Plasma Physics 2:

Dynamo Theory

IMPRS Lecture

Lindau, March 3-7, 2003

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Magnetic field structure in a stellar convection zone

The plan

- 1) Formation of magnetic structure
- 2) Physics of magnetic flux tubes
- 3) Magnetic structure and the dynamo

Branches of solar physics/astrophysics

Observation

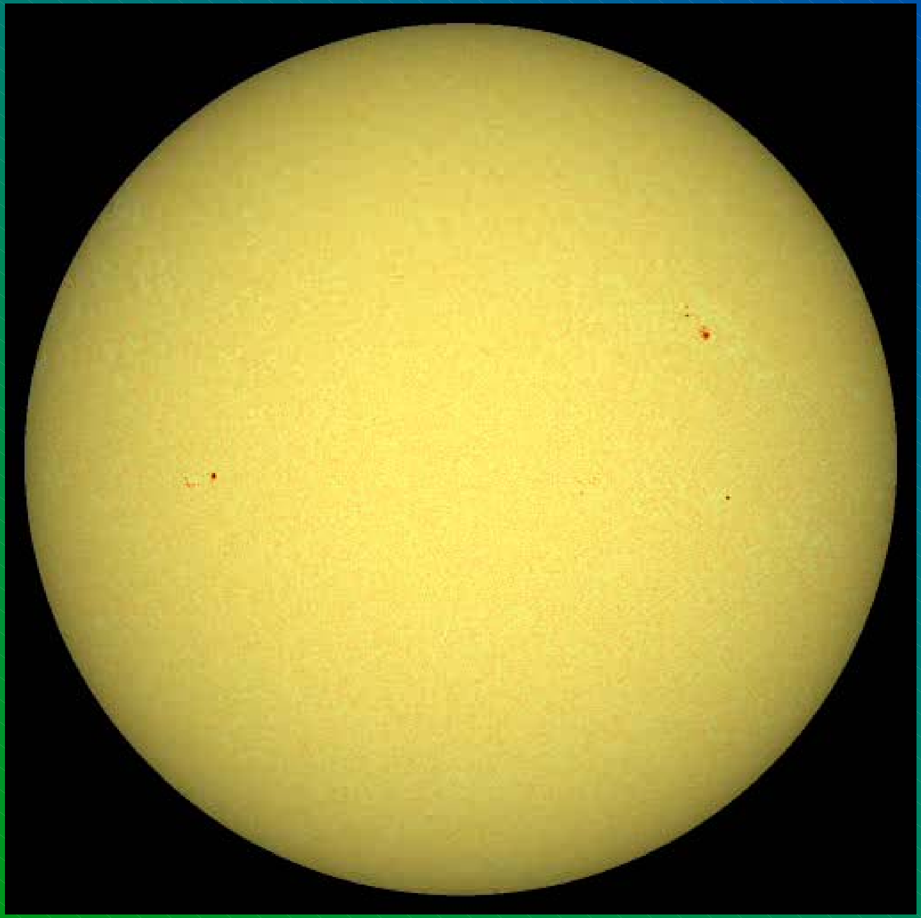
Theory

Experiment: Simulation

The plan

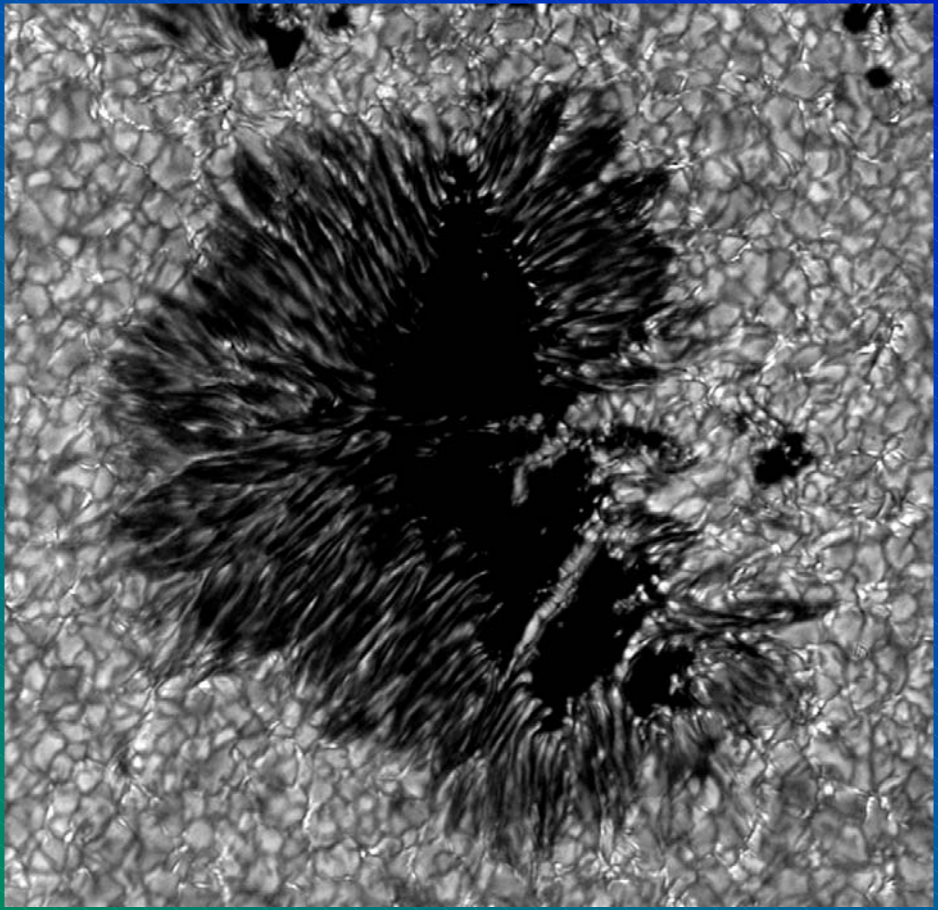
- 1) Formation of magnetic structure
- 2) Physics of magnetic flux tubes
- 3) Magnetic structure and the dynamo

Magnetic fields on the Sun



Sunspots 2001

~50,000 km



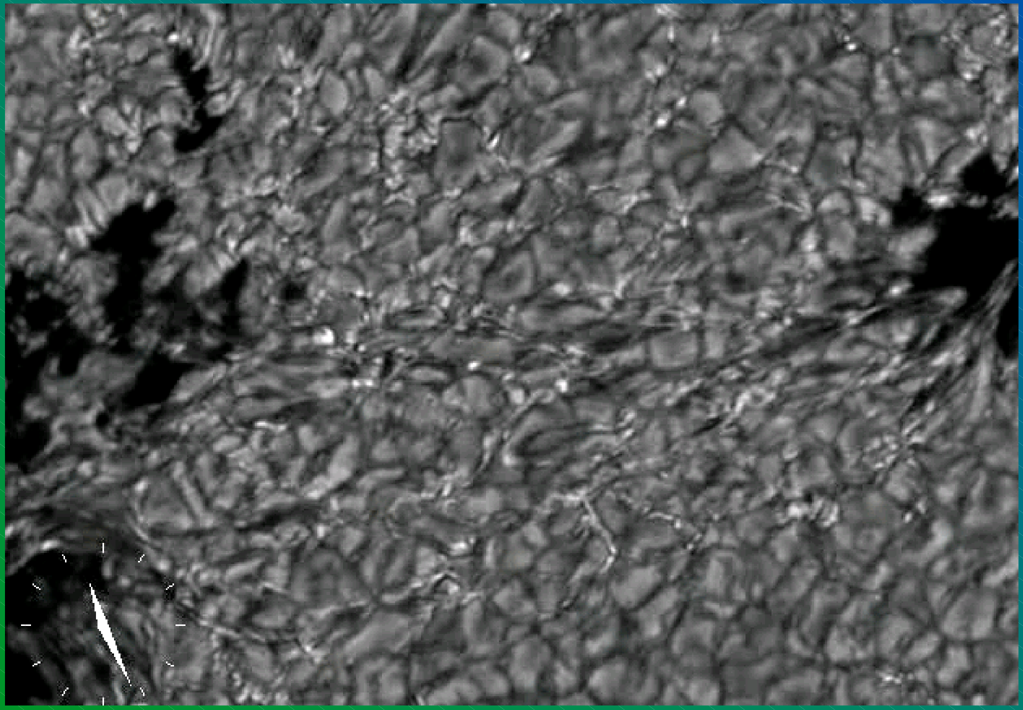
A big sunspot

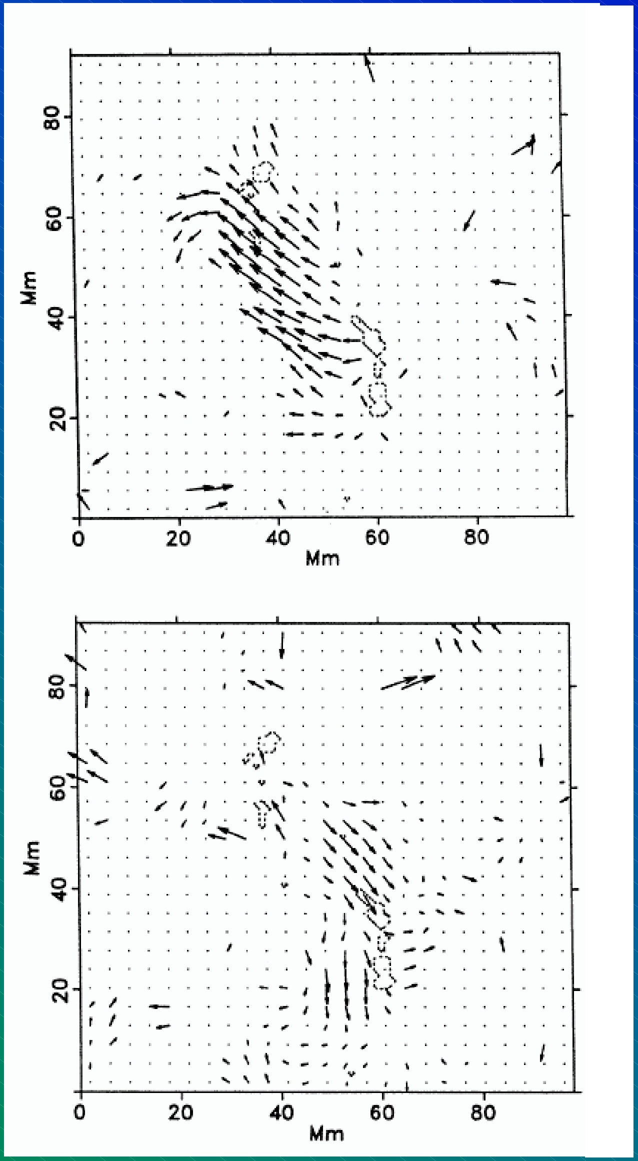
Magnetic flux emergence

Emergence of a large sunspot group

Magnetic flux emergence

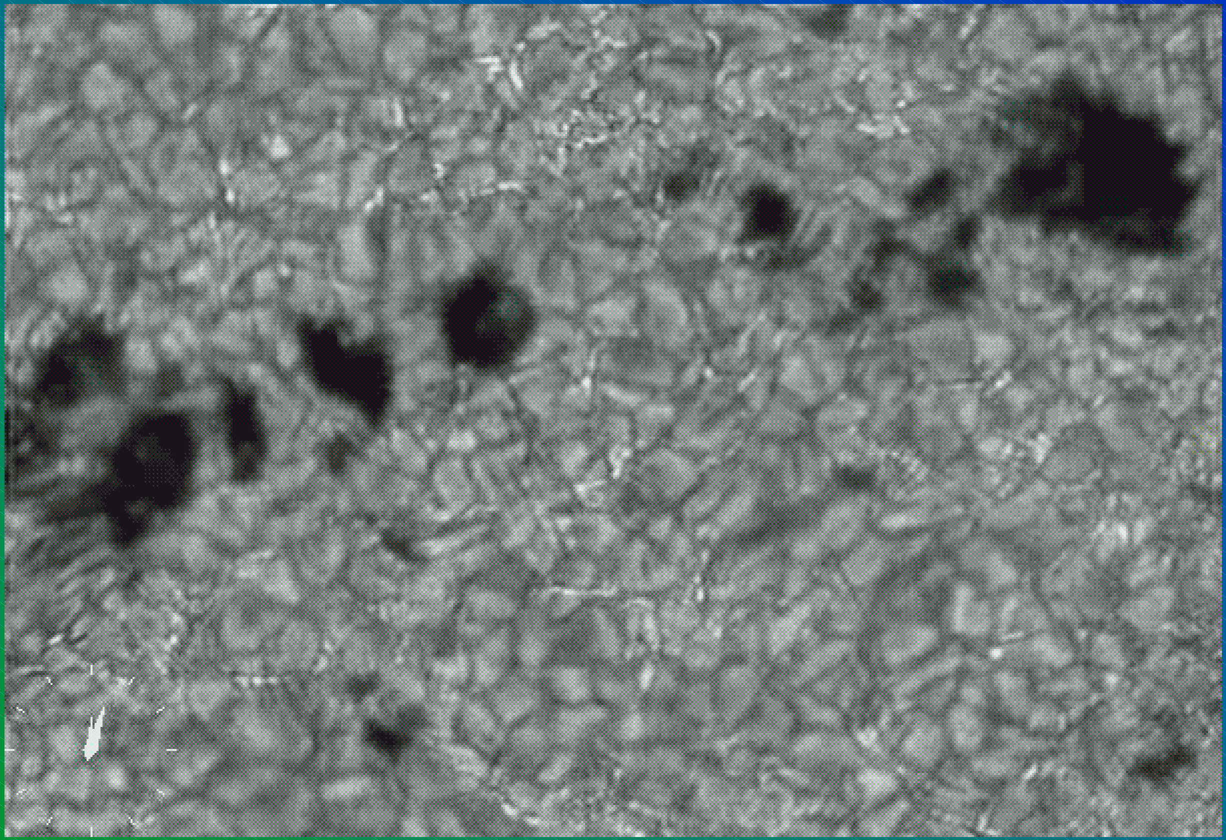
A flux emergence region



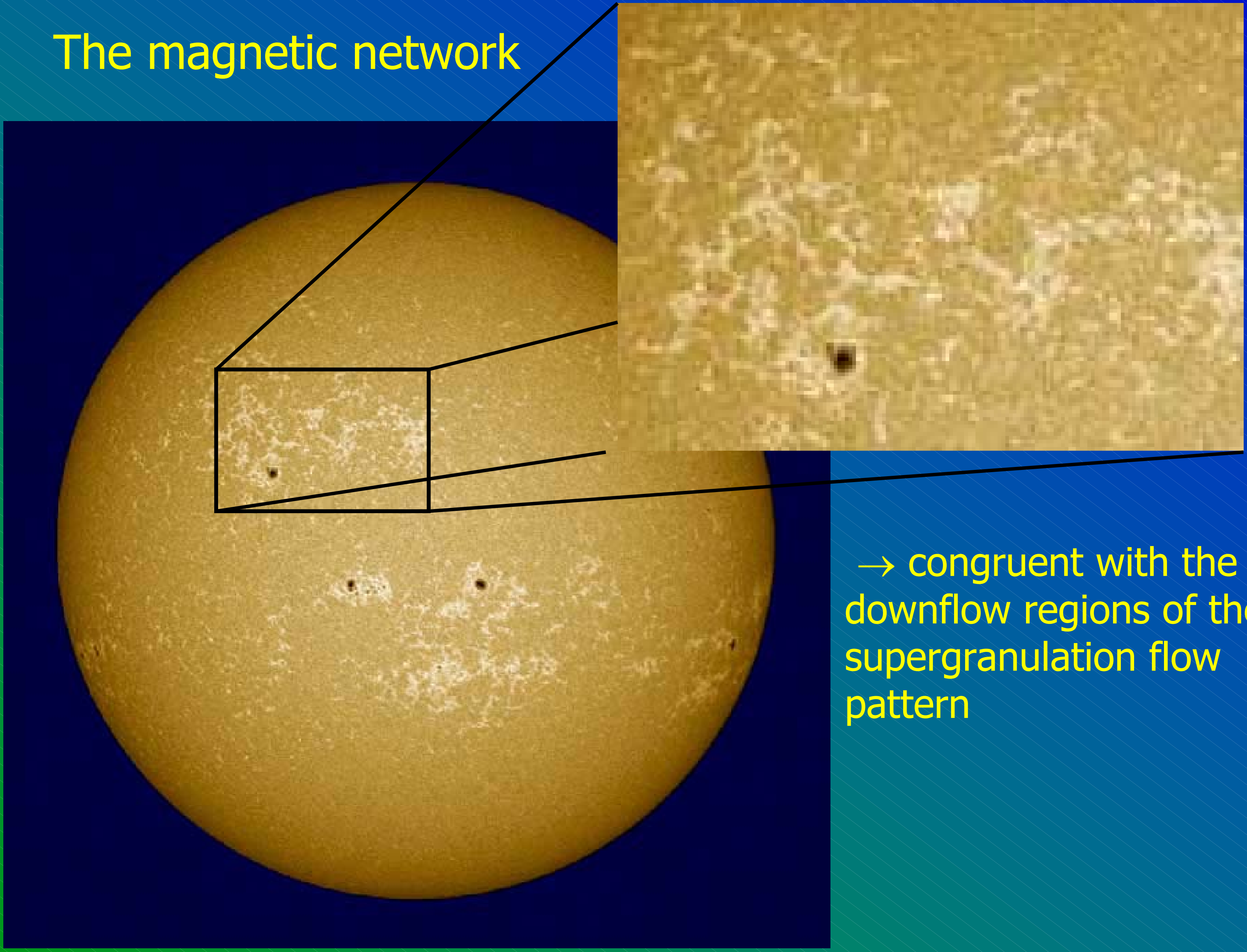


Motion of flux fragments
of different polaritiy

Granulation, sunspots, & small-scale magnetic field

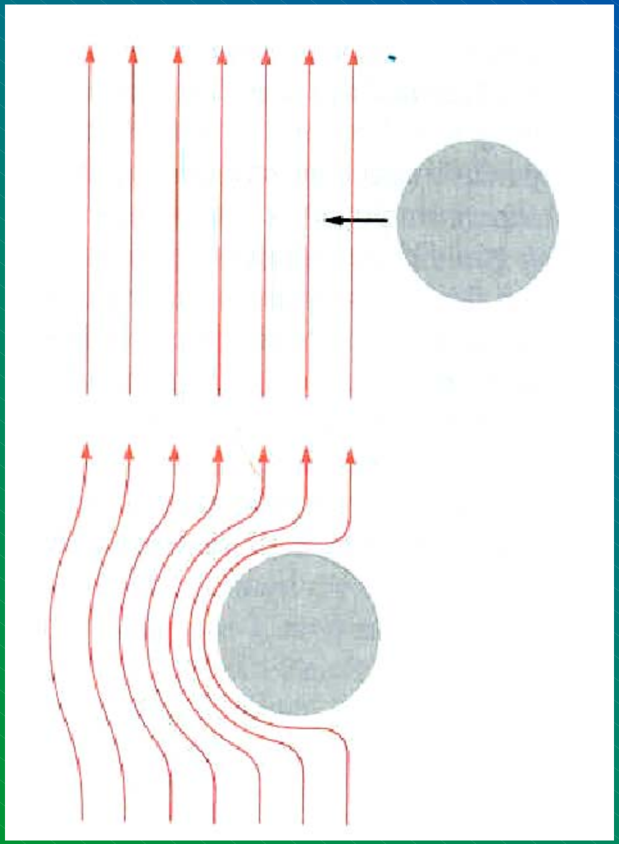


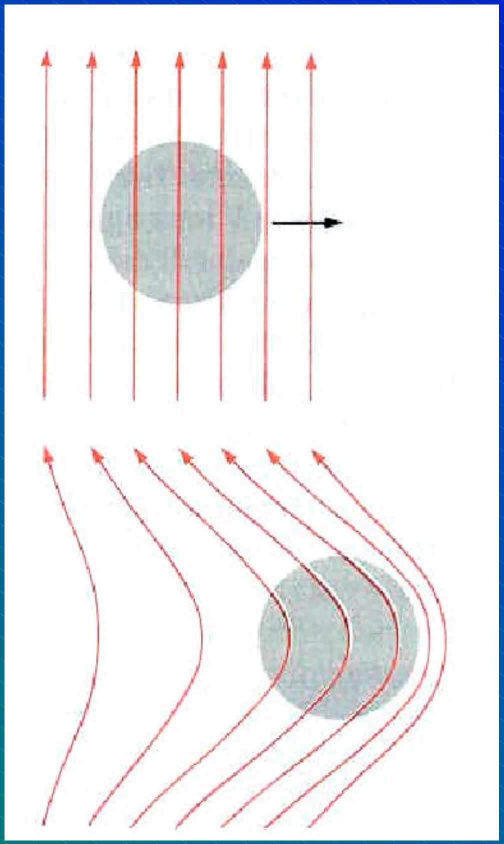
The magnetic network



→ congruent with the downflow regions of the supergranulation flow pattern

Good electrical conductors : "frozen field"



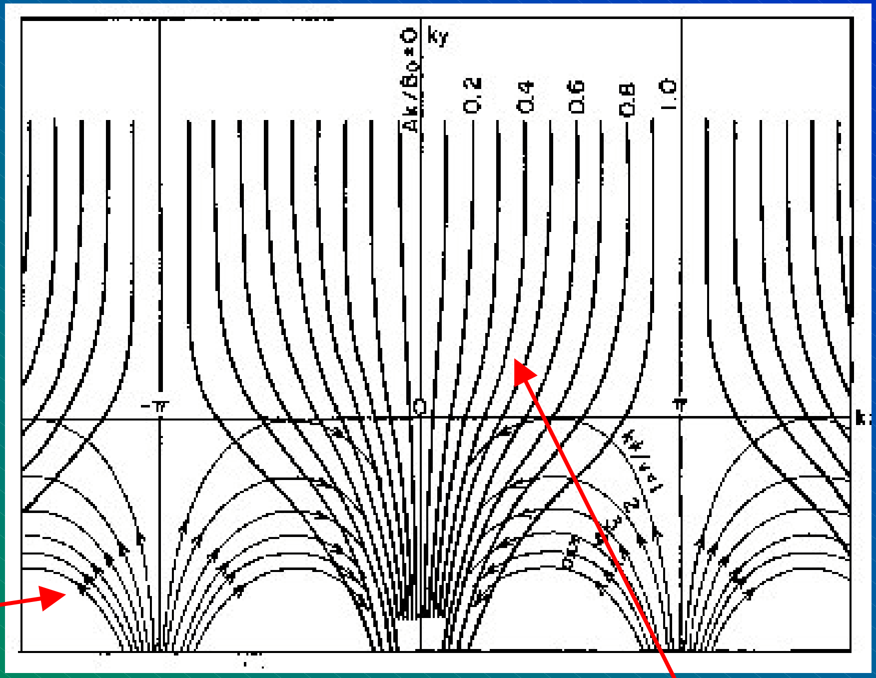


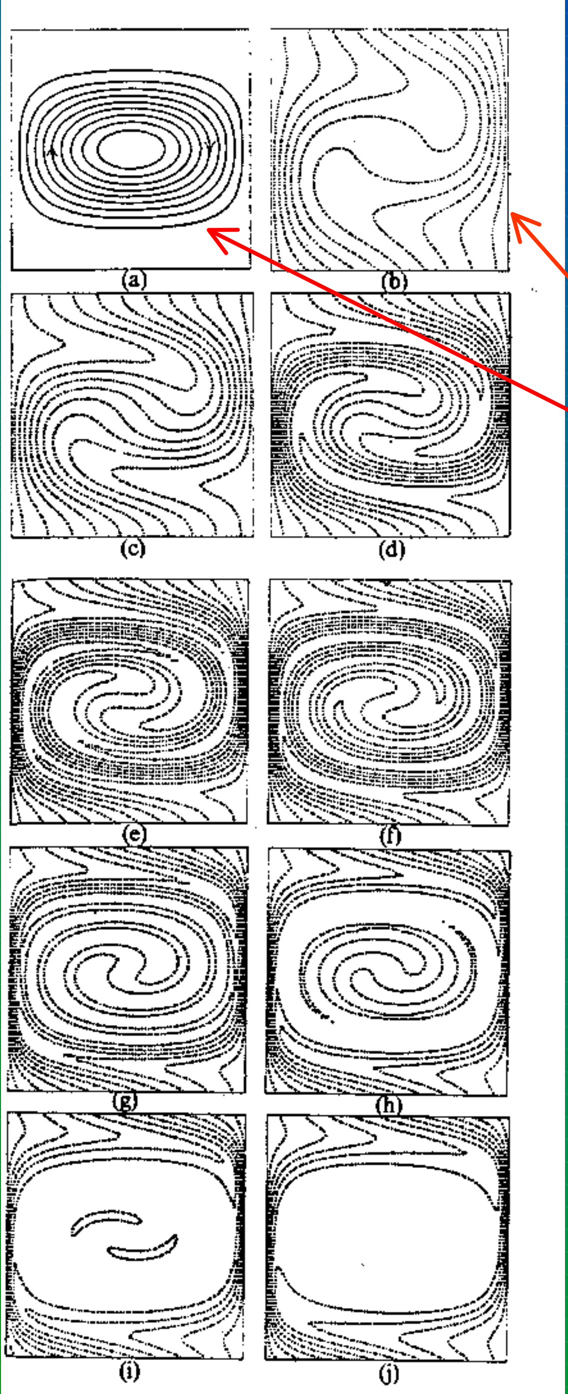
Initially field-free volumes remain field-free

Magnetic flux through a given volume remains constant

Flux expulsion and intermittency

- Non-uniform distribution of magnetic flux in the solar photosphere
- E.N. Parker (1963):
  "… *persistent motions tend to draw the field into concentrated sheets and filaments.*"





Simulation of flux expulsion

(N.O. Weiss, 1966)

b: final state for $Re_m = 40$

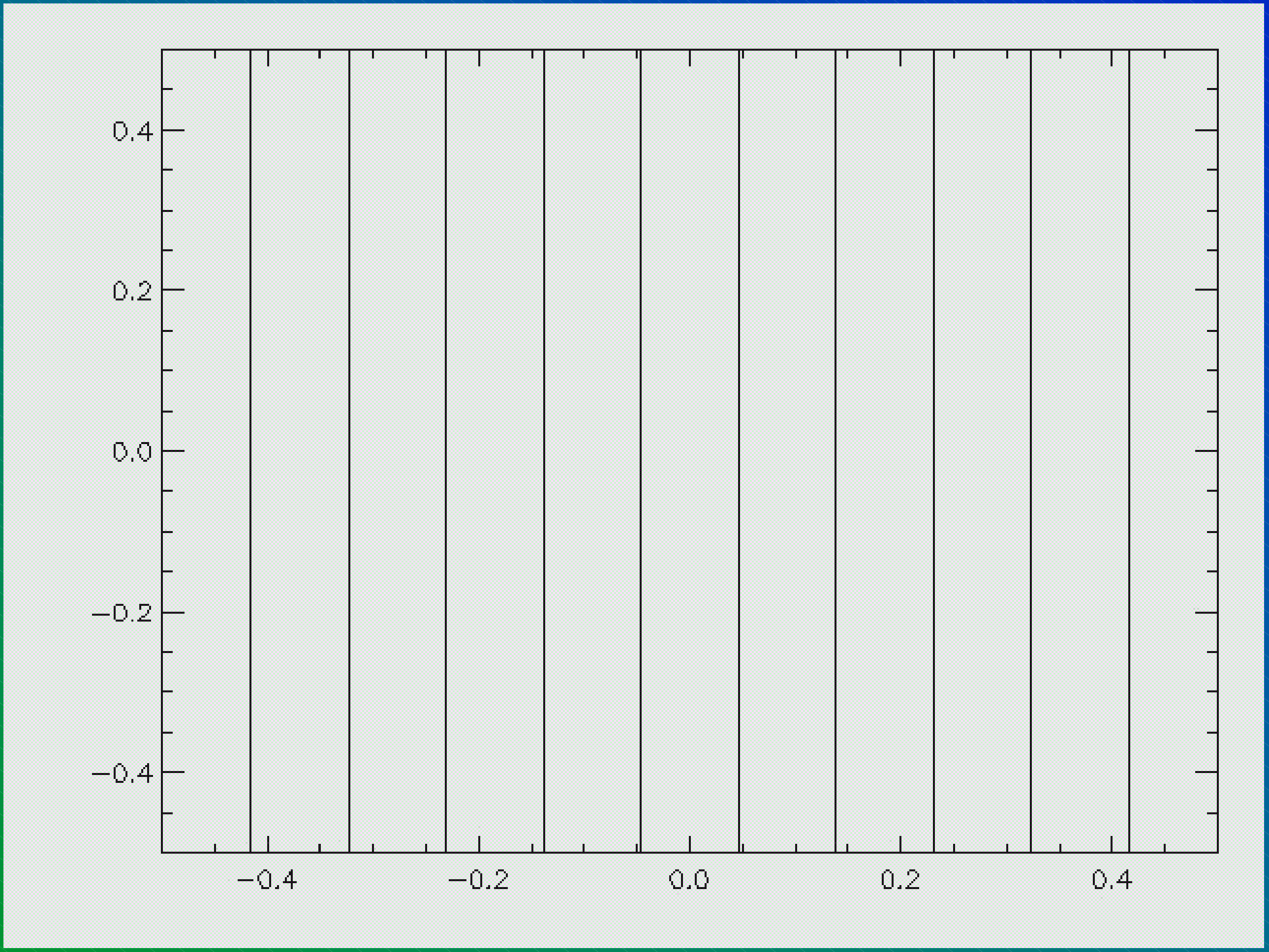
a: streamlines of the fixed velocity field

c-j: time evolution for $Re_m = 1000$

- evolution of an initially vertical magnetic field under the influence of a fixed flow field
- kinematic, 2D
- the magnetic flux is expelled from the area of closed streamlines and concentrated in narrow sheets

Flux expulsion and intermittency

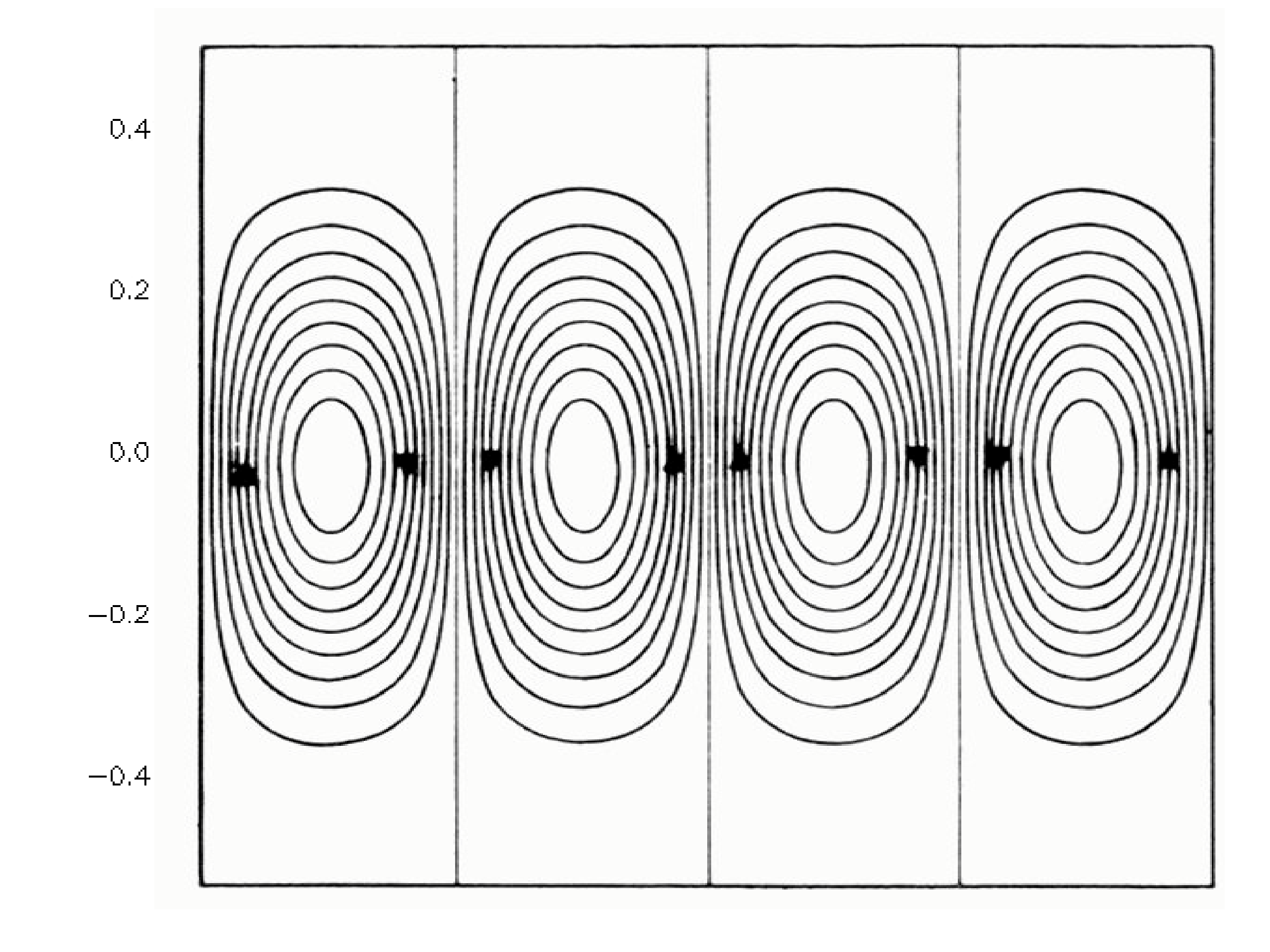
- N.O. Weiss (1964): *first simulations*



(Hupfer, 2001)

Flux expulsion and intermittency

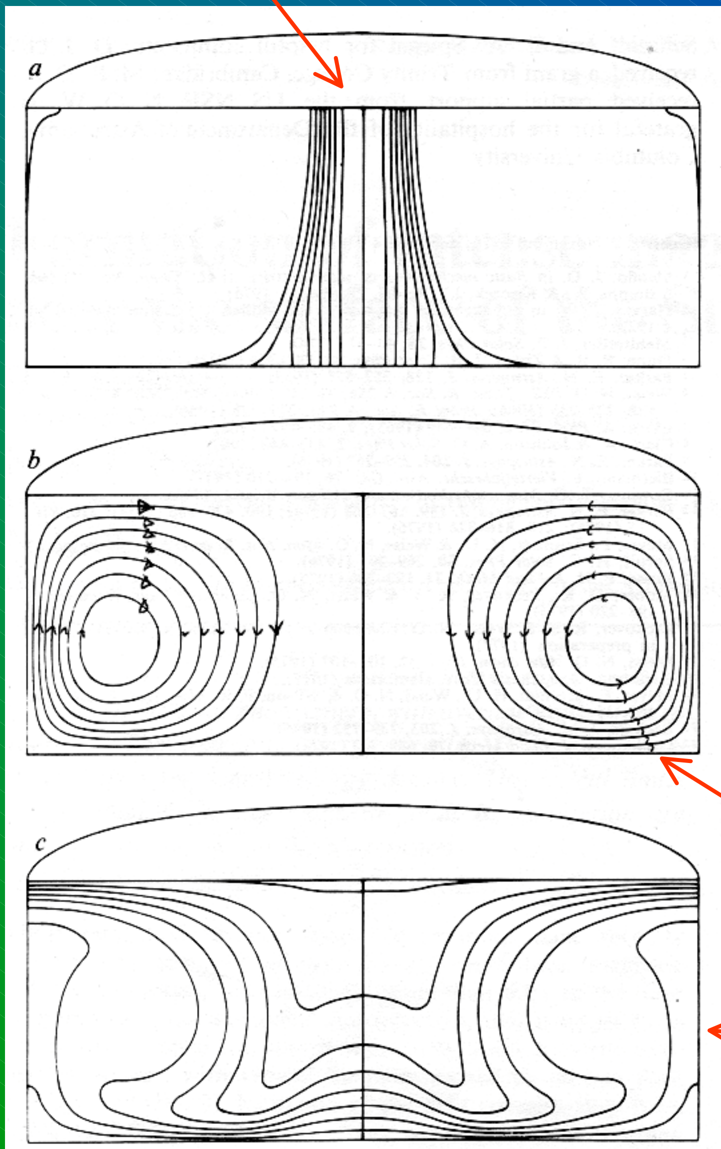
- N.O. Weiss (1964): *first simulations*



(Hupfer, 2001)

3D Flux expulsion

(Galloway, Proctor & Weiss, 1977)

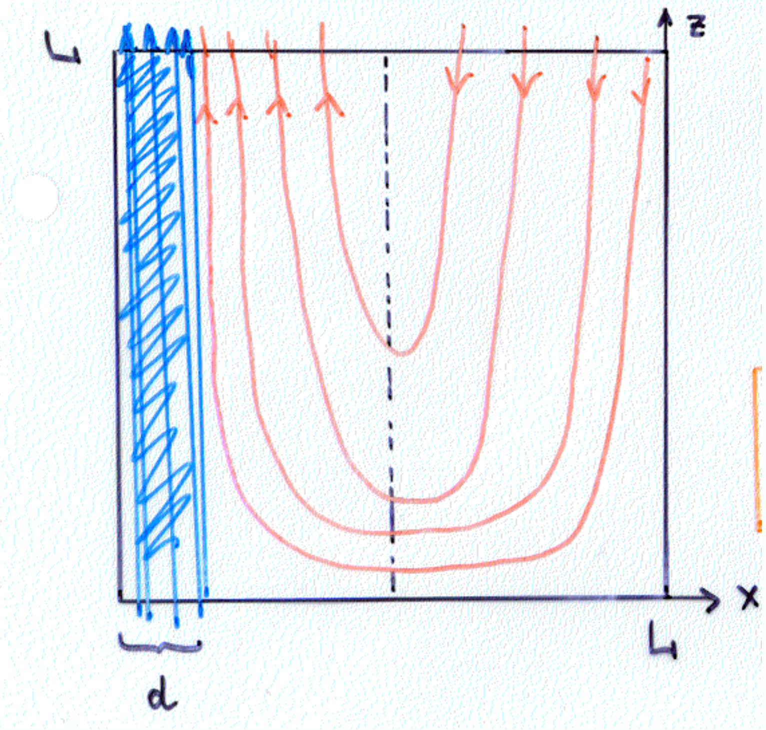


- axisymmetric thermal convection, Boussinesq approx. , dynamic
- flux concentration in cool downflow regions (flux tubes)
- final field strength dependent on ratio of viscosity to magn. diffusivity

Magnetic flux expulsion

Simple example: Kinematical flux expulsion

$$\mathbf{u} = U\left(-\sin\frac{\pi x}{L},\ 0,\ \frac{\pi z}{L}\cos\frac{\pi x}{L}\right),\quad \nabla\cdot\mathbf{u} = 0$$



Initial magnetic field: $\mathbf{B} = (0, 0, B_0)$

Induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

Magnetic field is compressed into a boundary layer, whose thickness is determined by the balance between advection and diffusion:

$$\nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} = 0$$

$$\rightarrow -\frac{\partial}{\partial x}(u_z B_x - u_x B_z) = \eta \frac{\partial^2 B_z}{\partial x^2}$$

Narrow boundary layer at $x \simeq 0$: $u_x \simeq -U\pi x/L$

$\partial B_z/\partial z \simeq 0$ along the z-axis $\rightarrow \partial B_x/\partial x \simeq 0 \rightarrow B_x \propto x^2$

$\rightarrow$ first term on l.h.s. negligible $\rightarrow$ equation for $B_z \equiv B(x)$:

$$-\frac{\pi U}{L}\frac{d}{dx}(xB) = \eta \frac{d^2 B}{dx^2}$$

$$-\frac{\pi U}{L}\frac{d}{dx}(xB) = \eta\frac{d^2B}{dx^2}$$

Integrate:

$$-\left(\frac{\pi U}{\eta L}\right)(xB) = \frac{dB}{dx} + C$$

[$C = 0$ since $dB/dx = 0$ for $x = 0$]

Introduce *magnetic Reynolds number*: $R_\mathrm{m} = UL/\eta$:

$$-\left(\frac{\pi R_\mathrm{m}}{L^2}\right)\int x\,\mathrm{d}x = \int\frac{\mathrm{d}B}{B}$$

Integrate:

$$-\frac{\pi R_\mathrm{m}x^2}{2L^2} = \ln\left[\frac{B}{B_\mathrm{m}}\right]$$

$$\rightarrow \text{Gaussian profile} \quad B(x) = B_\mathrm{m}\exp\left(-\frac{\pi R_\mathrm{m}x^2}{2L^2}\right)$$

Boundary layer width: $d \simeq L/R_\mathrm{m}^{1/2}$.

