

Introduction to Solar Physics

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Structure of lectures I

- Introduction and overview
- Core and interior: energy generation and standard solar model
- Solar radiation and spectrum
 - Solar spectrum
 - Radiative transfer
 - Formation of absorption and emission lines
- Convection: The convection zone and granulation etc.
- Solar oscillations and helioseismology
- Solar rotation

Structure of lectures II

- The solar atmosphere: structure
 - Photosphere
 - Chromosphere
 - Transition Region
 - Corona
 - Solar wind and heliosphere
- The magnetic field
 - Zeeman effect and Unno-Rachkovsky equations
 - Magnetic elements and sunspots
 - Chromospheric and coronal magnetic field
 - The solar cycle
 - Coronal heating
 - MHD equations & dynamo: see lectures by Ferriz Mas

Structure of lectures III: next time

- Explosive and eruptive phenomena
 - Flares
 - CMEs
 - Explosive events
- Sun-Earth connection
 - CMEs and space weather
 - Longer term variability and climate
- Solar-stellar connection
 - Activity-rotation relationship
 - Sunspots vs. starspots

The Sun: a brief overview



The Sun, our star

- The Sun is a normal star: middle aged (4.5 Gyr) main sequence star of spectral type G2
- The Sun is a special star: it is the only star on which we can resolve the spatial scales on which fundamental processes take place.
- The Sun is a special star: it provides almost all the energy to the Earth
- The Sun is a special star: it provides us with a unique laboratory in which to learn about various branches of physics.

The Sun: Overview



The Sun: a few numbers

- Mass = 1.99 10³⁰ kg (= 1 M_☉)
- Average density = 1.4 g/cm³
- Luminosity = $3.84 \ 10^{26} \text{ W}$ (= 1 L_{\odot})
- Effective temperature = 5777 K (G2 V)
- Core temperature = 15 10⁶ K
- Surface gravitational acceleration g = 274 m/s²
- Age = 4.55 10⁹ years (from meteorite isotopes)
- Radius = 6.96 10⁵ km
- Distance = 1 AU = 1.496 (+/-0.025) 10⁸ km
- 1 arc sec = 722±12 km on solar surface (elliptical Earth orbit)
- Rotation period = 27 days at equator (sidereal, i.e. as seen from Earth; Carrington rotation)

The Sun's Structure

Solar interior:

- Everything below the Sun's (optical) surface
- Divided into hydrogen-burning core, radiative and convective zones

Solar atmosphere:

 Directly observable part of the Sun.

- Divided into
- photosphere, chromosphere, corona, heliosphere



The solar surface

- Since solar material does not exhibit a phase transition (e.g. from solid or liquid to gaseous as for the Earth), a standard way to define the solar surface is through its radiation.
- The photons travelling from the core outwards make a random walk, since they are repeatedly absorbed and reemitted. The mean free path increases rapidly with radial distance from the solar core (as the density and opacity decrease).
- A point is reached where the average mean free path becomes so large that the photons escape from the Sun. This point is defined as the solar surface. It corresponds to optical depth τ = 1. Its height depends on λ.
- Often τ = 1 at λ=5000 Å is used as a standard for the solar surface.

Wide range of physical parameters

- The Sun presents a wide variety of physical phenomena and processes, between solar core and corona.
- E.g. Gas density varies by ≈ 30 orders of magnitude, temperature by 4 orders, relevant time scales from 10⁻¹⁰ sec to 10 Gyr



Different observational and theoretical techniques needed to study different parts of Sun, e.g. helioseismology & nuclear physics for interior, polarimetry & MHD for magnetism, etc. Solar physics in relation to other branches of physics



The Sun as a plasma physics lab.



Solar Tests of Gravitational Physics

- Curved light path in solar gravitational field → Test of General Relativity
- Red shift of solar spectral lines
 Test of EEP
- Oblate shape of Sun → Quadrupol moment of solar gravitational field: Test of Brans-Dicke theory (R. Mecheri)
- Comparison of solar evolution models with observations → Limits on evolution of fundamental constants
- Polarization of solar spectral lines: Tests gravitational birefringence → Tests of equivalence principle & alternative theories of gravity (O. Preuss)

The Sun and particle physics

- The fact that the rate of neutrinos measured by the Homestake ³⁷Cl detector is only 1/3 of that predicted by standard solar models was for > 30 years one of the major unsolved problems of physics.
- Possible resolutions:
 - Standard solar model is wrong
 - Neutrino physics is incomplete
- Recent findings from SNO and Superkamiokande: Problem lies with the neutrino physics
- Standard model of particle physics needs to be revised
- Nobel prize 2002 for R. Davies for discovery of the solar neutrino problem.

Solar physics and astrophysics

- Phenomena happening all over the universe can be best studied at close distances, where the relevant physical processes can be spatially resolved (same for solar planets).
- E.g. magnetic activity, is present on innumerable stars, in accretion disks, in jets, in the interstellar medium, etc., but the relevant spatial scales can generally not be resolved. The Sun provides a key.
- Radio astronomy, radiative transfer, spectropolarimetry, asteroseismolgy, etc. are techniques first developed to study the Sun

Which stars have magnetic fields or show magnetic activity?

- Best studied star: Sun
- F, G, K & M stars (outer convection zones) show magnetic activity & have
 B> fields of G-kG.
- Early type stars: Ap, Bp, (kG-100kG), Be (100G)
- White dwarfs have B ≈ kG-10⁹ G, no activity
- Not on diagram: pulsars



The Sun compared with active stars



Sun, Earth and planets

- Solar output affects the magnetospheres and atmospheres of planets
- Solar energy is responsible for providing a habitable environment on Earth
- Solar evolution and liquid water on Mars...





