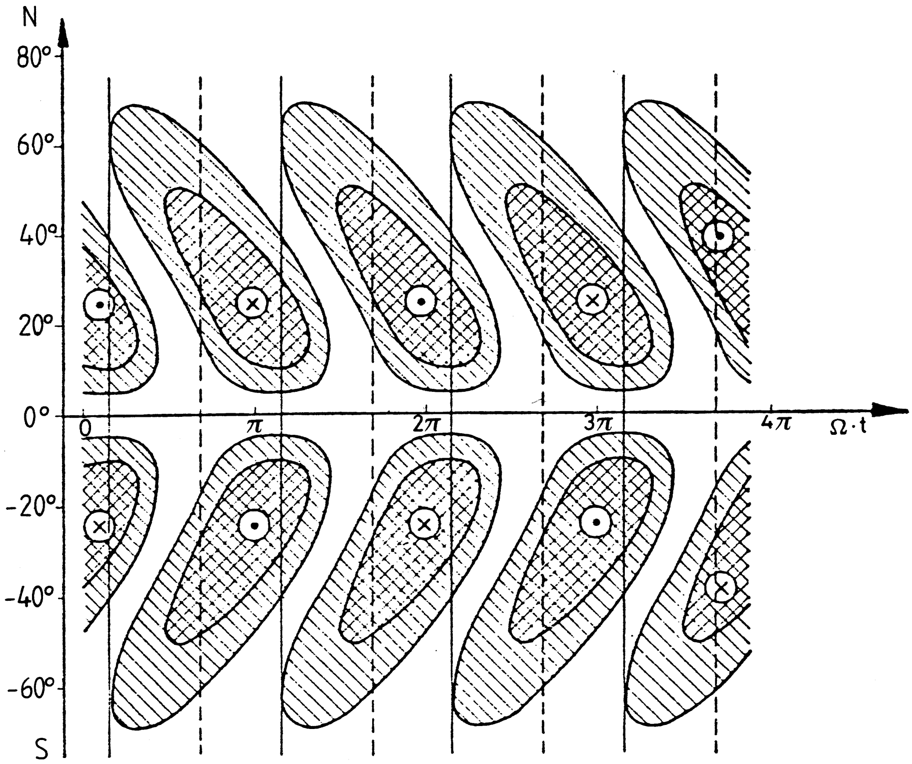
Exercise: find critical dynamo numbers for quadrupole, symmetric with respect to equator

1.7 Spherical $\alpha\Omega$ solutions



(Stix 1976)

theoretical butterfly diagram



(Krause and Steenbeck 1969)

1.8 Nonlinear effects

Linear theory:

exponential growth for $P > P_{\text{crit}}$,

thus often $P = P_{\text{crit}}$ used, further $B = |\boldsymbol{B}|$ not determined

nonlinear effects through Lorentz force or flux loss,

especially for $B \gtrsim B_{\text{eq}}$ with $B_{\text{eq}}^2/8\pi = 1/2\, \rho u_{\text{rms}}^2$

Lenz law: reduction of induction effect

heuristic approaches, partly backed by mean field theory

$$\langle F_{\text{Lor}} \rangle = \boldsymbol{j} \times \boldsymbol{B} + \langle \boldsymbol{j}' \times \boldsymbol{B}' \rangle \quad \longrightarrow \quad \boldsymbol{v} \quad \longrightarrow \quad \Omega$$

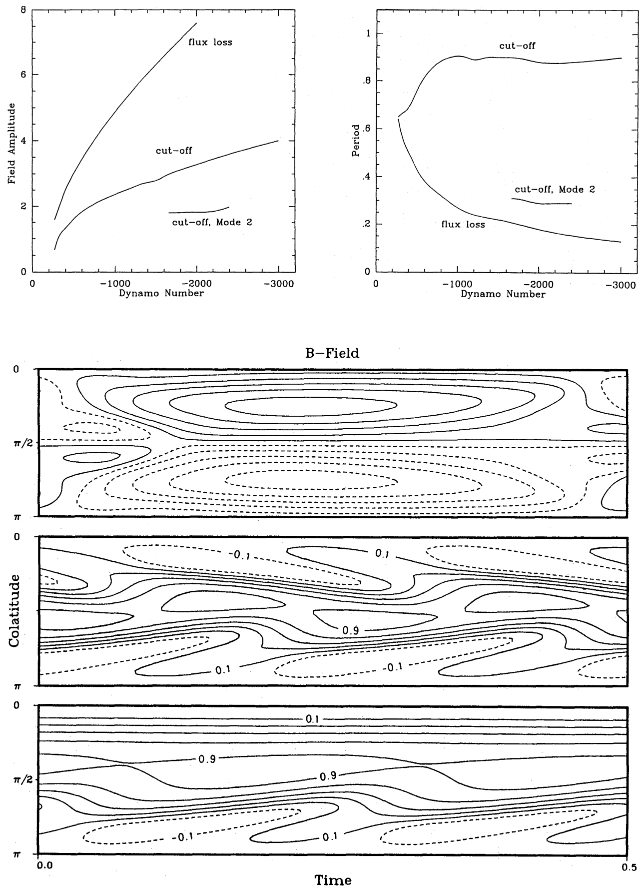
$$\boldsymbol{F}'_{\text{Lor}} = \boldsymbol{j} \times \boldsymbol{B}' + \boldsymbol{j}' \times \boldsymbol{B} \quad \longrightarrow \quad \boldsymbol{v}' \quad \longrightarrow \quad \alpha$$

α-quenching:

$\alpha = \alpha_0\, f(B)$, f decreasing with increasing B, often

$$f(B) = 1 - B^2/B_c^2 \quad \text{or} \quad f(B) = 1/(1 + B^2/B_c^2) \quad \text{or}$$

$$f(B) = B_c^3/B^3 \quad \text{with} \quad B_c \approx B_{\text{eq}} \quad \text{or} \quad B_c \approx B_{\text{eq}}/\sqrt{R_m}$$



Schmitt and Schüssler (1989)

Dynamical Ω-quenching:

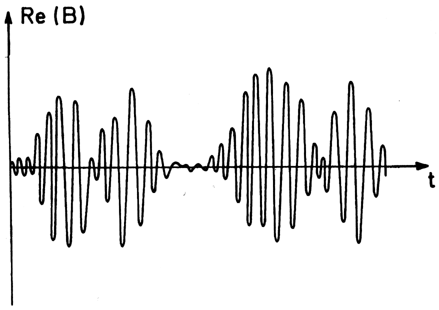
dynamical action of Lorentz force on differential rotation

truncated system

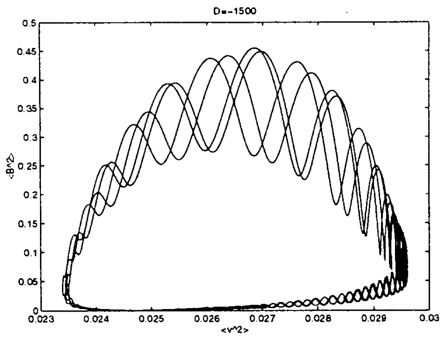
$$\dot{A} = B - A$$
$$\dot{B} = iP\Omega A - B$$
$$\dot{\Omega} = iAB - \nu\Omega$$

similar to Lorenz system

rich bifurcation structure for increasing P, chaotic solutions

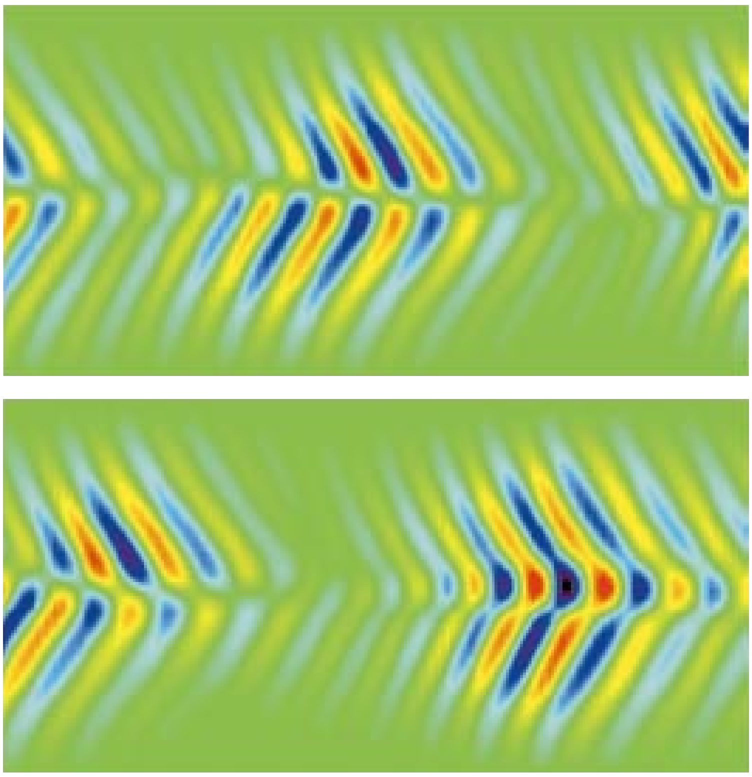


Cattaneo et al. (1984)



Tobias (1996)

2D PDE more regular



Weiss and Tobias (2000)

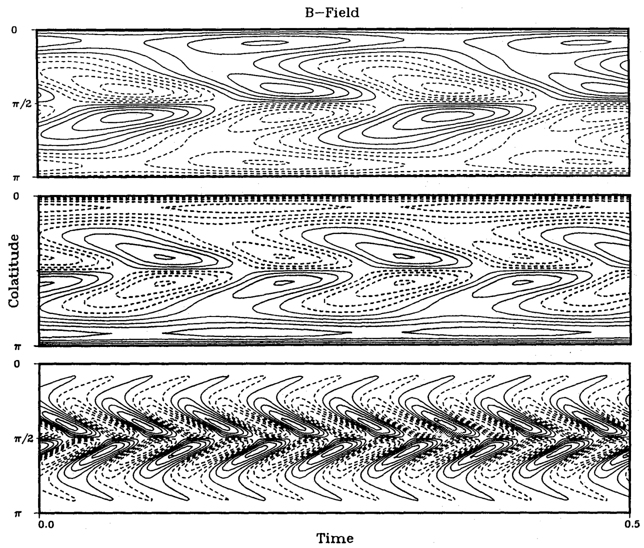
Flux loss due to magnetic buoyancy:

either by extra loss term in $\boldsymbol{B}_{\mathrm{tor}}$ equation, e.g.

$$\frac{B_0}{\tau}g(B) = \frac{B_0}{\tau}\begin{cases} -\mathrm{sgn}(B)(B^n - B_c^n) & \text{for} \quad |B| > B_c\,, \quad n = 2, 3 \\ 0 & \text{for} \quad |B| < B_c \end{cases}$$

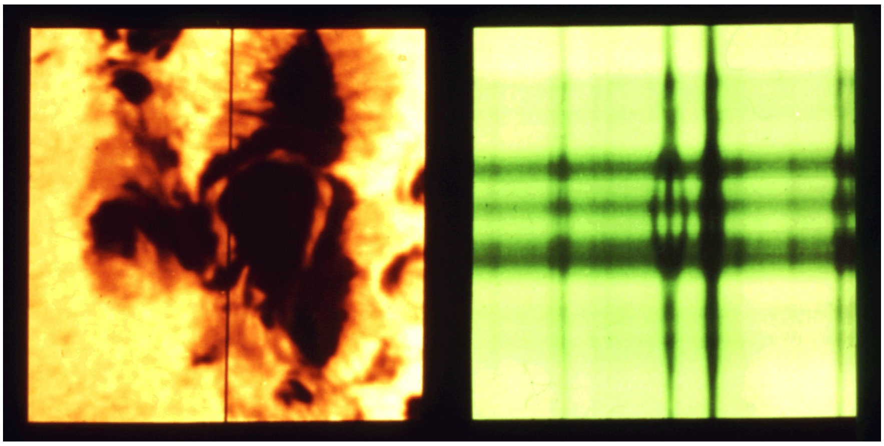
or

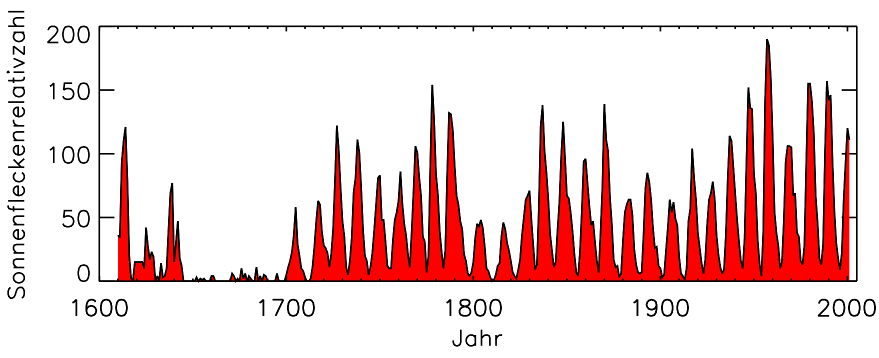
$$v_r = h(B^n) \quad \text{in} \quad \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \text{term}$$