Determine B_m by considering the total magnetic flux:

$$B_0 L = B_\mathrm{m} \int_0^\infty \exp\left(-\frac{\pi R_\mathrm{m} x^2}{2L^2}\right) \mathrm{d}x = B_\mathrm{m} L \, (2R_\mathrm{m})^{-1/2}$$

$$\rightarrow \boxed{B_\mathrm{m} \propto R_\mathrm{m}^{1/2}}$$

In a 3D (axisymmetric) case we have $B_\mathrm{m} d^2 \simeq B_0 L^2$ and thus $\boxed{B_\mathrm{m} \propto R_\mathrm{m}.}$

$R_\mathrm{m} \gg 1$: Dynamical effects limit the growth before B reaches B_m.
Balance beween Lorentz force and inertial force:

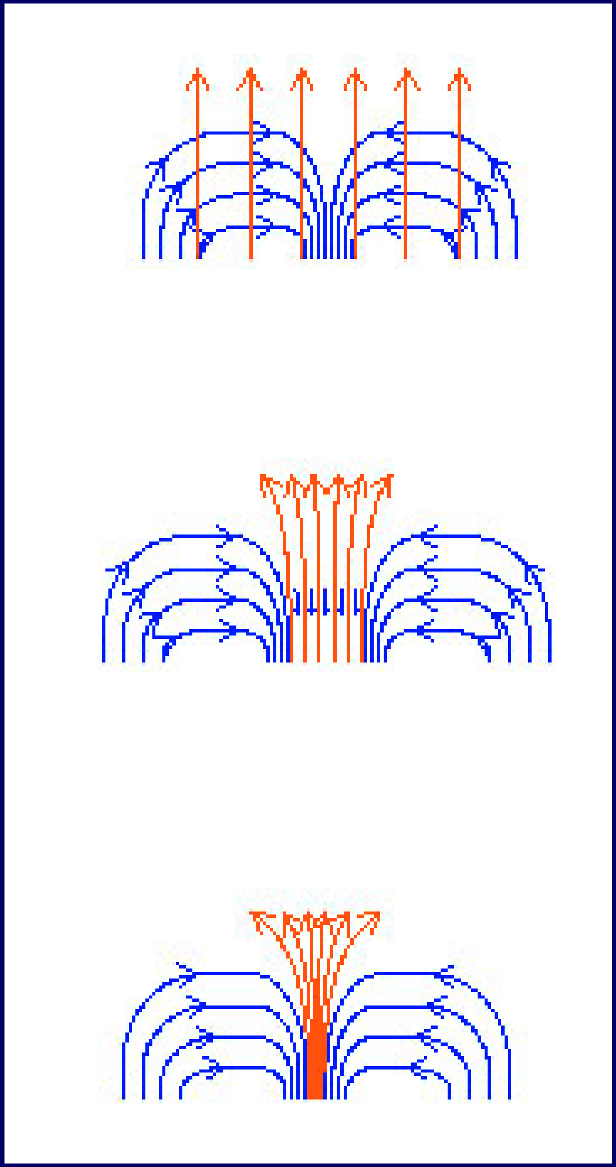
$$\frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B} \simeq \rho(\mathbf{u} \cdot \nabla \mathbf{u})$$

$$\frac{B^2}{4\pi} \simeq \rho U^2 \quad \rightarrow \quad \boxed{B \simeq B_\mathrm{eq} \equiv U\sqrt{4\pi\rho}}$$

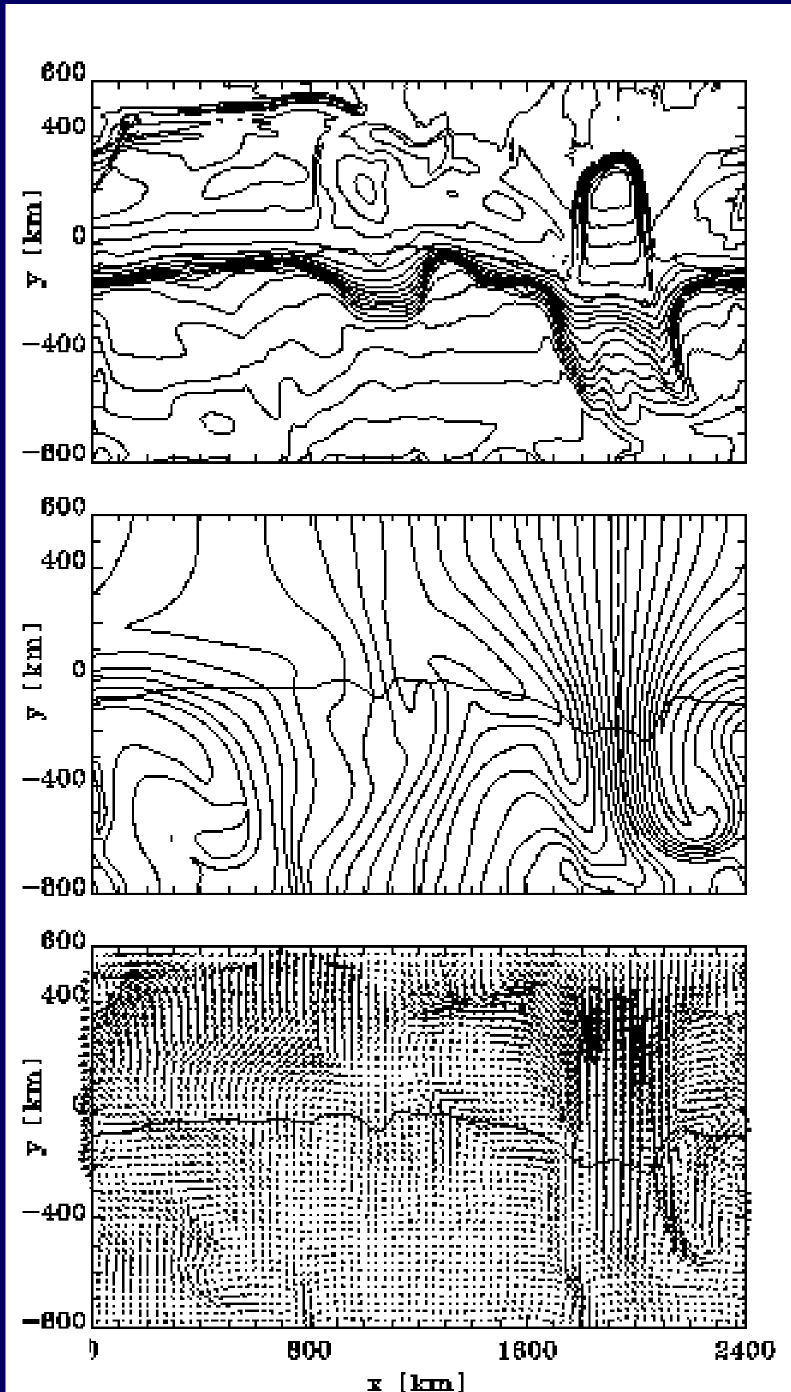
(equipartition field strength)

Solar photosphere: $U \simeq 2\,\mathrm{km/s}$, $\rho \simeq 3 \cdot 10^{-7}\,\mathrm{g/cm^3} \rightarrow B_\mathrm{eq} \simeq 400\,\mathrm{G}$.
Observation: $B \simeq 1500\,\mathrm{G}$!

Convective intensification



- Flux advection by horizontal flow (flux expulsion)
- Suppression of convection, cooling and downflow
- Evacuation, field intensification



Convective intensification

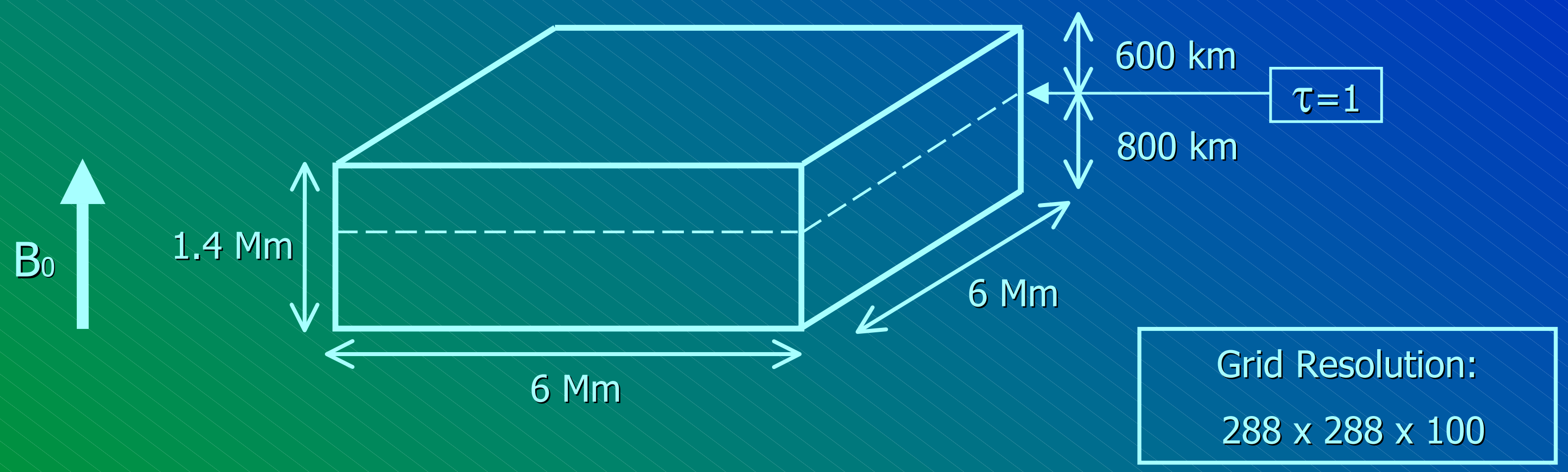
(Grossmann-Doerth, Sch., & Steiner, 1998)

- 2D, compressible
- radiation, ionization
- Stokes diagnostics
- 2400 x 1400 km^2
- 240 x 140 points (10 km hor. resol.)
- <B> = 100, 200, 400 G
- collapse + rebound

t = 100 s

3D Radiation MHD simulations

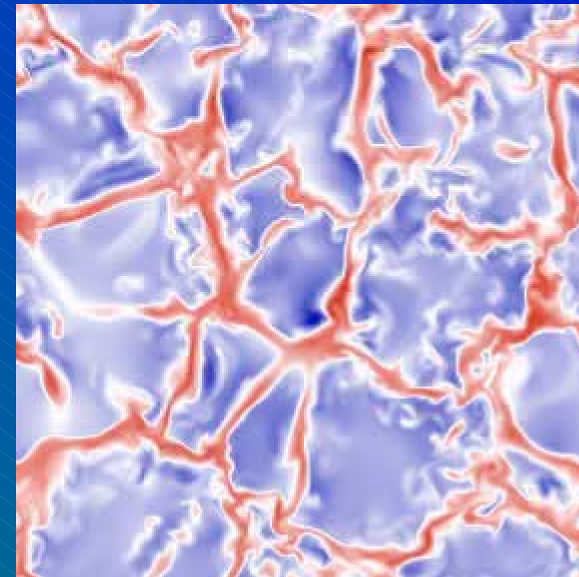
A. Vögler et al. (2002)



- start convection without magnetic field
- initially vertical magnetic field of B_0= 200 G introduced after convection has become quasi-stationary:

B_z

bright: up
dark: down



v_z

blue: up
red: down

$\langle \tau \rangle = 1$

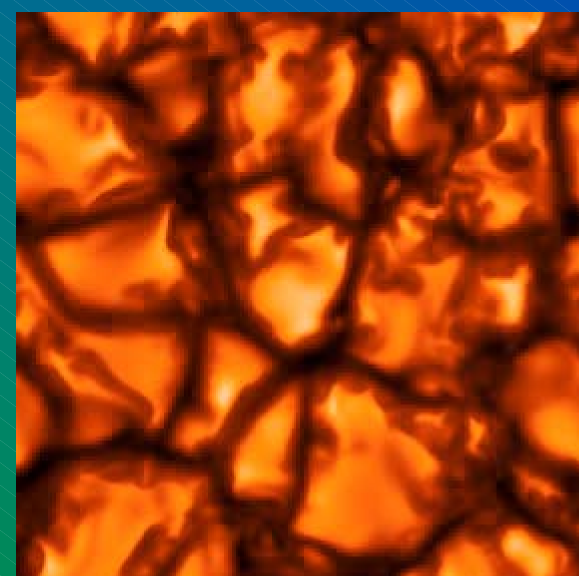
← 6000 km →

Plage run
B_0 = 200 G

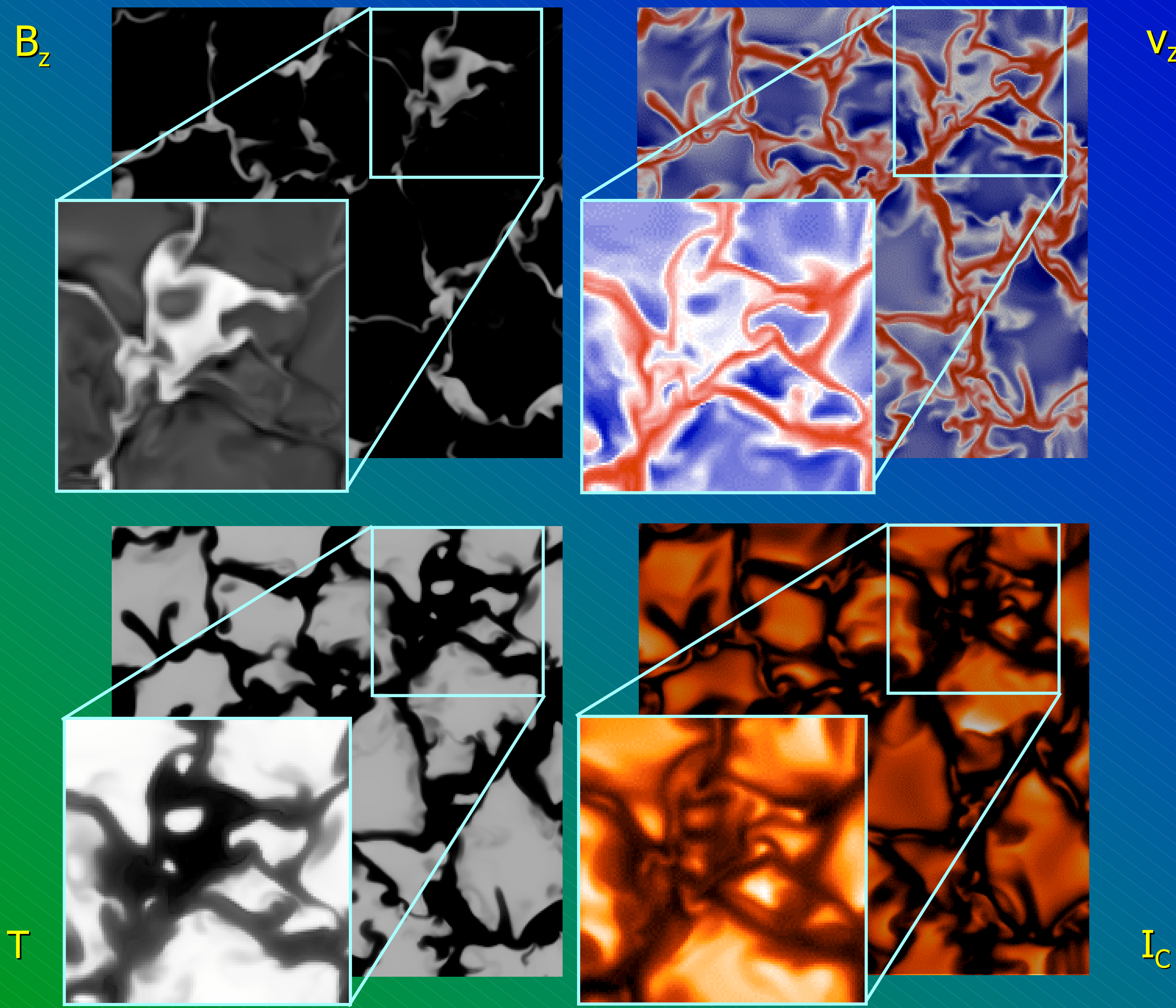
B_z

regions of
strong-field
colored

B > 500 ... 1000 ... 1500 G



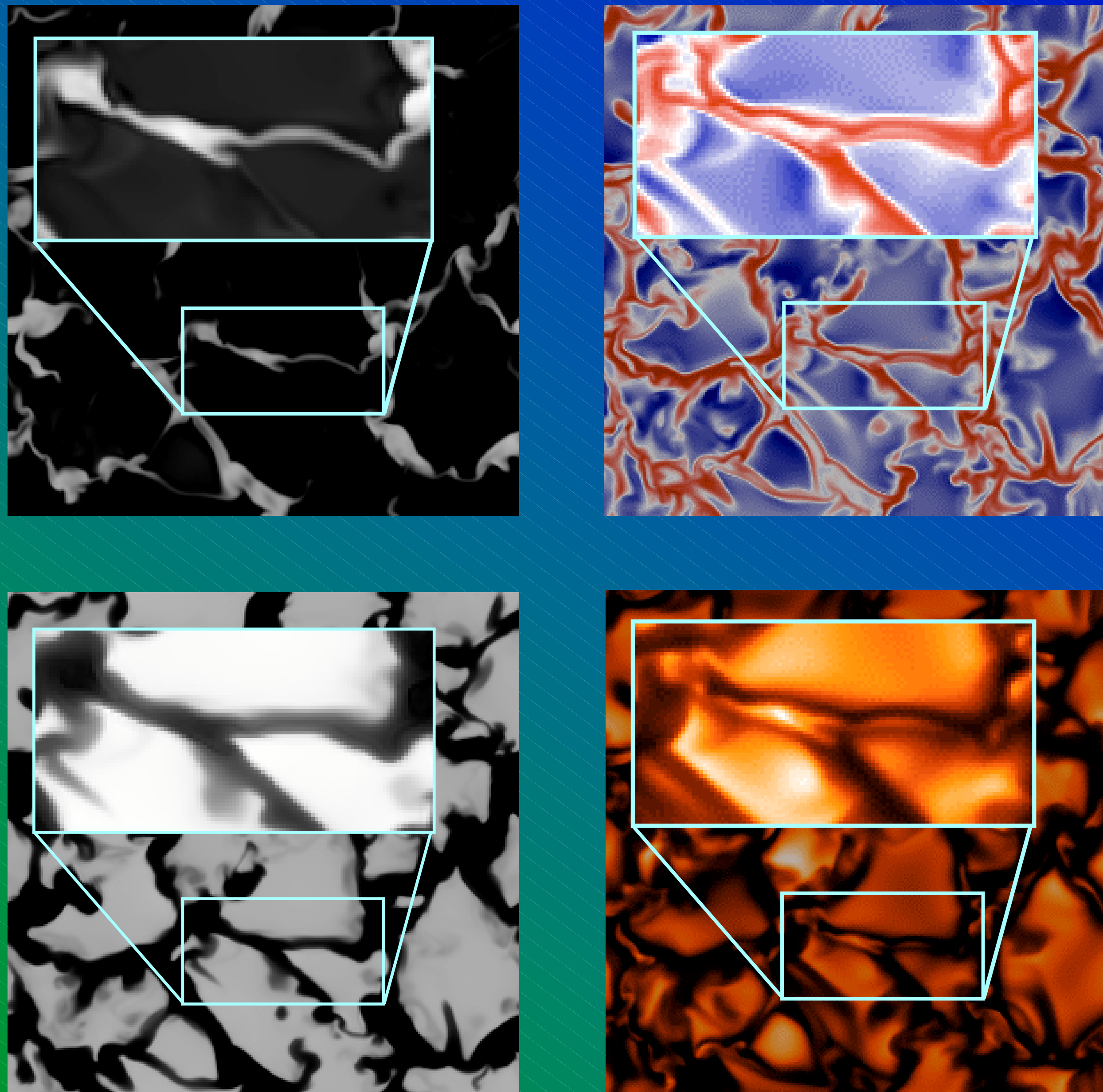
I_C



horizontal cuts near surface level

B_z

v_z

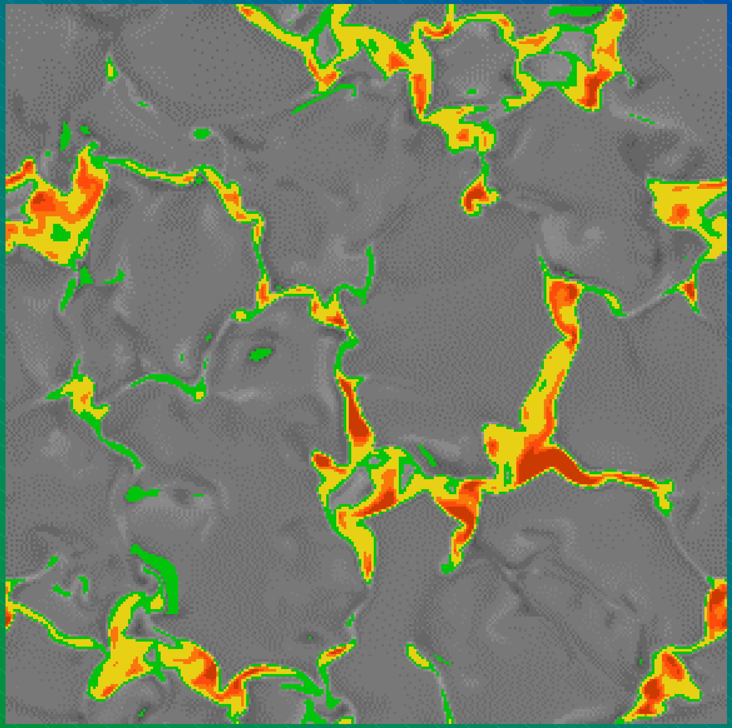


T

I_C

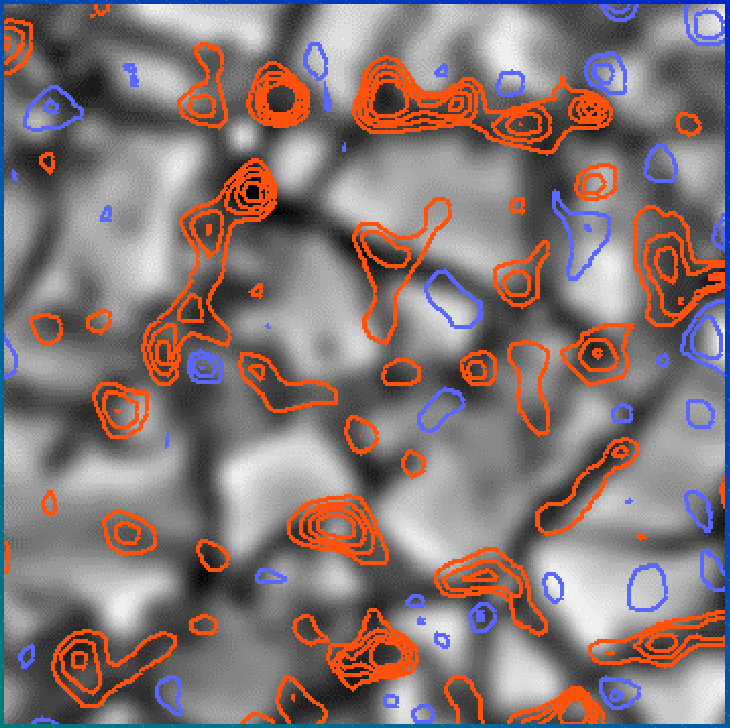
horizontal cuts near surface level

Simulation vs. observation



Simulation (20 km resolution)

(Vögler et al.)

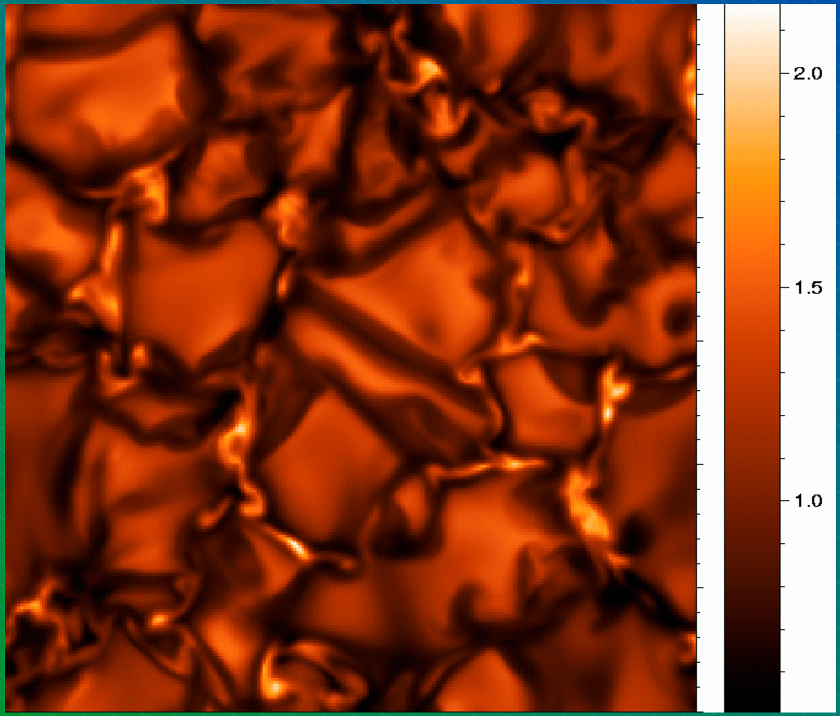


Observation (~250 km resolution)

German Vacuum Tower Telescope, Tenerife
(Dominguez Cerdena et al., 2003)

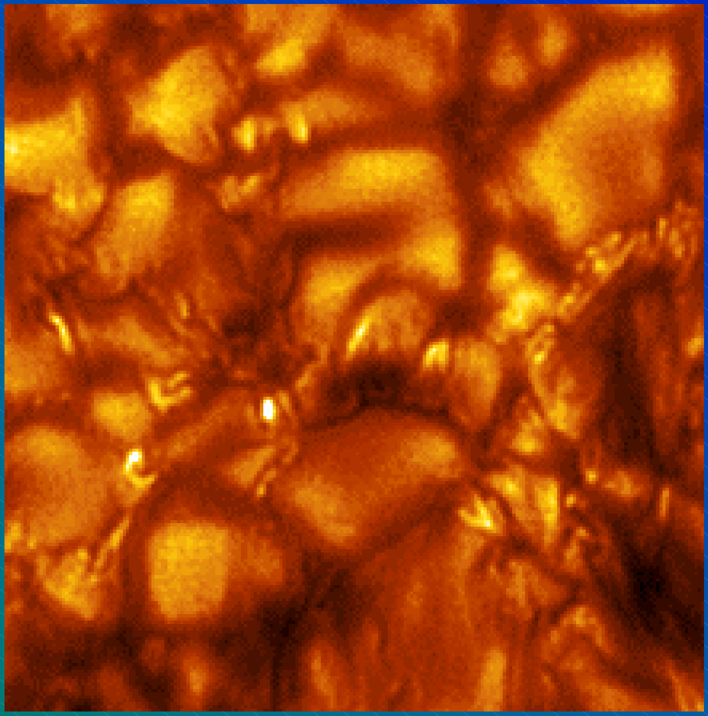
Vertical magnetic field component

Simulation vs. observation



Simulation (20 km resolution)

(Vögler, Shelyag et al.)



Observation (~100 km resolution)

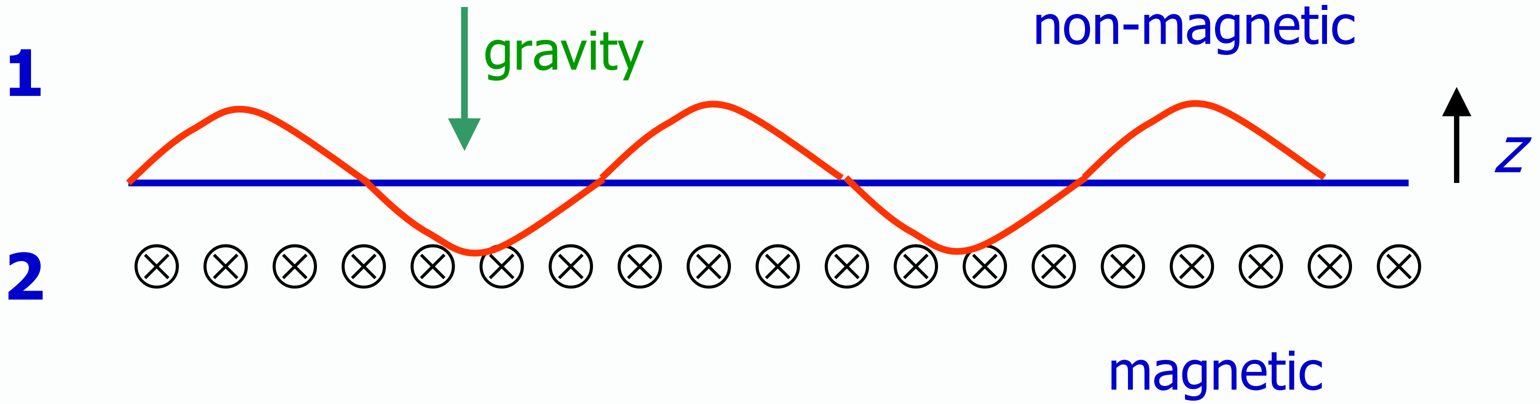
(new Swedish 1m telescope, La Palma, Scharmer et al. 2002)

"G band": molecular lines of CH

Magnetic Rayleigh-Taylor instability

Rayleigh-Taylor: dense fluid over light fluid (e.g., water over oil)

Here: Magnetic layer in magnetostatic equilibrium



Force balance:

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(p + \frac{B^2}{8\pi}\right) = -\rho g$$

$$z = z_0 : \quad p_1 = p_2 + \frac{B^2}{8\pi}$$

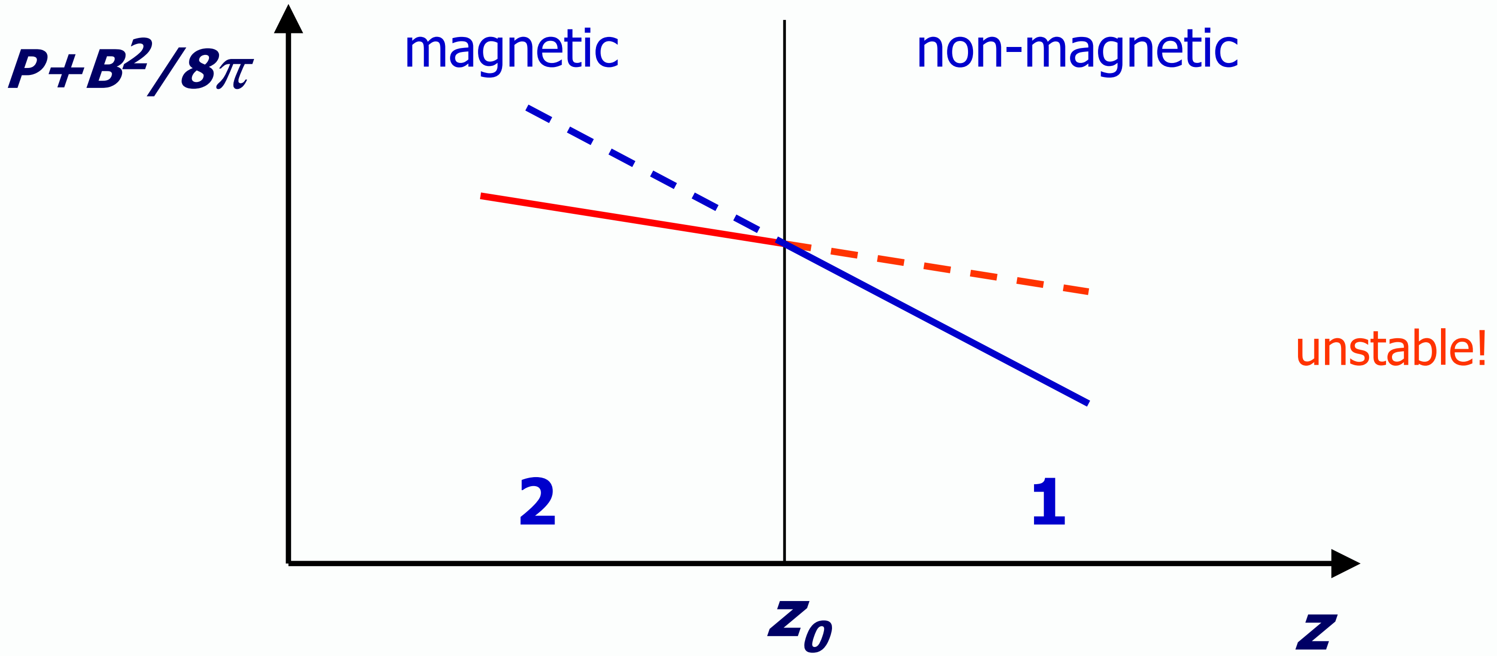
Let B=const. (except for the jump) and $T_1 = T_2 = T =$ const.

Pressure scale height: $H_\mathrm{p} = RT/\mu g$.

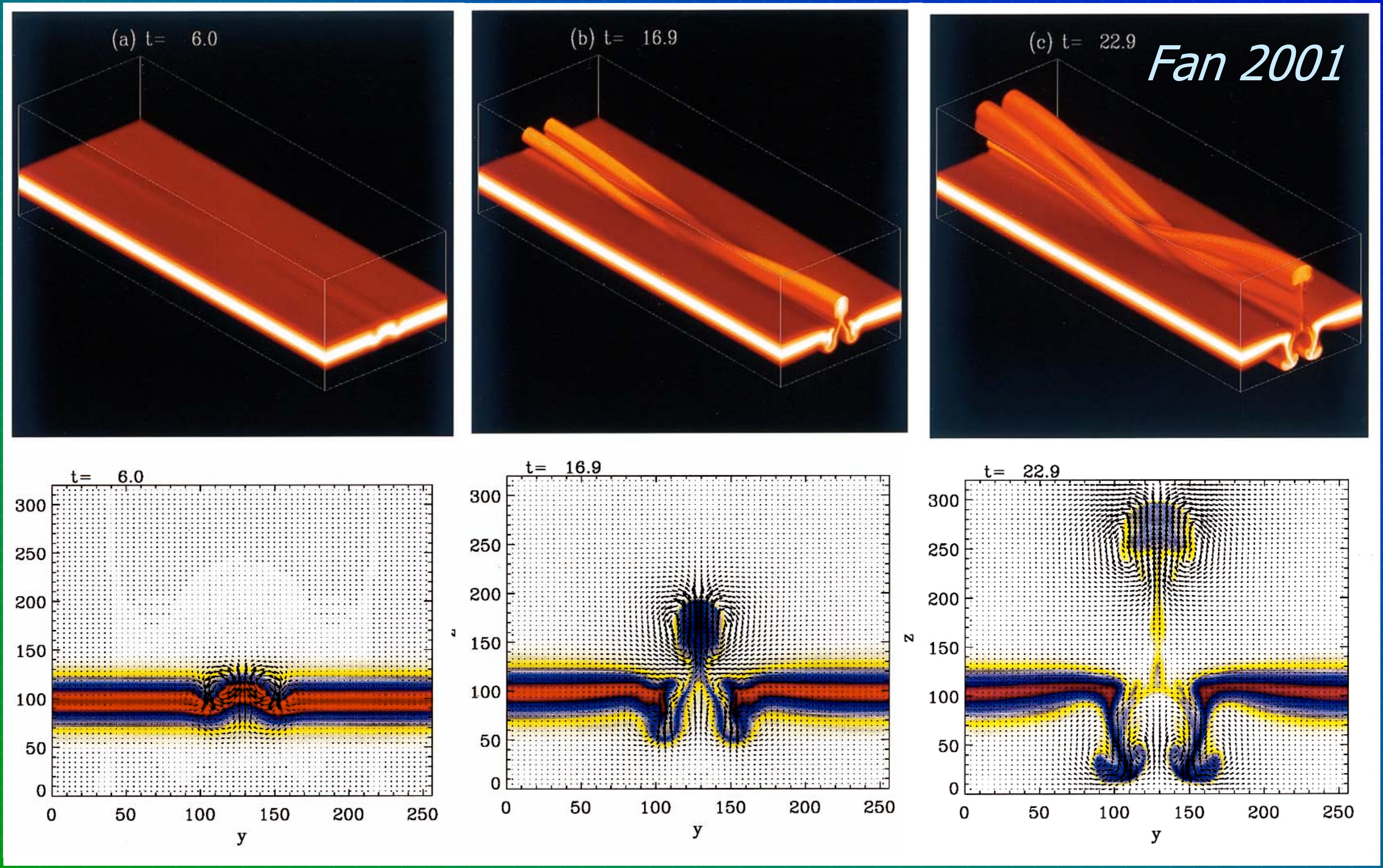
Pressure gradients at $z = z_0$:

$$\frac{\mathrm{d}p_1}{\mathrm{d}z} = -\frac{p_1}{H_\mathrm{p}} \quad \text{(non-magnetic region)}$$

$$\frac{\mathrm{d}p_2}{\mathrm{d}z} = -\frac{p_2}{H_\mathrm{p}} = -\frac{p_1}{H_\mathrm{p}} + \frac{B^2}{8\pi H_\mathrm{p}} > \frac{\mathrm{d}p_1}{\mathrm{d}z} \quad \rightarrow \text{instability!}$$



An unstable magnetic layer forming arched flux tubes

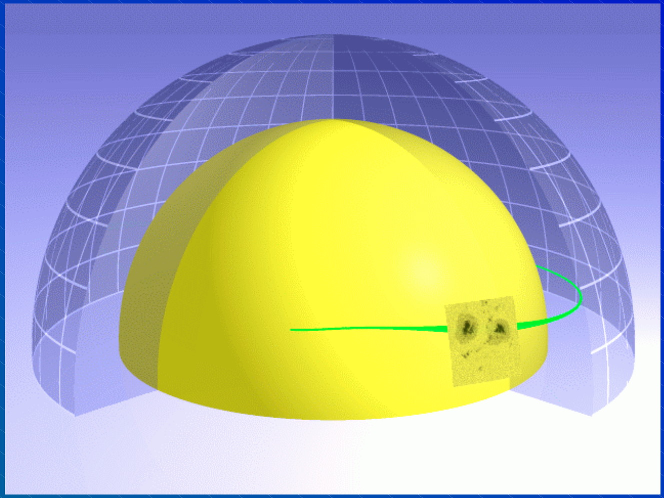


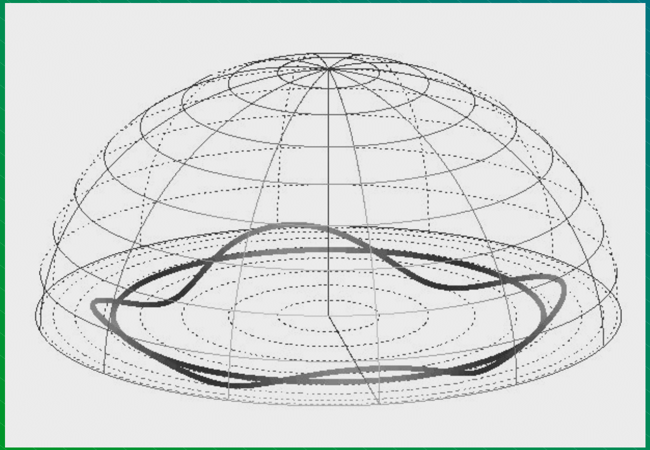
The plan

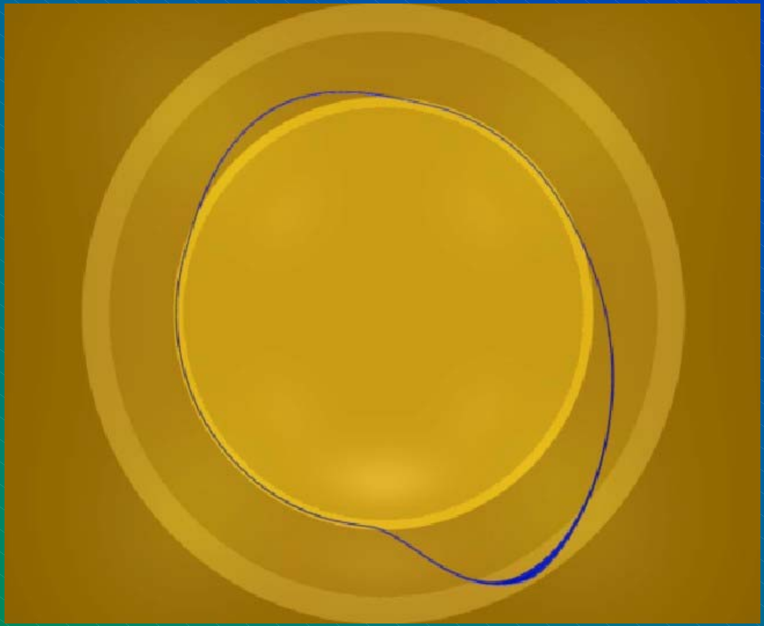
- 1) Formation of magnetic structure
- 2) Physics of magnetic flux tubes
- 3) Magnetic structure and the dynamo

Thin flux tube approximation

- All scales along field >> tube diameter
- Expansion perpend. to tube axis
- Truncate: neglect 2nd higher orders
- Quasi-1D approximation:
  String of mass elements
  moving in a 3D environment





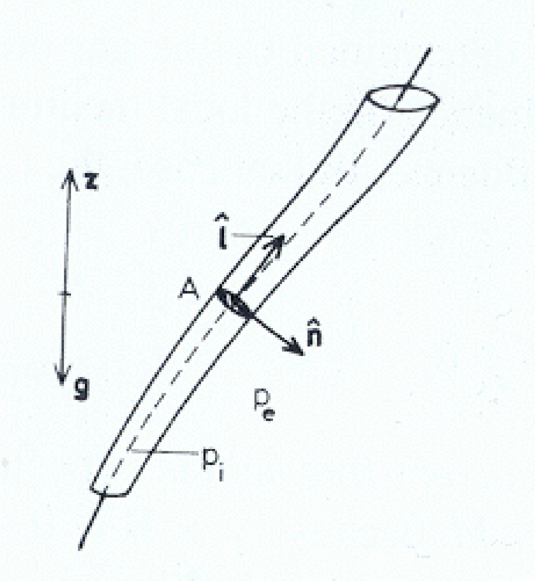


The Thin Flux Tube Approximation

Isolated magnetic flux tube: Bundle of magnetic field lines separated from the non-magnetic surrounding plasma by a tangential discontinuity (current sheet).

