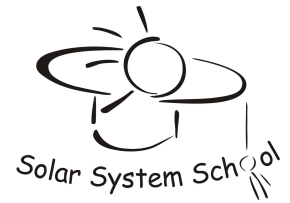




International Max Planck Research School
on Physical Processes in the Solar System and Beyond
at the Universities of Braunschweig and Göttingen



HYDRODYNAMICS

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31 January – 4 February 2005

Part I. Basic equations and concepts in Fluid Mechanics.

1. Kinematics of the continuum. Spatial and material descriptions.

- 1.1. Spatial (eulerian) and material (lagrangian) descriptions of the motion. Material derivative. Velocity and acceleration. Trajectories and streamlines.
- 1.2. Deformation and vorticity tensors. Physical interpretation.
- 1.3. Reynolds' transport theorem.

2. Fundamental equations in Continuum Mechanics.

- 2.1. Conservation of mass: continuity equation.
- 2.2. Long-range (volume) forces and contact (surface) forces. Stress tensor.
- 2.3. Momentum balance: equation of motion. Mechanical energy balance.
- 2.4. Angular momentum balance: symmetry of the stress tensor.
- 2.5. Conservation of energy and first principle of Thermodynamics.
- 2.6. Constitutive relations.

3. Viscous fluids. Navier-Stokes equation.

- 3.1. Hydrostatic pressure. Ideal fluid model. Euler's equation.
- 3.2. Stress tensor for a linearly viscous fluid. Newtonian fluid model. Coefficients of viscosity. Derivation of Navier-Stokes' equation.
- 3.3. Boundary conditions.
- 3.4. Scale analysis and dimensionless numbers.

4. Energy equation for a Newtonian fluid.

- 4.1. Second principle of Thermodynamics. Energy equation in the entropic representation. Concepts of adiabatic, isentropic and homoentropic motions.
- 4.2. Heat conduction. Entropy sources.
- 4.3. Alternative forms of expressing the energy equation.

5. Circulation and vorticity.

- 5.1. Circulation and vorticity. Vortex tubes. Some kinematic results.
- 5.2. Theorems of Kelvin and Helmholtz for ideal fluids.
- 5.3. Navier-Stokes' equation in terms of the vorticity. 2-D results.
- 5.4. Crocco's equation and Bernoulli's theorems.

Part II. Special topics.

6. The hydrodynamic equations in conservation form.

- 6.1. Momentum equation in conservation form. The momentum density flux tensor.
- 6.2. Energy equation in conservation form. The energy density flux vector.
- 6.3. Derivation of the jump relations across a discontinuity. Tangential discontinuities and shock fronts. Rankine-Hugoniot relations.

7. Chandrasekhar's adiabatic exponents in compressible hydrodynamics.

- 7.1. Definition of Chandrasekhar's adiabatic exponents as material response functions.
- 7.2. Physical interpretation of the adiabatic exponents Γ_1 , Γ_2 and Γ_3 .
- 7.3. Alternative representations of the energy equation in compressible hydrodynamics in terms of the material functions Γ_1 , Γ_2 and Γ_3 .

8. The virial theorem and astrophysical applications.

- 8.1. Derivation of the scalar virial theorem in Hydrodynamics. Interpretation of the various terms.
- 8.2. Some astrophysical applications of the virial theorem: Stars in hydrostatic equilibrium; restriction on the ratio of specific heats γ . Quasistatic contraction as possible energy source. Kelvin-Helmholtz time scale. Free-fall time scale. Derivation of the relationship between the pulsation period and the mean stellar density for pulsating stars.