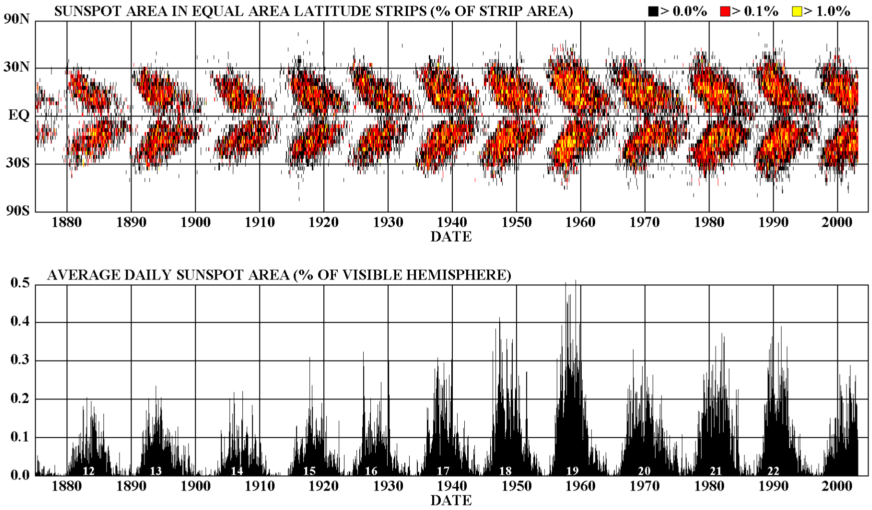
2. The solar cycle

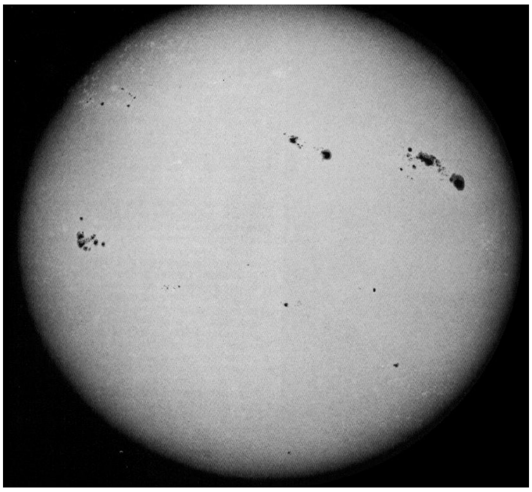
2.1 Observations

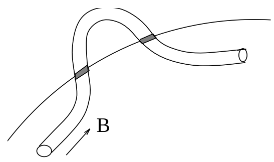


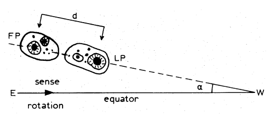


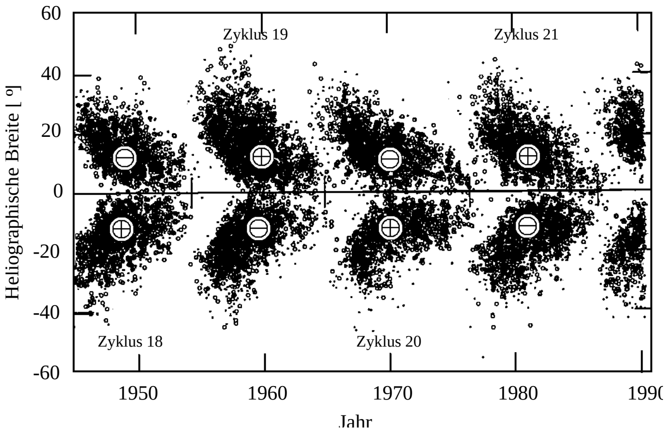
DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS

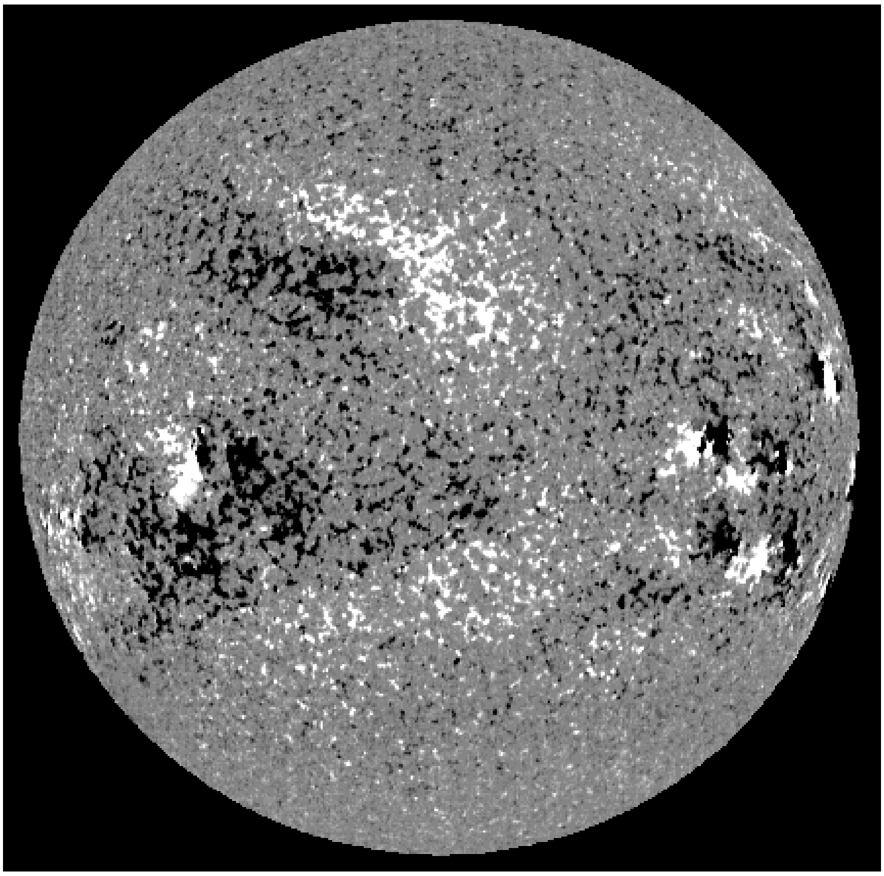








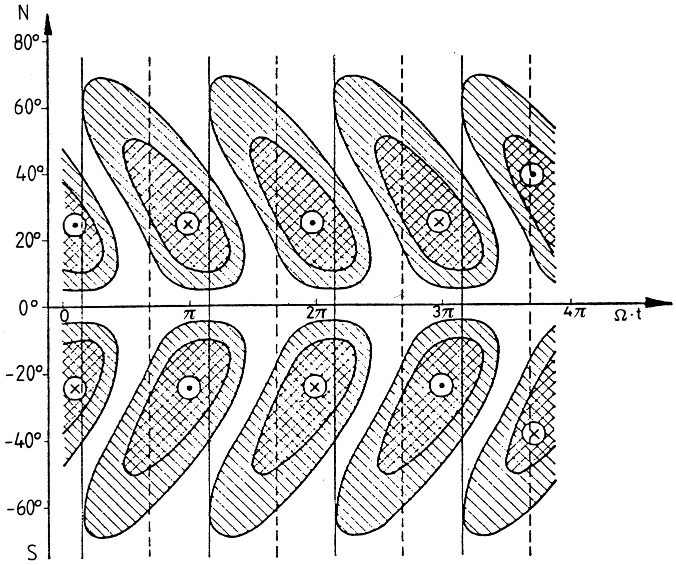




2.2 Classical dynamo models

$\alpha\Omega$-dynamo in convection zone , $\Omega(r)$ with $\partial\Omega/\partial r < 0$,