The Sun's core

- In the Sun's core mass is turned into energy.
- Nuclear reactions burn 7x10¹¹ kg/s of hydrogen into helium.
- Inside the core the particle density and temperature are so high, that individual protons ram into each other at sufficient speed to overcome the Coulomb

barrier, forming heavier He atoms and releasing

and releasing energy



Helium

Light

Nuclear reactions in cores of stars

- Sun gains practically all its energy from the reaction $4p \rightarrow \alpha + 2e^+ + 2v = {}^{4}\text{He} + 2e^+ + 2v$
- Two basic routes
 - p-p chain: yields about 99% of energy in Sun
 - CNO cycle : 1% of energy released in present day Sun (but dominant form of energy release in hotter stars)
- Both chains yield a total energy Q of 26.7 MeV, mainly in the form of γ-radiation Q_γ (which is absorbed and heats the gas) and neutrinos Q_γ (which escapes from the Sun).

Nuclear reactions of pp-chain

2 Protons

d=deuterium	[eee	Reaction	$Q'[{ m MeV}]$	$Q_{\nu}[{ m MeV}]$	Rate symbo
α=Helium	ppI	$p(p,e^+\nu)d$	1.177	0.265	$\lambda_{\rm pp}$
γ=radiation		3 He(3 He,2p) α	12.860		$\lambda_{ m pd} = \lambda_{33}$
v=neutrino	ppII	${}^{3}\mathrm{He}(\alpha,\gamma){}^{7}\mathrm{Be}$	1.586		λ_{34}
2 nd reaction		$^{\prime}\mathrm{Be}(\mathrm{e}^{-},\nu\gamma)^{\prime}\mathrm{Li}$ $^{7}\mathrm{Li}(\mathrm{p},\alpha)\alpha$	$0.049 \\ 17.346$	0.815	$\lambda_{e7} \lambda'_{17}$
replaces	ppIII	$^{7}\mathrm{Be}(\mathrm{p},\gamma)^{8}\mathrm{B}$	0.137		λ_{17}
step 3 of 1 st reaction		${}^{8}\mathrm{B}(\mathrm{,e}^{+}\nu){}^{8}\mathrm{Be}^{*}$ ${}^{8}\mathrm{Be}^{*}(\mathrm{,}\alpha)\alpha$	8.367 2.995	6.711	$\lambda_8 \\ \lambda_8'$

- 1st vs. 2nd + 3rd 87 : 13
- 2^{nd} vs. $3^{rd} \rightarrow 13$: 0.015

Nuclear reactions of CNO-cycle

C, N and O act only as catalysts: Basically the same things happens as with proton chain.

Table 2.2. Nuclear reactions of the CNO cycle. Energy val-	
ues according to Bahcall and Ulrich (1988) and Caughlan	
and Fowler (1988)	

Reaction	$Q'[{\rm MeV}]$	$Q_{\nu}[{ m MeV}]$	Rate symbol
$^{12}C(p,\gamma)^{13}N$	1.944		λ_{p12}
${}^{13}N(,e^+\nu){}^{13}C$	1.513	0.707	λ_{13}
${}^{13}C(p,\gamma){}^{14}N$	7.551		λ_{p13}
${}^{14}N(p,\gamma){}^{15}O$	7.297		λ_{p14}
${}^{15}O(,e^+\nu){}^{15}N$	1.757	0.997	λ_{15}
$^{15}N(p,\alpha)^{12}C$	4.966		λ_{p15}



Solar neutrinos

- Neutrinos, *v*, are produced at various stages of the pp-chain.
- Neutrinos are also produced by the reaction: p(p e⁻, v)d, socalled pep reaction. Being a 3-body reaction it is too rare to contribute to the energy, but does contribute to the number of v.

	Reaction	$Q'[{ m MeV}]$	$Q_{\nu}[{ m MeV}]$
ppI	$p(p,e^+\nu)d$	1.177	0.265
	$d(p,\gamma)^3$ He	5.494	
	$^{3}\mathrm{He}(^{3}\mathrm{He,2p})\alpha$	12.860	
ppII	${}^{3}\mathrm{He}(\alpha,\gamma)^{7}\mathrm{Be}$	1.586	
	$^{7}\mathrm{Be}(\mathrm{e}^{-},\nu\gamma)^{7}\mathrm{Li}$	0.049	0.815
	$^{7}\mathrm{Li}(\mathbf{p},\alpha)\alpha$	17.346	1997 - 1996 - 1997
ppIII	$^{7}\mathrm{Be}(\mathrm{p},\gamma)^{8}\mathrm{B}$	0.137	
iff, desid	${}^{8}\mathrm{B}(\mathrm{,e}^{+}\nu){}^{8}\mathrm{Be}^{*}$	8.367	6.711
	$^{8}\mathrm{Be}^{*}(,\alpha)\alpha$	2.995	<u> </u>

Solar neutrino spectrum



Solar neutrinos II

- Since 1968 the Homestake 37 Cl experiment has given a value of 2.1 ± 0.3 snu (1snu =1 v / 10³⁶ target atoms)
- Standard solar models predict: 7±2 snu
- Solar Neutrino Problem!
- In 1980s & 90s water based Kamiokande and larger Superkamiokande detectors found that approximately half the rare, high energy ⁸B v were missing.
- ⁷¹Ga experiments (GALLEX at Gran Sasso and SAGE in Russia) showed that the neutrino flux was too low, even including the $p(p,e^+\nu)d$ neutrinos.





Solar neutrinos III

Sensitivity of H_2O , ⁷¹Ga and ³⁷Cl to v increases ~ exponentially with increasing energy.

Homestake ³⁷Cl detector and (Super-) Kamiokande see mainly high-energy ν from rare β^+ -decay of ⁸B.