Thin magnetic flux tube: An isolated flux tube whose diameter is small compared to all other length scales of the system (scale heights, radius of curvature, wavelengths...)

→ Describe the flux tube by the values of the various physical quantities on the tube axis [space curve $\mathbf{r}(l)$, l: arc length].
→ zeroth/first order of an expansion perpendicular to the axis



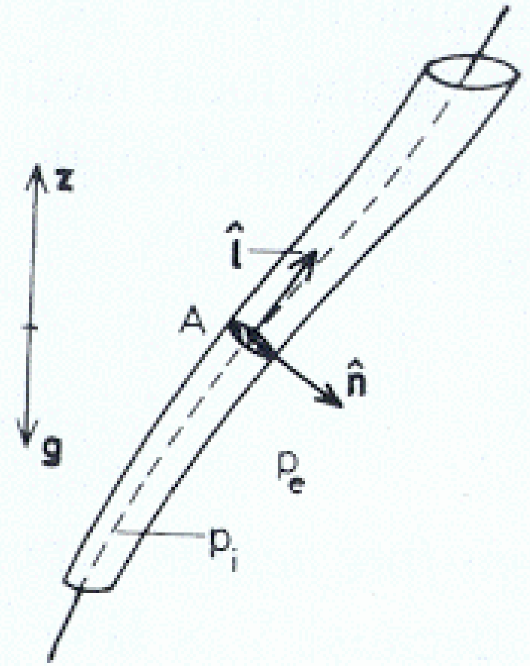
$$\hat{\mathbf{l}} = \frac{\partial \mathbf{r}}{\partial l}$$ unit tangent vector

$$\hat{\mathbf{n}} = R \frac{\partial \hat{\mathbf{l}}}{\partial l}$$ unit normal vector

$$\hat{\mathbf{b}} = \hat{\mathbf{l}} \times \hat{\mathbf{n}}$$ unit binormal vector

$$R = \left| \frac{\partial \hat{\mathbf{l}}}{\partial l} \right|^{-1}$$ radius of curvature

$$\frac{\partial \hat{\mathbf{b}}}{\partial l} = \hat{\mathbf{l}} \times \frac{\partial \hat{\mathbf{n}}}{\partial l} = R_t^{-1} \hat{\mathbf{n}}$$ radius of torsion



Consider the *Walén equation* (combined induction equation and equation of continuity → exercise!) and the equation of motion:

$$\frac{d}{dt}\left(\frac{\mathbf{B}}{\rho}\right) = \left(\frac{\mathbf{B}}{\rho}\cdot\nabla\right)\mathbf{u}$$

$$\rho\frac{d\mathbf{u}}{dt} = -\nabla p + \rho\mathbf{g} + \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B} + \mathbf{F}_D$$

With $\mathbf{B}(l) = B(l)\,\hat{\mathbf{l}}$ along the axis:

$$\frac{B}{\rho}\frac{d\hat{\mathbf{l}}}{dt} + \hat{\mathbf{l}}\frac{d}{dt}\left(\frac{B}{\rho}\right) = \frac{B}{\rho}(\hat{\mathbf{l}}\cdot\nabla)\mathbf{u} \equiv \frac{B}{\rho}\frac{\partial\mathbf{u}}{\partial l}$$

By scalar multiplication with $\hat{\mathbf{l}}$ and noting that a unit vector is perpendicular to its derivative we find:

$$\frac{d}{dt}\left(\frac{B}{\rho}\right) = \frac{B}{\rho}\,\hat{\mathbf{l}}\cdot\frac{\partial\mathbf{u}}{\partial l} = \frac{B}{\rho}\left(\frac{\partial\mathbf{u}\cdot\hat{\mathbf{l}}}{\partial l} - \mathbf{u}\cdot\frac{\partial\hat{\mathbf{l}}}{\partial l}\right)$$

With $\mathbf{u}\cdot\hat{\mathbf{l}} \equiv u_l$, $\mathbf{u}\cdot\hat{\mathbf{n}} \equiv u_n$:

$$\boxed{\frac{d}{dt}\left(\frac{B}{\rho}\right) = \frac{B}{\rho}\left(\frac{\partial u_l}{\partial l} - \frac{u_n}{R}\right)}$$

→ Walén equation in thin flux tube approximation.

Multiply with $\hat{\mathbf{n}}$ and $\hat{\mathbf{b}}$ → normal/binormal components of the time derivative of the tangent → change of the flux tube path in time:

$$\hat{\mathbf{n}} \cdot \frac{d\hat{\mathbf{l}}}{dt} = \frac{\partial u_n}{\partial l} + \frac{u_l}{R} + \frac{u_b}{R_t}$$

$$\hat{\mathbf{b}} \cdot \frac{d\hat{\mathbf{l}}}{dt} = \frac{\partial u_b}{\partial l} - \frac{u_n}{R_t}$$

(with $\hat{\mathbf{n}} = -\hat{\mathbf{l}} \times \hat{\mathbf{b}}$ and $\mathbf{u} \cdot \hat{\mathbf{b}} \equiv u_b$).

Lorentz force on the tube axis:

$$\begin{aligned}\frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B} &= \frac{1}{4\pi}[(\nabla B) \times \hat{\mathbf{l}} + B\,\nabla \times \hat{\mathbf{l}}] \times B\,\hat{\mathbf{l}} \\ &= \frac{1}{4\pi}[B(\nabla B \times \hat{\mathbf{l}}) \times \hat{\mathbf{l}} + B^2(\nabla \times \hat{\mathbf{l}}) \times \hat{\mathbf{l}}])\end{aligned}$$

Using $(\nabla \times \hat{\mathbf{l}}) \times \hat{\mathbf{l}} = R^{-1}\,\hat{\mathbf{n}}$ and

$$(\nabla B \times \hat{\mathbf{l}}) \times \hat{\mathbf{l}} = -\nabla B + \hat{\mathbf{l}}(\hat{\mathbf{l}} \cdot \nabla B) \equiv -(\nabla B)_\perp$$

$(\nabla B)_\perp$: projection of the gradient on the plane perpendicular to the tangential direction (the cross section of the tube).

$$\mathbf{F}_L \;\equiv\; \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B} \;=\; -\left[\nabla\left(\frac{B^2}{8\pi}\right)\right]_\perp + \frac{B^2}{4\pi R}\hat{\mathbf{n}}$$

Projections of $\mathbf{F}_L$ on the triad of unit vectors:

$$\begin{aligned}
\mathbf{F}_L\cdot\hat{\mathbf{l}} &= 0 \\
\mathbf{F}_L\cdot\hat{\mathbf{n}} &= -\frac{\partial}{\partial n}\left(\frac{B^2}{8\pi}\right) + \frac{B^2}{4\pi R} \\
\mathbf{F}_L\cdot\hat{\mathbf{b}} &= -\frac{\partial}{\partial b}\left(\frac{B^2}{8\pi}\right)
\end{aligned}$$

$(\hat{\mathbf{n}}\cdot\nabla\equiv\partial/\partial n,\, \hat{\mathbf{b}}\cdot\nabla\equiv\partial/\partial b)$.

$\rightarrow$ equation of motion:

$$\rho\frac{d\mathbf{u}}{dt}\cdot\hat{\mathbf{l}} \;=\; -\frac{\partial p}{\partial l} + \rho\,\mathbf{g}\cdot\hat{\mathbf{l}}$$

$$\rho\frac{d\mathbf{u}}{dt}\cdot\hat{\mathbf{n}} \;=\; -\frac{\partial}{\partial n}(p+\frac{B^2}{8\pi}) + \rho\,\mathbf{g}\cdot\hat{\mathbf{n}} + \frac{B^2}{4\pi R} + \mathbf{F}_D\cdot\hat{\mathbf{n}}$$

$$\rho\frac{d\mathbf{u}}{dt}\cdot\hat{\mathbf{b}} \;=\; -\frac{\partial}{\partial b}(p+\frac{B^2}{8\pi}) + \rho\,\mathbf{g}\cdot\hat{\mathbf{b}} + \;+\mathbf{F}_D\cdot\hat{\mathbf{b}}$$

Continuity of normal stress at the interface between flux tube and its environment:

$$p + \frac{B^2}{8\pi} = p_e$$

Thin tube: consider only the first derivatives in normal/binormal direction → already fixed by continuity condition above.

External hydrostatic equilibrium:

$$\nabla p_e = \rho_e \mathbf{g}$$

Resulting eqs. of motion:

$$\rho \frac{d\mathbf{u}}{dt} \cdot \hat{\mathbf{l}} = -\frac{\partial p}{\partial l} + \rho \mathbf{g} \cdot \hat{\mathbf{l}}$$

$$\rho \frac{d\mathbf{u}}{dt} \cdot \hat{\mathbf{n}} = (\rho - \rho_e)\, \mathbf{g} \cdot \hat{\mathbf{n}} + \frac{B^2}{4\pi R} + \mathbf{F}_D \cdot \hat{\mathbf{n}}$$

$$\rho \frac{d\mathbf{u}}{dt} \cdot \hat{\mathbf{b}} = (\rho - \rho_e)\, \mathbf{g} \cdot \hat{\mathbf{b}} + \mathbf{F}_D \cdot \hat{\mathbf{b}}$$

Perpendicular directions:
buoyancy force, curvature force & aerodynamic drag force.

Thin flux tube equations with rotation

$$\rho_i \frac{D\mathbf{U}}{Dt} \cdot \mathbf{l} = -\frac{\partial p_i}{\partial l} + \rho_i(\mathbf{g} - R\Omega^2 \mathbf{e}_R) \cdot \mathbf{l} + 2\rho_i(\mathbf{U} \times \boldsymbol{\Omega}) \cdot \mathbf{l}$$

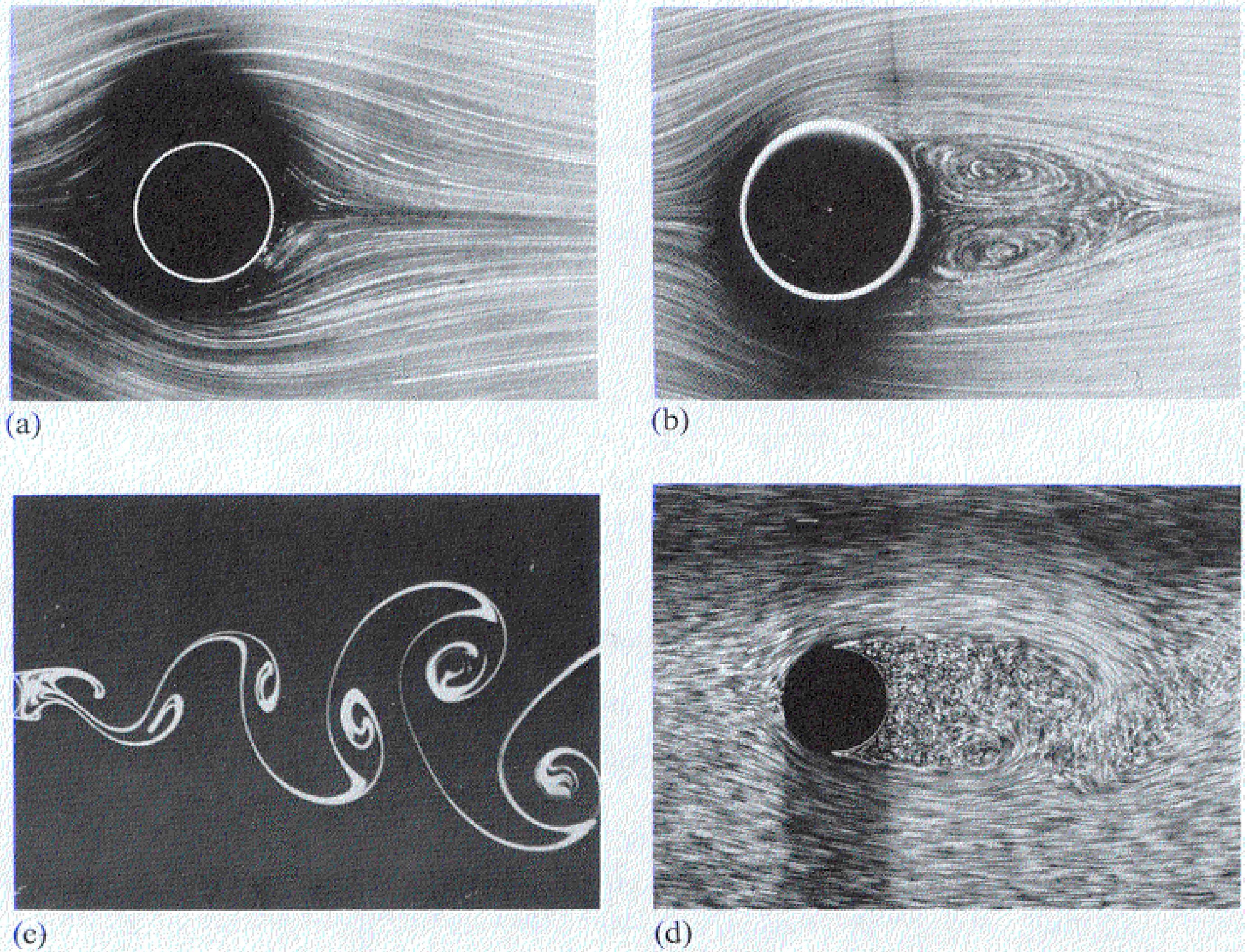
$$\rho_i \frac{D\mathbf{U}}{Dt} \cdot \mathbf{n} = \frac{B^2}{4\pi R_c} + (\rho_i - \rho_e)(\mathbf{g} - R\Omega_e^2 \mathbf{e}_R) \cdot \mathbf{n} +$$

$$+ \rho_i R(\Omega_e^2 - \Omega^2)\mathbf{e}_R \cdot \mathbf{n} + 2\rho_i(\mathbf{U} \times \boldsymbol{\Omega}) \cdot \mathbf{n} + \mathbf{F}_D \cdot \mathbf{n}$$

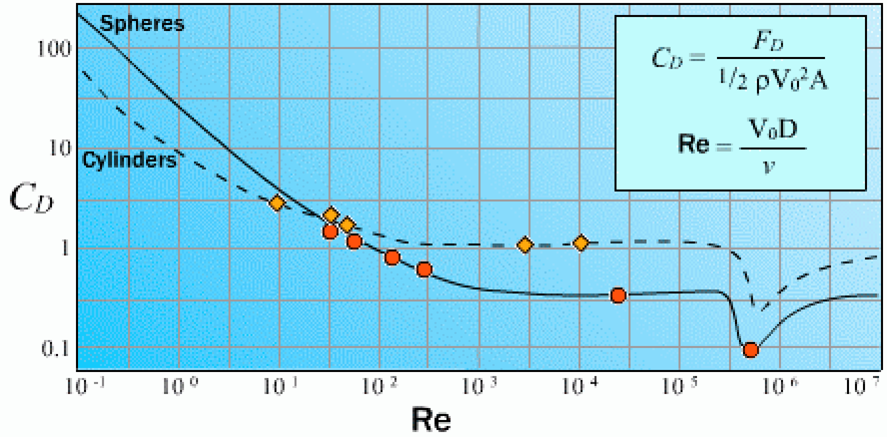
$$\rho_i \frac{D\mathbf{U}}{Dt} \cdot \mathbf{b} = (\rho_i - \rho_e)(\mathbf{g} - R\Omega_e^2 \mathbf{e}_R) \cdot \mathbf{b} +$$

$$+ \rho_i R(\Omega_e^2 - \Omega^2)\mathbf{e}_R \cdot \mathbf{b} + 2\rho_i(\mathbf{U} \times \boldsymbol{\Omega}) \cdot \mathbf{b} + \mathbf{F}_D \cdot \mathbf{b}$$

Aerodynamic drag force $\rightarrow$ pressure disturbance due to relative motion of the (locally cylindric) flux tube w.r.t. its environment:



Aerodynamic drag force $\rightarrow$ pressure disturbance due to relative motion of the (locally cylindric) flux tube w.r.t. its environment:



$$(\pi a^2)\,\mathbf{F}_D \;=\; C_D\,\rho_e\,a\,v_\perp^2\,\hat{\mathbf{k}}$$

C_D: drag coefficient, a: tube radius, and

$$v_\perp^2 \;=\; (\mathbf{v}\cdot\hat{\mathbf{k}})^2 \;=\; [\mathbf{v}-\hat{\mathbf{l}}(\mathbf{v}\cdot\hat{\mathbf{l}})]^2$$

$\mathbf{v} = \mathbf{v}_e - \mathbf{u}$: relative velocity between the flux tube and the surrounding fluid moving with velocity $\mathbf{v}_e$. $\hat{\mathbf{k}}$: unit vector in the direction of the component of $\mathbf{v}$ perpendicular to the tube axis.

Transversal forces per unit length of an tube in thermal equilibrium:

$$F_{\mathrm{C}} = \frac{B^2}{4\pi R} \cdot \pi a^2 = \frac{B^2 a^2}{4R}$$

$$F_{\mathrm{B}} = (\rho - \rho_e) g \cdot \pi a^2 = \frac{p_e - p}{H_p} \cdot \pi a^2 = \frac{B^2 a^2}{8 H_p}$$

$$F_{\mathrm{D}} = C_D \rho_e v_\perp^2 a$$

With $B_{\mathrm{eq}}^2 = 4\pi \rho_e v_c^2$:

$$\frac{F_{\mathrm{B}}}{F_{\mathrm{D}}} \simeq \left(\frac{a}{H_p}\right) \left(\frac{B}{B_{\mathrm{eq}}}\right)^2 \left(\frac{v_c}{v_\perp}\right)^2$$

$$\frac{F_{\mathrm{C}}}{F_{\mathrm{D}}} \simeq \left(\frac{a}{R}\right) \left(\frac{B}{B_{\mathrm{eq}}}\right)^2 \left(\frac{v_c}{v_\perp}\right)^2$$

$\rightarrow$ drag force dominates for sufficiently thin flux tubes

Buoyant rise of a horizontal flux tube: $\mathbf{g} \cdot \hat{\mathbf{n}} = g$, $R \to \infty$

$T = T_e$ & $p < p_e \to \rho < \rho_e \to$ upward buoyancy force!

Stationary state: buoyancy = drag

$$(\rho - \rho_e) g \cdot \pi a^2 + C_D \rho_e v^2 a = 0$$

$$v^2 = \left(\frac{p_e - p}{p_e}\right)\left(\frac{g\pi a}{C_D}\right) = \left(\frac{\pi}{C_D}\right)\left(\frac{B^2}{8\pi\rho_e}\right)\left(\frac{a}{H_p}\right)$$

$$\left(\frac{v}{v_\mathrm{A}}\right)^2 = \left(\frac{\pi}{2C_D}\right)\left(\frac{a}{H_p}\right)$$

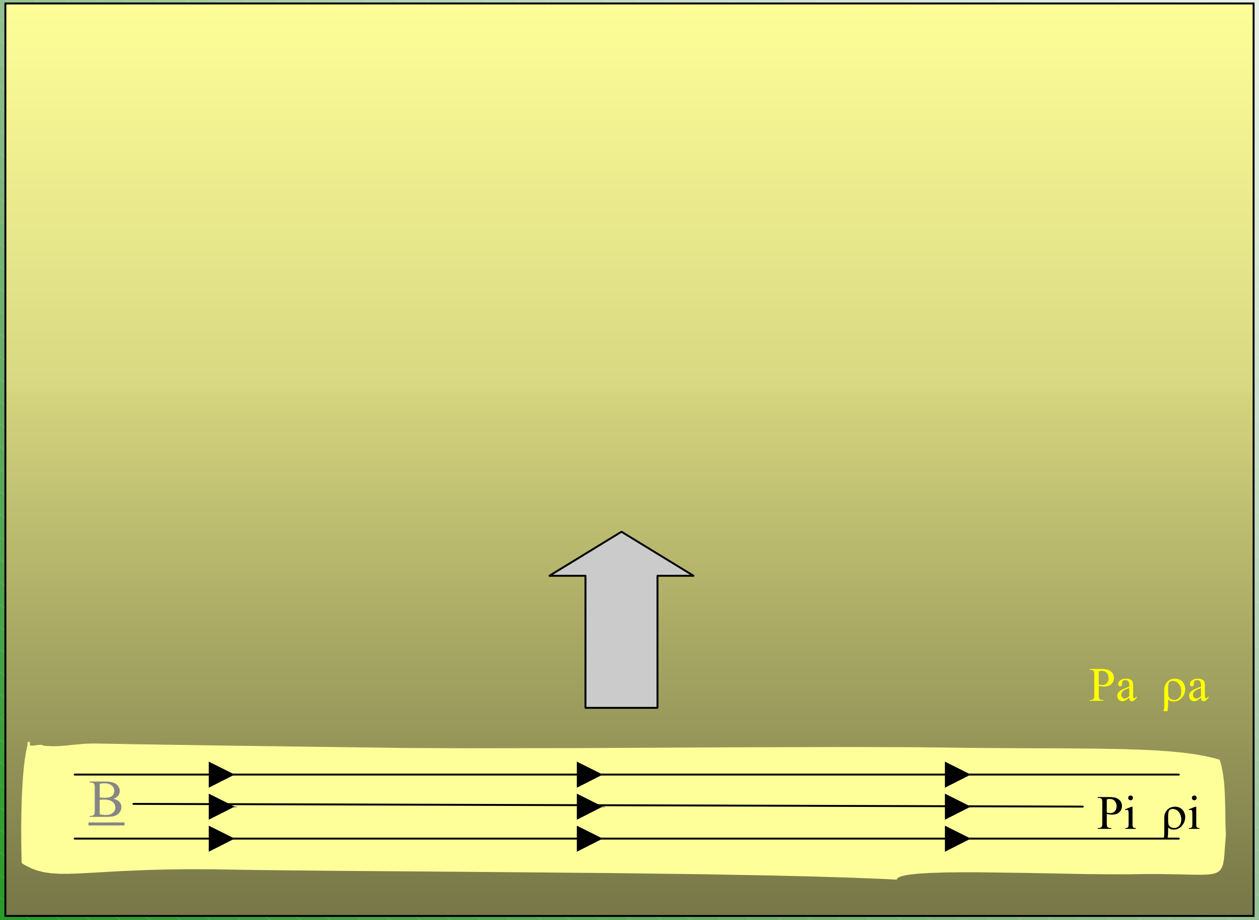
Equipartition field: $v_\mathrm{A} = v_c \simeq 100\,\mathrm{m/s}$

$H_p \simeq 6 \cdot 10^4\,\mathrm{km}$, $a \simeq 6 \cdot 10^3\,\mathrm{km}$ ($\Phi_m = 10^{22}\,\mathrm{Mx}$)

$\to v \simeq 30\,\mathrm{m/s} \to$ rise time through the convection zone (200,000 km) is about 2 months $\ll$ 11 years

$\to$ *magnetic flux storage problem*

Magnetic buoyancy of a flux tube



Pressure equilibrium

$$\mathbf{Pa = Pi + B^2/8\pi}$$

B ≠ **0** ⇒ Pi < Pa

⇒ ρi < ρa

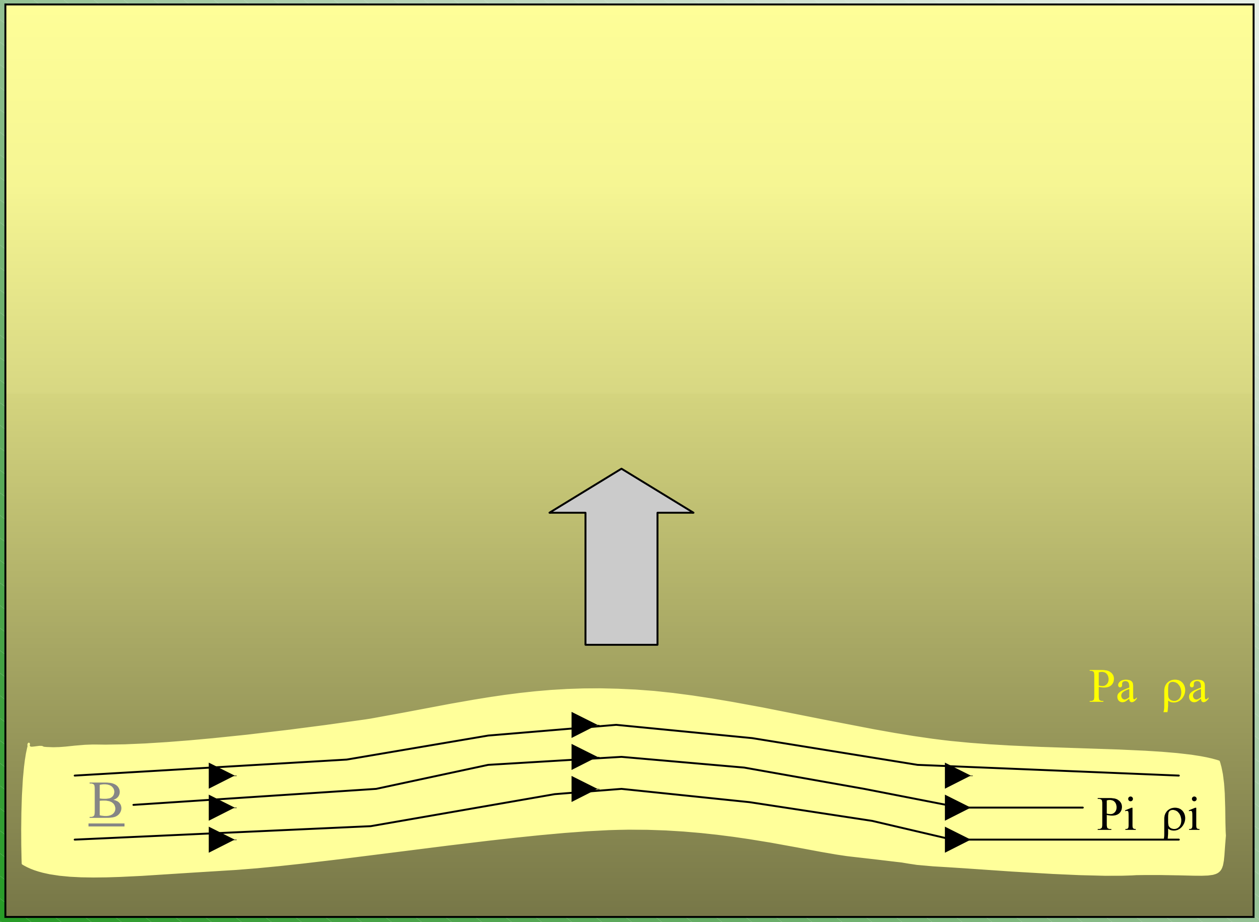
⇒ **buoyancy**

Pa ,ρa **external pressure, density**

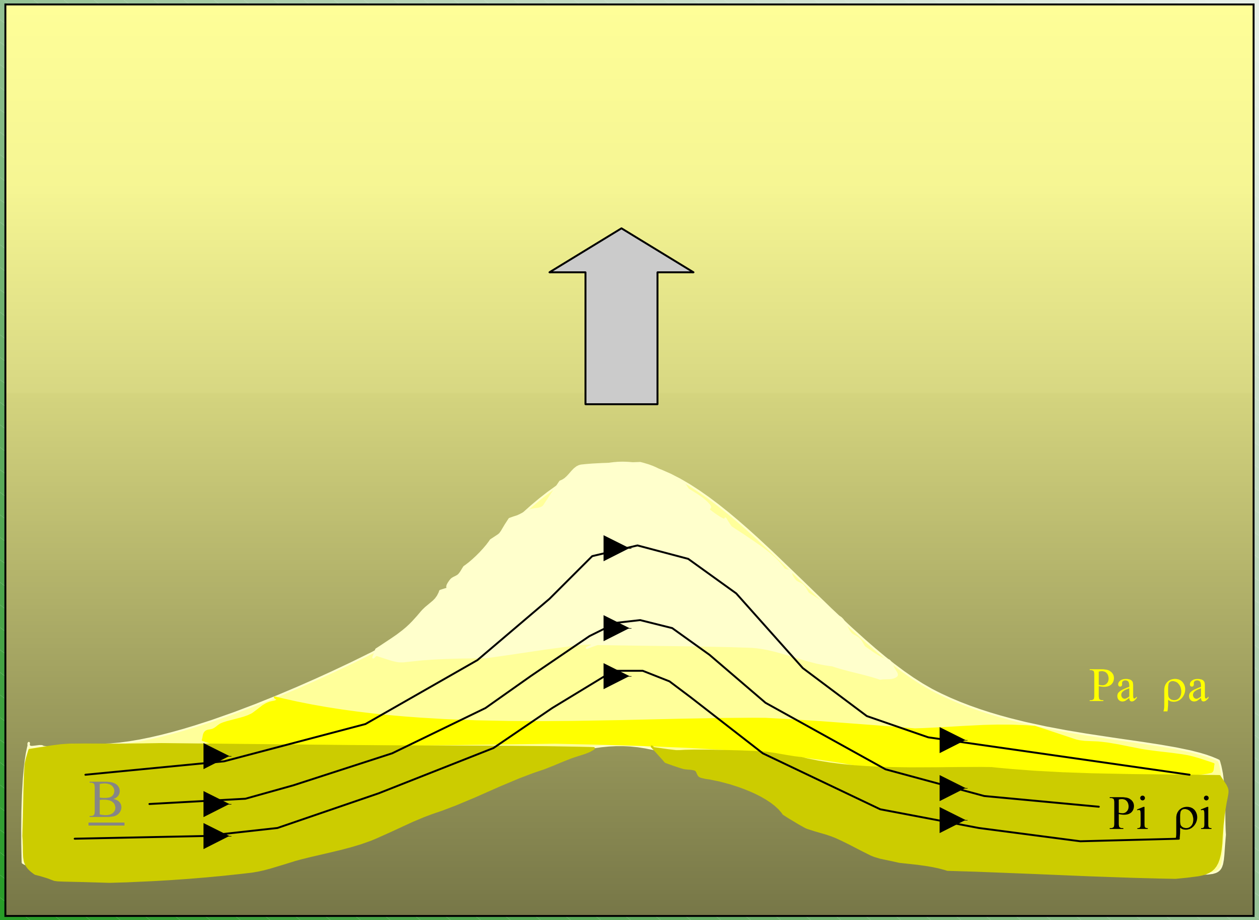
Pi ,ρi **internal pressure, density**

Parker instability

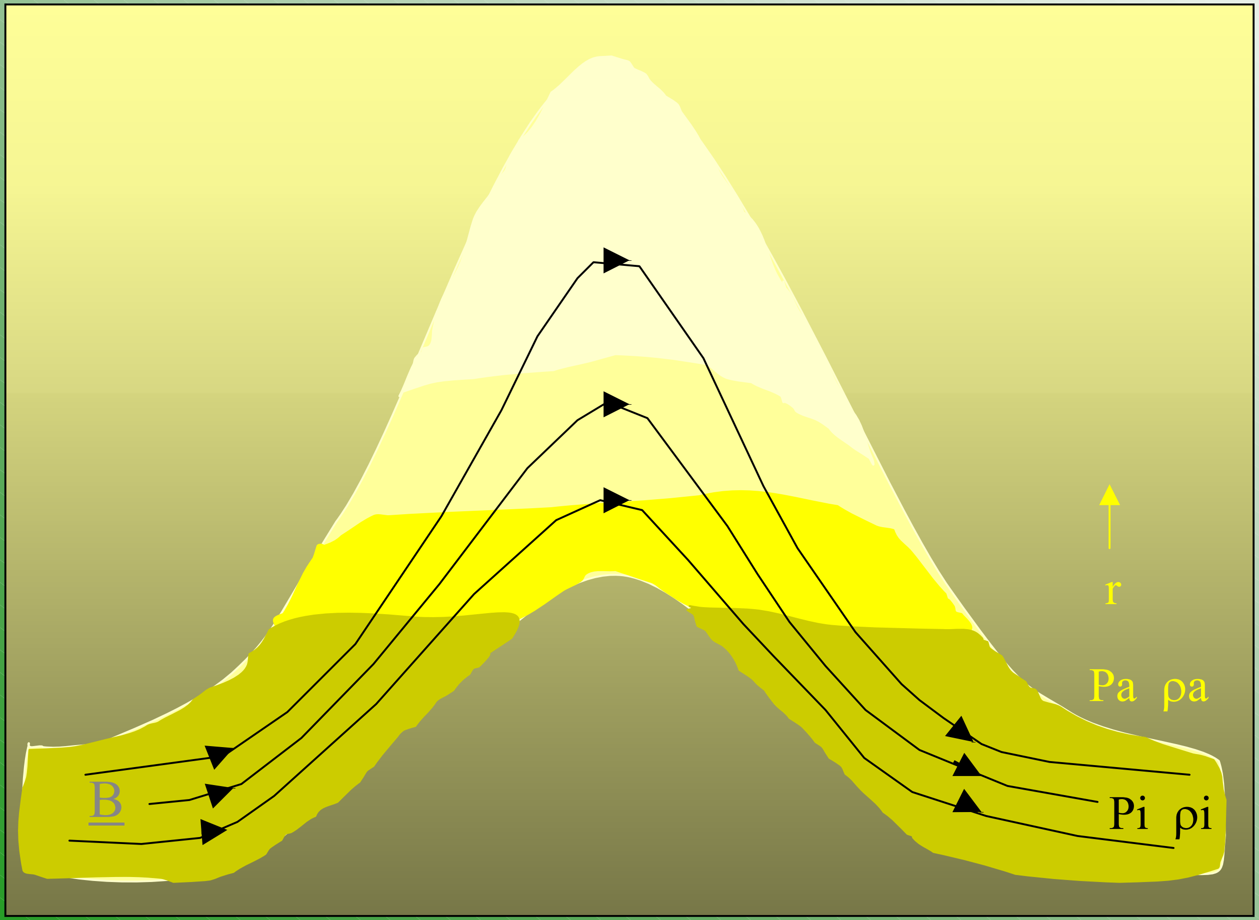
Magnetic buoyancy of a flux tube



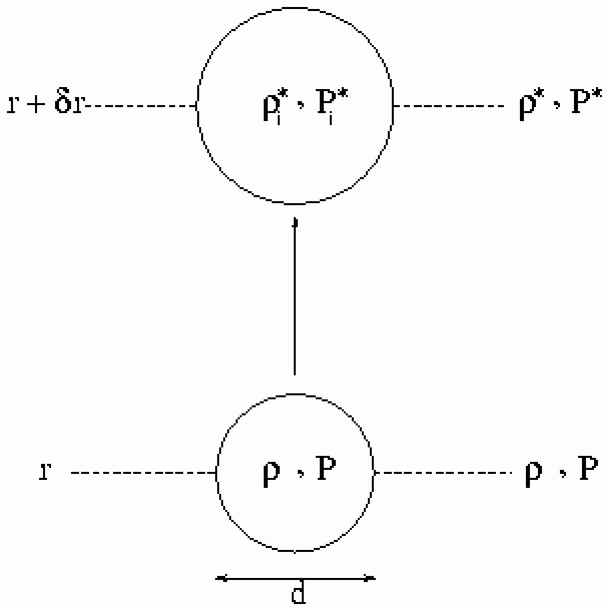
Magnetic buoyancy of a flux tube



Magnetic buoyancy of a flux tube



Convective instability



Instabilität: $\rho_i^* < \rho^*$

$$\rho_i^* - \rho^* = \left[\left(\frac{d\rho}{dr}\right)_{\rm ad} - \frac{d\rho}{dr}\right]\delta r < 0$$

$$p_i^* = p^* \;\rightarrow\; \frac{dT}{dr} < \left(\frac{dT}{dr}\right)_{\rm ad} \;\rightarrow\; \nabla > \nabla_{\rm ad} \;\text{ with }\; \nabla = \frac{d\ln T}{d\ln p}$$

Stellar stratification

$$\nabla \equiv \frac{\mathrm{d}\ln T}{\mathrm{d}\ln p} \qquad \text{logarithmic temperature gradient}$$

adiabatic stratification : $\nabla = \nabla_{\mathrm{ad}} = \frac{\gamma - 1}{\gamma}$

super/subadiabatic : $\nabla = \nabla_{\mathrm{ad}} + \delta$

$\delta > 0$: convectively unstable

$\delta < 0$: convectively stable

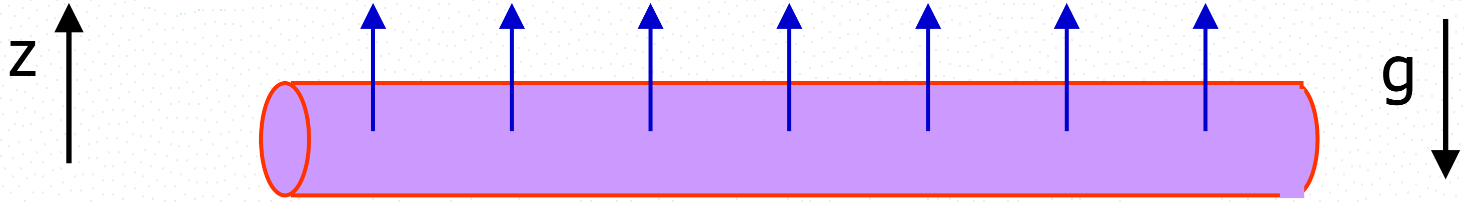
hydrostatic equilibrium : $\frac{\mathrm{d}p}{\mathrm{d}z} = -\varrho g \equiv -\frac{p}{H_{\mathrm{p}}}$

pressure scale height : $H_{\mathrm{p}} = \frac{p}{\varrho g} = \frac{RT}{\mu g}$

$$\frac{1}{H_\varrho} \equiv \frac{\mathrm{d}\ln\varrho}{\mathrm{d}z} = \frac{\mathrm{d}\ln p}{\mathrm{d}z} \cdot \frac{\mathrm{d}\ln\varrho}{\mathrm{d}\ln p} = \frac{1}{H_{\mathrm{p}}}\left(1 - \frac{\mathrm{d}\ln T}{\mathrm{d}\ln p}\right) = \frac{1 - \nabla}{H_{\mathrm{p}}}$$

Stability of a horizontal flux tube

Equilibrium: $\varrho_0 = \varrho_{e0}$; $\quad p_{e0} = p_0 + B_0^2/8\pi$



z_1: constant vertical displacement
external pressure at the new position: $p_{e0} + p_{e1}$

$$p_{e1} = -\frac{p_{e0}\, z_1}{H_{pe}}$$

external density at the new position: $\varrho_{e0} + \varrho_{e1}$

$$\varrho_{e1} = -\frac{\varrho_{e0}\, z_1 (1 - \nabla)}{H_{pe}}$$

total pressure perturbation : $\quad p_{e1} = \dfrac{B_0 B_1}{4\pi} + p_1$

Mass & magnetic flux conservation:

$$\frac{B}{\varrho} = \text{const.} \quad \rightarrow \quad \frac{B_1}{B_0} = \frac{\varrho_1}{\varrho_0}$$

Insert into equation for total pressure perturbation:

$$\gamma \frac{\varrho_1}{\varrho_0} = \frac{p_1}{p_0} = -\frac{p_{e0}}{p_0}\frac{z_1}{H_{pe}} - \frac{B_0^2}{4\pi p_0}\frac{\varrho_1}{\varrho_0}$$

With $\beta \equiv 8\pi p_0/B_0^2$ we have:

$$\frac{\varrho_1}{\varrho_0}\left(\gamma + \frac{2}{\beta}\right) = -\frac{(p_0 + B_0^2/8\pi)}{p_0}\frac{z_1}{H_{pe}} = -\left(1 + \frac{1}{\beta}\right)$$

$\beta \gg 1$ in the deep solar convection zone $\rightarrow$

$$\frac{\varrho_1}{\varrho_0} = -\left(\frac{1 + 1/\beta}{\gamma + 2/\beta}\right)\frac{z_1}{H_{pe}} \simeq \left[\frac{1}{\gamma} + \frac{1}{\beta\gamma}\left(1 - \frac{2}{\gamma}\right) + O(\beta^{-2})\right]\frac{z_1}{H_{pe}}$$

Perturbation of the buoyancy force:

$$F_{\rm B1} = (\varrho_{\rm e1} - \varrho_1)g = \left\{-\varrho_{\rm e0}(1-\nabla) + \varrho_0\left[\frac{1}{\gamma} + \frac{1}{\beta\gamma}\left(1-\frac{2}{\gamma}\right)\right]\right\}\frac{gz_1}{H_{\rm pe}}$$

$$= \left[-1 + \frac{\gamma-1}{\gamma} + \delta + \frac{1}{\gamma} + \frac{1}{\beta\gamma}\left(1-\frac{2}{\gamma}\right)\right]\frac{\varrho_0 g z_1}{H_{\rm pe}}$$

$$= \left[\beta\delta + \frac{1}{\gamma}\left(1-\frac{2}{\gamma}\right)\right]\frac{z_1}{H_{\rm p0}H_{\rm pe}}\left(\varrho_0 g H_{\rm p0}\right)\frac{B_0^2}{8\pi p_0}$$

$$= \left[\beta\delta + \frac{1}{\gamma}\left(1-\frac{2}{\gamma}\right)\right]\frac{B_0^2 z_1}{8\pi H_{\rm p0}H_{\rm pe}}$$

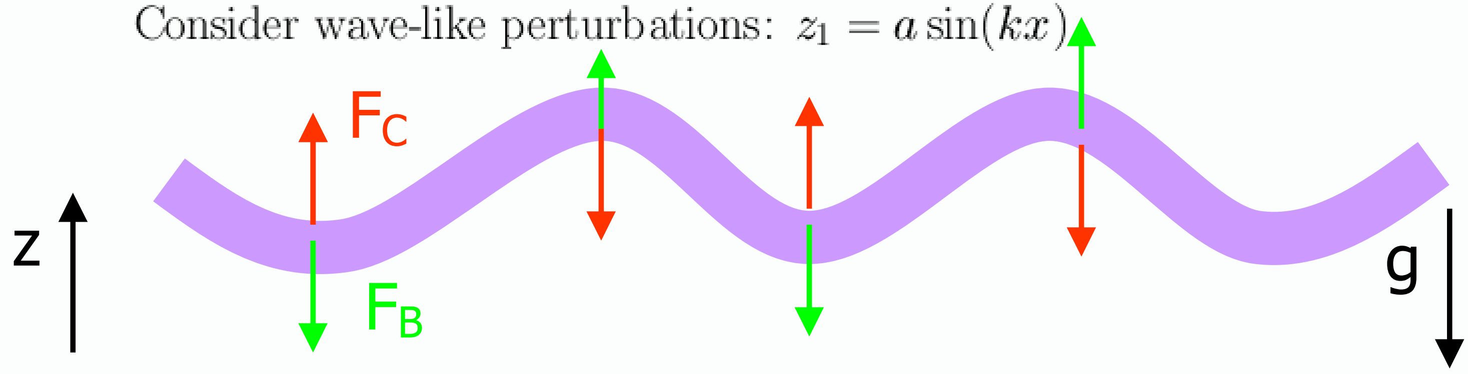
For stability: [...] $< 0 \rightarrow$

$$\beta\delta < -\frac{1}{\gamma}\left(1-\frac{2}{\gamma}\right) \quad \rightarrow$$

$$\beta\delta < 0.12 \quad \text{for } \gamma = 5/3$$

→ Strong fields (smaller β) stable in a superadiabatic layer ($\delta > 0$)!?