$\alpha \sim \cos\vartheta$, $\eta_T = 10^{10}\ \mathrm{cm^2s^{-1}}$

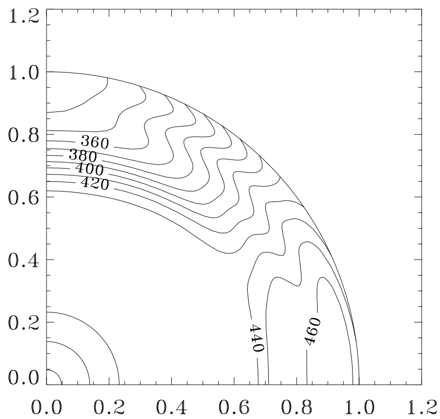


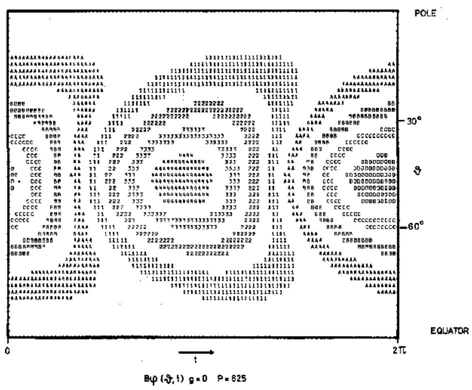
theoretical butterfly diagram $B_\varphi(\vartheta, t)$ in good accordance with observations

Steenbeck and Krause (1969)

2.3 Difficulties of convection zone models

- Intermittency: magnetic flux in small-scale structures embedded in field-free plasma (flux tubes)
- Polarity rules: strictly obeyed $\longrightarrow B \approx 10^5$ G (Schüssler, 1993)
- Magnetic buoyancy: rise time $\ll$ cycle length (Parker, 1975)
- Rotation law: from helioseismology, e.g. Tomzyck et al. (1995)
- Resulting butterfly diagram (Köhler, 1973)





2.4 Overshoot layer dynamos

Favourable dynamo site:

storage, reduced turbulent diffusivity, rotation, dynamic α-effect

- Dynamo action of magnetostrophic waves (Schmitt, 1985):

magnetic field layer unstable due to magnetic buoyancy

$\longrightarrow$ excitation of magnetostrophic waves in a fast rotating fluid

$v_A^2/v_{\rm rot} \approx v_{\rm mw} \ll v_A \ll v_{\rm rot} \ll v_S$

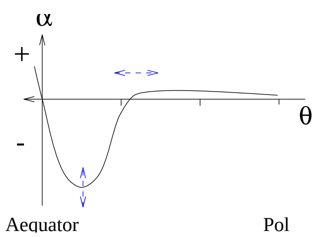
mw are helical and induce an electromotive force

$\longrightarrow$ electric current parallel to toroidal magnetic field

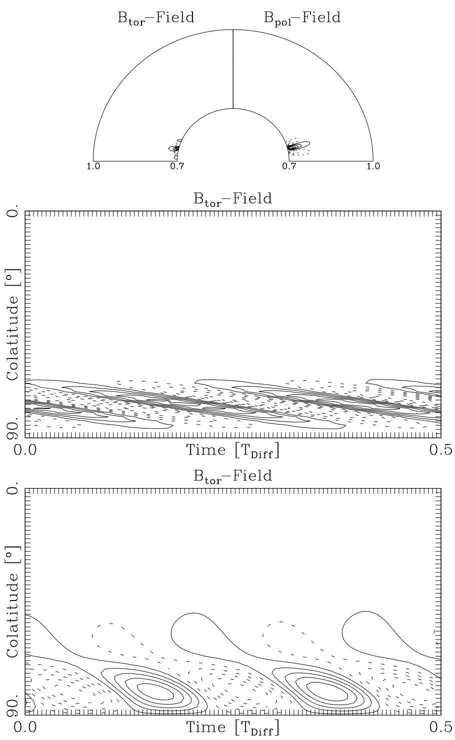
$\equiv$ dynamic α-effect: $\alpha \langle \boldsymbol{B} \rangle_{\rm tor} = \langle \boldsymbol{v} \times \boldsymbol{b} \rangle_{\rm tor}$

not based on convection, applicable to strong fields

superposition of most unstable waves:



- Dynamo model



Schmitt (1993)

difficulties: overlapping wings, parity, phase $B_\varphi - B_r$

- Flux tube instability: $B > B_{\text{threshold}}$ (Ferriz-Mas et al., 1994)

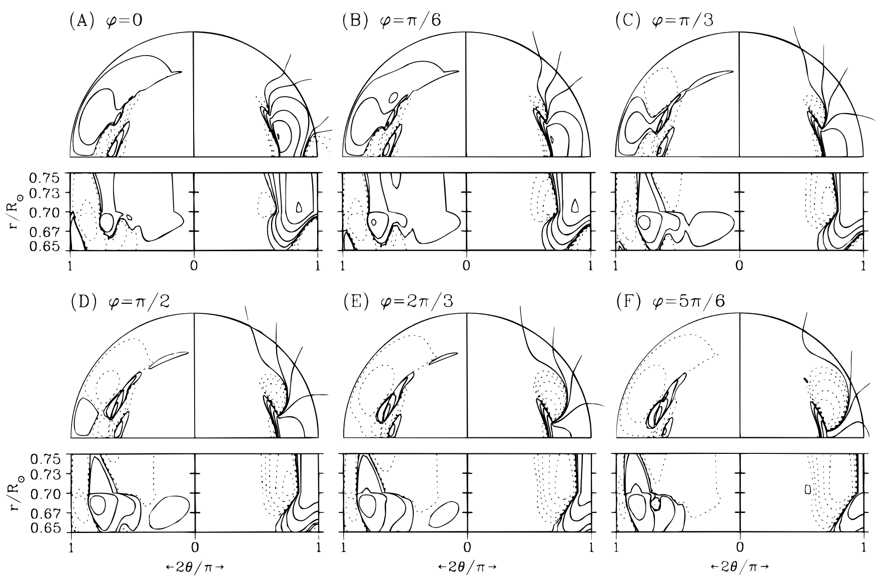
2.5 Interface dynamos

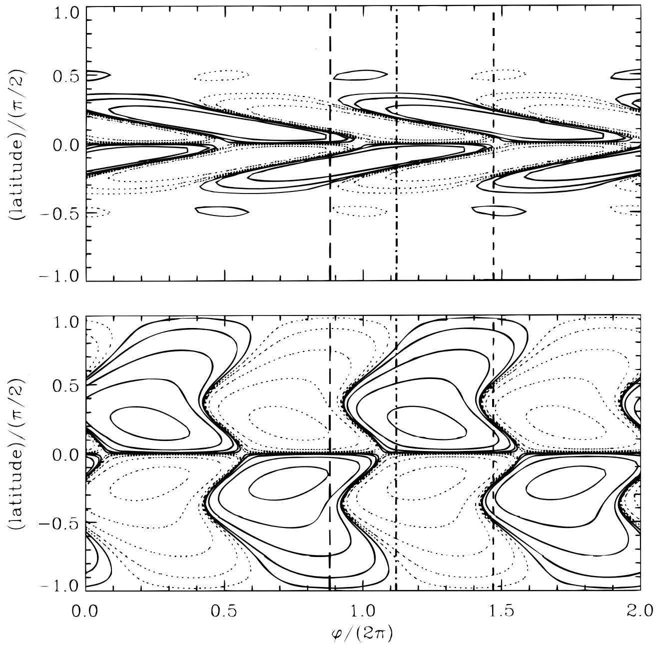
Parker (1993):

convection zone: η_T large, α

overshoot layer: η_T small, $\partial\Omega/\partial r$, most flux

dynamo on interface layer





Charbonneau and MacGregor (1997)