C. (ve.e")37

σ (m²) 10⁻⁻

10

All states to 7.4 Me

- Branching ratios between the various chains: central for predicting exact v-flux detectable by ³⁷Cl & H₂O
- Branching ratios depend very sensitively on T(r=0), while total v-flux depends only linearly on luminosity.
- Even ⁷¹Ga experiments sensitve largely to high energy *v*.

Solar Neutrinos IV

Possible solutions to solar neutrino problem:

- Standard solar model is incorrect (5-10% lower temperature in core gives neutrino flux consistent with Homestake detector).
- Neutrino physics is incomplete (i.e. the standard model of particle physics is wrong!)
- Nuclear physics describing the pp-chain is incorrect
- Nuclear physics describing interaction between neutrino and ³⁷Cl is incorrect (Kamiokande & 740 June 1997) Nuclear State (Kamiokande) Nuclear Physics (Kamiokande) Nuclear Physics
 - ⁷¹Ga showed that this wasn't the problem)

Resolution of neutrino problem

- SNO (Sudbury Neutrino Observatory) in Sudbury, Canada uses D_2O and can detect not just the electron neutrino, but also μ and τ neutrinos
- The neutrinos aren't missing, e⁻ neutrinos produced in the Sun just convert into μ and τ neutrinos
- The problem lies with the neutrino physics.
- The neutrino has a small rest mass (10⁻⁸ m_e), which allows it to oscillate between the three flavours: e⁻ neutrino, μ neutrino and τ neutrino (proposed 1969 by russian theorists: Bruno Pontecorvo and Vladimir Gribov, ... but nobody believed them)
- Confirmation by measuring anti-neutrinos from power plant (with Superkamiokande).

Resolution of neutrino problem II

- Lesson learnt: neutrinos have a multiple personality problem (J. Bahcall)
- Other lesson learnt: the "dirty" and difficult solar model turned out to be correct, the clean and beautiful standard theory of particle physics turned out to be wrong, or at least incomplete (J. Bahcall)
- 2002: Raymond Davis got Nobel prize for uncovering the neutrino problem

Standard solar model

- Ingredients: Conservation laws and material dependent equations
 - Mass conservation
 - Hydrostatic equilibrium (= momentum conservation in a steady state)
 - Energy conservation
 - Energy transport
 - Equation of state
 - Expression for entropy
 - Nuclear reaction networks and reaction rates → energy production
 Opacity ______
- Assumptions: standard abundances, no mixing in core or in radiative zone, hydrostatic equilibrium, i.e. model passes through a stage of equilibria (the only time dependence is introduced by the reduction of H and the build up of He in the core).

Equations describing solar interior

Mass conservation :
$\partial r = 1$
$\frac{\partial m}{\partial m} = \frac{1}{4\pi\rho r^2}$
$\rho = \text{gas density}$
Hydrostatic equilibrium :
∂P _ Gm
$\frac{\partial m}{\partial m} = -\frac{1}{4\pi^2}$
P = pressure
G = Gravitational constant
Energy balance:
$\partial L = c T \partial S$
$\frac{\partial m}{\partial m} = \mathcal{E} - T \frac{\partial T}{\partial T}$
ε = energy generation per unit mas
T = temperature

S = entropy

Equation of state : $\rho = \rho(P,T)$ or (for an ideal gas) : $P_G = \frac{\rho \Re T}{\mu}$ P_G = gas pressure \Re = gas constant μ = mean molecular weight Energy transport :

$F = F_{\rm R} + F_{\rm C} + F_{\rm cond} = \frac{L}{4\pi}$

- F = total energy flux $F_{\text{p}} = \text{radiative flux}$
- $F_{\rm R}$ = radiative flux $F_{\rm c}$ = convective flux
- $F_{...,t} =$ conductive flux
- L = luminosity





Evolution of Sun's luminosity



Faint young Sun paradox

- According to the standard solar model the Sun was approximately 30% less bright at birth than it is today
- Too faint to keep the Earth free of ice!
- Problem: Albedo of ice is so high that even with its current luminosity the Sun would not be able to melt all the ice away.
- Obviously the Earth is not covered with ice...
- So: Where is the mistake?

Possible resolution of the faint young Sun paradox

- The Earth's atmosphere was different 4 Gyr ago. More methane and other greenhouse gases. Higher insulation meant that even with lower solar input the Earth remained ice-free.
- As the Sun grew brighter life grew more abundant and changed the atmosphere of the Earth, reducing the greenhouse effect.
- Problem: what about Mars? Could it have had liquid water 4Gyr ago if Sun were so faint?
- Alternative: Sun was slightly more massive (1.04-1.07M, at birth and lost this mass (enhanced solar wind) in the course of time (Sackmann & Boothroyd 2003, ApJ). A more massive star on the ZAMS emits more light. Also agrees w. Mars data



Solar radiation and spectrum

The Sun in white light: Limb darkening

- In the visible, the Sun's limb is darker than the centre of the solar disk (Limb darkening)
- Since intensity ~ Planck function, $B_v(T)$, T is lower near limb.
- Due to grazing incidence we see higher near limb: T decreases outward



Limb darkening vs. λ

Upper fig.: short λ: large limb darkening; long λ: small limb darkening

departure from straight line: limb darkening is more complex than $I(\theta) \sim \cos(\theta)$



The Sun in the EUV: Limb brightening

In the EUV, the Sun's limb is brighter than the centre of the solar disk (Limb brightening)

Since the solar atmosphere is optically thin at these wavelengths, intensity ~ thickness of layer contributing to it. Due to geometrical effects this layer appears thicker near limb (radiation comes from roughly the same height everywhere).



The Sun in the EUV: Limb brightening

Limb brightening in optically thin lines does not imply that the Sun's temperature increases outwards (although by chance it does in these layers....)



Solar irradiance spectrum

Spectrum is similar to, but not equal to Planck function → Radiation comes from layers with diff. temperatures.