Consider wave-like perturbations: $z_1 = a\sin(kx)$



Simplified treatment ($\rightarrow$ correct stability result):
hydrostatic equilibrium along the loop $\rightarrow$

$$\frac{\varrho_1}{\varrho_0} = \frac{1}{\gamma}\,\frac{p_1}{p_0} = -\frac{z_1}{\gamma H_{\rm p0}}$$

Perturbation of the buoyancy force:

$$F_{\rm B1} = (\varrho_{\rm e1} - \varrho_1)g = \left[-\frac{\varrho_{\rm e0}(1-\nabla)}{H_{\rm pe}} + \frac{\varrho_0}{\gamma H_{\rm p0}}\right] g z_1$$

$$= \ldots$$

$$= \left(\beta\delta + \frac{1}{\gamma}\right)\frac{B_0^2 z_1}{8\pi H_{\rm p0}^2}$$

Magnetic curvature force:

$$\frac{B^2}{4\pi R} = \frac{B^2}{4\pi} \frac{z_1''}{(1+z_1'^2)^{3/2}}$$

At loop maximum ($x = \pi/2k$):

$$z_1'(x) = ka\cos(kx) = 0$$
$$z_1''(x) = -k^2 a \sin(kx) = -k^2 a$$

Sum of buoyancy force and curvature force:

$$F_{B1} + F_{C1} = \left[\left(\beta\delta + \frac{1}{\gamma}\right)\frac{B_0^2}{8\pi H_{p0}^2} - \frac{B_0^2 k^2}{4\pi}\right] a$$

$$= \left[\left(\beta\delta + \frac{1}{\gamma}\right) - 2(kH_{p0})^2\right]\frac{aB_0^2}{8\pi H_{p0}^2}$$

For stability: [...] $< 0 \rightarrow$

$$\beta\delta < -\frac{1}{\gamma} + 2(kH_{p0})^2$$

k=0:

$$\beta\delta < -\frac{1}{\gamma}\left(1 - \frac{2}{\gamma}\right) \rightarrow$$

$\beta\delta < 0.12$ for $\gamma = 5/3$

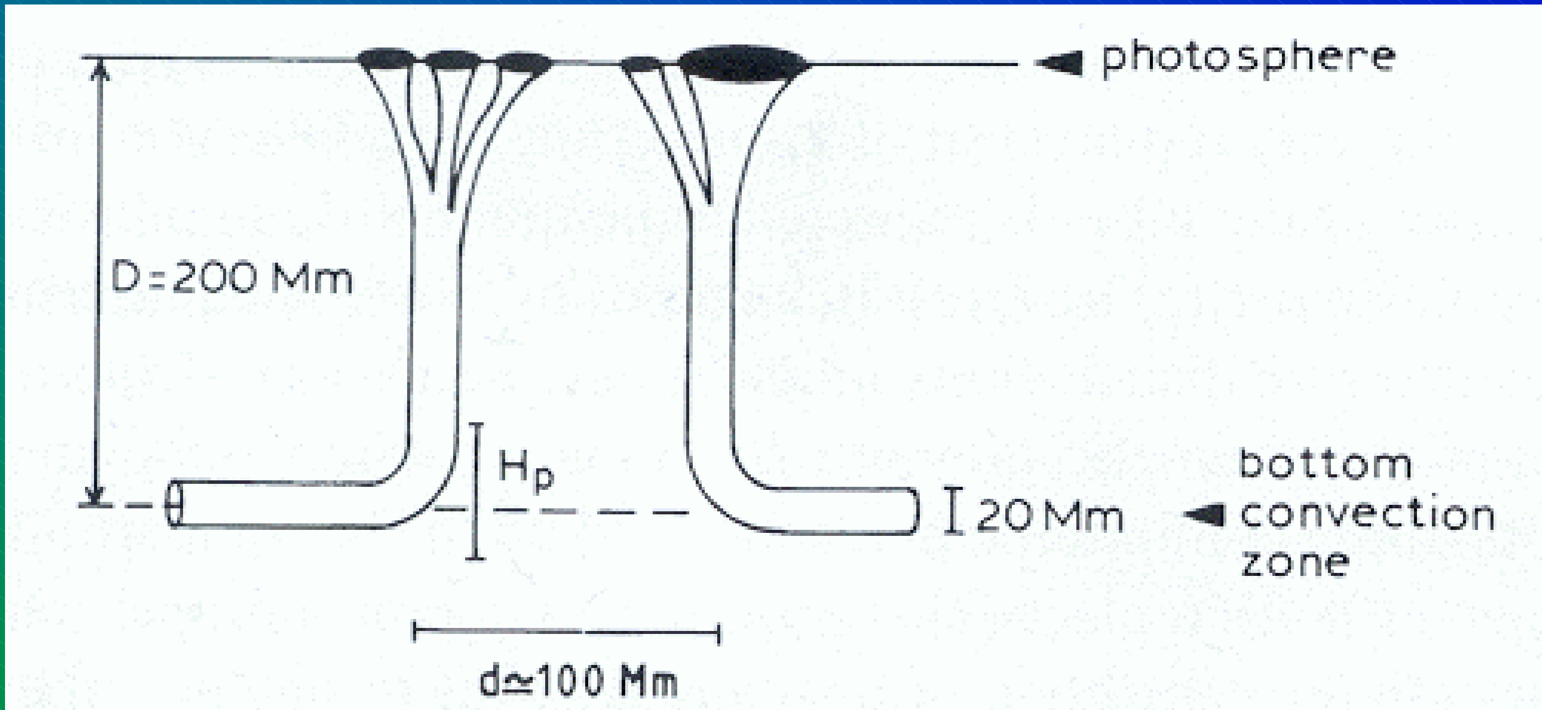
for $k \rightarrow 0$ (long wavelength): $\beta\delta < -1/\gamma = -0.6$

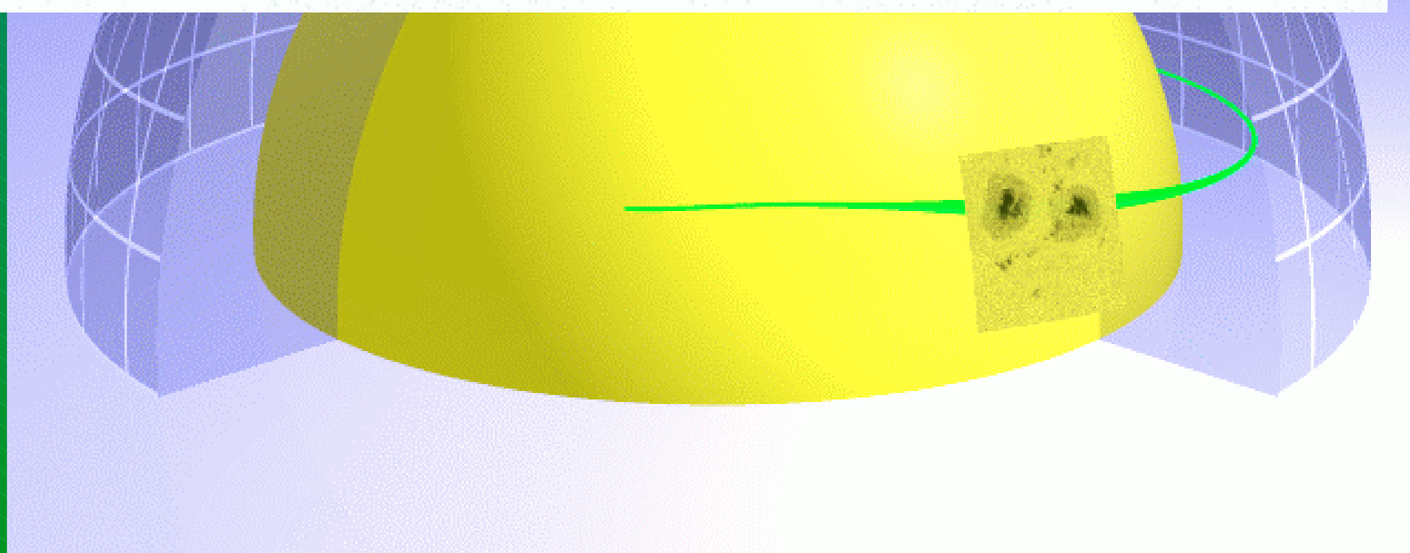
All flux tubes are unstable in a superadiabatic layer ($\delta > 0$)!!

The plan

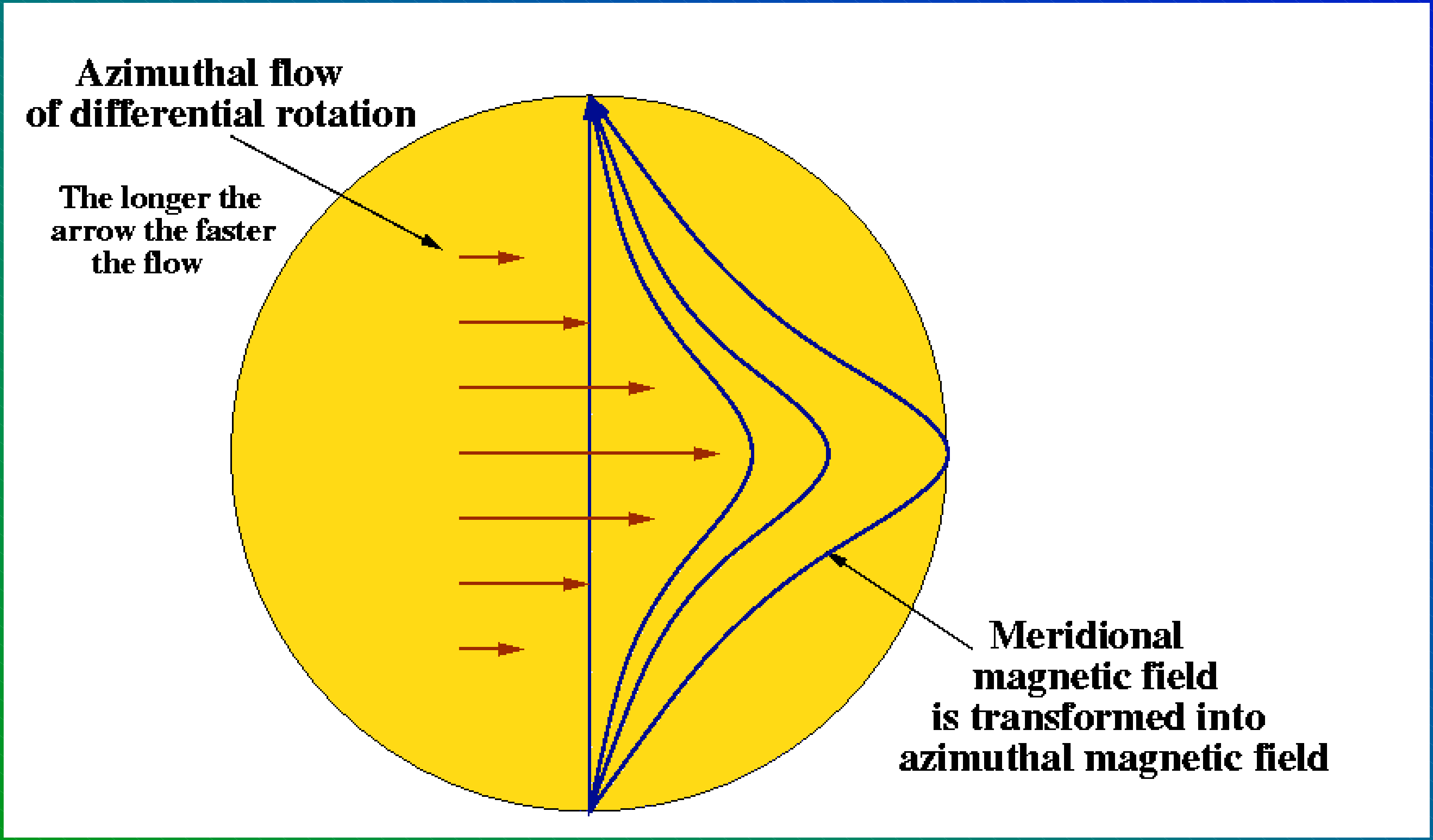
- 1) Formation of magnetic structure
- 2) Physics of magnetic flux tubes
- 3) Magnetic structure and the dynamo

from?

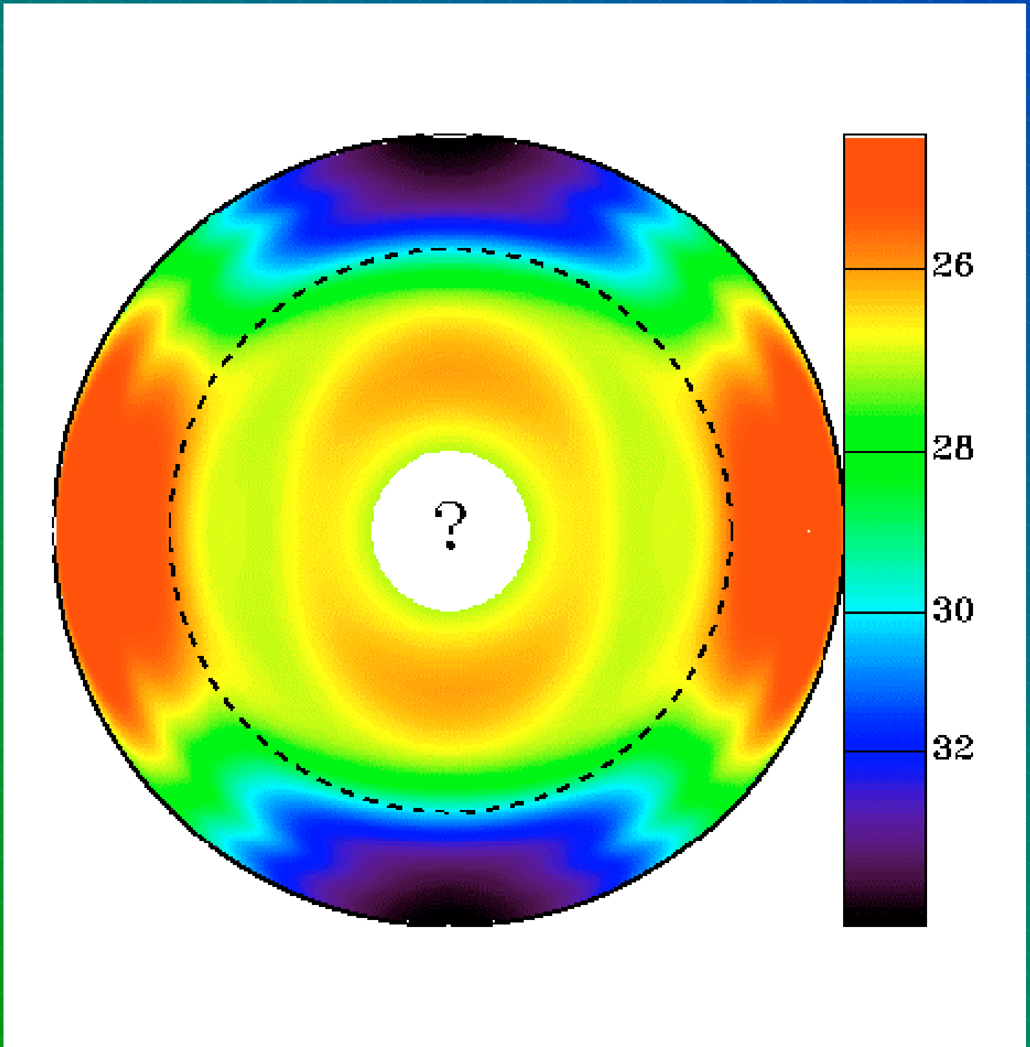




Differential rotation generates azimuthal (toroidal) magnetic field

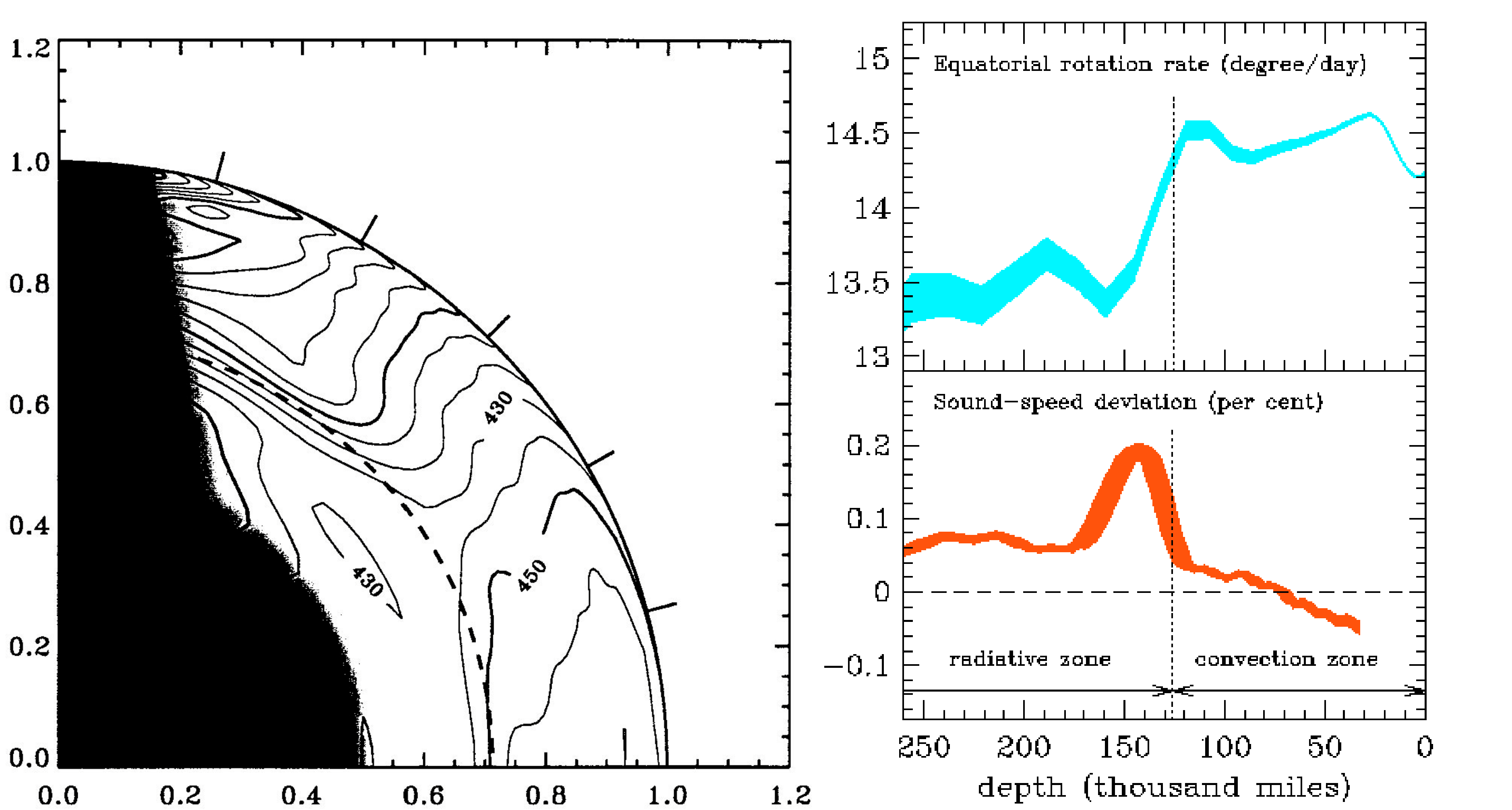


Internal rotation of the Sun as determined by helioseismology

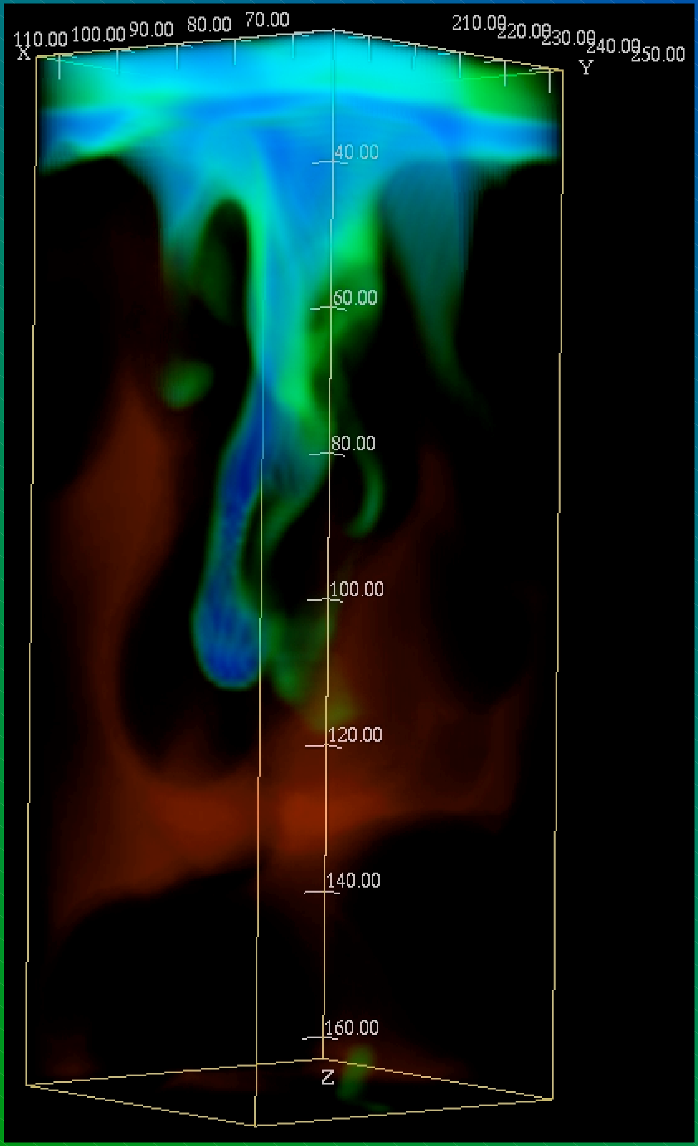


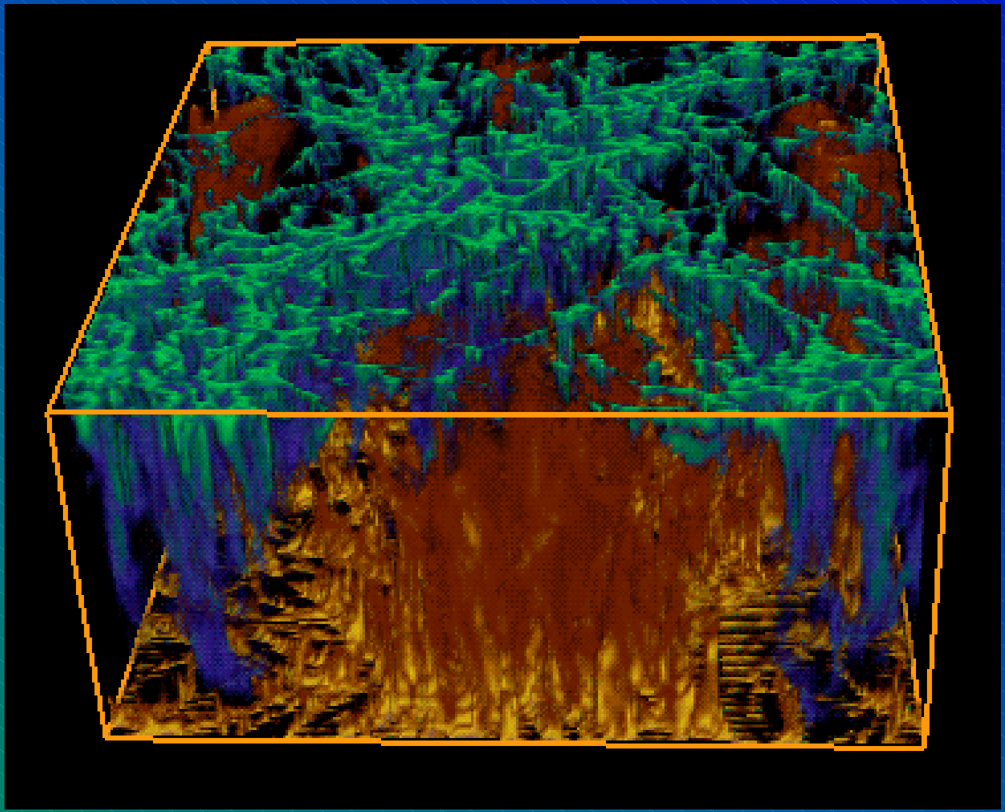
- Convection zone rotates similar to surface
- Core rotates nearly rigidly
- Steep transition at the bottom of the convection zone; width ~2% R_{sun}
- Region of strongest shear ➔ Dynamo!

Internal rotation of the Sun as determined by helioseismology



Plumes: coherent structures in high-Ra convection

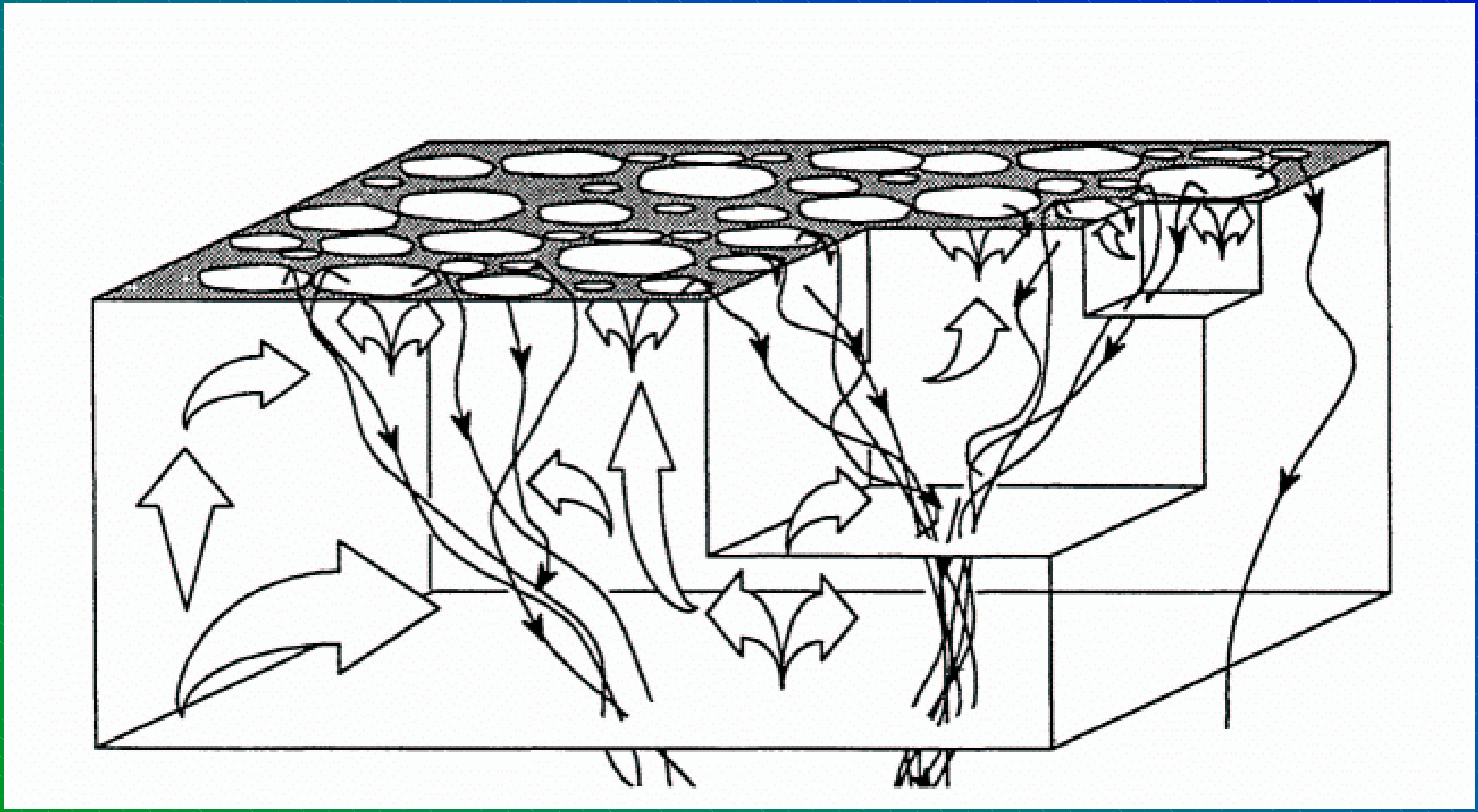




Kerr 1996

Nordlund & Stein 1998

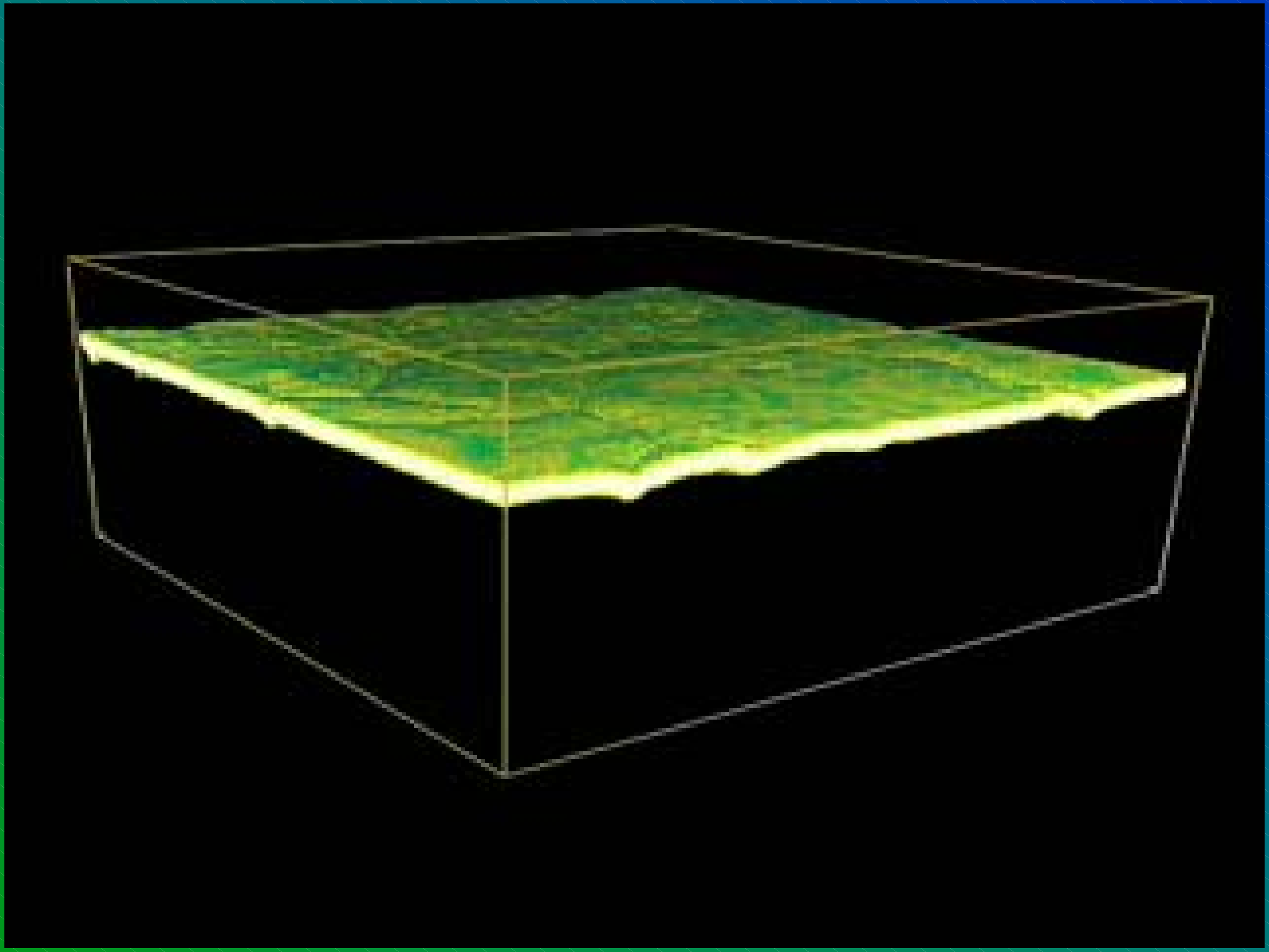
Hierarchical downflow structure in solar convection



Spruit et. al. 1990

Downward "pumping" of magnetic flux by convection

(Brummell 2001)

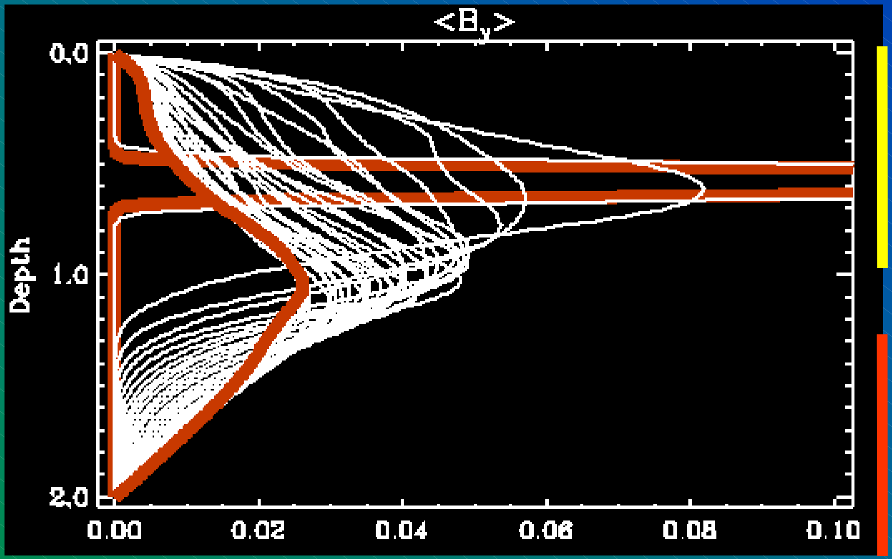


Yellow - white:
strong field

green - blue:
weak field

Downward "pumping" of magnetic flux by convection

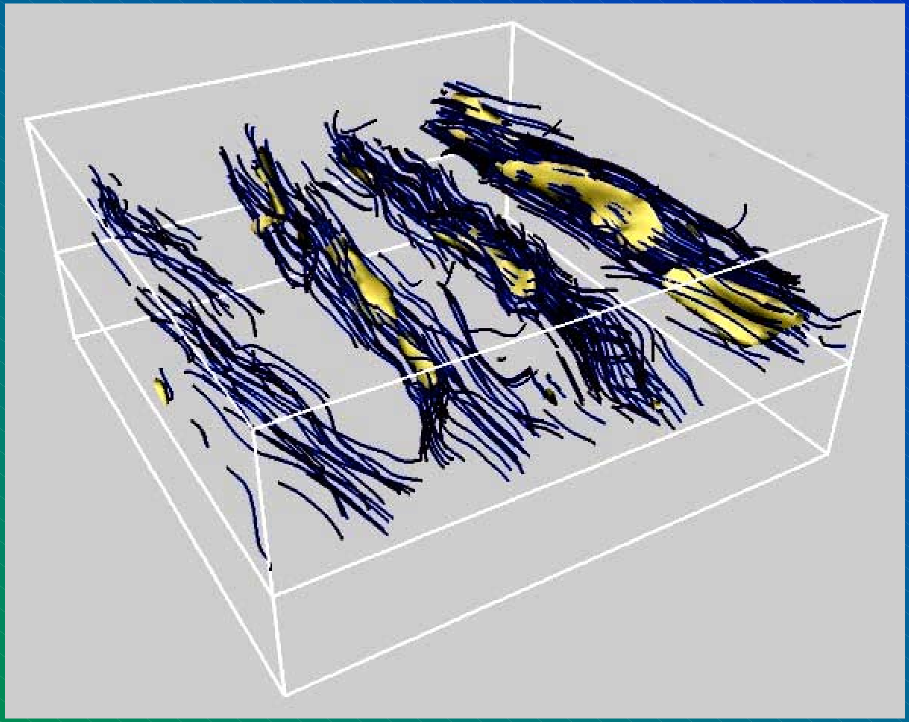
(*Tobias et al. 2001*)



convectively unstable

stable layer

Strands of toroidal field generated by downward pumping and differential rotation



(*Dorch & Nordlund 2001*)

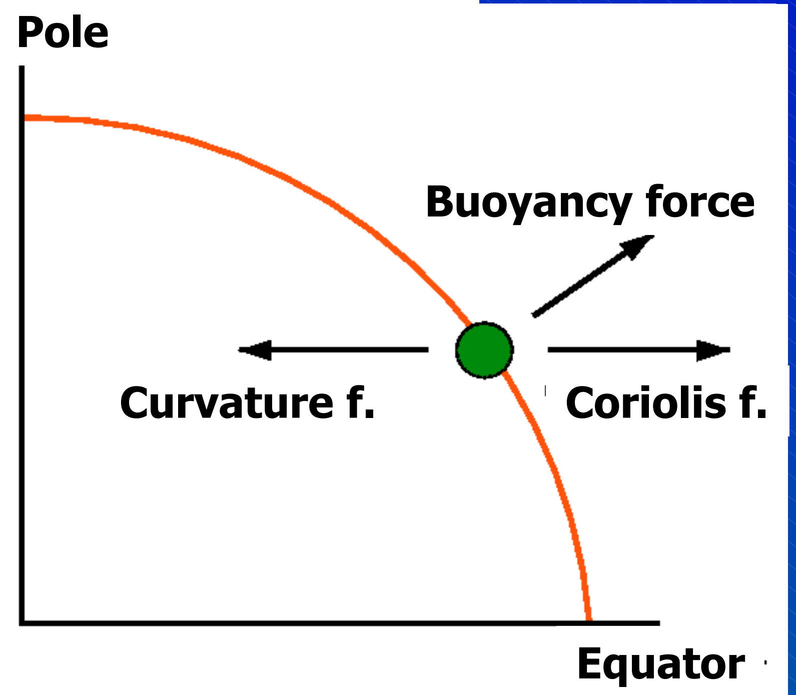
- Assume magnetic flux to be stored within the overshoot layer in form of individual toroidal flux tubes

- Storage requires *mechanical equilibrium*

Buoyancy force : $\mathbf{F}_B = \Delta\rho\,\mathbf{g}$

Magnetic force : $\mathbf{F}_M = -\frac{B^2}{4\pi\varpi}\,\mathbf{e}_\varpi$

Rotational force : $\mathbf{F}_\Omega = \rho_i\varpi(\Omega_i^2 - \Omega_e^2)\,\mathbf{e}_\varpi$



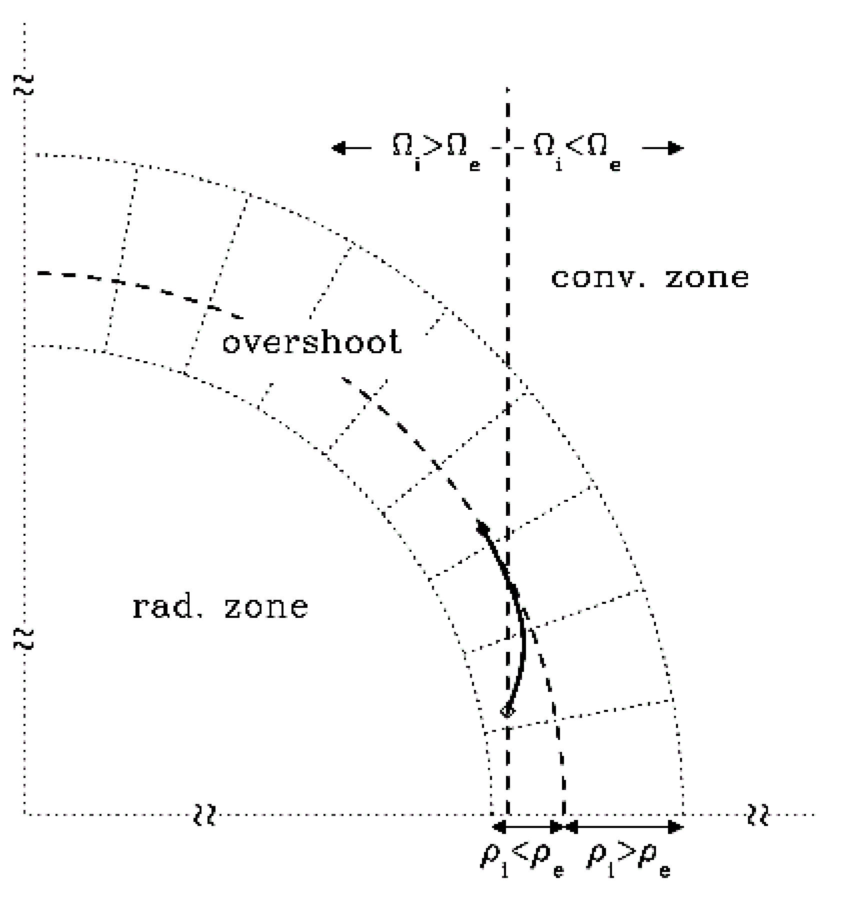
- Outside equatorial plane:

  - $\mathbf{F}_B = \mathbf{0} \quad \rightarrow \quad \Delta\rho = 0, \quad T_i < T_e$
  - $\mathbf{F}_M + \mathbf{F}_\Omega = \mathbf{0} \quad \rightarrow \quad \Omega_i^2 = \Omega_e^2 + v_A^2/\varpi^2$

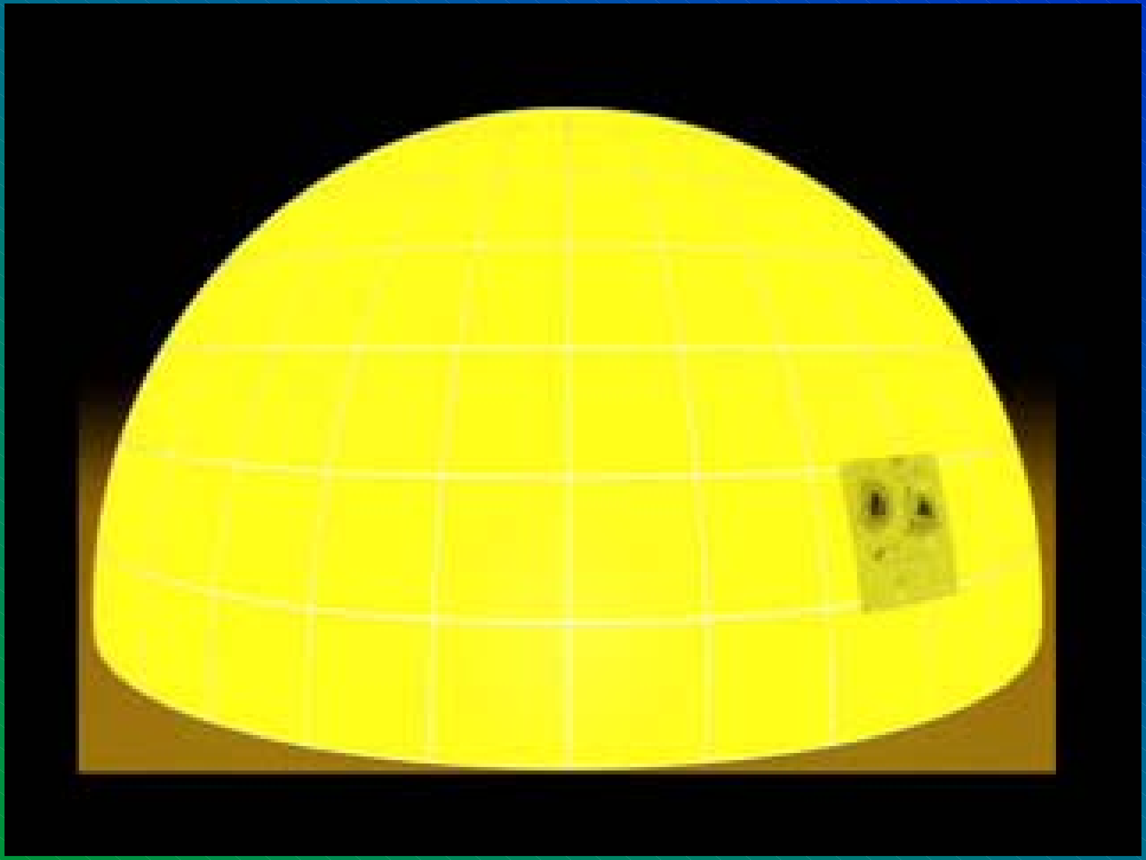
$\Rightarrow$ No equilibrium without rotation !