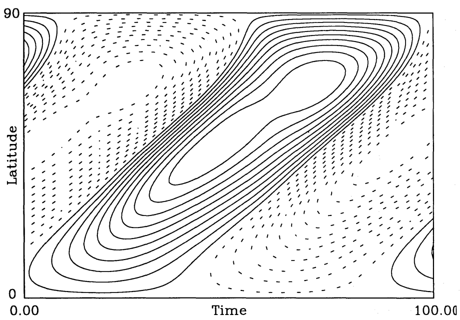
influences of $\partial\Omega/\partial\vartheta$, $\partial\Omega/\partial r$-profile, α-profile

2.6 Flux transport dynamos

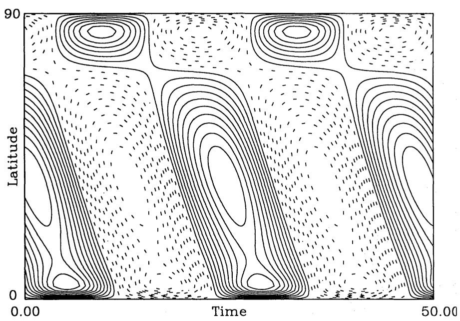
Durney (1995), Choudhuri et al. (1995),

Dikpati and Charbonneau (1999)

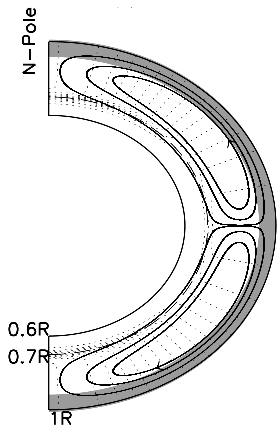
- regeneration of poloidal field through tilt
  of decaying bipolar active regions
  (Babcock, 1961; Leighton, 1969)
- rotational shear in tachocline
- transport of magnetic flux by meridional circulation

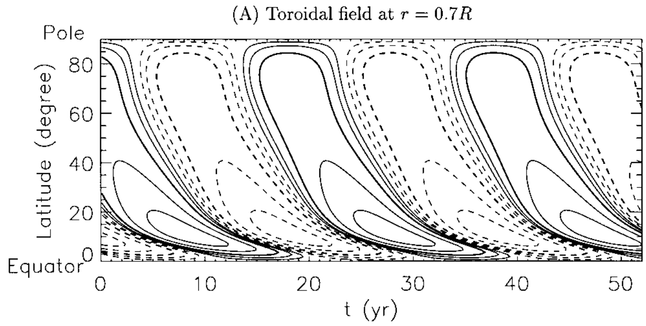


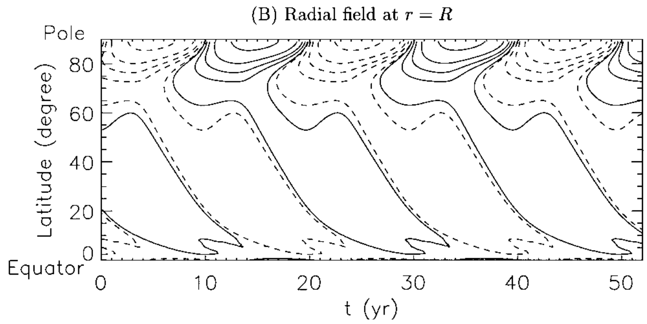
without



with meridional circulation

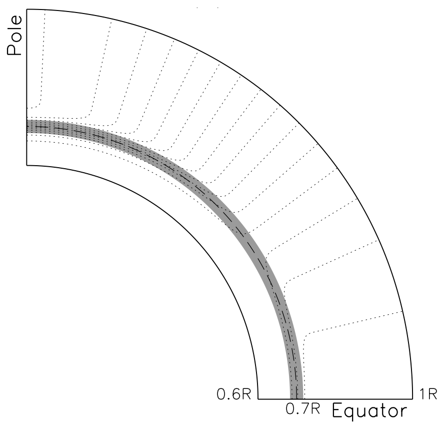




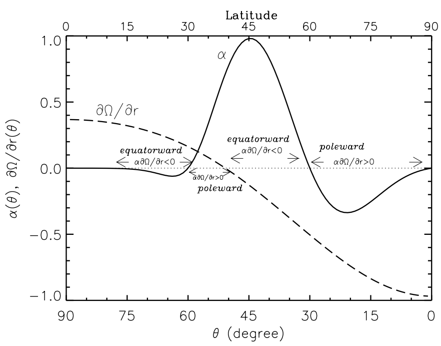


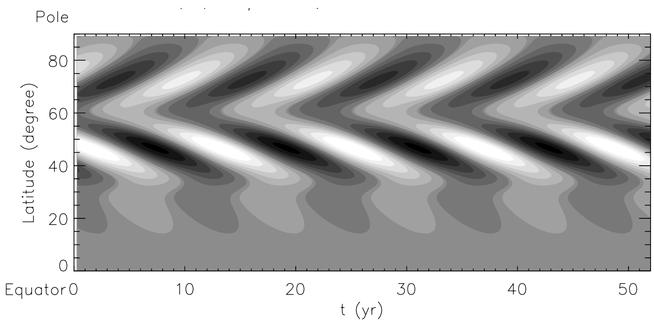
2.7 Overshoot layer dynamo with meridional circulation

Dikpati and Gilman (2001)

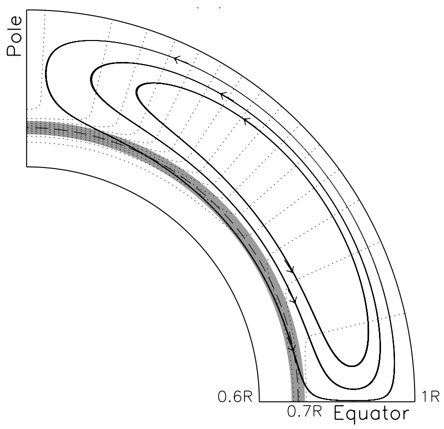


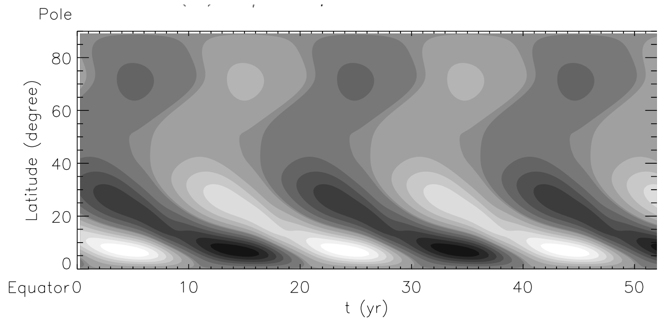
α-effect due to shear instability





without
merid. circ.