Often used temperature measure for st

measure for stars: Effective temp: σT^4_{eff} = Area under flux curve



The solar spectrum: continua with absorption and emission lines

- The solar spectrum changes in character at different wavelengths.
- X-rays: Emission lines of highly ionized species
 EUV: Emission lines of neutral to multiply ionized species plus recombination continua
- species plus recombination continua
 UV: stronger recombination continua and absorption
- UV: stronger recombination continua and absorption lines
- Visible: H⁻ b-f continuum with absorption lines
- FIR: H⁻ f-f continuum, increasingly cleaner (i.e. less lines, except molecular bands)
- Radio: thermal and, increasingly, non-thermal continua













Formation of the solar spectrum

Continuum

- Spectral energy distribution
- Centre to limb variation
- Spectral lines
 - Absorption lines
- Emission lines
- Radiative transfer

Radiative transfer: optical depth

- Axis *z* points in the direction of light propagation
- Optical depth: $\Delta \tau_v = -\kappa_v \Delta z$ where κ_v is the absorption coefficient and v is the frequency of the radiation. Light only knows about the τ_{y} scale and is unaware of z $\tau_{v} = -\int \kappa_{v}(z) dz$ Integration:
- (note that the scales are floating, no constant of integration is fixed)

Optical depth and solar surface

- Radiation escaping from the Sun is emitted mainly at values of $\tau_v \approx 1$.
- At wavelengths at which κ_{v} is larger, the radiation comes from higher layers in the atmosphere.
- In solar atmosphere κ_v is small in visible and near IR, but large in UV and FIR \rightarrow We see deepest in visible and NIR, but sample higher layers at shorter and longer wavelengths.



λ...

 λ_{μ}^{-1}

-100 λum

Height of τ = 1, brightness

Radiative Transfer Equation

Equation of radiative transfer:

$\mu dI_v/d\tau_v = I_v - S_v$

where I_v is the intensity (i.e. the measured quantity) and S_v is the source function. $\mu = \cos\theta$ is only important for non-vertical rays.

- S_v = emissivity ε_v divided by absorption coefficient κ_v
- The physics is hidden in ε_v and κ_v , i.e. in τ_v and S_v
- These quantities depend on temperature, pressure, elemental abundances, and frequency or wavelength

Formal solution of RT equation

For $S_v = 0$ the solution of the RTE in a slab with (optical) boundaries τ_{v2} and τ_{v1} is :

 $I_{v}(\tau_{v2}) = I_{v}(\tau_{v1}) \exp(-\tau_{v2} + \tau_{v1})$

For general case formal solution reads (formal soln. assumes that we already know S_y)

 $I_{v}(\tau_{v2}) = I_{v}(\tau_{v1}) \exp(-\tau_{v2} + \tau_{v1}) + \int S_{v} \exp(-\tau_{v}) d\tau_{v2}$

- 1st term describes radiation that enters through lower boundary (only absorption, no emission in slab), 2nd term describes radiation emitted in slab.
- In a stellar atmosphere $\tau_{v1} = \infty$, so that only the 2nd term survives (lower boundary is unimportant).

When is an emission line formed, when an absorption line?

- Continua are formed deeper in a stellar atmosphere than spectral lines at the same wavelengths.
- A line is in absorption if S, decreases with height, i.e. if the absorption at greater heights dominates over emission
- A line is in emission if S_v increases with height, i.e. if the emission at greater heights dominates over absorption

Statistical equilibrium

- In general both S_{u} and κ_{u} require a computation of the full statistical equilibrium for the species being considered.
- This implies computing how much each atomic/ionic/molecular level is populated, i.e. solving rate equations describing transitions to and from each considered level.
- Requires detailed knowledge of atomic, ionic. molecular structure and transitions.

The assumption of LTE

- In the solar interior and photosphere (i.e. where density is large and collisions are common) we can assume Local Thermodynamic Equilibrium (LTE)
- Thermodynamic equilibrium (TE): a single temperature everywhere (blackbody) \rightarrow Radiation emerging from object follows the Planck function B_{ν} .
- **LTE:** Each layer of the solar atmosphere has its own temperature \rightarrow Replace S_v by $B_v(T)$ in the RTE and its solution.
- Problem of knowing S_v is reduced to knowing $T(\tau)$ in the atmosphere.
- In addition, the statistical equilibrium can be solved simply by considering the Saha-Boltzmann equilibrium \rightarrow basically *T* and n_{ρ} need to be known.

Elemental abundances

Photospheric values

- Logarithmic (to base 10) abundances of the 32 lightest elements on a scale on which H has an abundance of 12
- Heavier elements all have low abundances
- Note that in general the solar photospheric abundances are very similar to those of meteorites, with exception of Li, with is depleted by a factor of 100.

1	H	12.00	2001 - 11
2	He	10.93 ± 0.004	
3	Li	1.10 ± 0.10	3.31 ± 0.04
4	Be	1.40 ± 0.09	1.42 ± 0.04
5	В	2.55 ± 0.30	2.79 ± 0.05
6	C	8.52 ± 0.06	_
7	N	7.92 ± 0.06	de la la construcción de la constru
8	0	8.83 ± 0.06	and a the next of
9	F	4.56 ± 0.3	4.48 ± 0.06
10	Ne	8.08 ± 0.06	-
11	Na	6.33 ± 0.03	6.32 ± 0.02
12	Mg	7.58 ± 0.05	7.58 ± 0.01
13	Al	6.47 ± 0.07	6.49 ± 0.01
14	Si	7.55 ± 0.05	7.56 ± 0.01
15	Р	5.45 ± 0.04	5.56 ± 0.06
16	S	7.33 ± 0.11	7.20 ± 0.06
17	Cl	5.5 ± 0.3	5.28 ± 0.06
18	Ar	6.40 ± 0.06	8 30 - DANIS
19	K	5.12 ± 0.13	5.13 ± 0.02
20	Ca	6.36 ± 0.02	6.35 ± 0.01
21	Sc	3.17 ± 0.10	3.10 ± 0.01
22	Ti	5.02 ± 0.06	4.94 ± 0.02
23	V	4.00 ± 0.02	4.02 ± 0.02
24	Cr	5.67 ± 0.03	5.69 ± 0.01
25	Mn	5.39 ± 0.03	5.53 ± 0.01
26	Fe	7.50 ± 0.05	7.50 ± 0.01
27	Co	4.92 ± 0.04	4.91 ± 0.01
28	Ni	6.25 ± 0.04	6.25 ± 0.01
29	Cu	4.21 ± 0.04	4.29 ± 0.04
30	Zn	4.60 ± 0.08	4.67 ± 0.04
31	Ga	2.88 ± 0.10	3.13 ± 0.02
32	Ge	3.41 ± 0.14	3.63 ± 0.04

