International Max Planck Research School
on Physical Processes in the Solar System and Beyond
at the Universities of Braunschweig and Göttingen

HYDRODYNAMICS

Antonio Ferriz Mas, University of Vigo

31 January – 4 February 2005

Part I. Basic equations and concepts in Fluid Mechanics.

1. **Kinematics of the continuum. Spatial and material descriptions.**
   - 1.1. Spatial (eulerian) and material (lagrangian) descriptions of the motion. Material derivative. Velocity and acceleration. Trajectories and streamlines.
   - 1.2. Deformation and vorticity tensors. Physical interpretation.
   - 1.3. Reynolds' transport theorem.
2. **Fundamental equations in Continuum Mechanics.**
   - 2.1. Conservation of mass: continuity equation.
   - 2.2. Long-range (volume) forces and contact (surface) forces. Stress tensor.
   - 2.3. Momentum balance: equation of motion. Mechanical energy balance.
   - 2.4. Angular momentum balance: symmetry of the stress tensor.
   - 2.5. Conservation of energy and first principle of Thermodynamics.
   - 2.6. Constitutive relations.
3. **Viscous fluids. Navier-Stokes equation.**
   - 3.1. Hydrostatic pressure. Ideal fluid model. Euler's equation.
   - 3.2. Stress tensor for a linearly viscous fluid. Newtonian fluid model. Coefficients of viscosity. Derivation of Navier-Stokes' equation.
   - 3.3. Boundary conditions.
   - 3.4. Scale analysis and dimensionless numbers.
4. **Energy equation for a Newtonian fluid.**
   - 4.1. Second principle of Thermodynamics. Energy equation in the entropic representation. Concepts of adiabatic, isentropic and homoentropic motions.
   - 4.2. Heat conduction. Entropy sources.
   - 4.3. Alternative forms of expressing the energy equation.
5. **Circulation and vorticity.**
   - 5.1. Circulation and vorticity. Vortex tubes. Some kinematic results.
   - 5.2. Theorems of Kelvin and Helmholtz for ideal fluids.
   - 5.3. Navier-Stokes' equation in terms of the vorticity. 2-D results.
   - 5.4. Crocco's equation and Bernoulli's theorems.

Part II. Special topics.

6. The hydrodynamic equations in conservation form.

6.1. Momentum equation in conservation form. The momentum density flux tensor.

6.2. Energy equation in conservation form. The energy density flux vector.

6.3. Derivation of the jump relations across a discontinuity. Tangential discontinuities and shock fronts. Rankine-Hugoniot relations.

7. Chandrasekhar's adiabatic exponents in compressible hydrodynamics.

7.1. Definition of Chandrasekhar's adiabatic exponents as material response functions.

7.2. Physical interpretation of the adiabatic exponents Γ_1, Γ_2 and Γ_3.

7.3. Alternative representations of the energy equation in compressible hydrodynamics in terms of the material functions Γ_1, Γ_2 and Γ_3.

8. The virial theorem and astrophysical applications.

8.1. Derivation of the scalar virial theorem in Hydrodynamics. Interpretation of the various terms.

8.2. Some astrophysical applications of the virial theorem: Stars in hydrostatic equilibrium; restriction on the ratio of specific heats γ. Quasistatic contraction as possible energy source. Kelvin-Helmholtz time scale. Free-fall time scale. Derivation of the relationship between the pulsation period and the mean stellar density for pulsating stars.