– Flux tubes created with $T_i = T_e$ and $\Omega_i = \Omega_e$:

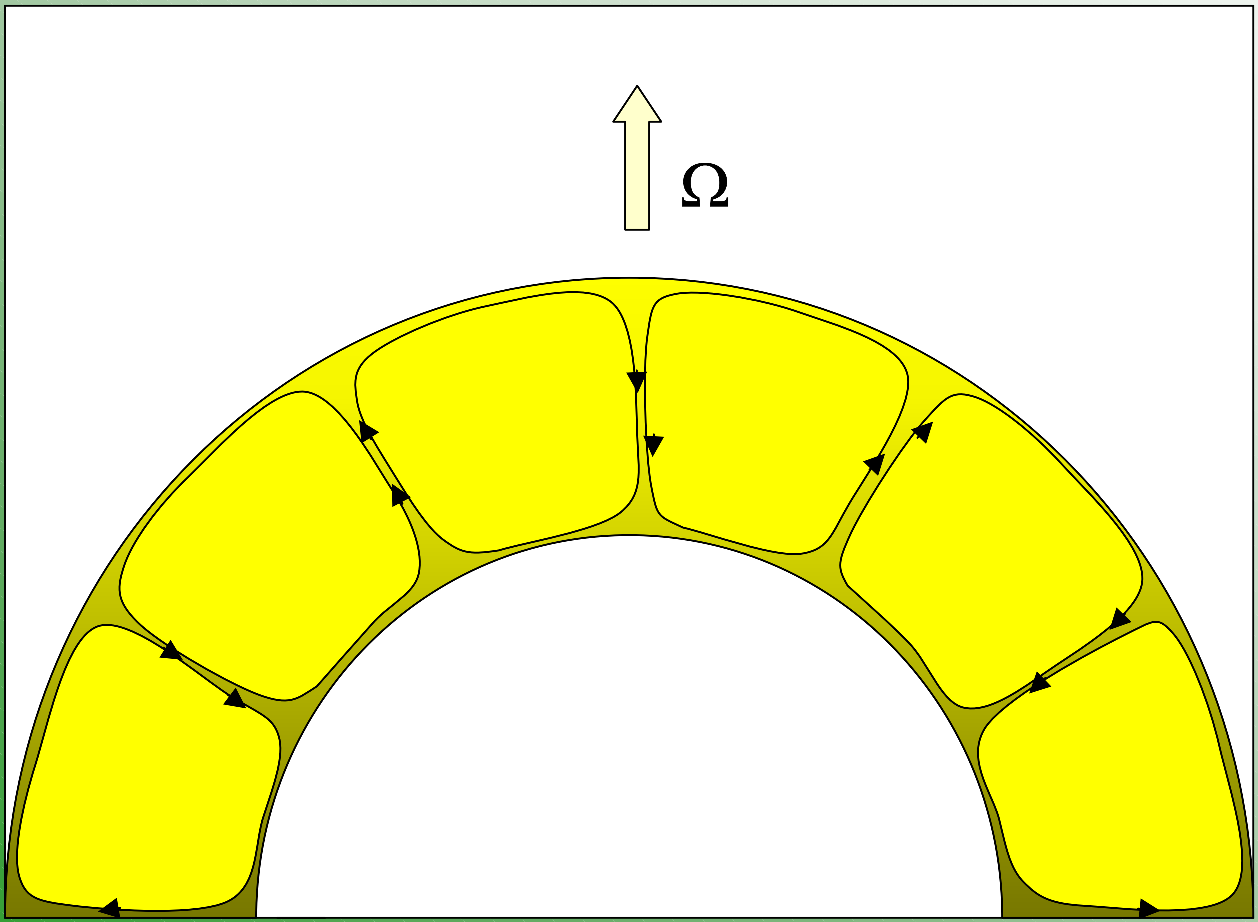
→ approach equilibrium position via damped buoyancy and inertial oscillations



How does the magnetic field come to the surface?