Element Photosphere Meteorites

Elemental abundances: the FIP effect

- In TR and corona abundances differ from photospheric values depending on the first ionization potential (FIP)
- of element. Elements with low FIP have enhanced
- abundance Apparently, acceleration through the chromosphere is more efficient for these elements (which are ionized in chromosphere, while high FIP elements are net)

are not).



Spectrum synthesis by 1-D
radiative equilibrium model
LAMPANARA AND MUMARA ALARDANARA
A A A A A A A A A A A A A A A A A A A

Diagnostic power of spectral lines

- Doppler shift of line: (net) flows in the LOS direction.
- Line width: temperature and turbulent velocity
- Equivalent width: elemental abundance, temperature (via ionisation and excitation balance)
- Line depth: temperature and temperature gradient
- Line asymmetry: inhomogenieties in the solar atmosphere.

Heights of formation: on which layers do lines give information ?





The convection zone

- Through the outermost 30% of solar interior, energy is transported by convection instead of by radiation
- In this layer the gas is convectively unstable. The unstable region ends just below the solar surface. I.e. the visible signs of convection are actually due to overshooting.
- Due to this, the time scale changes from the time scale for a random walk of the photons through the radiative zone (due to high density, the mean free path in the core is well below a millimeter) to the convective transport time:

 $t_{\text{radiative}} \sim 10^6 \text{ years } > t_{\text{convective}} \sim \text{months}$



Surface convection: granulation

- Typical size: 2 Mm
- Lifetime: 6-8 min
- Velocities: 1 km/s (but peak velocities > 10 km/s, i.e. supersonic)
- Brightness contrast: 20% in visible continuum (under ideal conditions)
- All quantities show a continuous distribution of values
- At any one time 10⁶ granules on sun.



Surface convection: Supergranulation

- 1 hour average of MDI Dopplergrams (averages out oscillations).
- Dark-bright: flows towards/away from observer.
- No supergranules visible at disk centre: velocity is mainly horizontal
- Size: 20-30 Mm, lifetime: days, horiz. speed: 400 m/s, no contrast in visible

Observing convection

Observing granulation

- Continuum images and movies at high spatial resolution: gives sizes, lifetimes and evolution (splitting and dissolving granules), contrasts
- Spectral lines (also at low spatial resolution). Line bisectors, line widths and convective blue shifts: contrasts, area factors, stratification

Observing supergranulation

- Images and movies in cores of chromospheric spectral lines, or magnetograms: observe the magnetic field at the edges of the supergranules instead of the supergranules directly
- Helioseismic techniques

Line bisectors



Supergranules seen by SUMER

- Si I 1256 Å full disk scan by SUMER in 1996
- Bright network indicates location of magnetic network
- Darker cells: supergranules



Supergranules & magnetic field

- Why are supergranules seen in chromospheric and transition region lines?
- Supergranules are related to the magnetic network.
- Network magnetic fields are concentrated at edges of supergranules.



Illustration of convectively stable and unstable situations

Convectively stable







(dp/dz)_{adiab} : adiabatic gradient

Onset of convection II

Rewriting in terms of temperature and pressure:

 $\nabla_{ad} = (d \log T/d \log P)_{ad}$ $\nabla_{rad} = (d \log T/d \log P)_{rad} = \text{gradient in an}$ atmosphere with radiative energy transport
Schwarzschild's convective instability criterion: $\nabla_{ad} < \nabla_{rad}$ $T_{\text{ADIABATISCH}}$ $T_{\text{ADIABATISCH}}$ $T_{\text{ADIABATISCH}}$ $T_{\text{ADIABATISCH}}$ $T_{\text{ADIABATISCH}}$ $T_{\text{ADIABATISCH}}$ $T_{\text{ADIABATISCH}}$ $T_{\text{ADIABATISCH}}$

Why an outer convection zone?

- Why does radiative grad exceed adiabatic gradient?
- Mainly: radiative gradient becomes very large due to ionization of H and He below the solar surface.
- Expression for radiative gradient (for Eddington approximation):

 ∇_{rad} = (3F_r /16 σ g) (κ_{qr} P_g / T⁴)

- F_r = radiative flux (≈ constant)
- σ = Stefan-Boltzmann constant
- g = gravitational acceleration (≈ constant)
- κ_{gr} = absorption coefficient per gram. As H and He become ionized with depth, κ_{gr} increases rapidly, leading to large radiative gradient.

Ionisation of H and He

- Ionisation balance is described by Saha's equation: degree of ionisation depends on T and n_a
- H ionisation happens just below solar surface
- He \rightarrow He⁺ + e⁻ happens 7000 km below surface
- He⁺ \rightarrow He⁺⁺ + e⁻ happens 30'000 km below surface
- Since H is most abundant, it provides most electrons (largest opacity) and drives convection most strongly
- At still greater depth, other elements also provide a minor contribution.