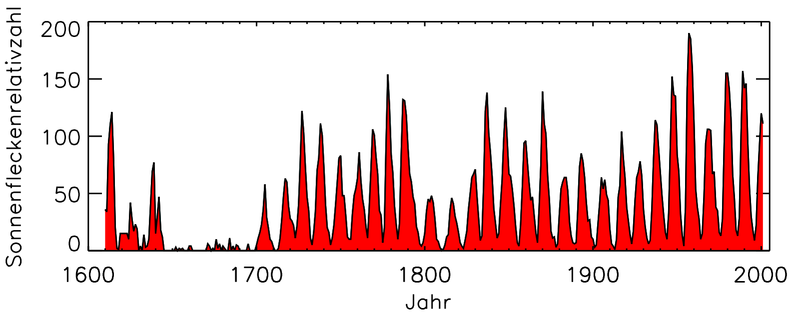


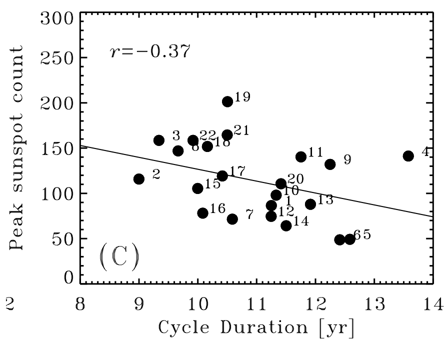
deep α-effect favours dipolar, high α-effect quadrupolar solutions

3. Long-term variations

3.1 Observations

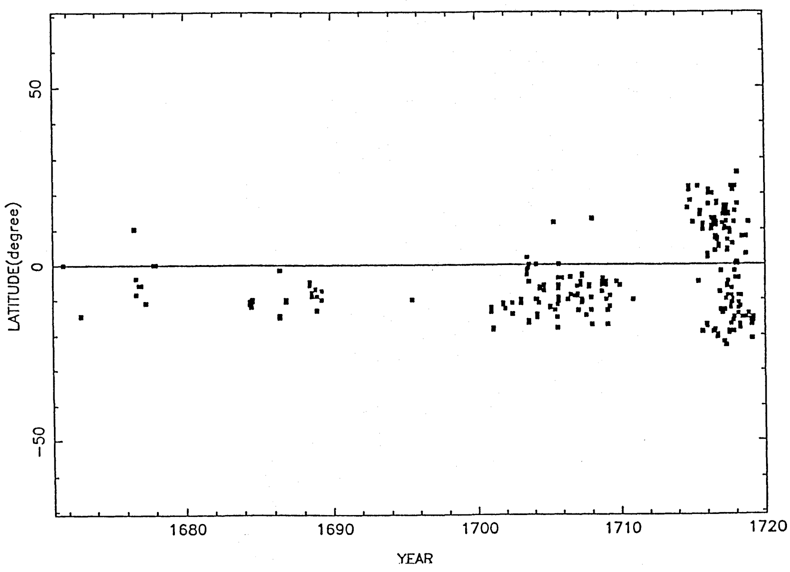
- Variation in cycle length and strength





odd-even effect (Gnevychev-Ohl)

• Maunder minimum 1630–1710, grand minima



Ribes and Nesme-Ribes (1993)

oscillatory? asymmetric?

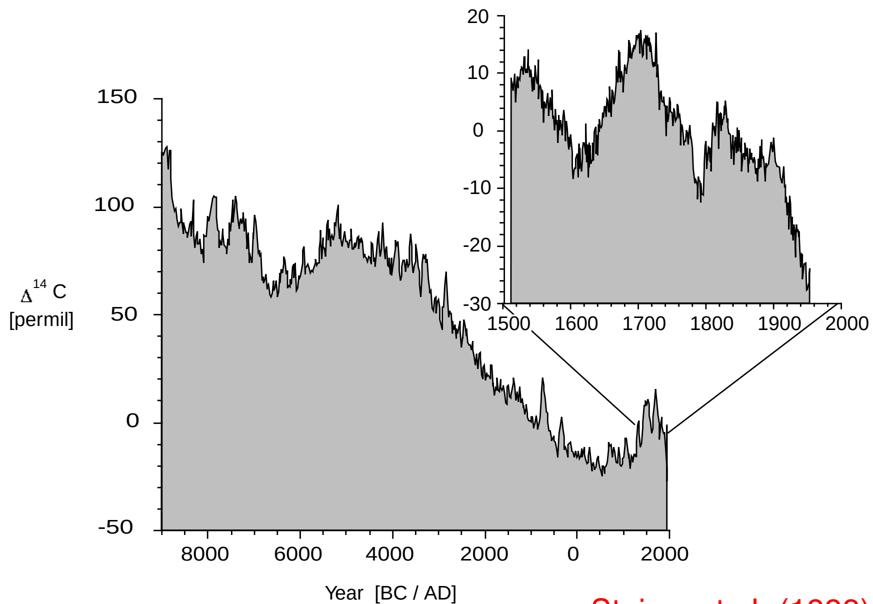
• Cosmogenic isotopes: ^{14}C, ^{10}Be

formed by cosmic rays as spallation products in the atmosphere

flux of galactic cosmic rays anticorrelated with solar activity

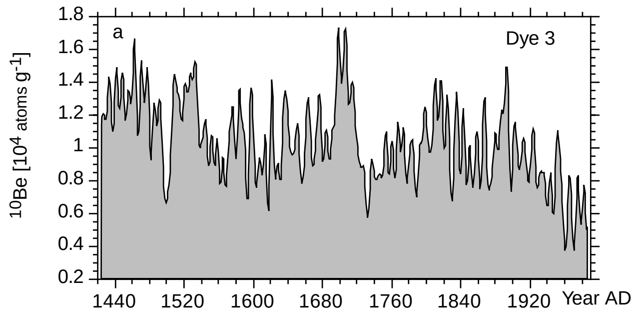
long term trend due to geomagnetic field

^{14}C: tree rings, 30-year convolution, long-term trend



Stuiver et al. (1998)

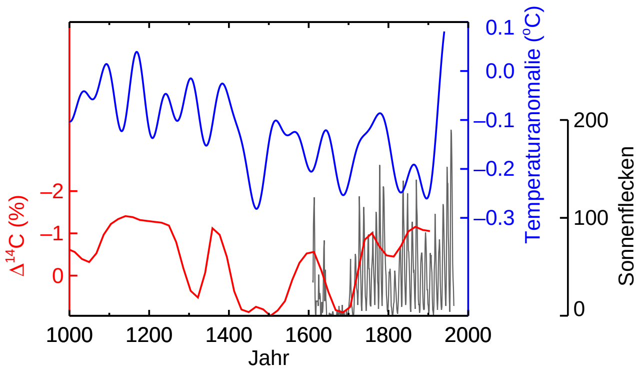
^{10}Be: ice cores, precipitation, 2-year convolution, 11-year cycles