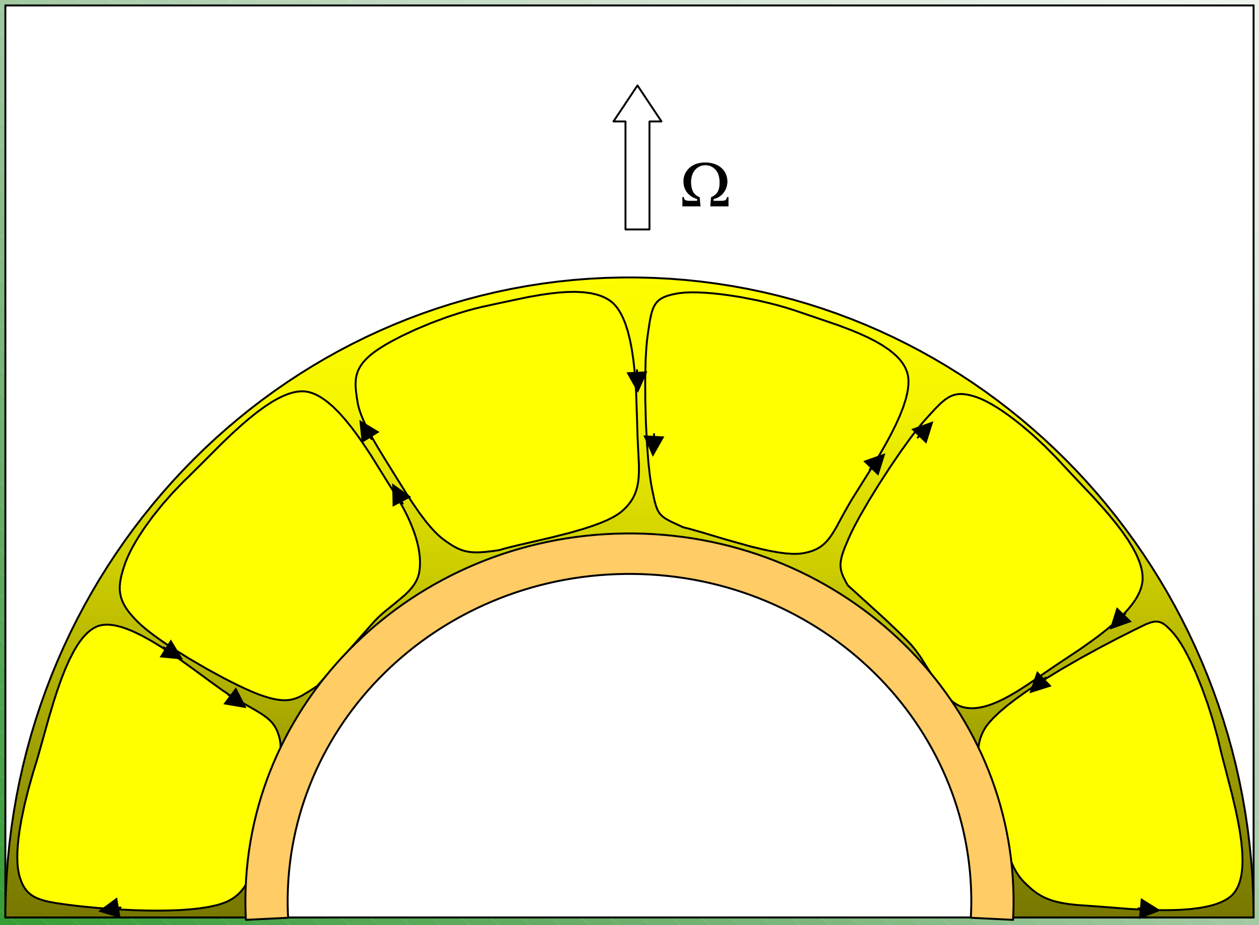
Origin of sunspots



1

•Convection zone

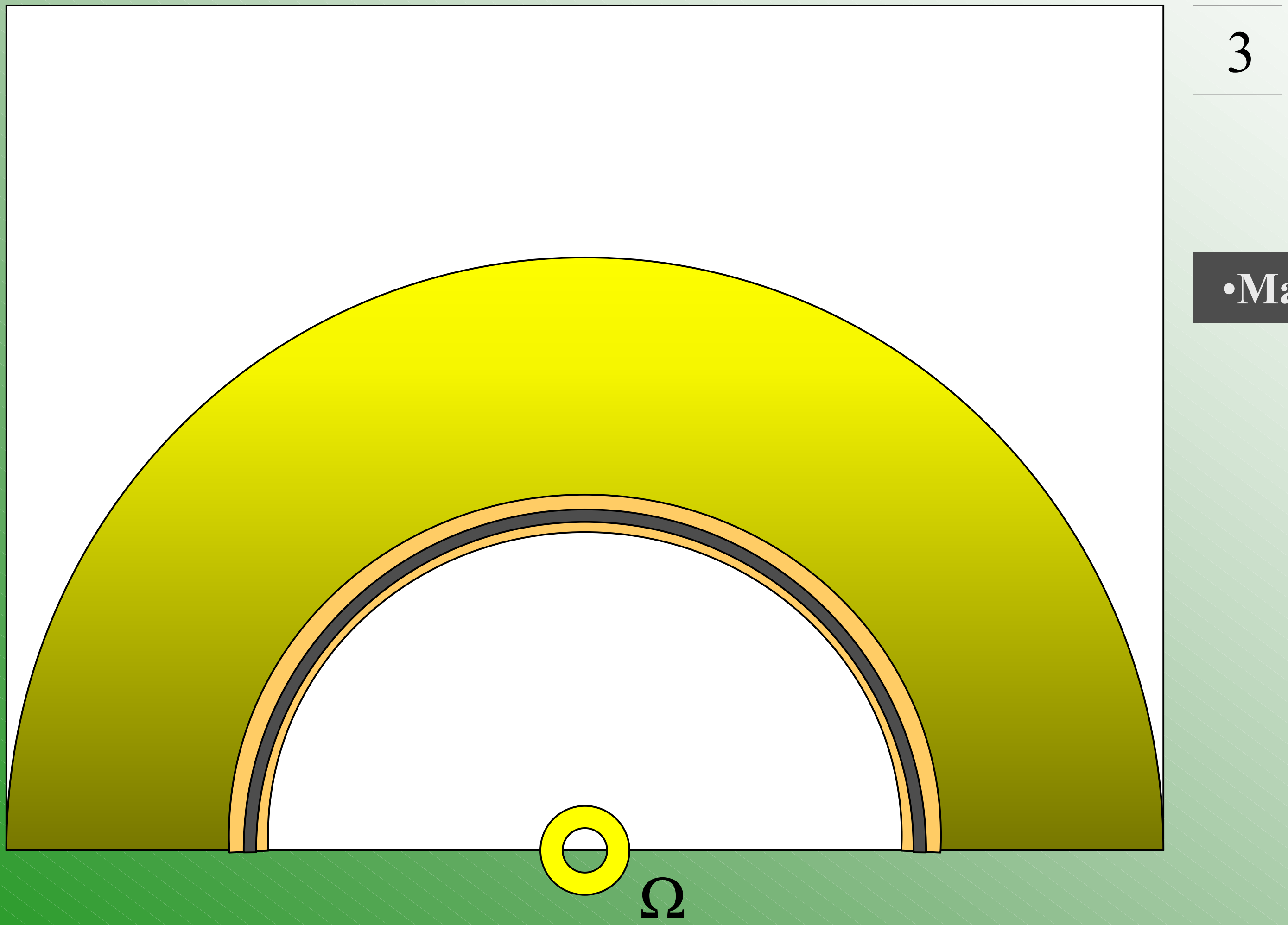
Origin of sunspots



2

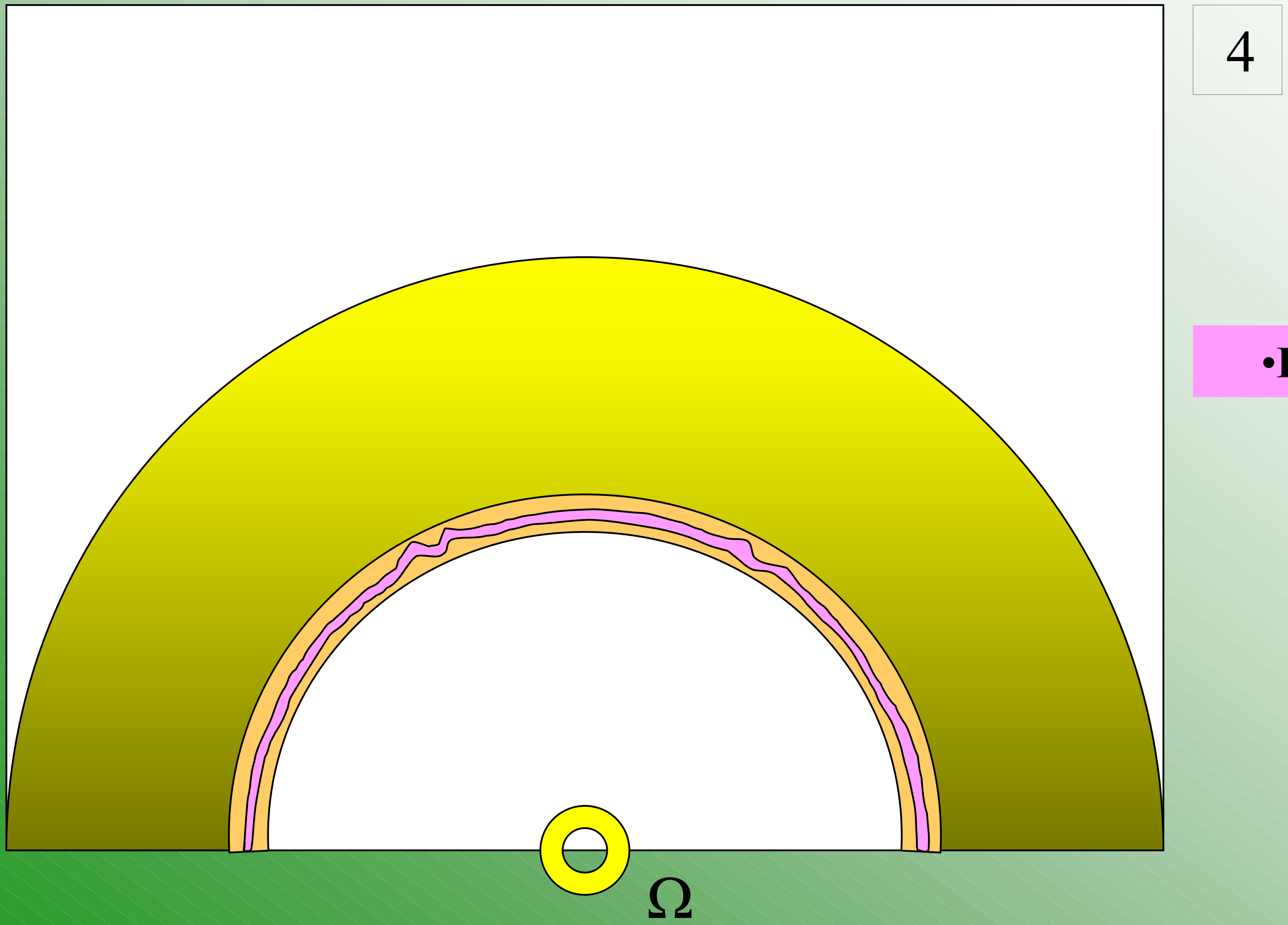
•**Overshoot layer**

Origin of sunspots



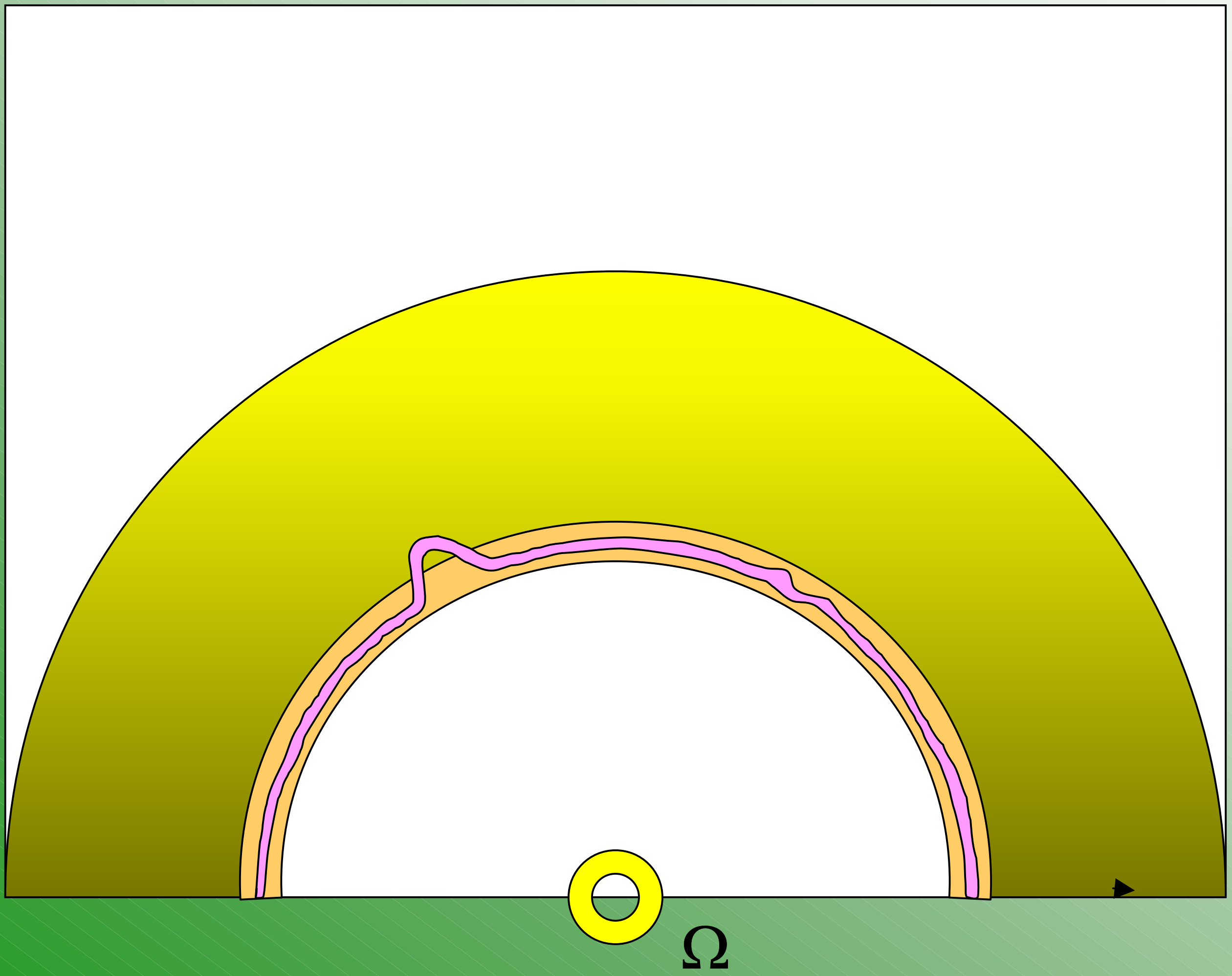
•Magnetic flux tube

Origin of sunspots



•**Parker instability**

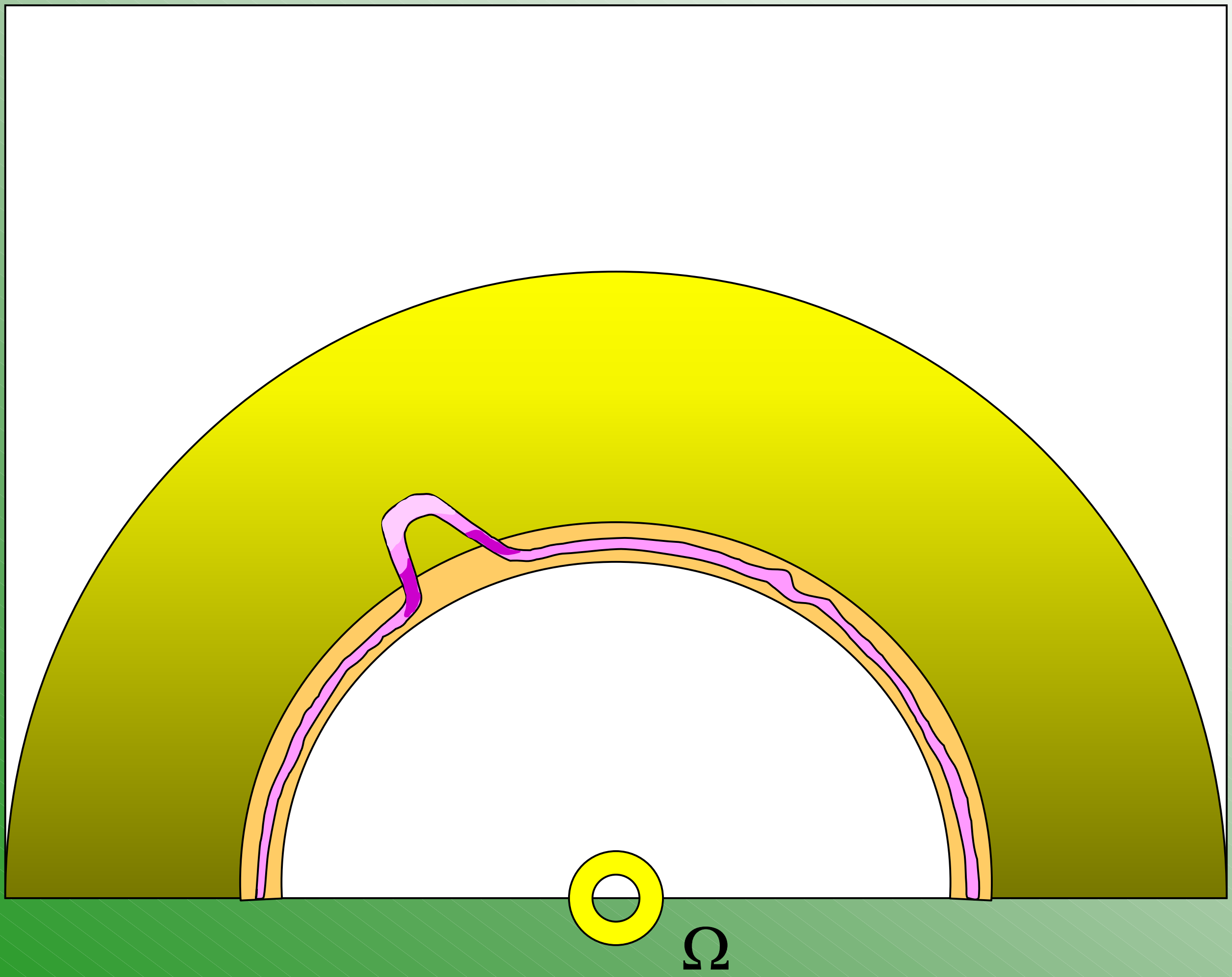
Origin of sunspots



5

•**Magnetic buoyancy**

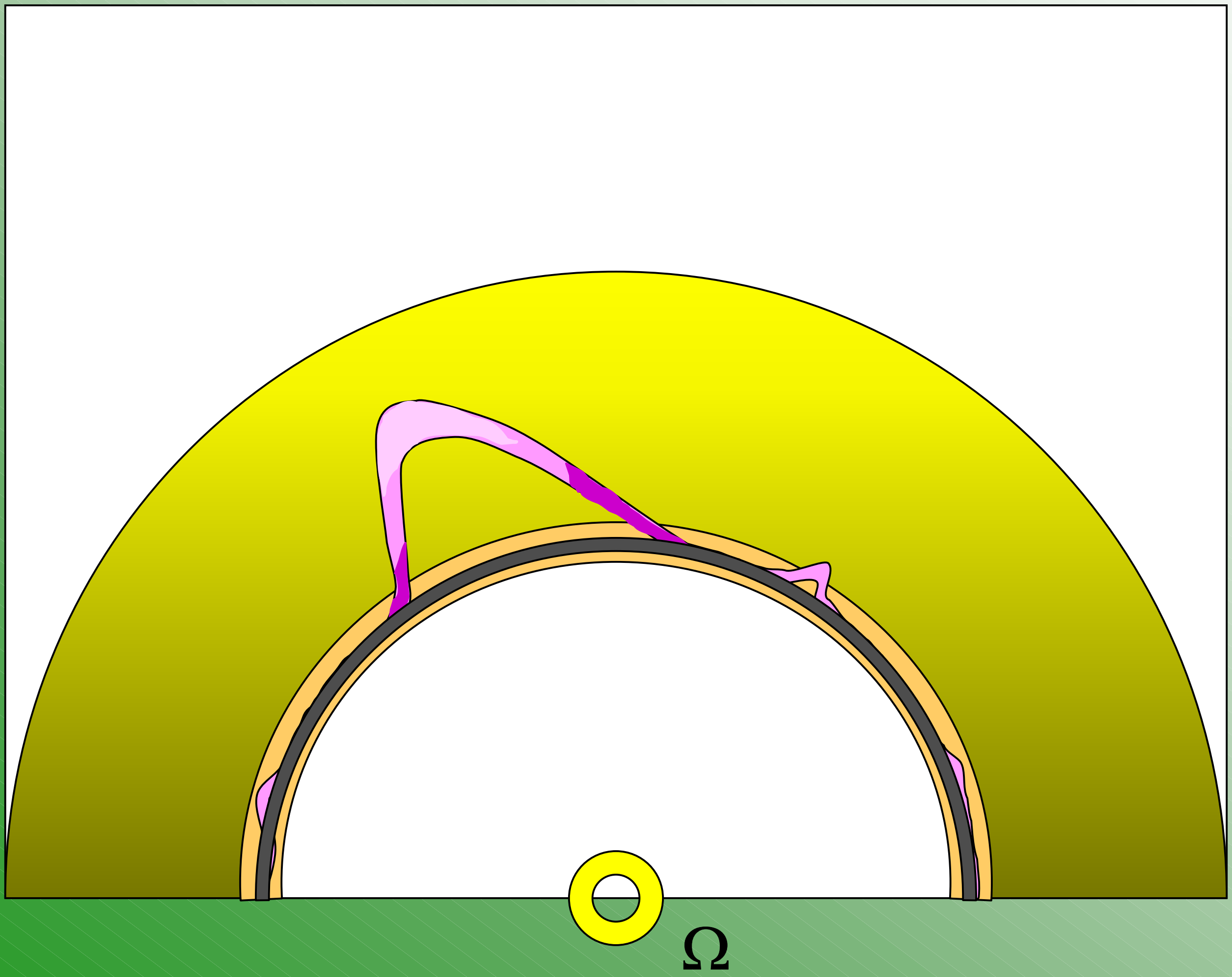
Origin of sunspots



6

•**Magnetic buoyancy**

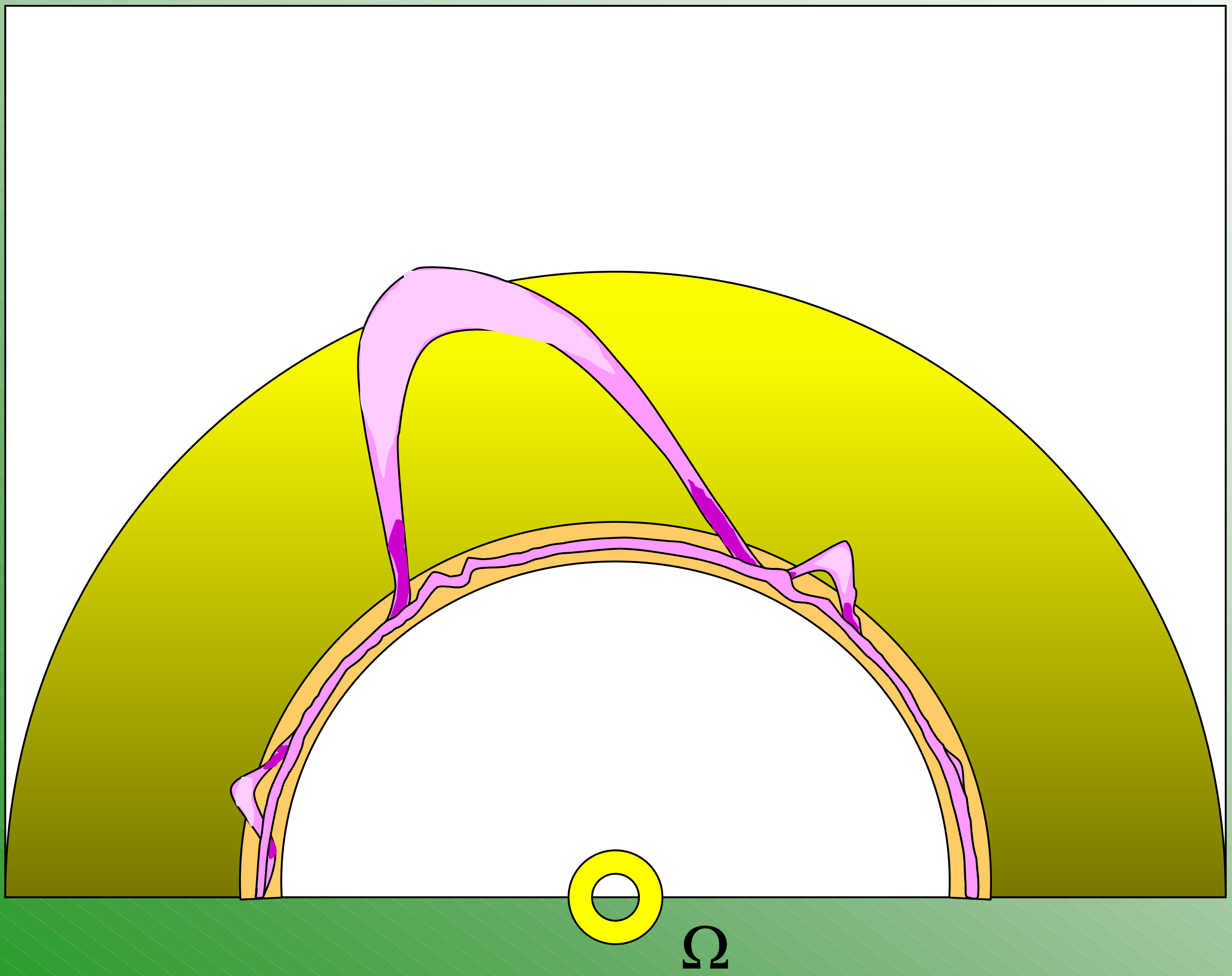
Origin of sunspots



7

• **Tube expansion and decreasing field strength**

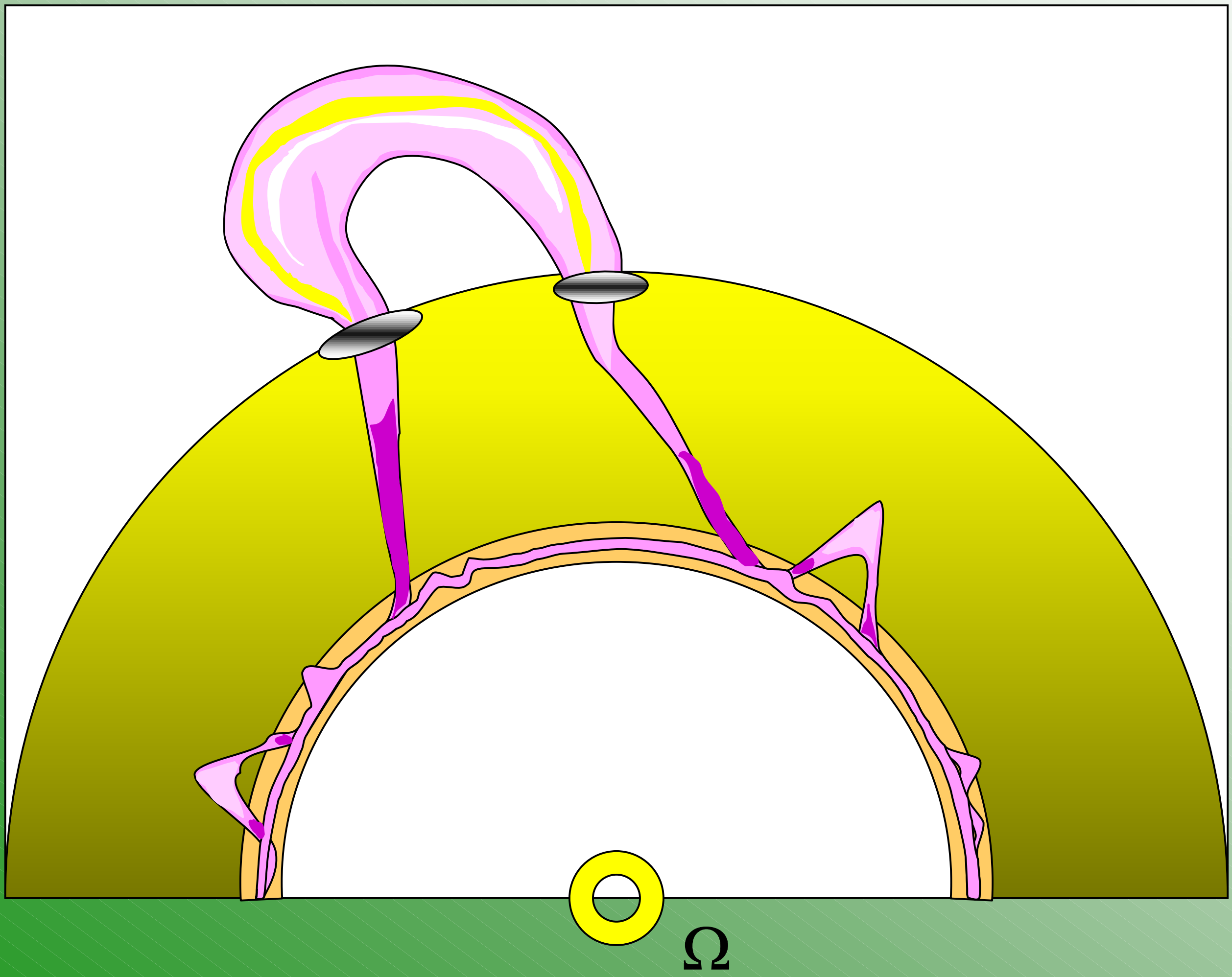
Origin of sunspots



8

•**Eruption at the solar surface**

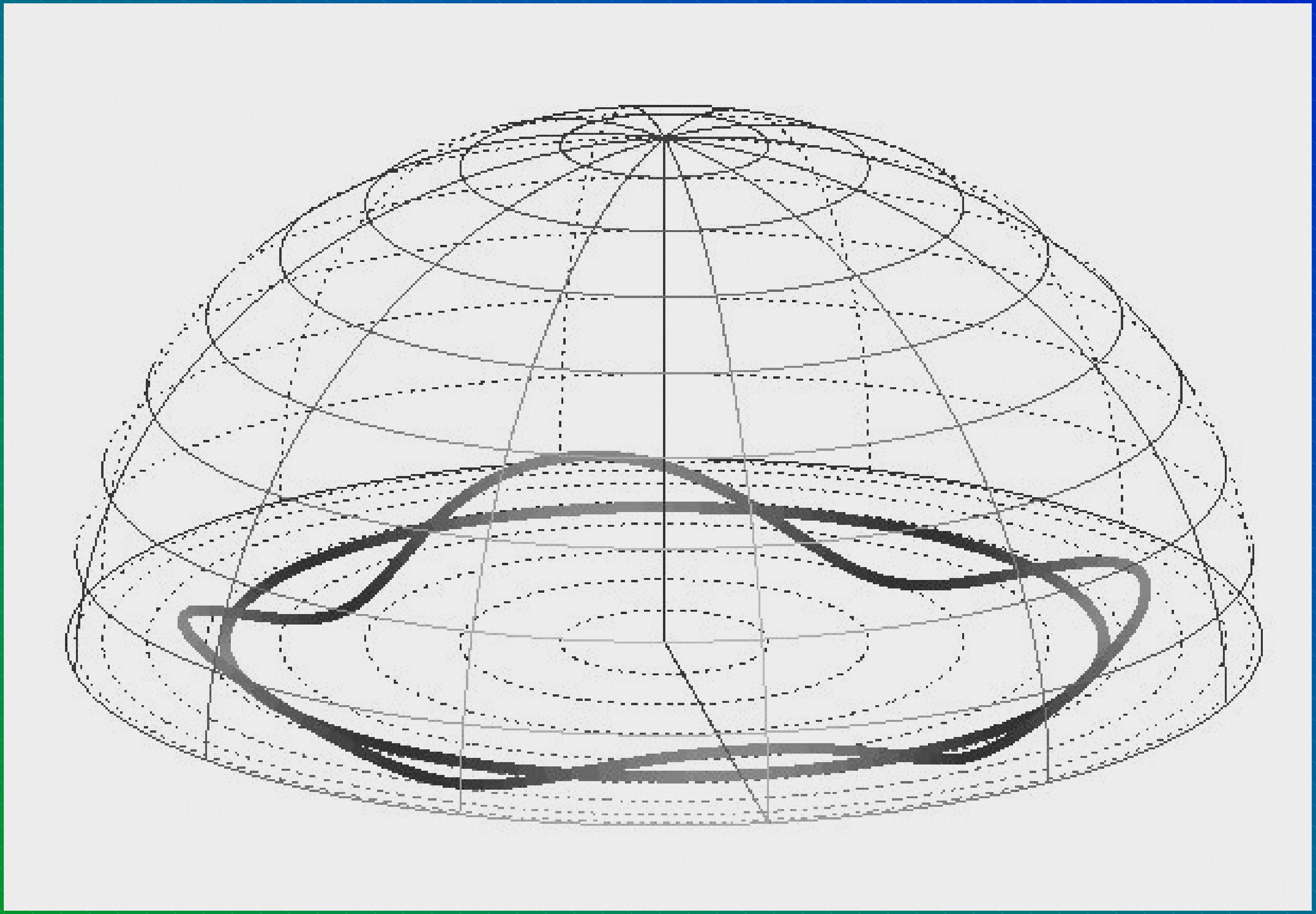
Origin of sunspots



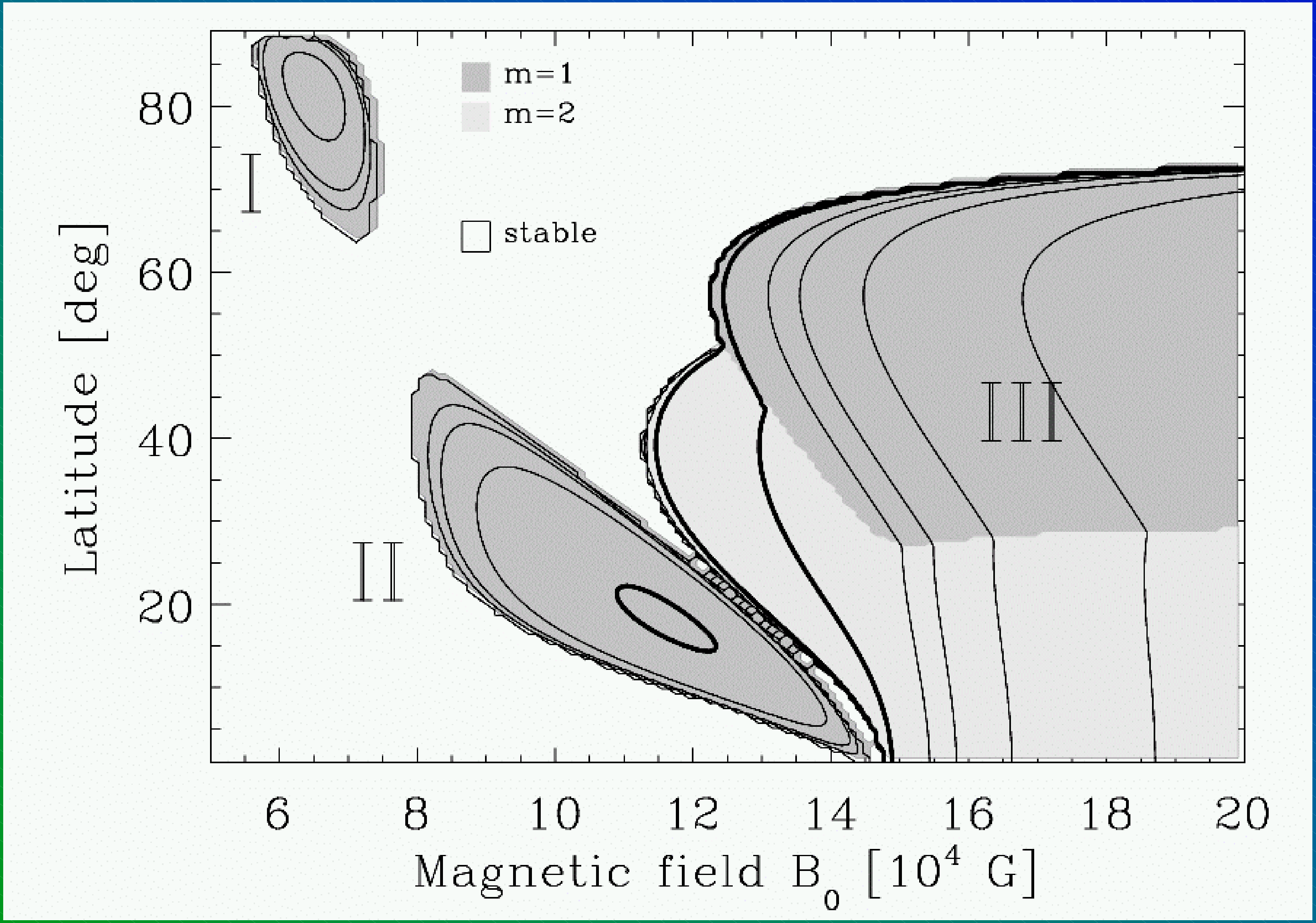
9

•Formation of a bipolar sunspot pair/group

Undulatory instability in spherical geometry



Undulatory instability in spherical geometry



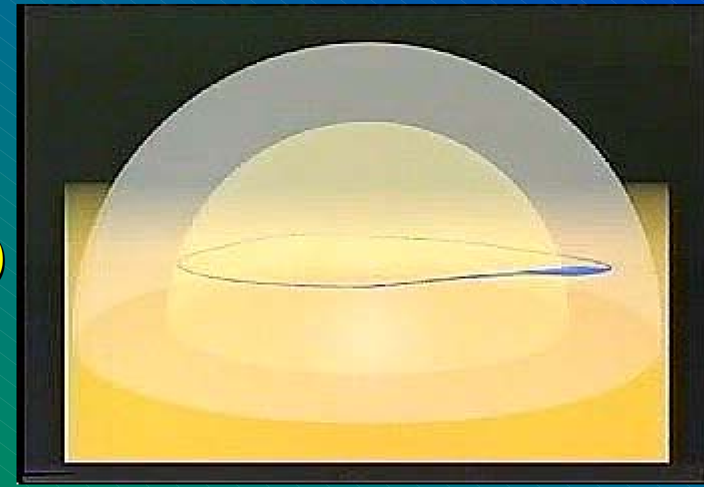
Emergence of thin flux tubes

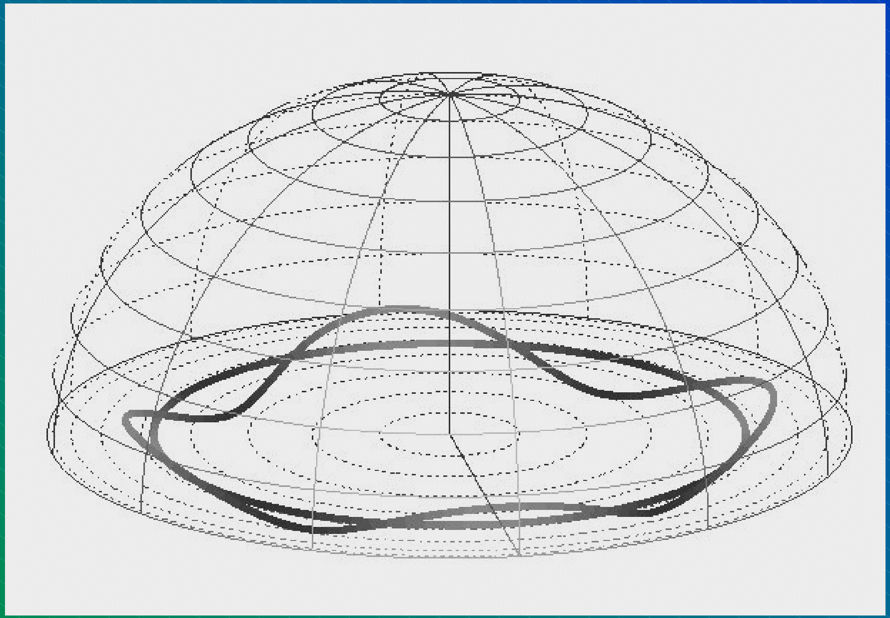
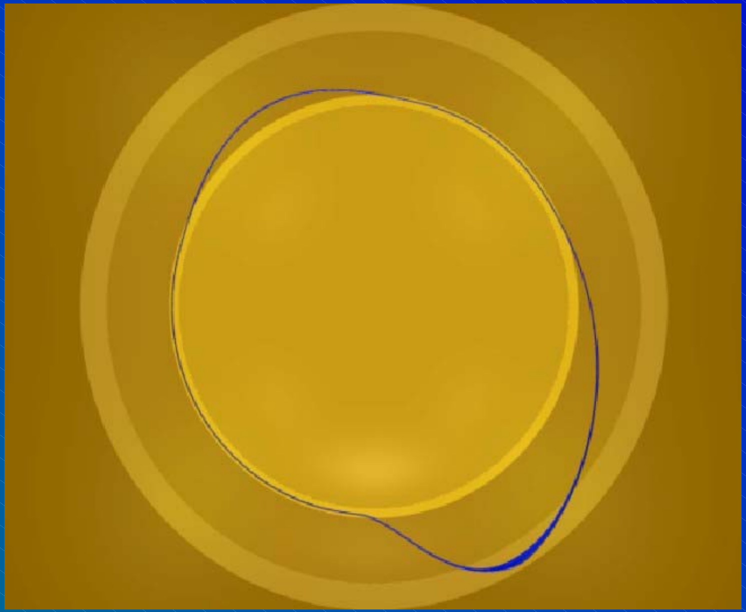
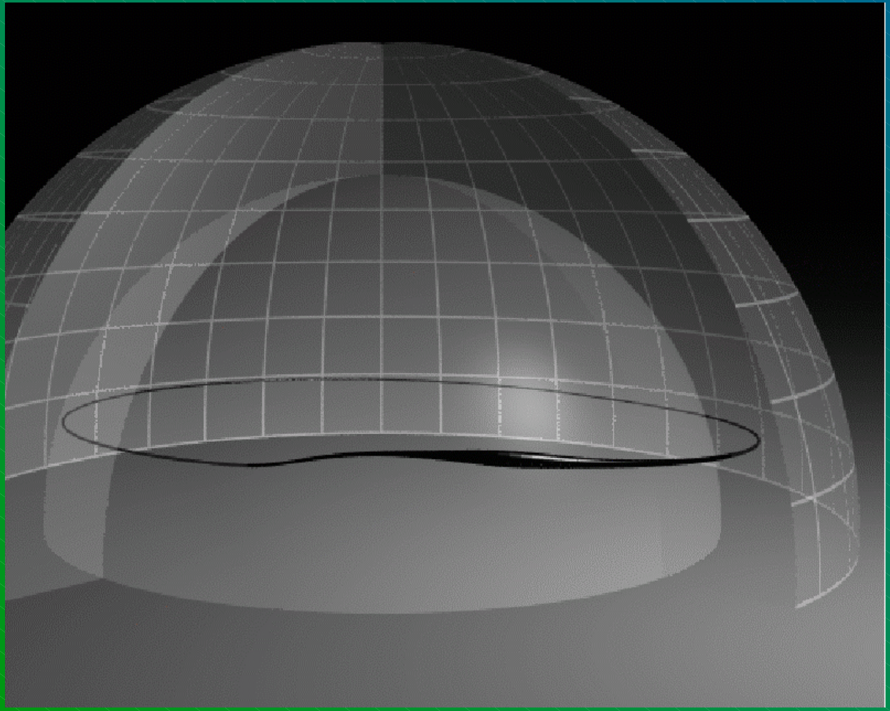
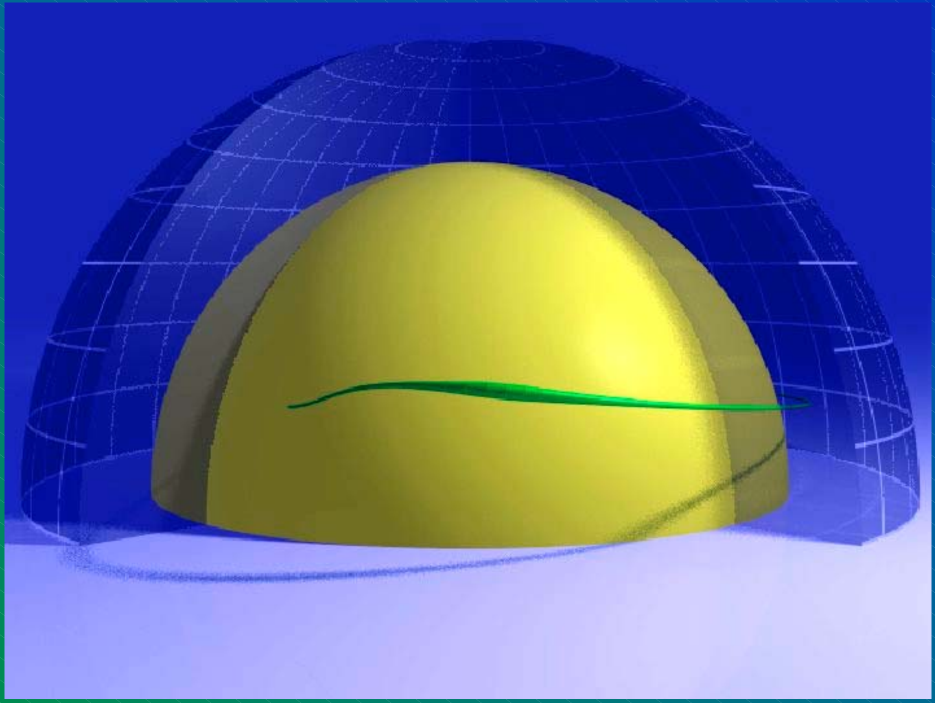
- Toroidal flux tube stored in the subadiabatic overshoot layer
- Undular (Parker) instability
- Rise of one or two flux loops throu

- Simul ved featu (cohe instal s,…)

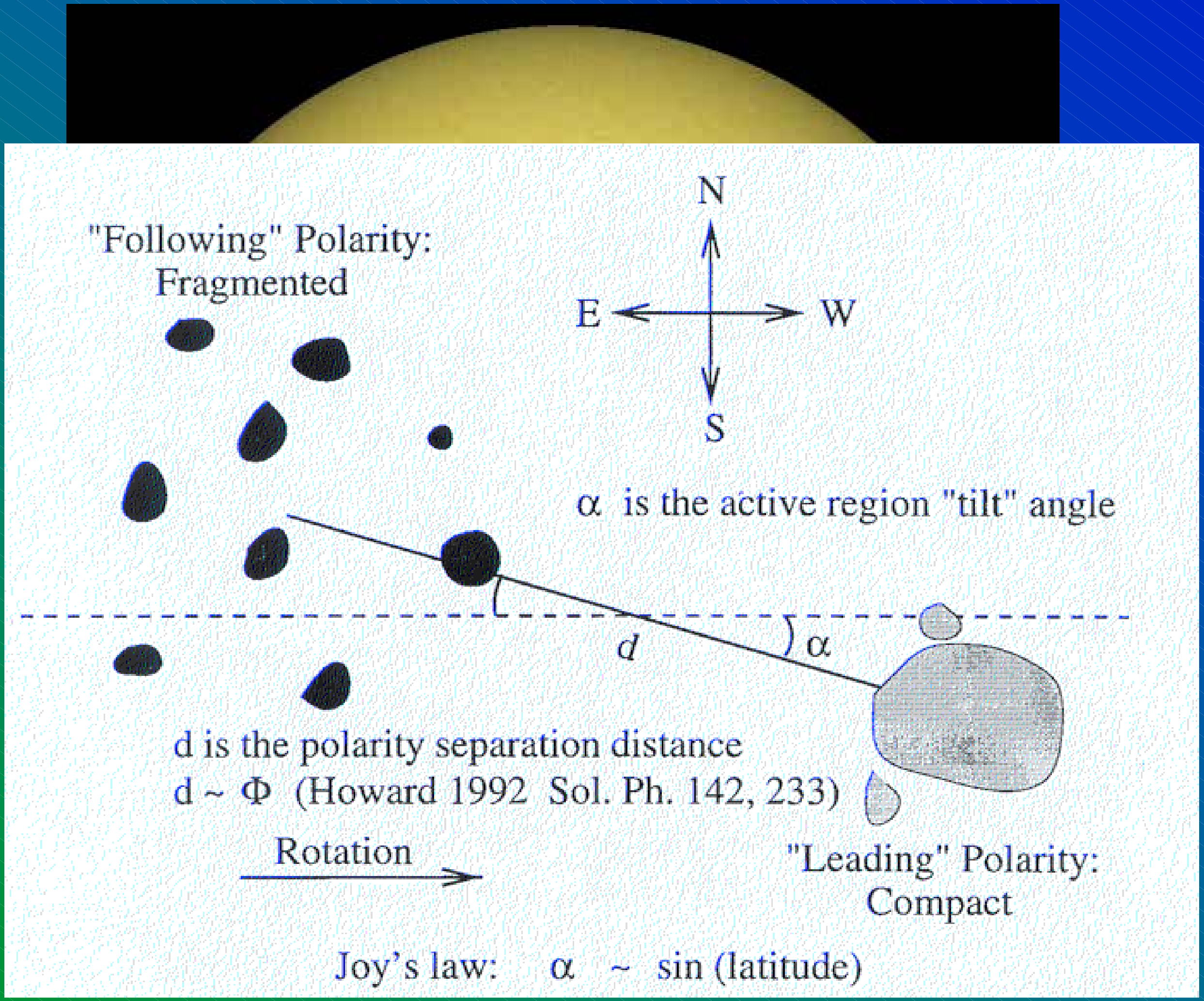
provided that the initial field is ~10^5 G

Caligari et al. (1995)

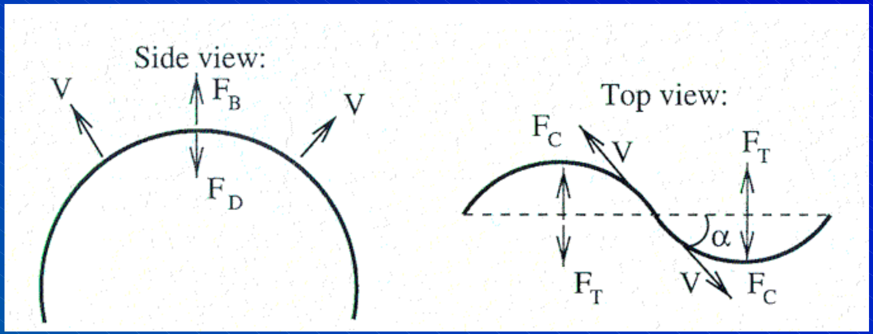


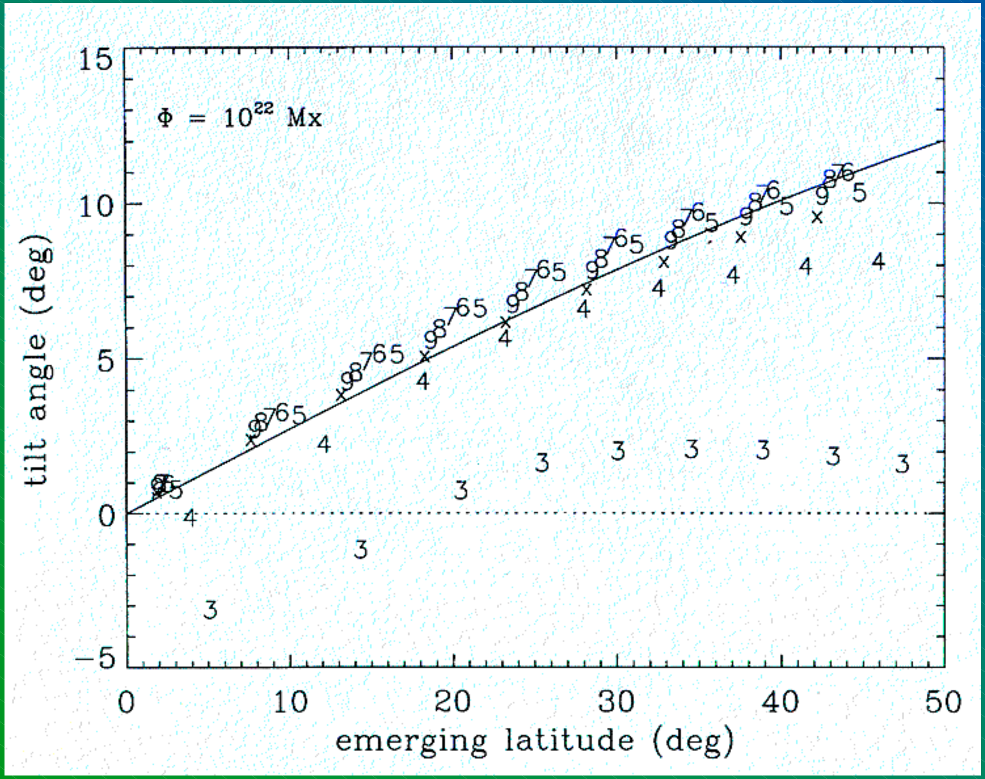
Tilt angle of sunspot groups



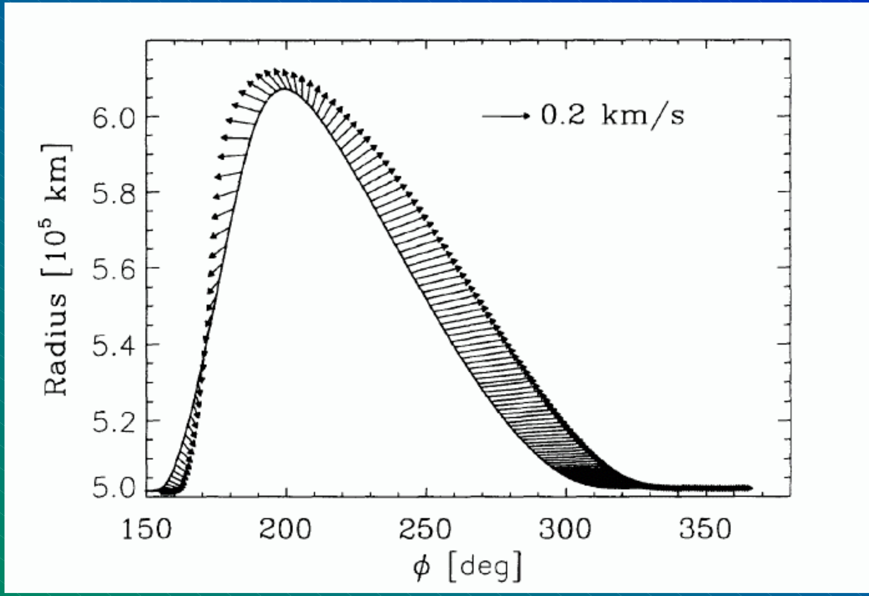
Origin of the tilt angle and numerical simulation results



Coriolis force

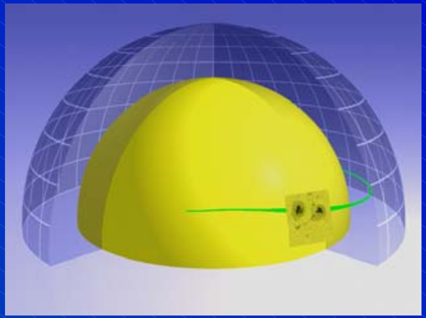


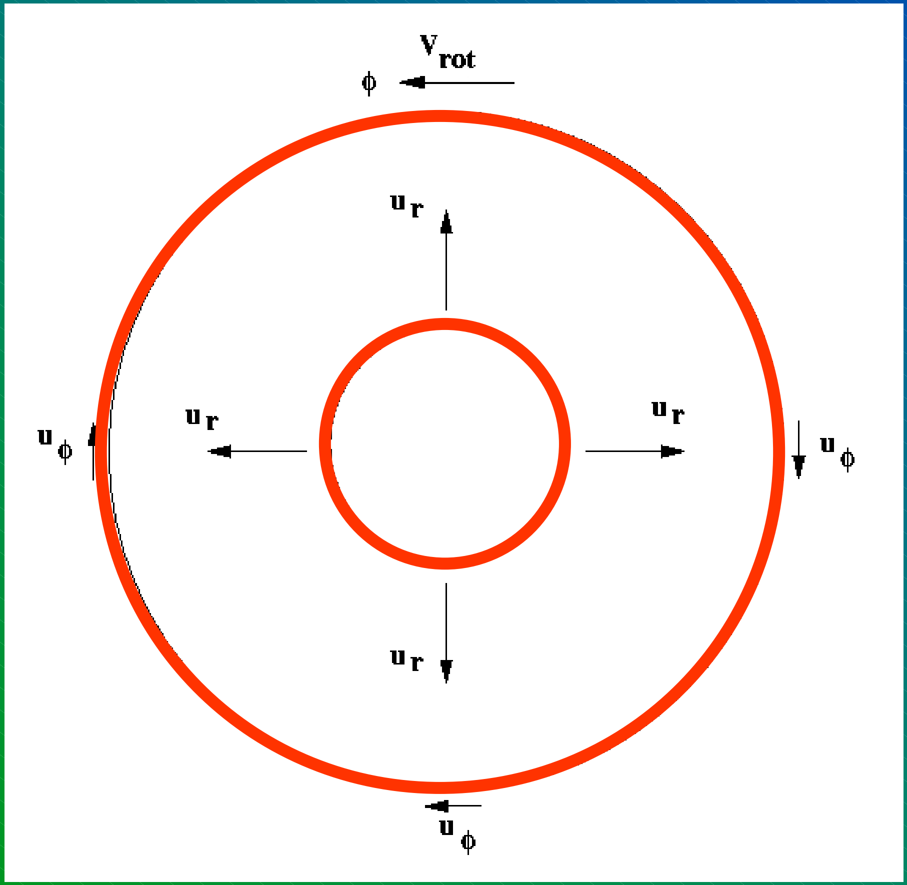
Tilt angle as f(latitude)
initial field strength: *n* Tesla



Expanding motion

The effect of rotation





Buoyantly expanding flux ring

$$\mathbf{F}_{\text{Coriolis}} = 2\rho\Omega\,(u_\phi, -u_R, 0)$$

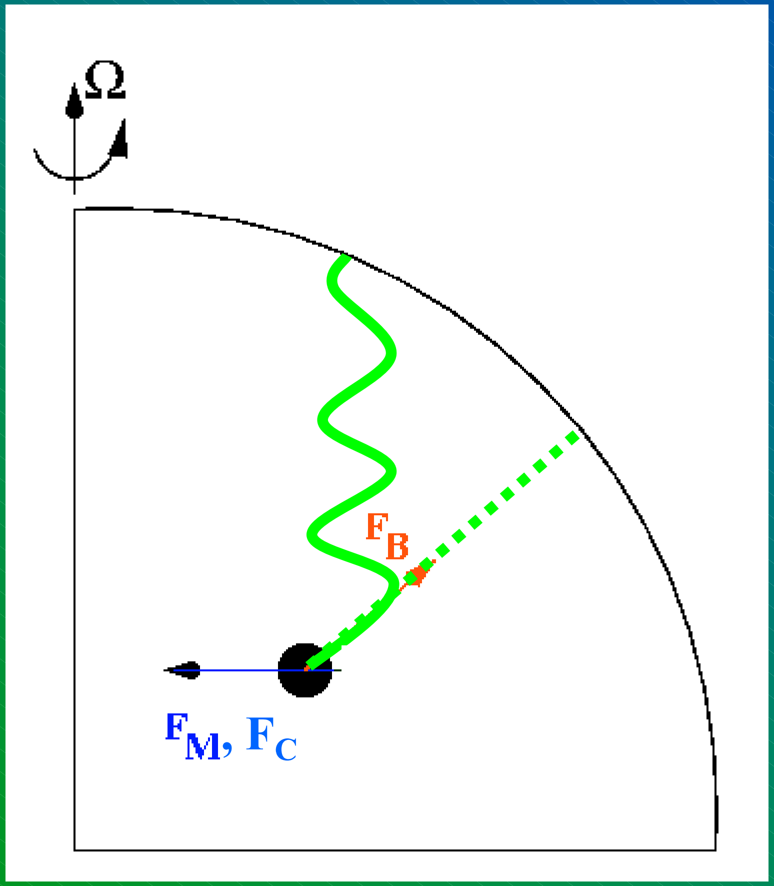
➔ restoring force

$$\dot{u}_\phi = -2\,\Omega\,u_R$$
$$\dot{u}_R = 2\,\Omega\,u_\phi$$
$$\rightarrow u_R, u_\phi \propto \sin(2\,\Omega\,t)$$

➔ inertial oscillations perpendicular to the axis of rotation

➔ amplitude depends on the strength of the buoyancy force

The effect of rotation



Forces on a rising magnetic flux tube: cut through a meridional plane

➔ component of buoyancy force parallel to rotation axis unbalanced

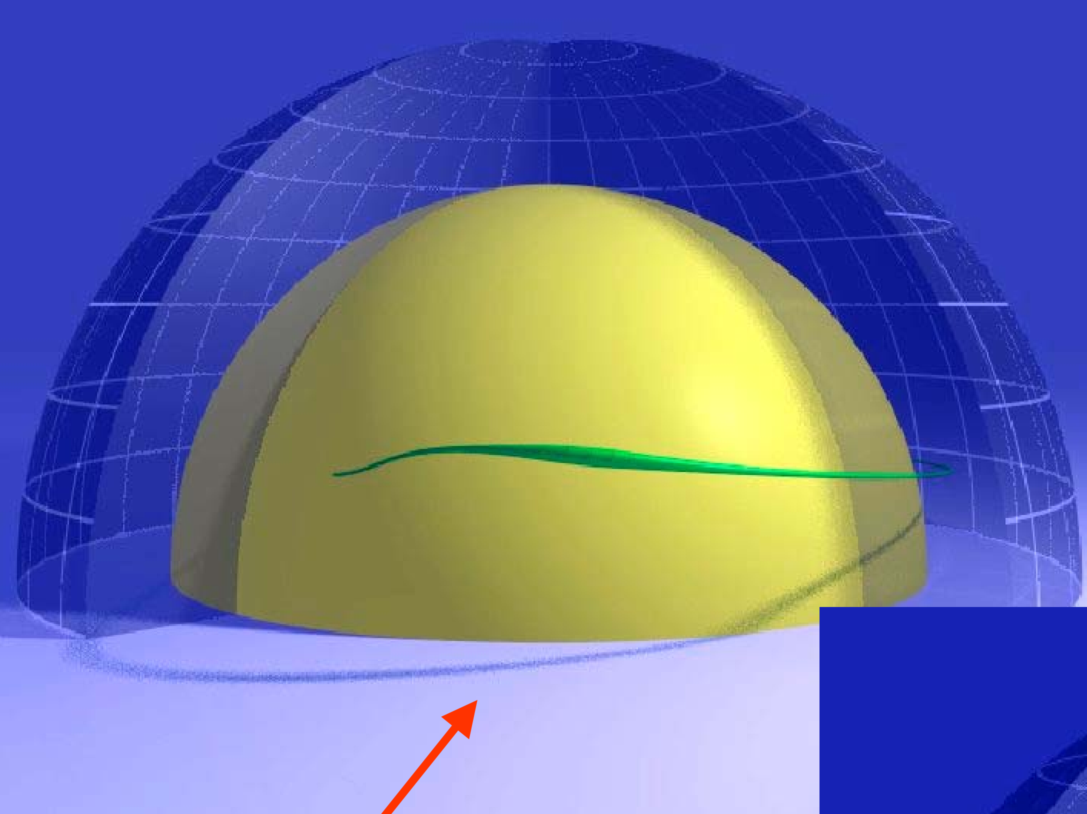
$$\frac{|\ \mathbf{F}_C\ |}{|\ \mathbf{F}_B\ |} = \frac{2\,\rho\,\upsilon_{\mathrm{rise}}\,\Omega}{B^2\,/\,(8\pi H_p)} \propto \frac{2\Omega H_p}{\upsilon_A}$$

➔ 1 / "magnetic Rossby number"

(Schüssler & Solanki, 1992)

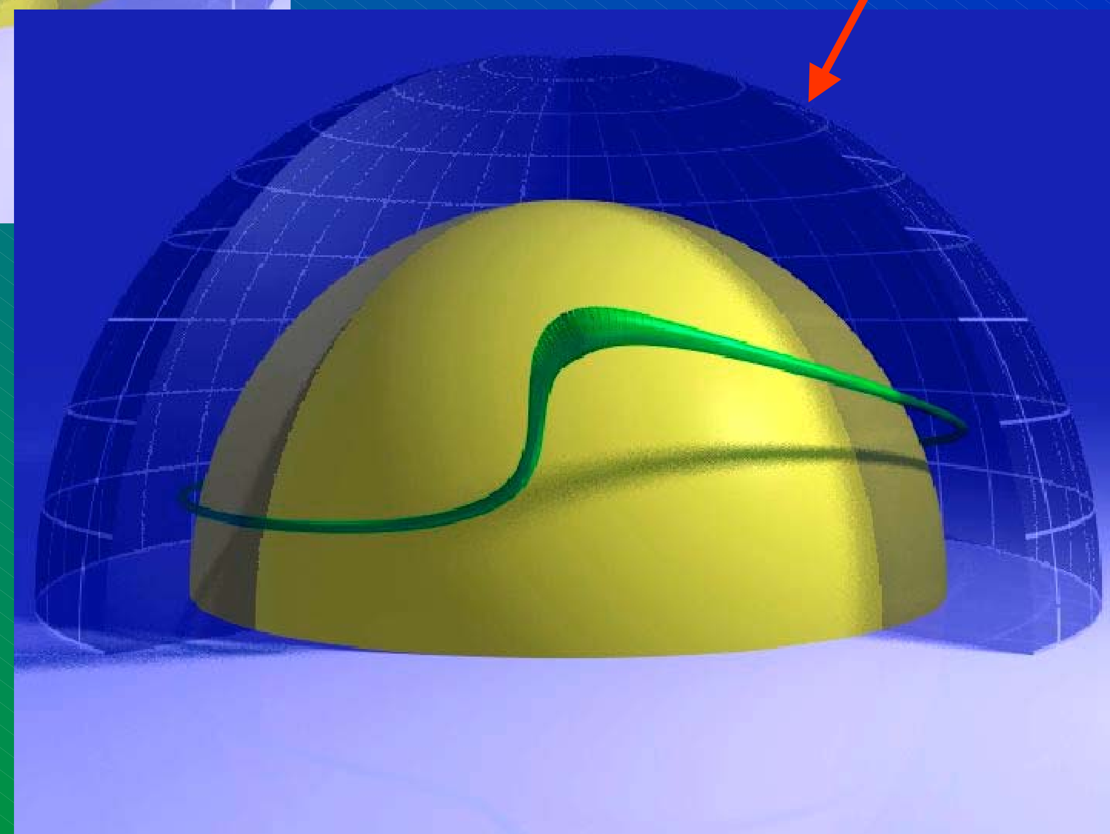
➔ loop deflected to higher latitudes if rise time > 1/Ω