Radiative, adiabatic & actual gradients



The mixing length

- As a gas packet rises, diffusion of particles and thermal exchange with surroundings causes it to lose its identity and to stop from moving on.
- Length travelled up to that point: mixing length *l*.
- Often used parameterization of *l*: $l = \alpha H_p$
 - *H_p* = pressure scale height
 - α = mixing length parameter, typically
 1-2 (determined empirically)



Why is mixing length interesting

- Gives an idea of the size scale of a convection cell (height or depth of cell given by mixing length)
- horizontal extent of cell cannot be much bigger than mixing length due to mass conservation!

History of mixing length

- Mixing length was introduced by L. Prandl, who was director at Max Planck Institut für Strömungsforschung in Göttingen.
- It allowed him to describe convection in a simple, but powerful way.
- Even today, most stellar interior models (and many atmospheric models) use the mixing length to describe the effects of convection.

Convective overshoot

- Due to their inertia, the packets of gas reaching the boundary of the CZ pass into the convectively stable layers, where they are braked & finally stopped.
- overshooting convection
- Typical width of overshoot layer: order of H_n
- This happens at both the bottom and top boundaries of the CZ and is important:
 - top boundary: Granulation is overshooting material. $H_p \approx$ 100 km in photosphere
 - bottom boundary: the overshoot layer allows B-field to be stored → seat of the dynamo?

Convection simulations

- 3-D hydrodynamic simulations reproduce a number of observations and provide new insights into solar convection.
- These codes solve for mass conservation, momentum conservation (force balance, Navier-Stokes equation), and energy conservation including as many terms as possible.
- Problem: Simulations can only cover 2-3 orders of magnitude in length scale (due to limitations in computing power), while the physical processes on the Sun act over at least 6 orders of magnitude.
- Also, simulations can only cover a part of the size scale of solar convection, either granulation, supergranulation, or larger scales, but not all.

Convection simulations II

- Simulations do not achieve the solar Reynolds number (R_e = vl/v) of 10¹⁰, where v = typical velocity, l = typical length scale, v = kinematic viscosity (R_e ~ ratio of viscous to advection time scales).
- For comparison with observations it is important that the simulations describe the surface layers well, i.e. code needs to consider:
 - radiative transport of energy. Only few simulation codes do this properly.
 - partial ionization of many elements
 - as low a viscosity as numerically possible
- The role of radiation is primarily to transport energy. At the solar surface the energy transported by radiation becomes comparable to that by convection.

Simulations of solar granulation





Solution of Navier-Stokes equation etc. describing fluid dynamics in a box (6000 km² x 1400 km) containing the solar surface. Realistic looking granulation is formed.





Granule evolution

- Granules die in two ways:
 - dissolve: grow fainter and smaller until they disappear (small granules)
- split: break into two smaller granules (large granules)
- Granules are born in two ways:
 - as fragments of a large splitting granule
 - appearing as small structures and growing
- Initially most granules grow in size, some keep growing until they become unstable (see next slide) and split, others stop growing and start shrinking until they disappear (all within 5-10 min).

Granule evolution II

- Why do large granules split and small granules get squeezed out of sight?
- Granules are overshooting convection structures, with an upflow in their centre, a radial horizontal flow over the whole granule and a downflow in the surrounding lanes.
- The upflow builds up density and excess pressure above the granule → pressure gradient relative to flanks of granule.
- Pressure gradient = force.
- Gas is accelerated sideways.
- Above intergranular lane: horizontally flowing gas meets gas from opposite granule → build up pressure excess which decelerates flow.

Granule evolution III

- Consider mass conservation: A larger granule will build up a larger pressure above its centre because more mass needs to be accelerated horizontally.
- At some point the pressure becomes so large that the upflow is quenched. The centre of the granule cools and a new downflow lane forms there. The granule splits

Relation between granules and supergranules

- Downflows of granules keep going down to bottom of simulations. However, the intergranular lanes break up into individual narrow downflows.
- I.e. topology of flow reverses with depth:
 At surface: isolated upflows, connected downflows
 At depth: connected upflows, isolated downflows
- Idea put forward by Spruit et al. 1990: At increasingly greater depth the narrow downflows from different granules merge, forming a larger and less fine-meshed network that outlines the supergranules.

Increasing size of convective cells with depth



Figure 7 Flow lines showing the merging of the downdrafts on successively larger scales (schematic). The boxes cut out illustrate how the same process occurs on (in this illustration) three different scales.

Convection on other stars

- F, G, K & M stars posses outer convection zones and show observable effects of convection (also WDs)
- Observations are difficult since surfaces cannot be resolved.
- Use line bisectors: independent of spatial resolution
- A,F stars show inverse bisectors: granulation has different geometry.



Oscillations and helioseismology

5-minute oscillations

- The entire Sun vibrates from a complex pattern of acoustic waves, with a period of around 5 minutes
- The oscillations are best seen as Doppler shifts of spectral lines, but also as intensity variations.
- Identified as acoustic waves, called p-modes
- Spatio-temporal properties of oscillations best revealed by 3-D Fourier transforms.





Solar Eigenmodes

- The p-modes show a distinctive dispersion relation (k-ω diagram: k~ ω²)
- Important: there is power only in certain ridges, i.e. for a given k² (= k_x² + k_y²), only certain frequencies contain power.
- This discrete spectrum suggests the oscillations are trapped, i.e. eigenmodes of the Sun.