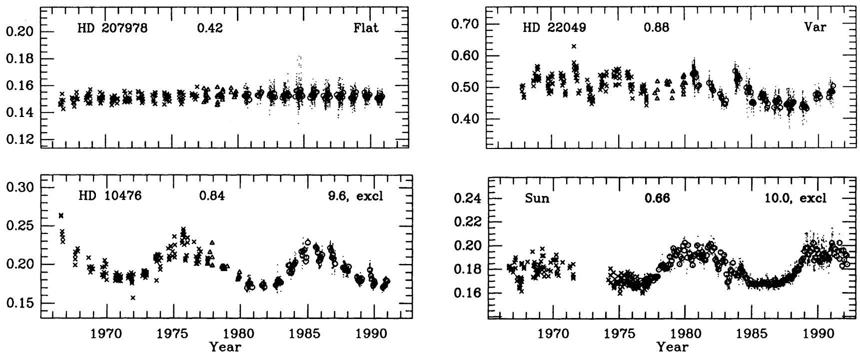
Beer et al. (1994)

cycle possibly continued during Maunder minimum

Dalton minimum, Maunder minimum, Spörer minimum, Wolf minimum, medieval maximum: potentially correlated with climate



• Cool star activity: star spots, Ca index, X-ray emission



0.25 : 0.25 : 0.5

Wilson (1978), Baliunas et al. (1995)

fast rotating stars more active

• Origin of long-term modulation of solar cycle hardly understood

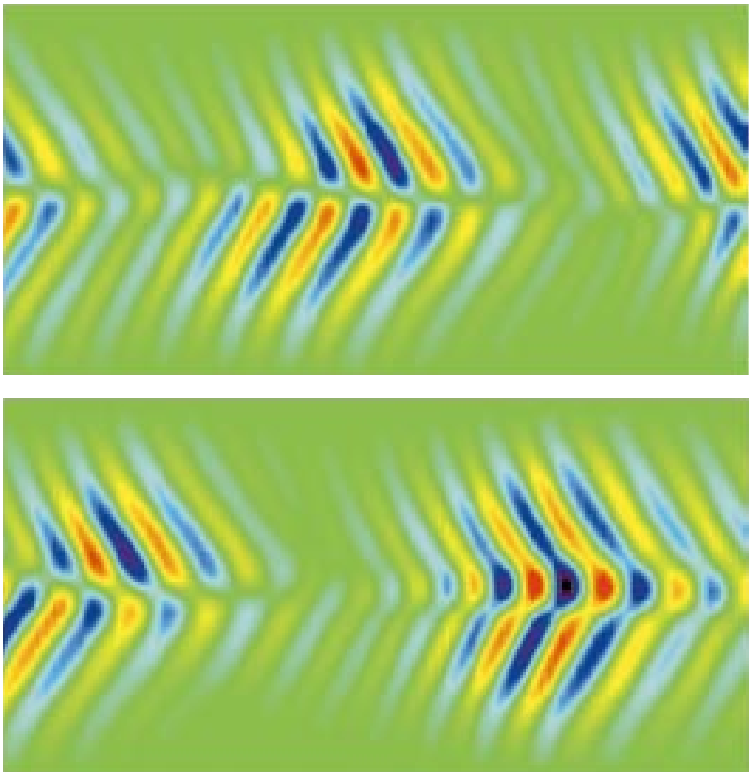
– modulation of differential rotation

– stochastic fluctuations of the α-effect

– variation of meridional circulation

– on-off intermittency

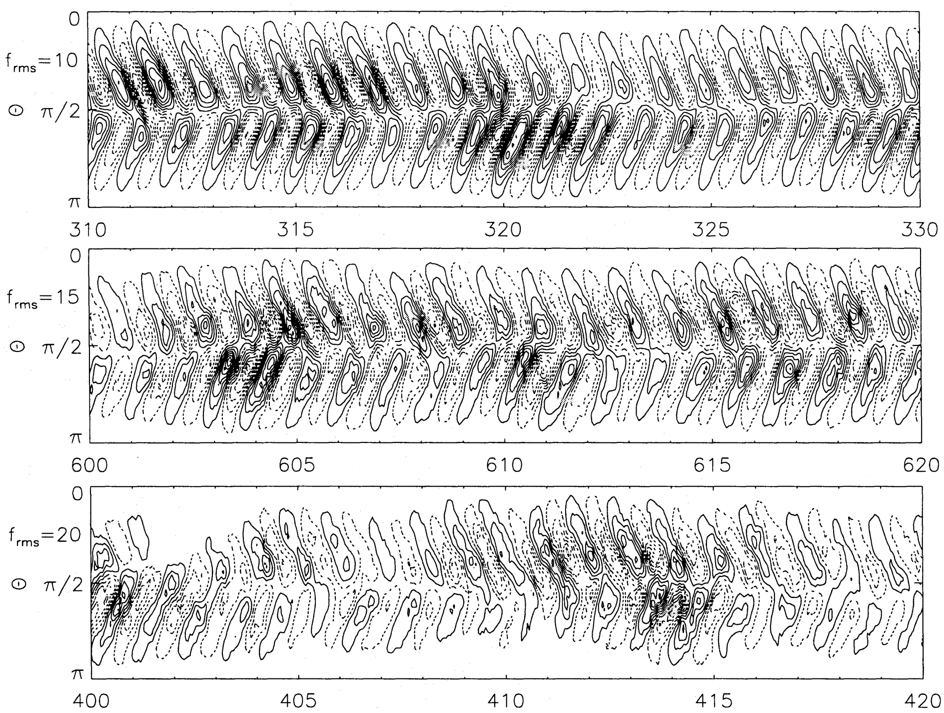
3.2 Modulation of differential rotation



Weiss and Tobias (2000)

3.3 Stochastic fluctuations of the α-effect

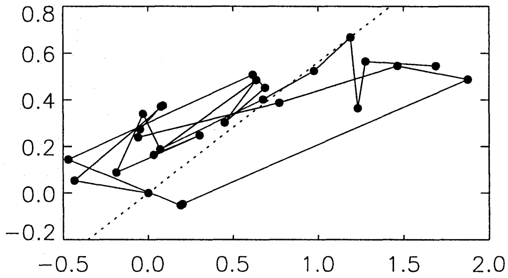
$\alpha = \alpha_0(r, \vartheta) + \delta\alpha(r, \vartheta, t)$