➔ Sun: buoyancy dominates for B>10^5 G



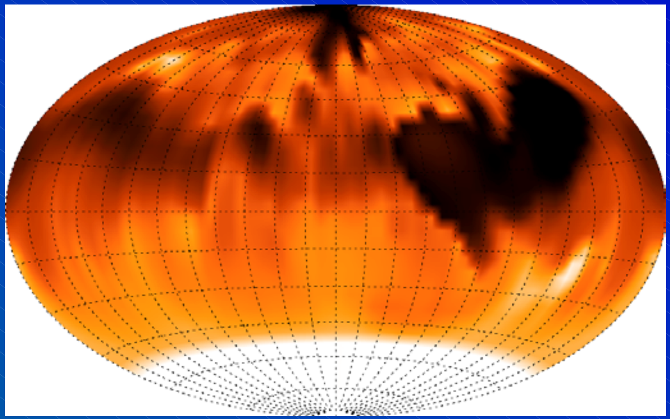
B = 1 Tesla

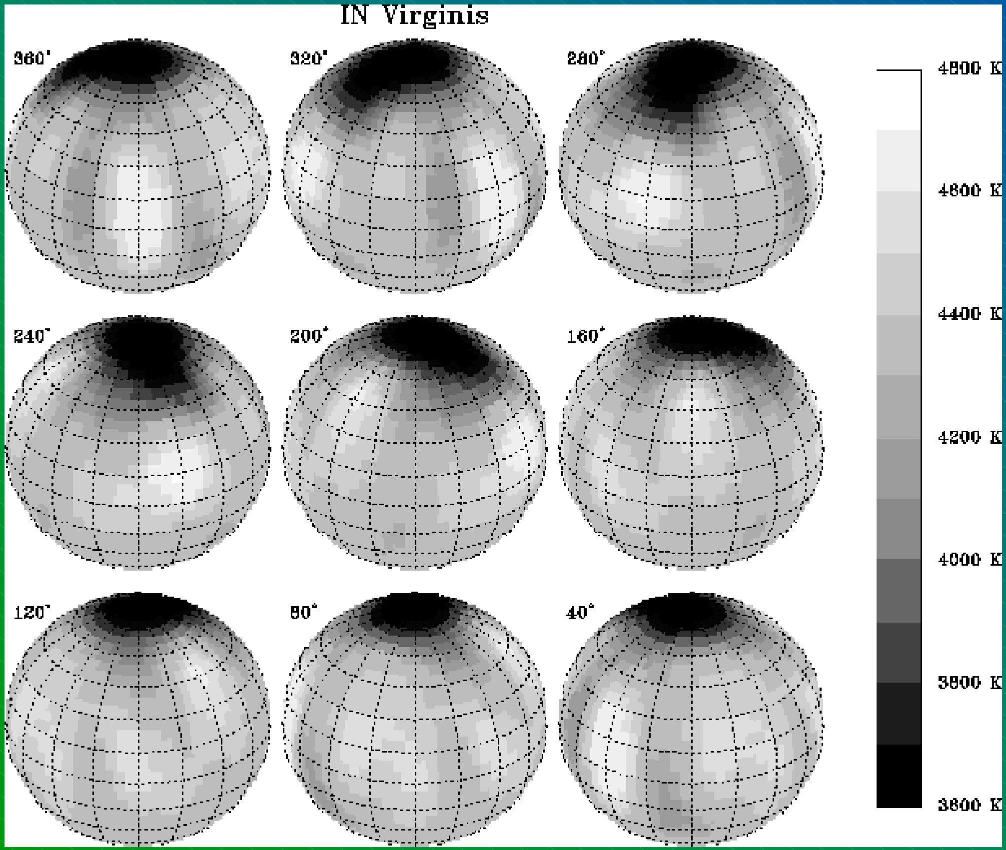
B = 10 Tesla

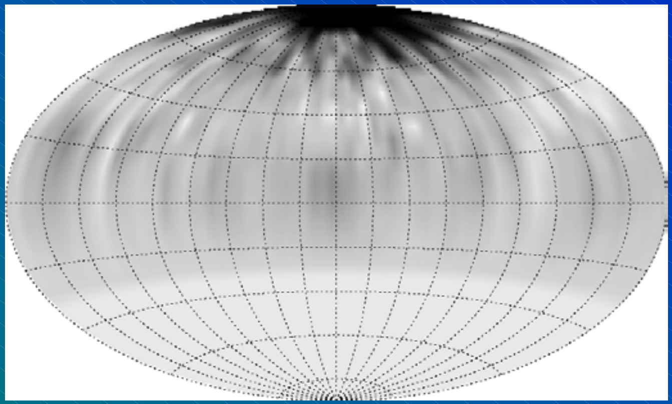


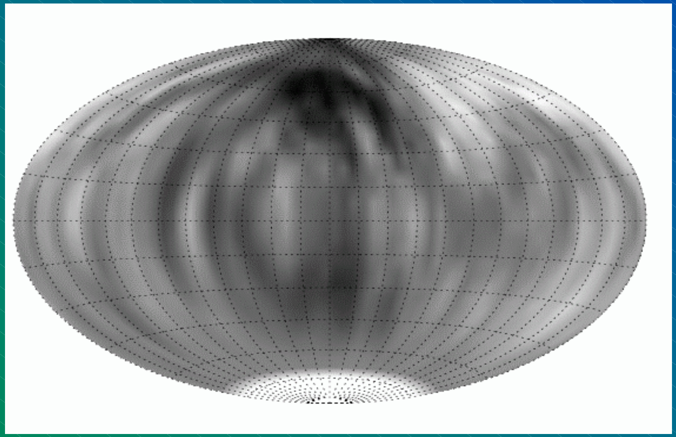
Doppler imaging: "starspot" maps

Rapidly rotating stars show spots at high latitudes, often polar spots

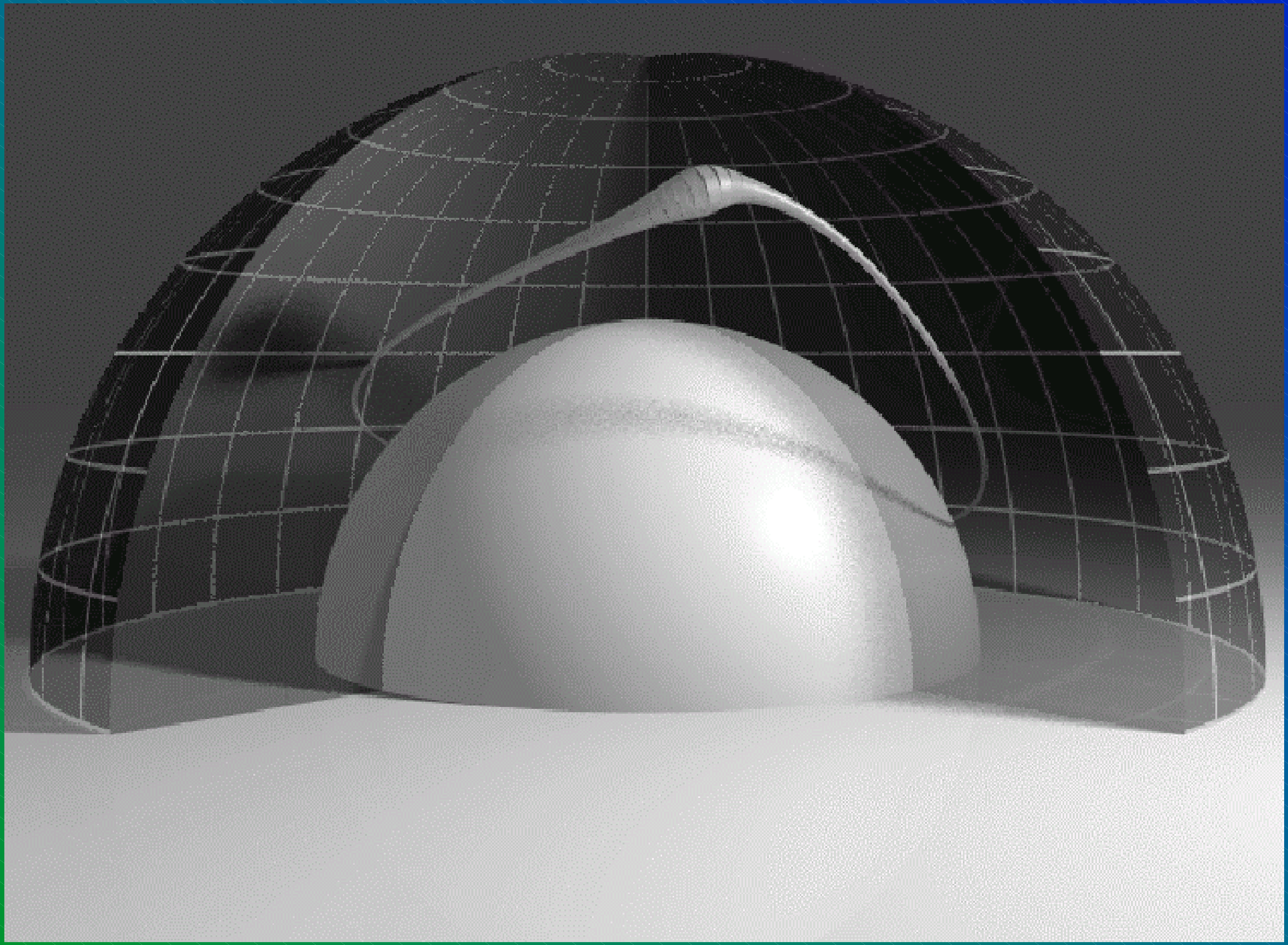




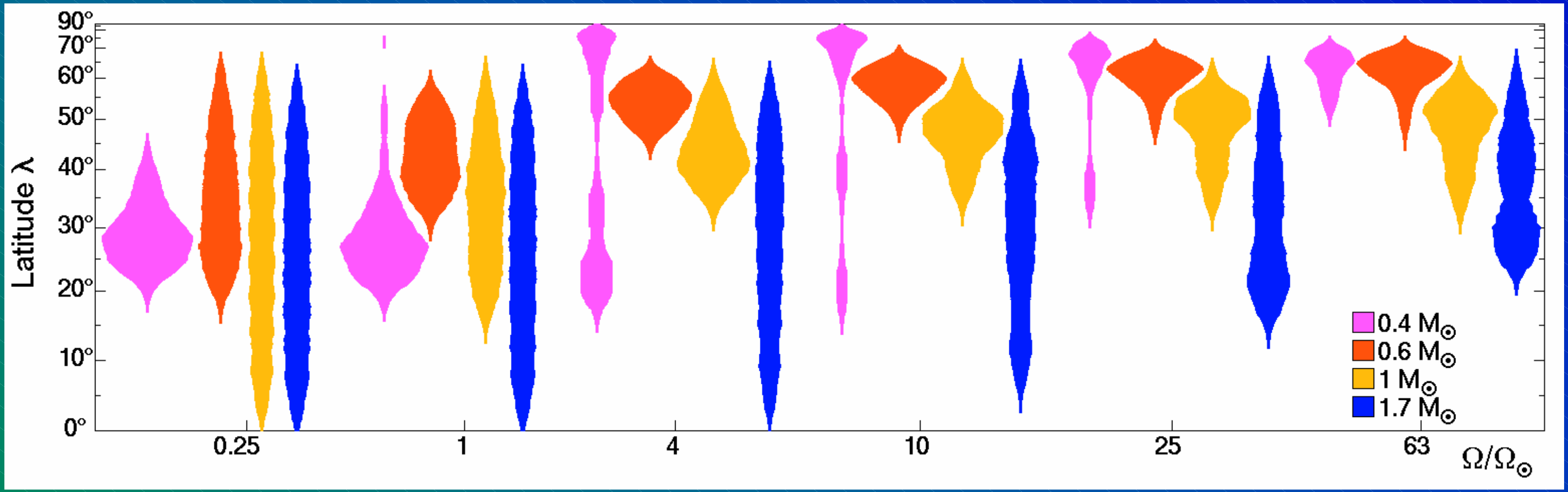




Emerging flux tube in a rapidly rotating subgiant

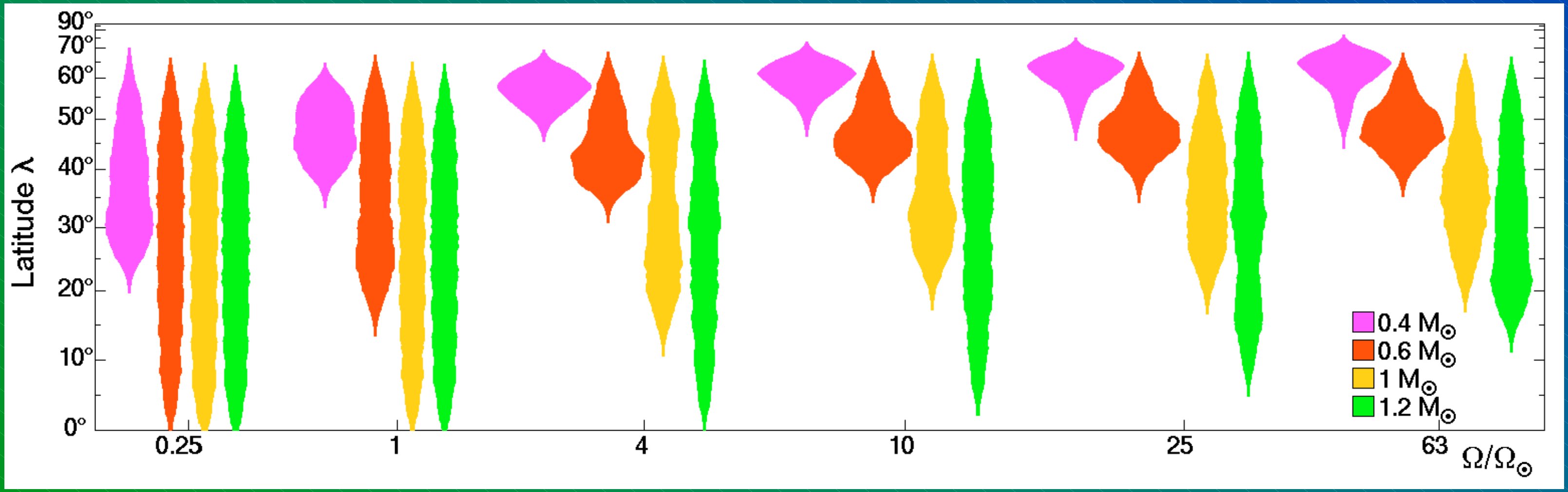


Simulation results for various stellar models & rotation rates



Young (pre-main-sequence) stars ↑

↓ Main-sequence stars

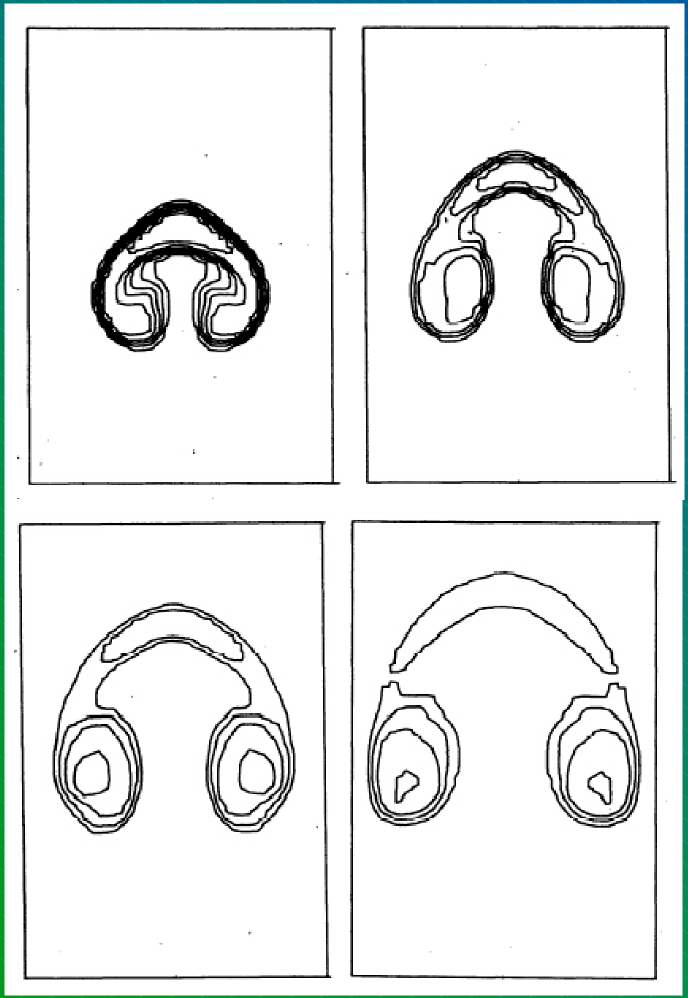


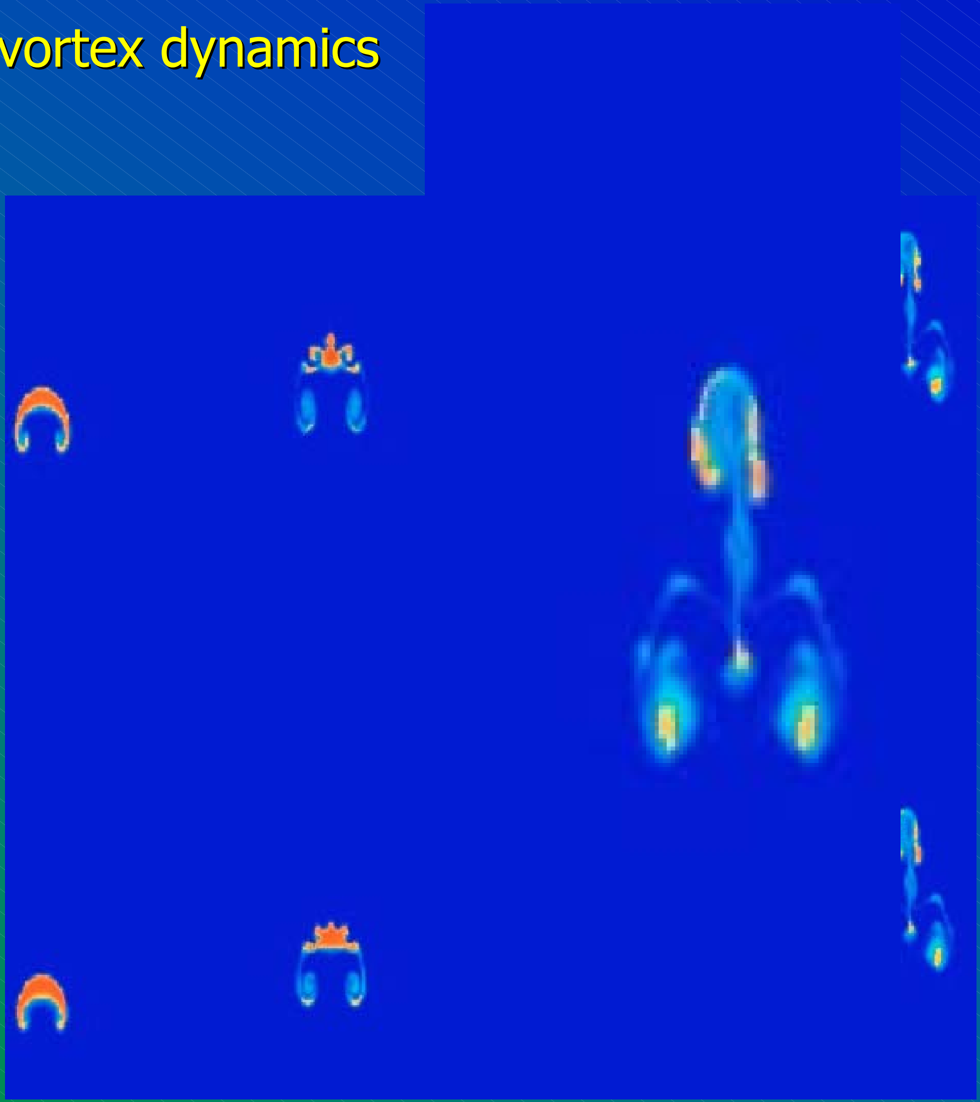
Latitude distributions of emerging flux tubes

2D: Fragmention… avoided by twist

- Dynamical fragmentation / vortex dynamics

Sch. (1979)



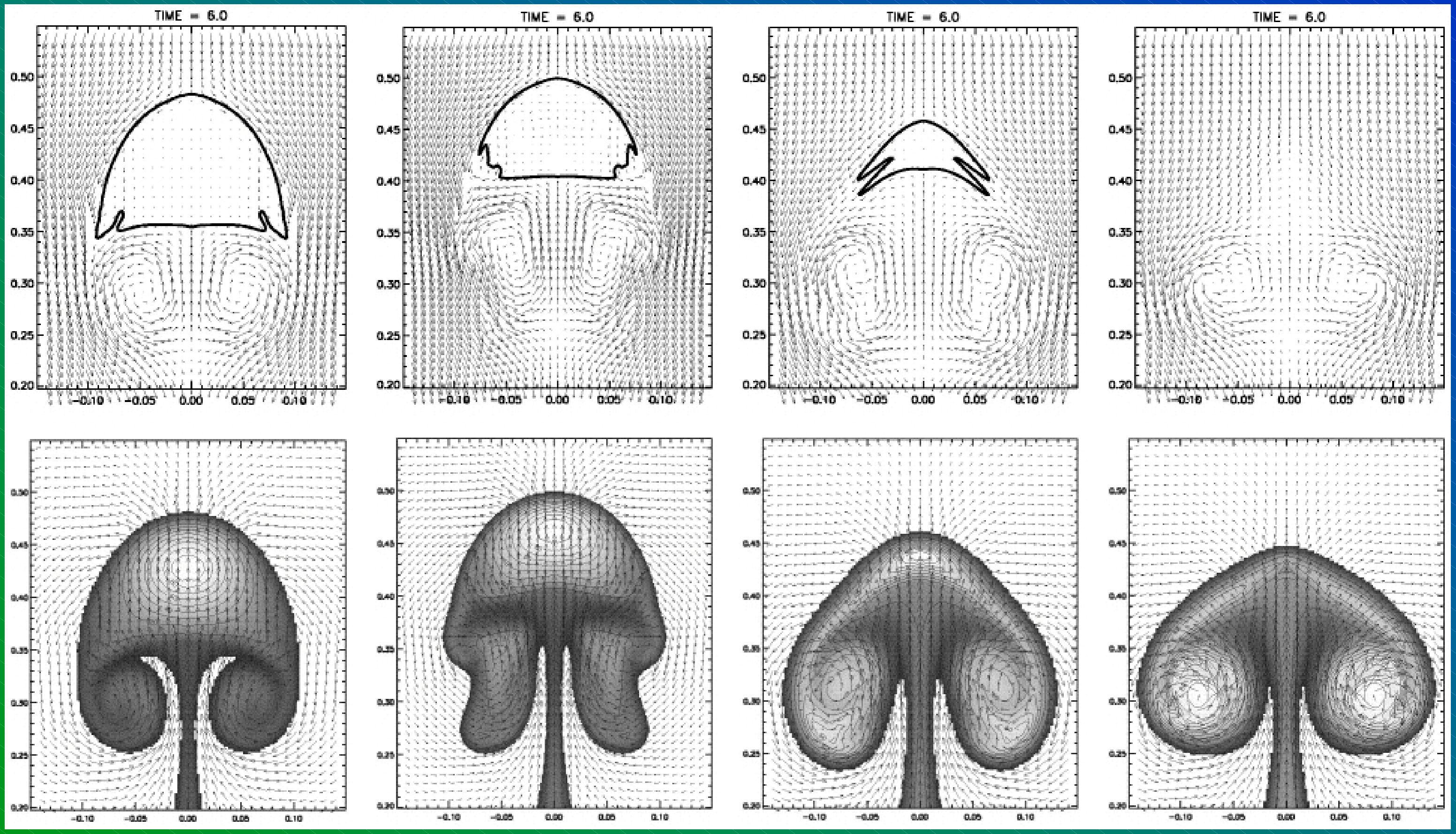


Dorch & Nordlund (1998)

2D: Fragmention avoided by twist

- A sufficiently twisted field maintains the coherence of the tube

Increasing twist

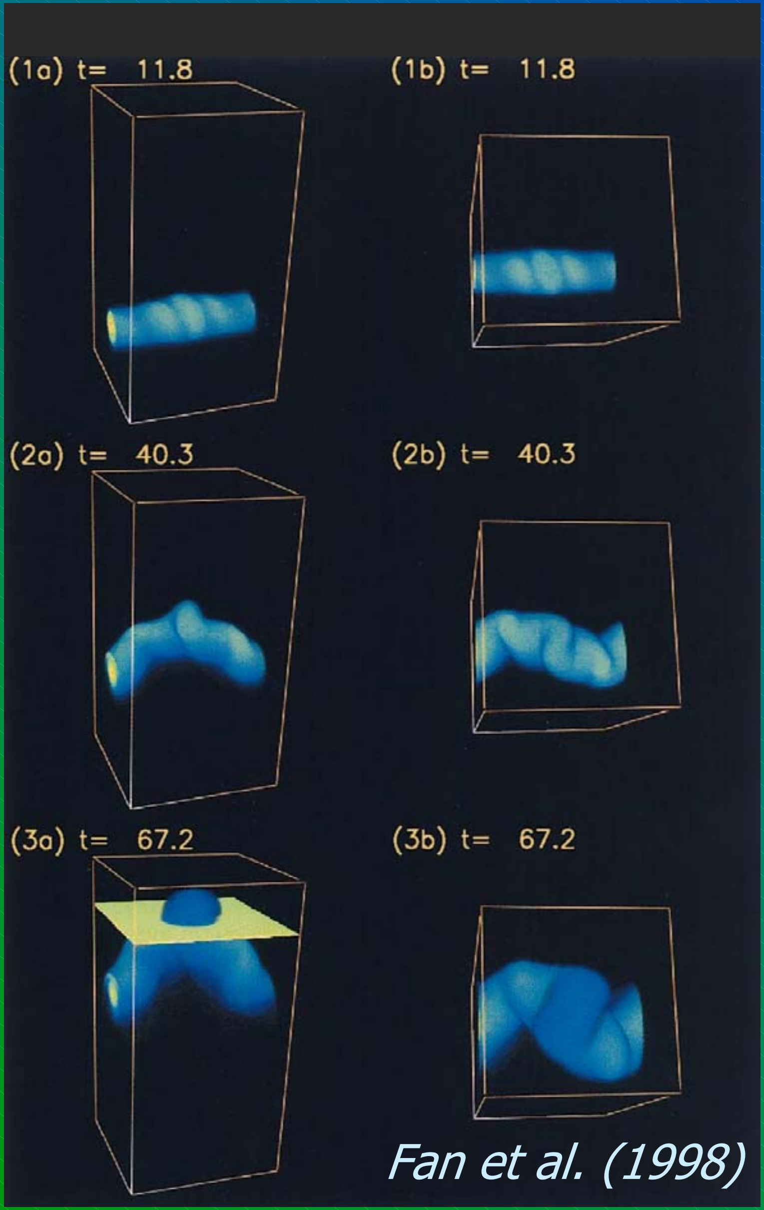


Equipartion line

Field strength

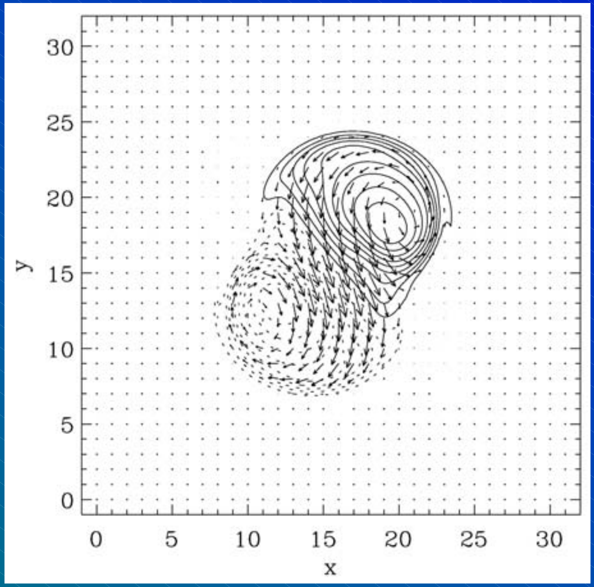
Emonet & Moreno-Insertis (1998)

3D: kink instability



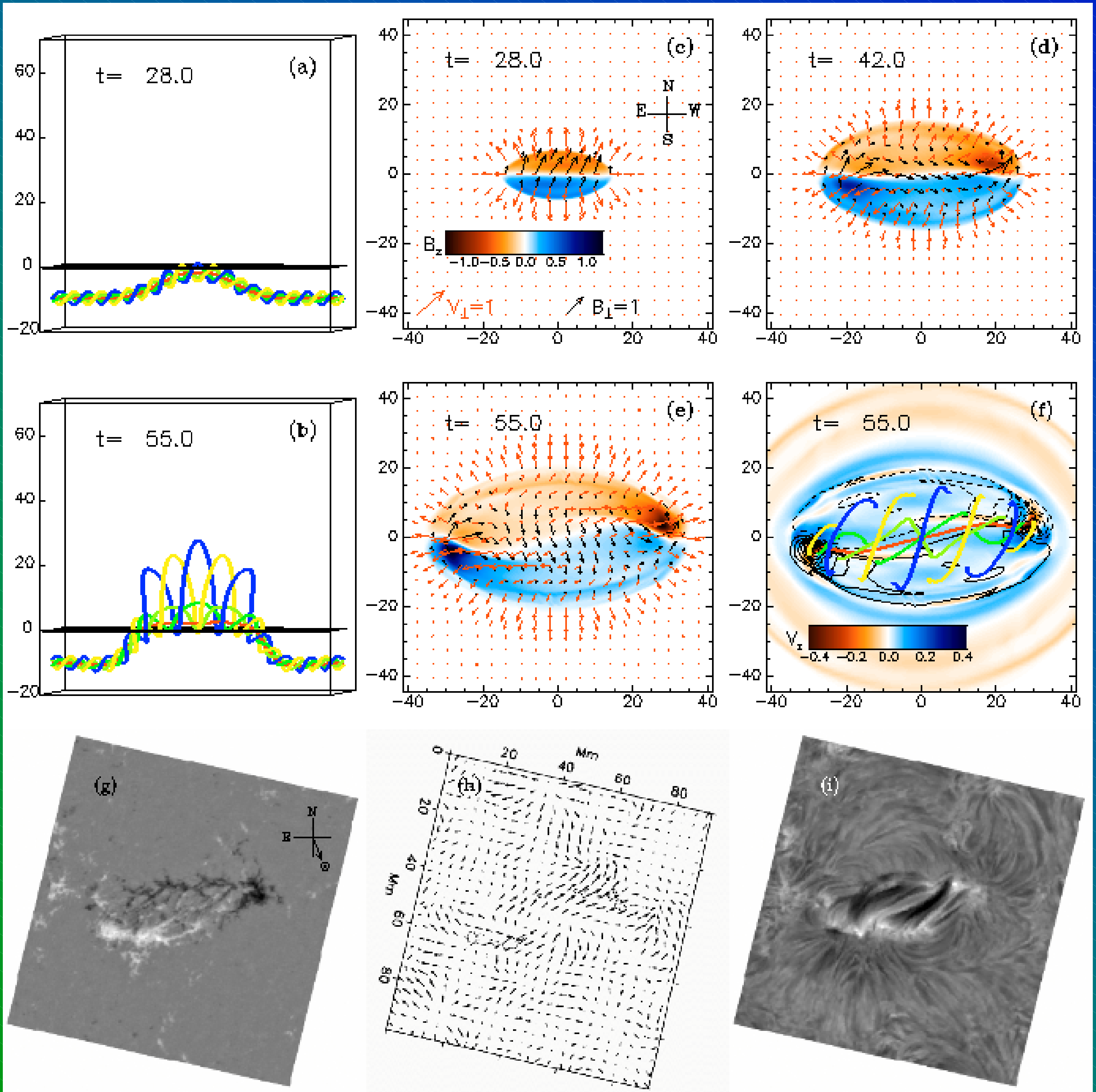
Fan et al. (1998)

"Sigmoid" shape after emergence



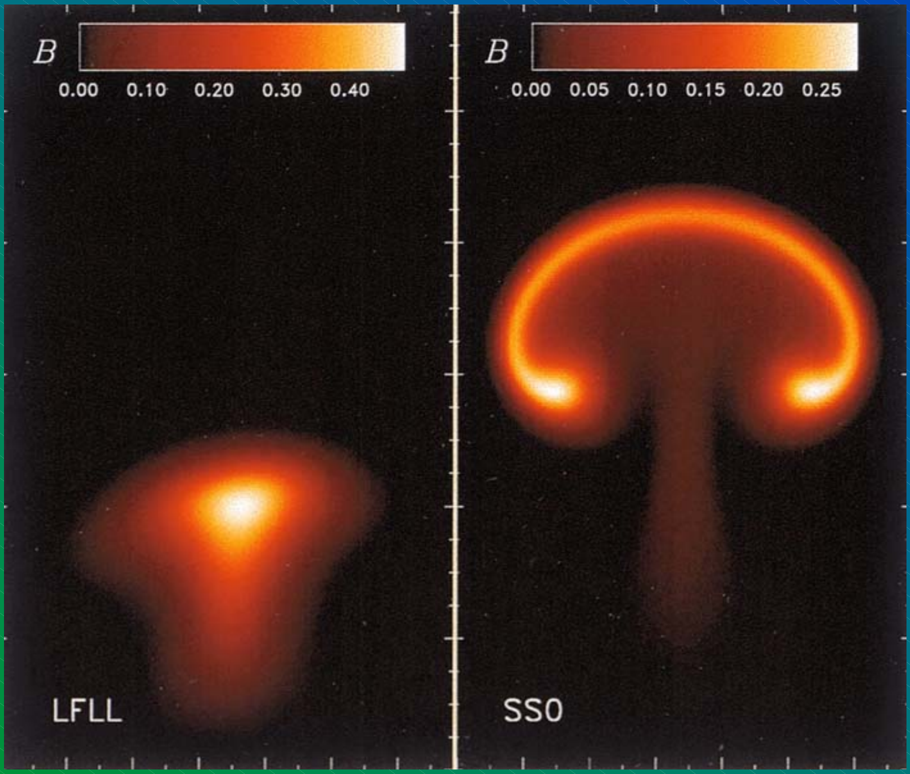
X-ray picture (from Yohkoh satellite)

3D: kink instability



Fan et al. (2001)

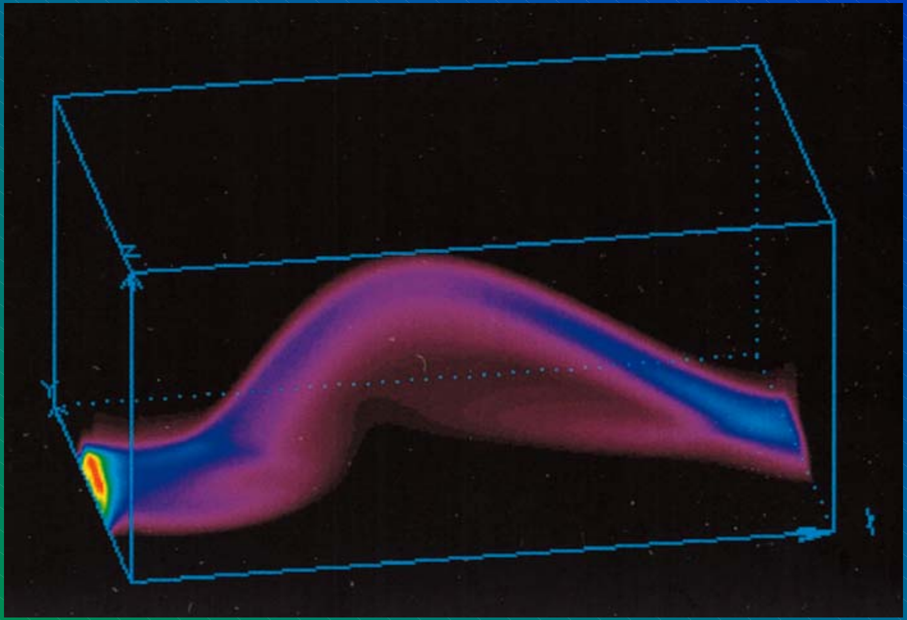
3D: rotation stabilizes, even without twist



With rotation

Non-rotating

Characteristic asymmetry due to the Coriolis force



Abett et al. (2001)

A case for strong fields

- Dynamics of rising flux tubes:
  agreement with the properties of sunspots requires:

  ➔ criterion for Parker instability satisfied ($B > B_{crit}$)
  ➔ flux tubes remains coherent while rising through c.z.
  ➔ low-latitude eruption
  ➔ correct tilt angle of sunspot groups
  ➔ …

- Only satisfied if the field strength is ~10 T (10^5 G)
  in the dynamo region at the bottom of the conv. zone

How can a field of 10 Tesla (10^5 G) be generated ?

- Convective motions are too weak:

$$\frac{e_{\rm mag}}{e_{\rm kin}} = \frac{B^2/8\pi}{\rho\, v_{\rm t}^2/2} \simeq 100$$

- The energy of the differential rotation in the shear layer is too small by a factor 10
- With both effects only about 1 Tesla can be reached
- What remains: potential energy of the stratification!

Origin of the 10^5 G (10 Tesla) field

Stretching by differential rotation:

$$\tau \approx \frac{\Delta B}{B} \cdot \frac{d}{\Delta v} \cong \frac{10^5}{10^2} \cdot \frac{10^4\ \text{km}}{0.1\ \text{km/s}} \cong 3\ \text{yr}$$

Dynamical considerations:

Flux tubes: drag force vs. magnetic tension force
➔ tube radius < 100 km

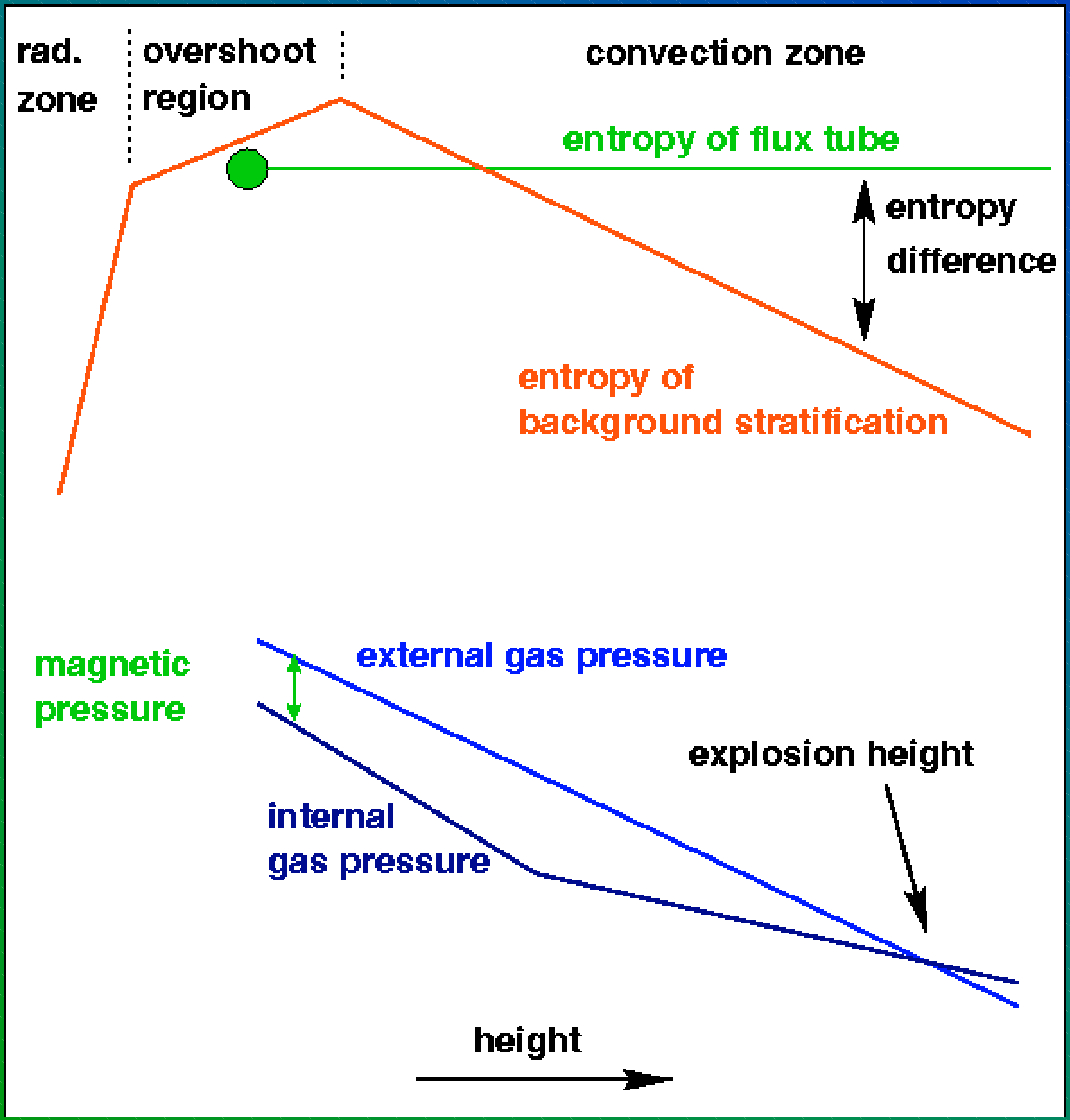
Energetics:

10^{24} Mx magnetic flux at 10^5 G field strength : $E_{mag} \sim 10^{39}$ erg

100 m/s diff. Rotation over 10^4 km shear layer: $E_{kin} \sim 10^{38}$ erg

➔ strong cyclic variation of shear layer, unless rapidly replenished

"Explosion" of magnetic flux tubes:

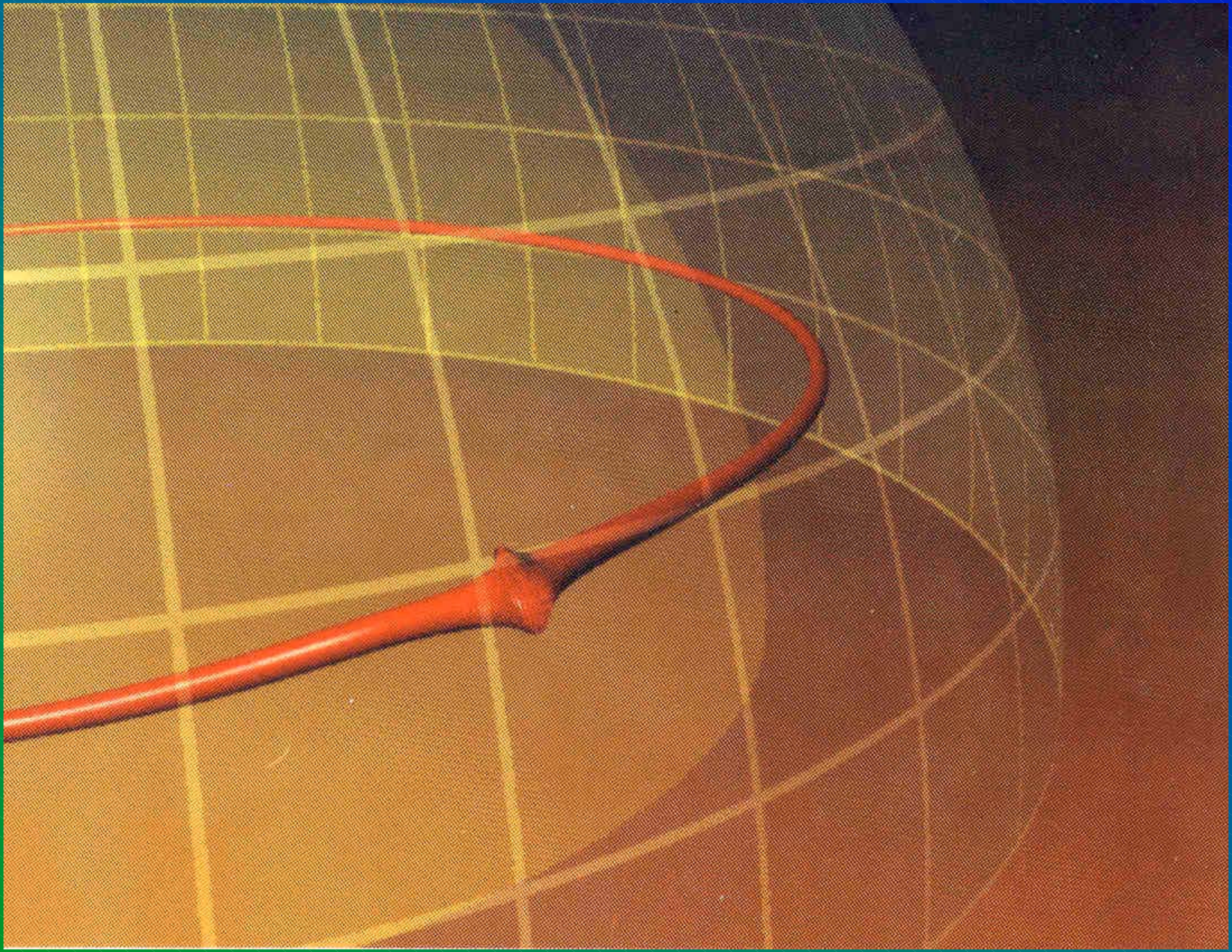


- isentropic flux tube
- rise in superadiabatic convection zone
- hydrostatics along field lines
- violation of pressure equilibrium