Global oscillations

- The Sun's acoustic waves bounce from one side of the Sun to the other, causing the Sun's surface to oscillate up and down. They are reflected at the solar surface.
- Modes differ in the depth to which they penetrate: they turn around because sound speed ($C_S \sim T^{1/2}$) increases with depth (refraction)
- p-modes are influenced by conditions inside the Sun. E.g. they carry info on sound speed
- By observing these oscillations on the surface we can learn about the structure of the solar interior

Description of solar eigenmodes

- Eigen-oscillations of a sphere are described by spherical harmonics
- Each oscillation mode is identified by a set of three parameters:
 - n = number or radial nodes
 - *l* = number of nodes on the solar surface
 - m = number of nodes passing through the poles (next slide)



Illustration of spherical harmonics

l = total number of nodes (in images: *l* = 6) = degree *m* = number of nodes connecting the "poles"



Spherical harmonics

Let v(θ, φ, t) be the velocity, e.g. as measured at the solar surface over time t. Then:

$$v(\theta, \varphi, t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm}(t) Y_l^m(\theta, \varphi)$$

The temporal dependence lies in a_{im} , the spatial dependence in the spherical harmonic Y_i^m .

 $Y_l^m(\theta, \varphi) = P_l^{|m|}(\theta) \exp(im\varphi)$ $P_l^{|m|}(\theta) = \text{associated Legendre Polynomial}$

- Due to the normalization of the spherical harmonic, the Fourier power is given by $F(a)F(a)^*$
- Here F(a) is the Fourier transform of the amplitude a_{im}

More examples and a problem with identifying spherical harmonics

General problem: Since we see only half of the Sun, the decomposition of the sum of all oscillations into spherical harmonics isn't unique.

This results in an uncertainty in the deduced *l* and *m*



Interpretation of *k-ω* or *v-l* diagram

- At a fixed *l*, different frequencies show significant power. Each of these power ridges belongs to a different order *n* (*n* = number of radial nodes), with *n* increasing from bottom to top.
- Typical are small values of *n*, but intermediate to large degree *l*.



A few observational remarks

- 10⁷ modes are present on the surface of the Sun at any given time (and interfering with each other).
- Typical amplitude of a single mode: < 20 cm/s</p>
- Total velocity of all 10⁷ modes: a few 100 m/s
- Accuracy of current instruments: better than 1 cm/s
- Frequency resolution ~ length of time series (Heisenberg's uncertainty principle) ~ lowest detectable frequency
- Longer time series are better
- Gaps in time series produce side lobes (i.e. spurious peaks in the power spectrum)
- Highest detectable frequency ~ cadence of obs.

Accuracy of frequency measurements

- Plotted are identified frequencies and error bars (yellow; 1000 σ for blue freq., 100σ for red freq. below 5 mHz and 1σ for higher freq.)
- Best achievable freq. resolution: a few parts in 10⁵; limit set by mode lifetime ~100 d



Frequency vs. amplitude

- Frequencies are the important parameter, more so than the amplitudes of the modes or of the power peaks.
- The amplitudes depend on the excitation, while the frequencies do not. They carry the main information on the structure of the solar interior.
- p-modes are excited by turbulence, which excites all frequencies. However, only at Eigenfrequencies of the Sun can eigenmodes develop.
- Frequencies (being more constant) are also measured with greater accuracy.

The measured low-*l* eigenmode signal

- Sun seen as a star: Due to cancellation effects, only modes with l=0,1,2 are visible \rightarrow simpler power spectrum.
- Low *l* modes are important for 2 reasons:
 - They reach particularly deep into the Sun (see cartoon on earlier slide)
 - These are the only modes measurable on other Sun-like stars.
- These modes are sometimes called "global" modes.
- The different peaks of given *l* correspond to different *n* values (n = 15...25 are typical).





due to random re-excitation of the oscillation mode by turbulence



Types of oscillations

- Solar eigenmodes can be of 2 types:
 - p-modes, where the restoring force is the pressure, i.e. normal sound waves
 - g-modes, where the restoring force is gravity (also called buoyancy modes)
- So far only p-modes have been detected on the Sun with certainty.
- They are excited by the turbulence associated with the convection, mainly the granulation near the solar surface (since there the convection is most vigorous).
- Being p-modes, they travel with the sound speed C_S . They dwell longest where C_S is lowest. Since $C_S \sim T^{1/2}$, this is at the solar surface.

p-modes vs. g-modes

- p-modes propagate throughout the solar interior, but are evanescent (later slide) in the solar atmosphere
- g-modes propagate in the radiative interior and in the atmosphere, but are evanescent in the convection zone (their amplitude drops exponentially there, so that very small amplitudes are expected at the surface). Convection means buoyancy instability; oscillations require stability.
- g-modes are expected to be most sensitive to the very core of the Sun, while p-modes are most sensitive to the surface
- Current upper limit on solar interior g-modes lies below 1 cm/s.

Solar oscillations: simple treatment

 Equations describing radial structure of adiabatic oscillations, neglecting any perturbations to the gravitational potential, are:

$$\frac{1}{r^2}\frac{d}{dr}(r^2\xi_r) - \frac{\xi_r}{c^2} + \frac{1}{\rho_0}\left(\frac{1}{c^2} - \frac{l(l+1)}{r^2\omega^2}\right)P_1 = 0$$
$$\frac{1}{\rho_0}\frac{dP_1}{dr} + \frac{g}{\rho_0c^2}P_1 - (\omega^2 - N^2)\xi_r = 0$$

- where $N^2 = g\left(\frac{1}{\Gamma P_0}\frac{dP}{dr} \frac{1}{\rho_0}\frac{d\rho_0}{dr}\right) =$ Brunt Vaisala frequency
- Here ξ_r is radial displacement and P_1 is pressure perturbs. Quantities with subscript 0 refer to the unperturbed Sun. Γ is the adiabatic exponent: $\Gamma = \left(\frac{d \ln P}{d \ln \rho}\right)$

Solar oscillations: simplified treatment II

 Analytical solutions of these equations for an isothermal atmosphere are readily obtained:

$$\xi_r(r) \sim \rho_0^{-1/2} \exp(ik_r r)$$
$$P_1(r) \sim \rho_0^{1/2} \exp(ik_r r)$$

■ These oscillations are trapped in the body of the Sun. Since the time scale on which the atmosphere reacts to disturbances is low, waves which are travelling in the solar interior are evanescent in the atmosphere → They are present only for discrete frequencies (similar to the bound states in atomic physics).