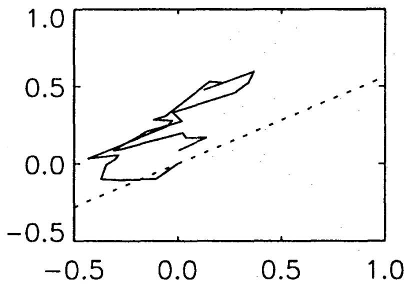
Ossendrijver, Hoyng, Schmitt (1996)

log Amplitude vs Phase shift

Sun

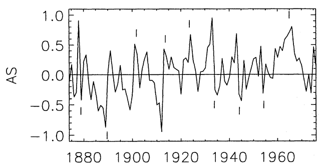


model

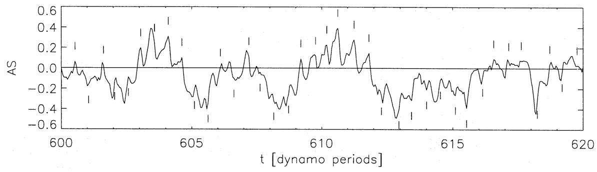


Asymmetry

Sun



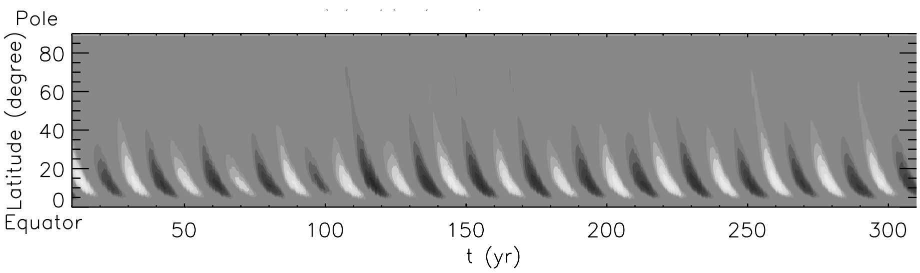
model

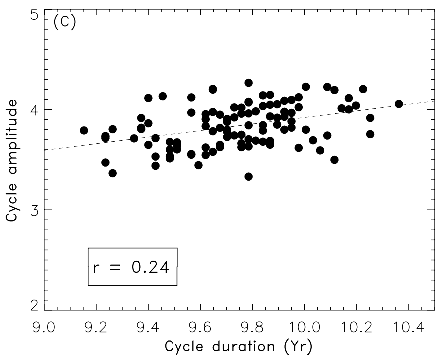


3.4 Variation of meridional circulation

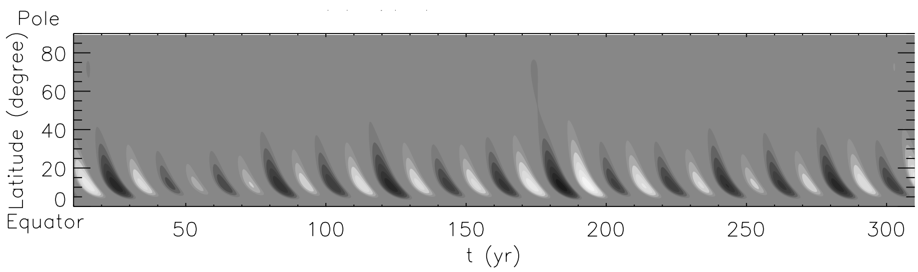
Charbonneau and Dikpati (2000)

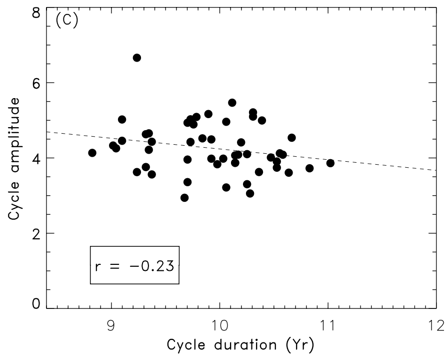
$\delta\Psi$





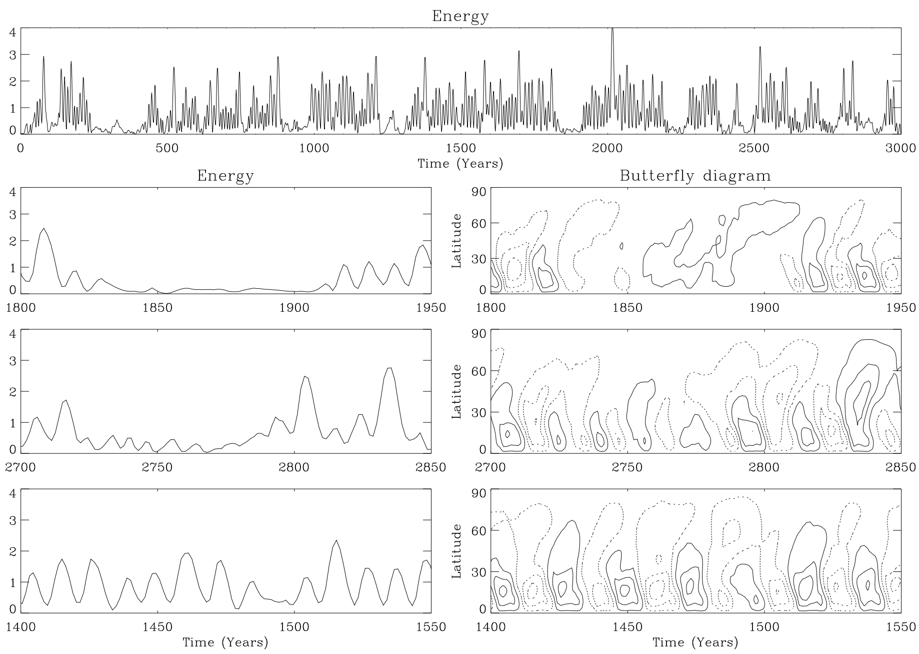
$\delta\alpha$





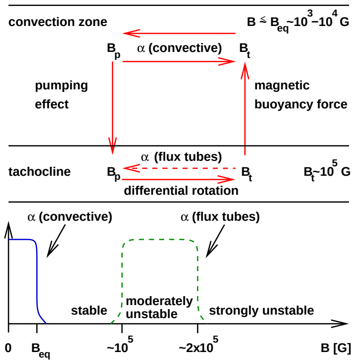
3.5 On-off intermittency

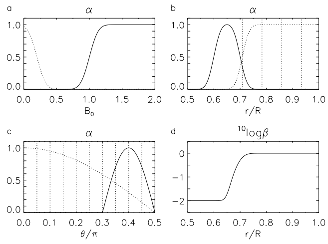
- overshoot layer dynamo driven by flux tube instability
- lower threshold in field strength for dynamo action
- random fluctuations due to magnetic fields in convection zone

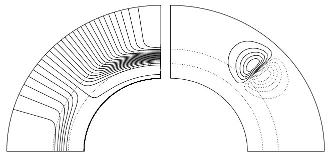


Schmitt et al. (1996)

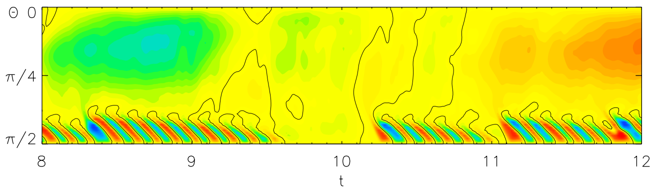
interface dynamo / flux tube dynamo







differential rotation and downflow



Ossendrijver (2000)