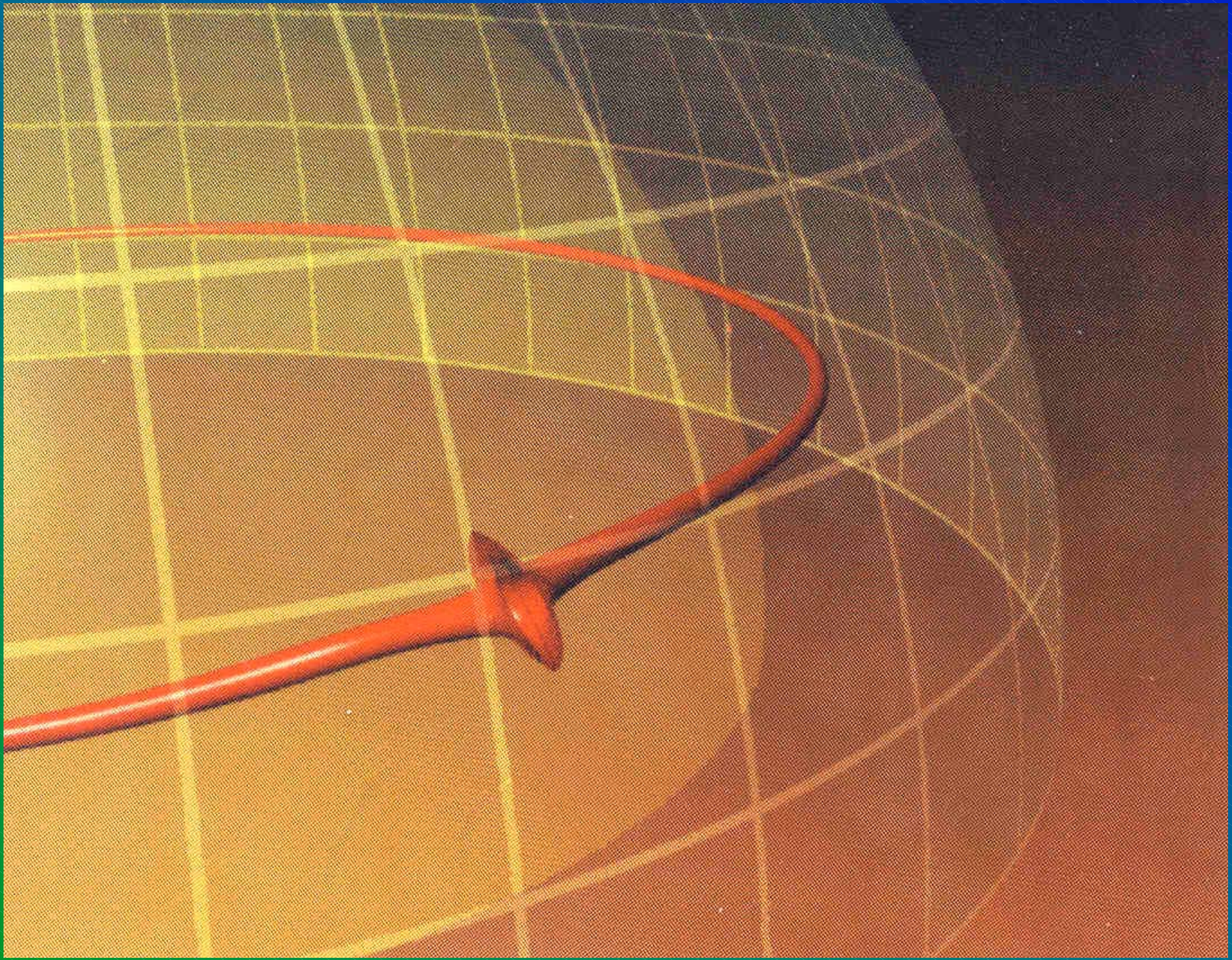
$$\frac{B^2}{8\pi} + p_i = p_e$$

- sudden decrease of field strength near summit

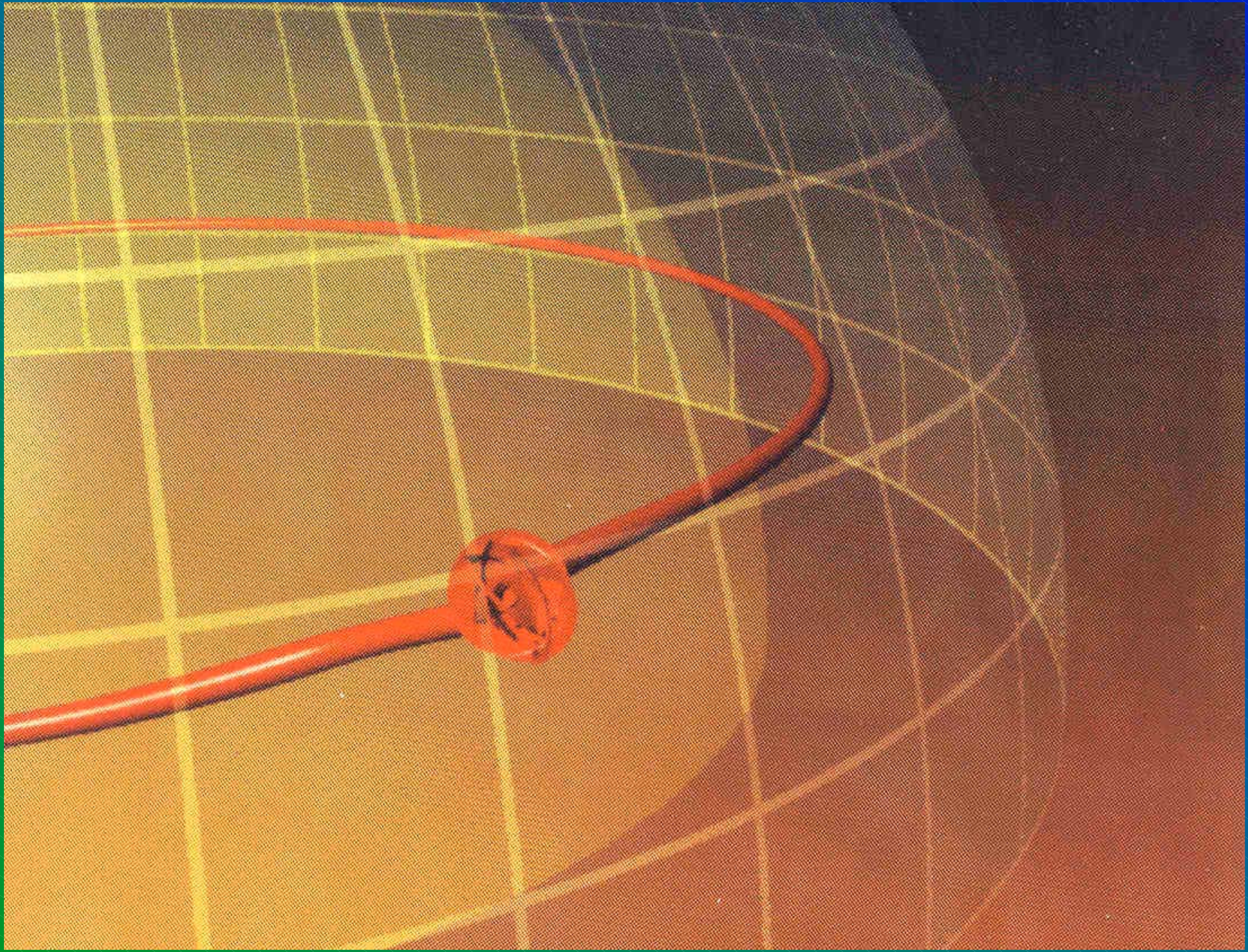
“Explosion” of a thin flux tube



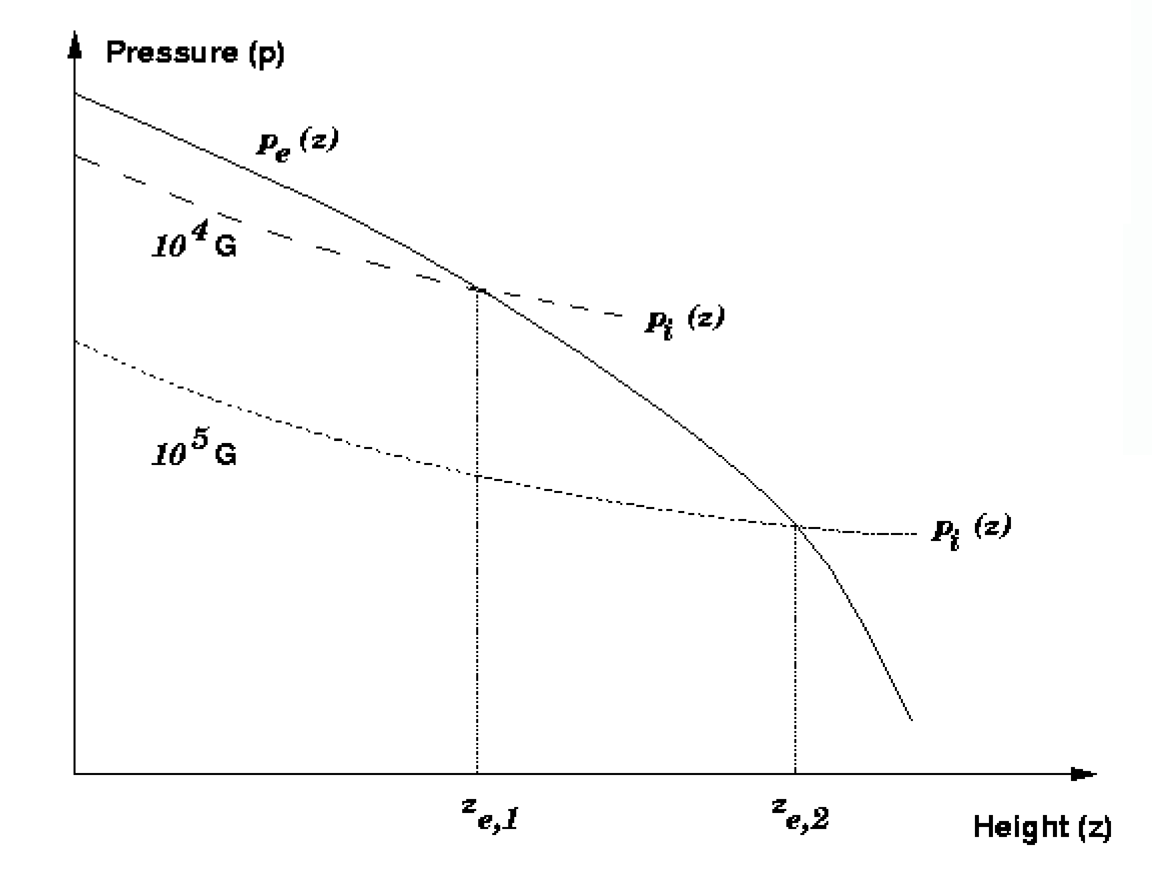
“Explosion” of a thin flux tube



“Explosion” of a thin flux tube



“Explosion” of magnetic flux tubes:



$$p_i(z) = p_{i,0}\left(1 - \frac{\nabla_{\rm ad} z}{H_{i,0}}\right)^{1/\nabla_{\rm ad}}$$

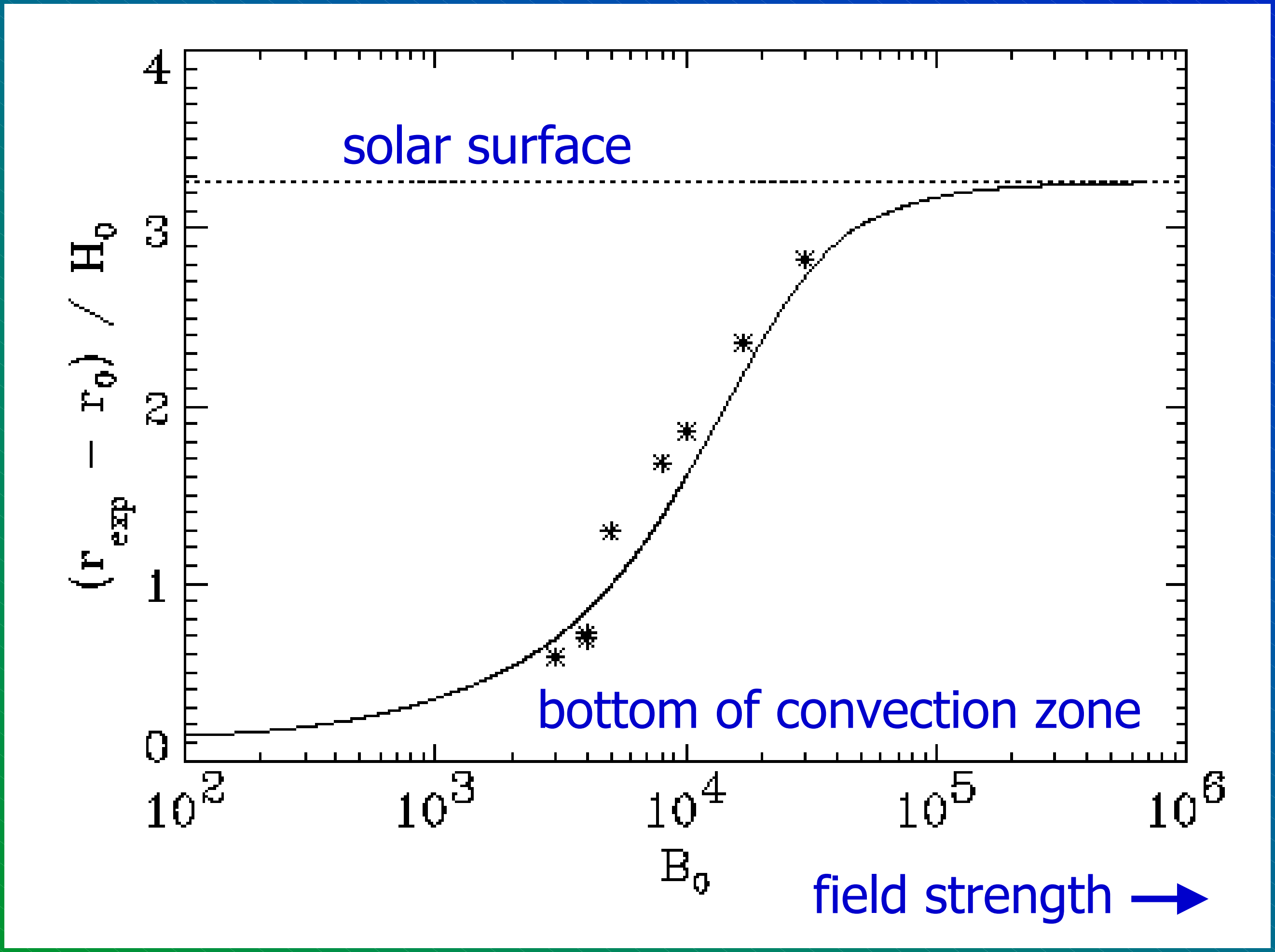
$$p_e(z) = p_{e,0}\left(1 - \frac{\nabla z}{H_{e,0}}\right)^{1/\nabla}$$

$$p + \frac{B^2}{8\pi} = p_e$$

$$\Delta T \equiv T_i - T_e = T_{i,0} - T_{e,0} + \frac{\mu g}{\mathcal{R}}(\nabla - \nabla_{\rm ad}) z$$

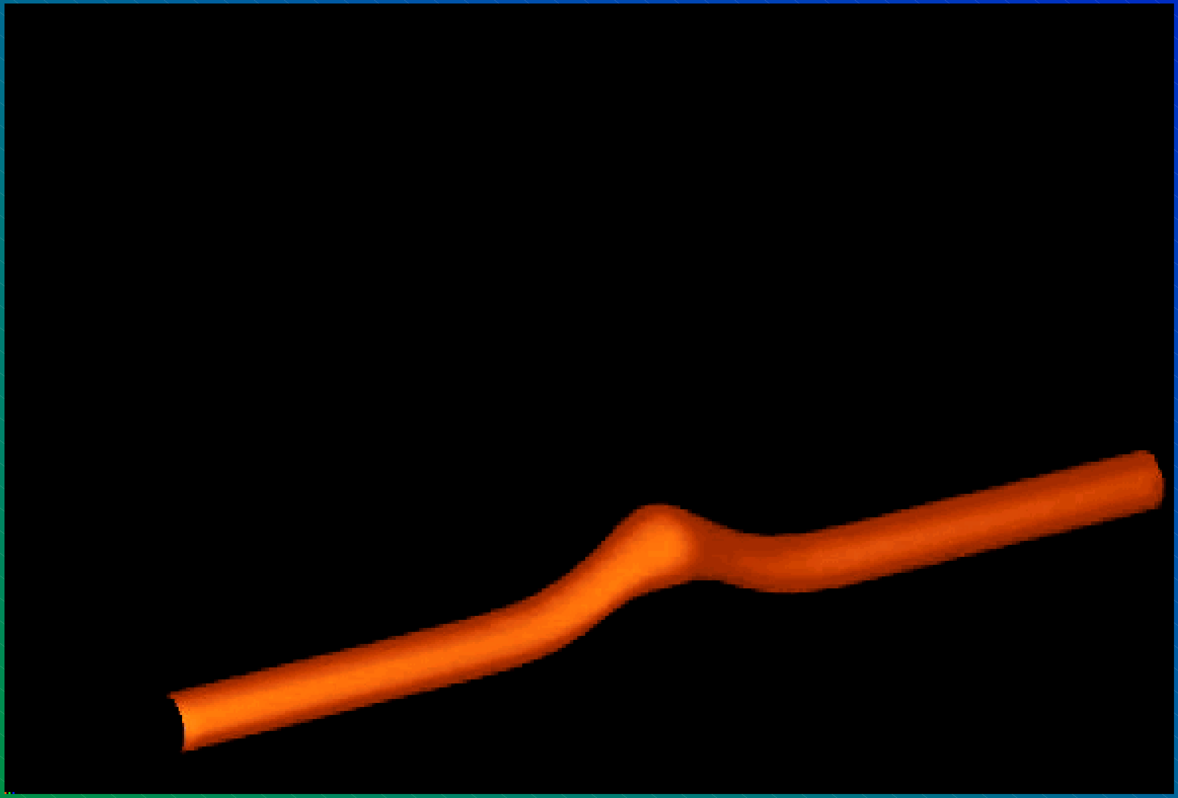
- sudden decrease of field strength near tube summit

Explosion height as f(initial field strength)



solid line: analytical prediction
asterisks: numerical simulation

Explosions also occur in 3D simulations

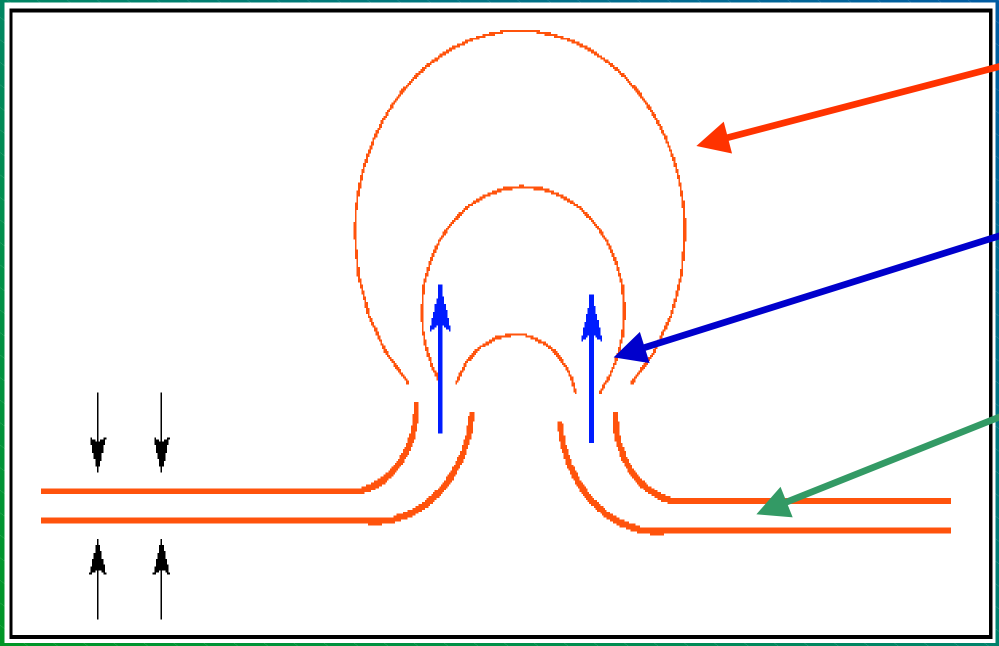


Resolution: 120×240×120
(Rempel, 2001)

Relevance of the explosion process: Field amplification by conversion of potential energy

➔ source of weak field within convection zone

➔ field amplification at base of convection zone:

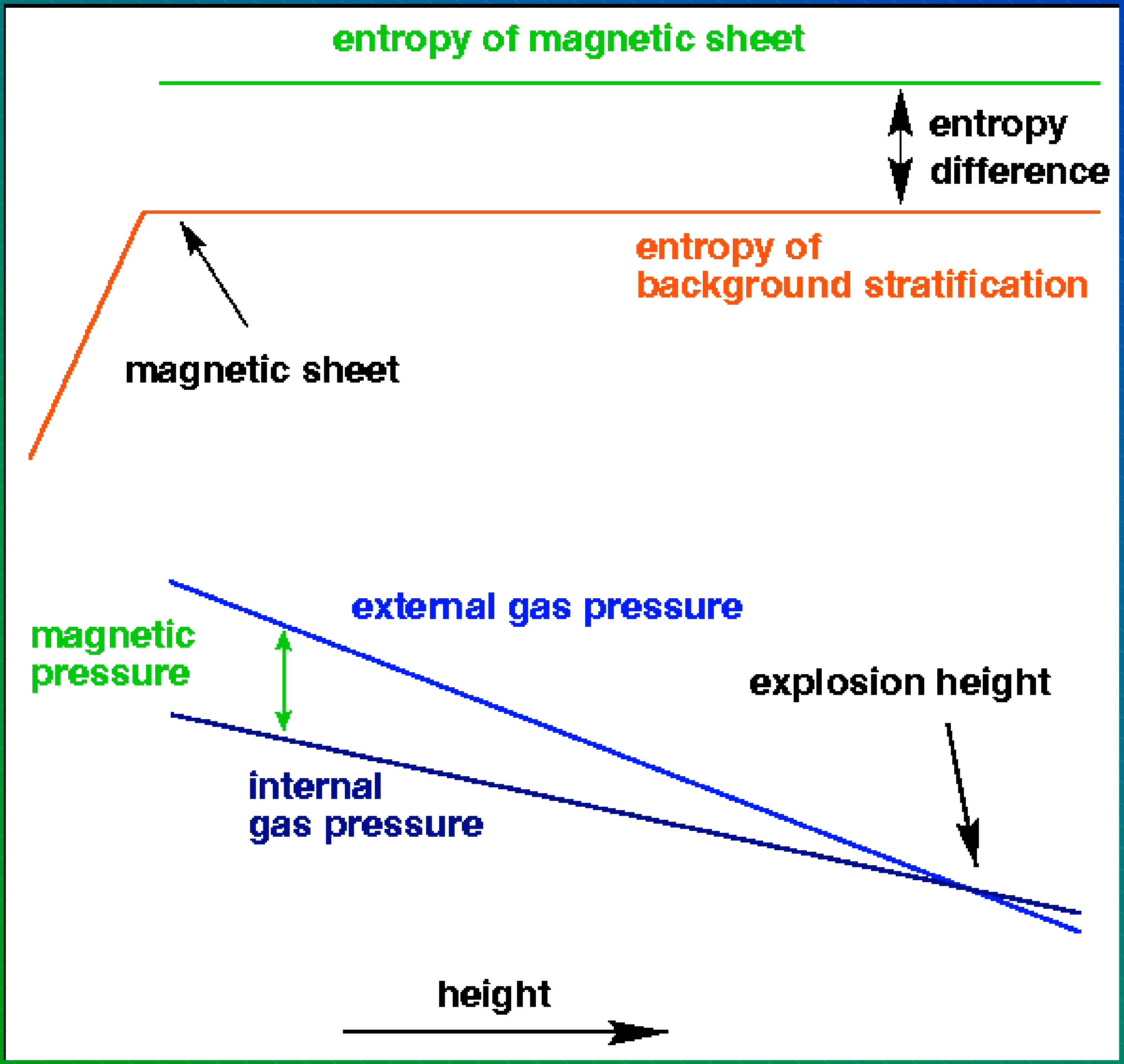


- exploded part no longer dynamically relevant
- outflow of buoyant high-entropy plasma
- decrease of gas pressure in the non-exploded part of flux tube

➔ magnetic field intensification

numerical simulations required

Numerical model



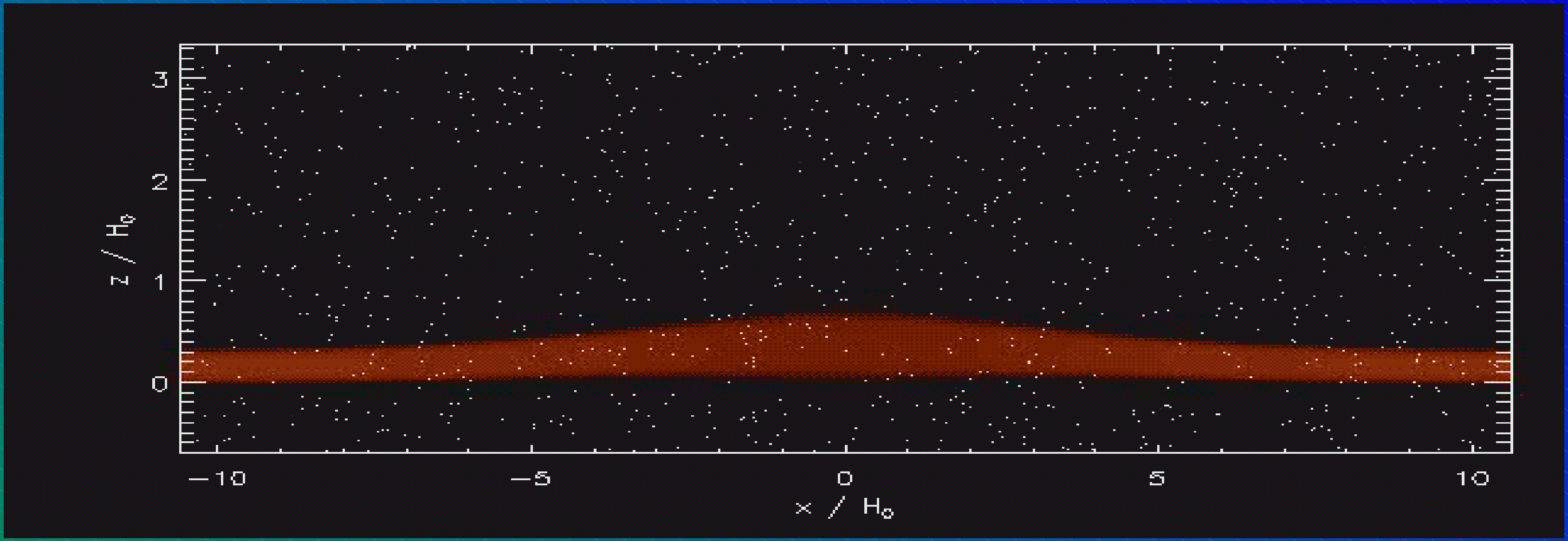
- 2D magnetic sheet
- adiabatic background stratification
- $\beta \approx 100 - 1000$

<u>basic characteristics:</u>

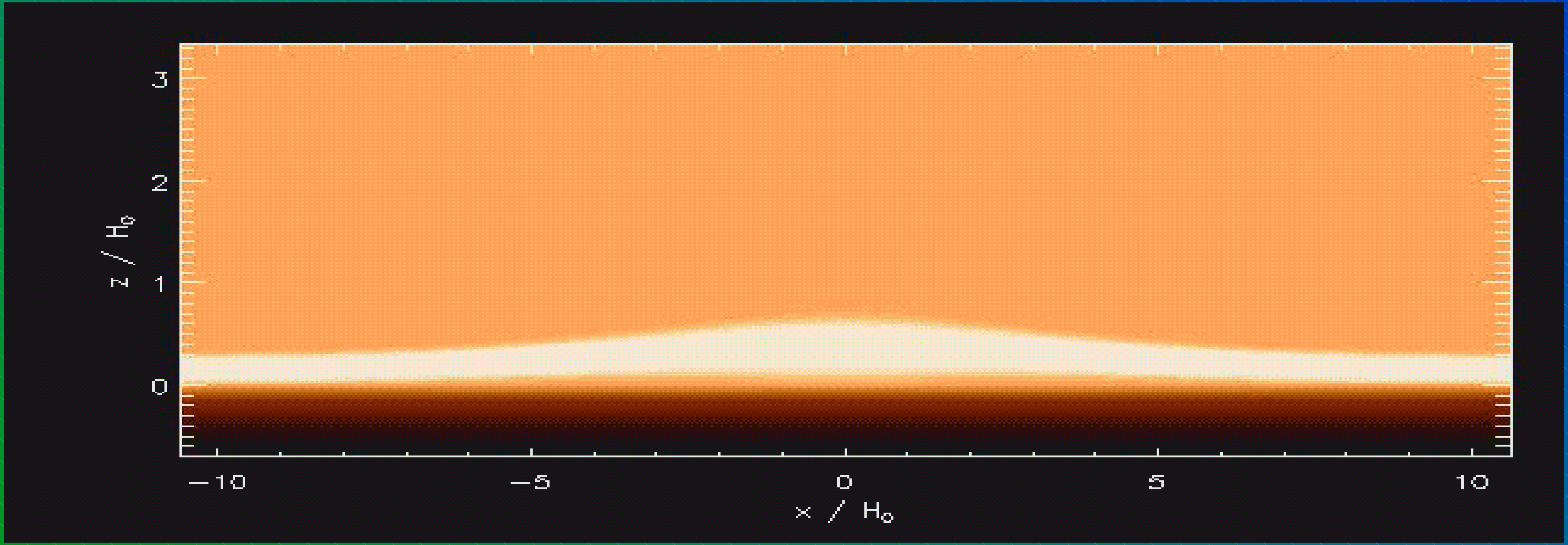
- entropy difference
- stable storage of magnetic sheet

<u>numerics (VAC-Code):</u>

- Riemann-solver
- resolution up to 512^2
- hydrostatic correction



magnetic field strength and velocity

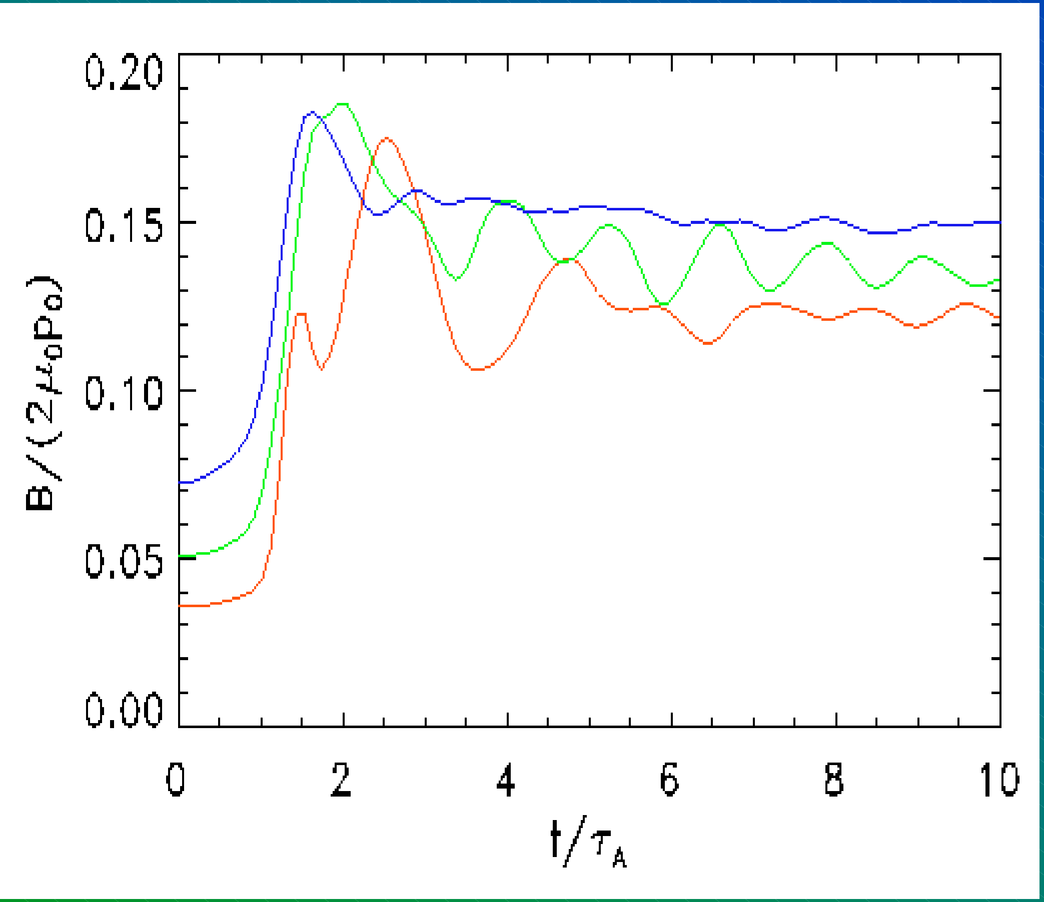


entropy

(Rempel & Sch., 2001)

512 × 512

Time evolution of magnetic field strength

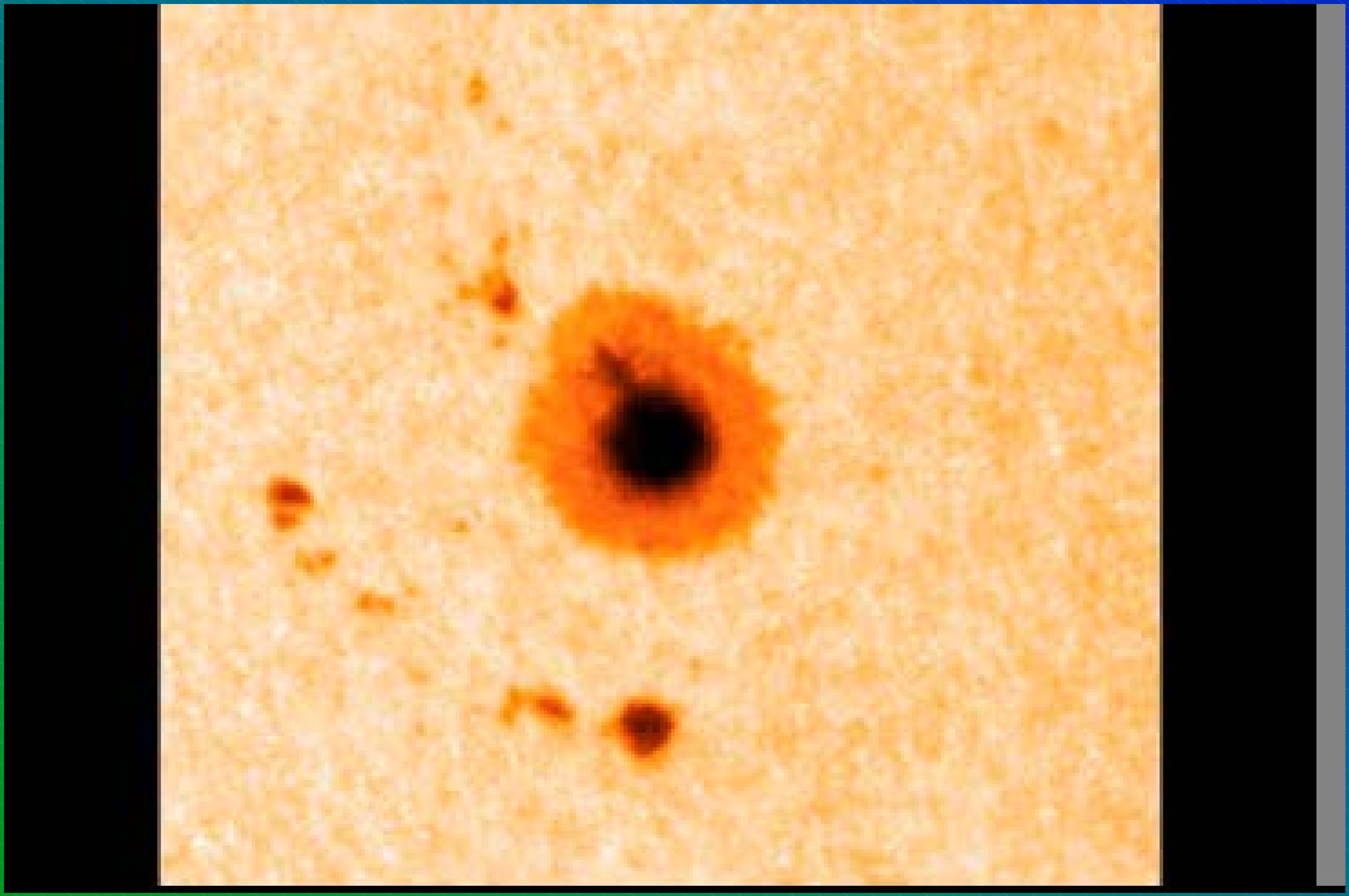


- formation of "stumps": $t < \tau_A$
- amplification: $\tau_A < t < 2\,\tau_A$
- saturation: $2\,\tau_A < t$
- asymptotic field strength determined by entropy difference

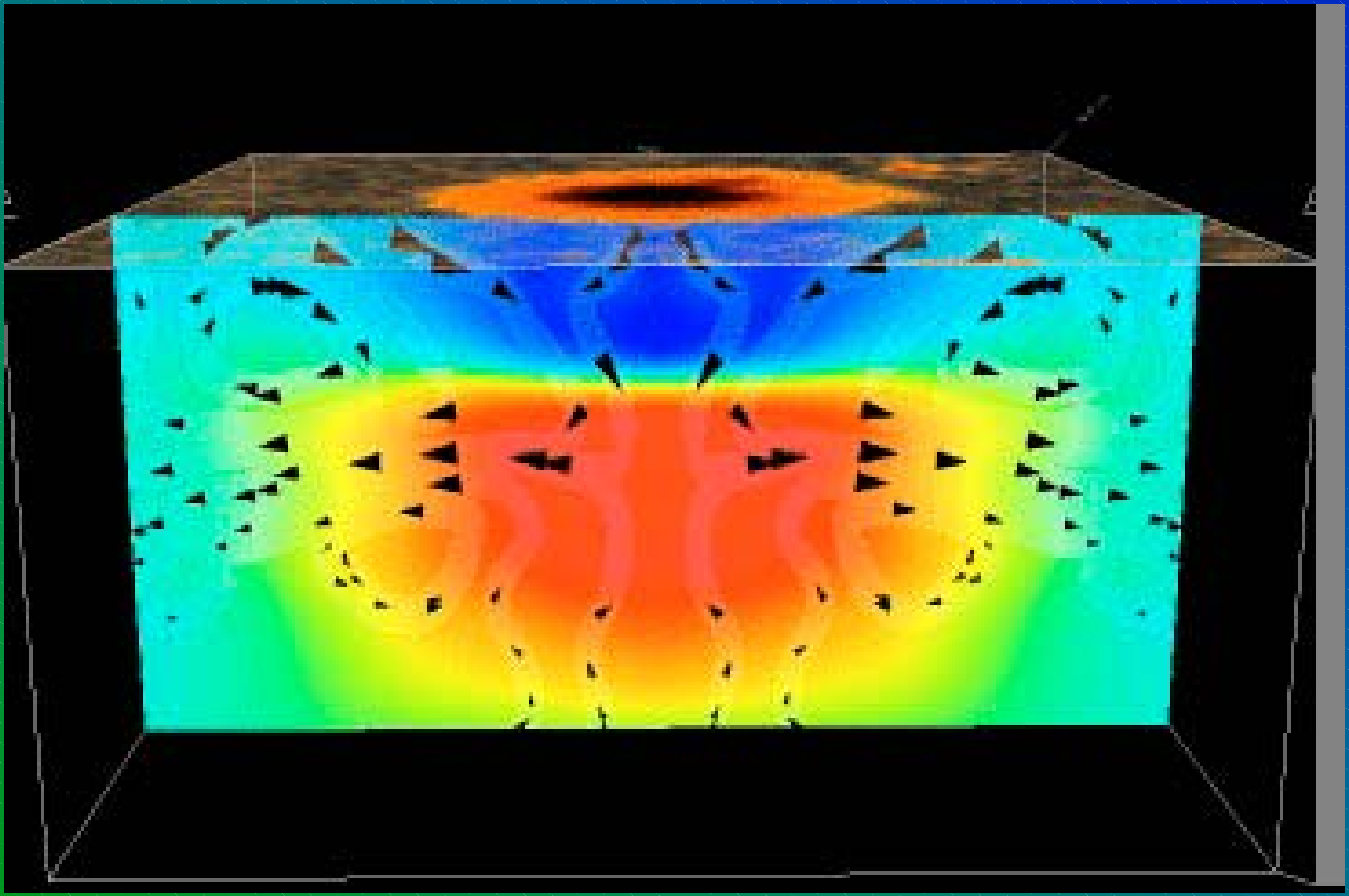
Scaling to solar parameters:
➔ amplification to 10^5 G (10 T) possible within ~ 6 months

$\tau_A = L / v_A$: horizontal Alfvén travel time

Temperature and flows below a sunspot



Temperature and flows below a sunspot



The end…