Regimes of oscillation

- In regimes of acoustic and gravity waves $k_r^2 > 0$, while in regime of evanescent waves $k_r^2 < 0$ (exponential damping). The solid lines show $k_r^2 = 0$.
- Evanescent waves occur when the period is so long that the whole (exponentially stratified) medium has time to adapt to the perturbation, achieving a new equilibrium. Therefore the wave does not propagate, but rather the medium as a whole oscillates.



Deducing internal structure from solar oscillations

- Global helioseismology: Gives mainly the radial dependence of solar properties, although latitudinal dependence can also be deduced (ask R. Mecheri).
 - Radial structure of sound speed
 - Structure of differential rotation
- Local helioseismology: Allows in principle 3-D imaging of solar interior. E.g. time-distance helioseismology does not measure frequencies, but rather the time that a wave requires to travel a certain distance (relatively new)

Global helioseismology

- Use frequencies of many modes.
- Basically two techniques for deducing information on the Sun's internal structure
 - Forward modelling: make a model of the Sun's internal structure (e.g. standard model discussed earlier), compute the frequencies of the eigenoscillations of the model and compare with observations
 - Inverse technique: Deduce the sound speed and rotation by inverting the oscillations (i.e. without any comparison with models)
- Note that forward modelling is required in order to first identify the modes. Only after that can inversions be carried out.

Testing the standard solar model: results of forward modelling

- Relative difference between C_S² obtained from inversions and from standard solar model plotted vs. radial distance from Sun centre.
- Typical difference: 0.002 → good!
 Typical error bars from inversion:
- 0.0002 → poor!
- Problem areas:
 solar core
- solar core
 bottom of CZ
 - solar surface



Variation of the solar rotation velocity with period of 1.3 years



Local excitation of wave by a flare



Clear example of wave being triggered. The wave is not travelling at the surface, but rather reaching the surface further out at later times. Note how it travels ever faster. Why?

Local helioseismology

- Does not build upon measuring frequencies of eigenmodes, but rather measures travel times of waves through the solar interior, between two "bounces" at the solar surface (for particular technique of time-distance helioseismology).
- The travel time

between source and first bounce depends on the structure of C_s below the surface. By considering waves following different paths inhomogeneous distributions of C_s can be determined.



Local helioseismology II

- Temperature and velocity structures can be distinguished, since a flow directed with the wave will affect it differently than a flow directed the other way (increase/decrease the sound speed).
- By considering waves passing in both directions it is possible to distinguish between T and velocity.
- At right: 1st images of convection zone of a star!



Time-Distance Helioseismology of a sunspot

- Subsurface structure of sunspots
- Sunspots are good targets, due to the large temperature contrast.
- Major problem: unknown influence of the magnetic field on the waves.



Time-Distance Helioseismology of a sunspot II



Subsurface Structure of Sunspots

- How can sunspots last for several weeks without flying apart?
- Models: inflows
- Observations at surface: outflows (moat flow)
- Strong inward flows right underneath the surface
- Sunspots surprisingly shallow: become warmer than surroundings already some 4000 km below surface
- Results to be confirmed (unkown influence of B)

Seeing right through the Sun

- A technique, called twoskip far-side seismic holography, allows images of the far side of the Sun to be made.
- Waves from front go to back and then return.
- Acoustic waves speed up in active regions (hotter in subsurface layers)
- The delay of the sound waves is about 12 sec in a total travel time of 6 hours





Helioseismology instruments

Needed:

- uninterrupted, long time series of observations
- Either high velocity sensitivity, or high intensity sensitivity (and extremely good stability)
- Low noise
- Spatial resolution better than 1" (for local helioseismology)
- Instruments are either:
 - Ground based global networks (GONG+, BiSON)
 - Space based instruments in special full-Sun orbits (advantage of lower noise relative to ground-based networks; MDI, GOLF, VIRGO on SOHO, HMI on SDO)
 - Usually filter instruments with high spectral or intensity fidelity

Instruments and projects

- Ground (networks of 3 or more telescopes aimed at reducing the length and number of data gaps)
 - GONG+
 - BISON
 - TON
- Space (uninterrupted viewing, coupled with lack of noise introduced by the atmosphere)
 - SOHO MDI, GOLF and VIRGO (running)
 - SDO HMI (being built)
 - Solar Orbiter VIM (planned)

Asteroseismology

First reliable detection of oscillations on the near solar analogue, α Centauri, and other Sun-like stars. Note the shift in the pmode frequency range to lower values for α Centauri, which is older than the Sun (note also factor 10³ difference in ν scale)



Projects

- Major asteroseismic Space missions:
 - COROT
 - Kepler
- Ground based:
 - ESO 3.6m (HARPS)
 - ESO VLT (UVES)
 - Networks of smaller Telescopes



Solar rotation

- Solar rotation
- The Sun rotates differentially, both in latitude (equator faster than poles) and in depth (more complex).
- Standard value of solar rotation: Carrington rotation period: 27.2753 days (the time taken for the solar coordinate system to rotate once).
- Sun's rotation axis is inclined by 7.1° relative to the Earth's orbital axis (i.e. the Sun's equator is inclined by 7.1° relative to the ecliptic).

Discovery of solar rotation

- Galileo Galilei and Christoph Scheiner noticed already that sunspots move across the solar disk in accordance with the rotation of a round body
- Sun is a rotating sphere
- Movie based on Galileo Galilei's historical data



Surface differential rotation

- Poles rotate more slowly than equator.
- Surface differential rotation from measurements of:
 - Tracers, such a sunspots or magnetic field elements (always indicators of the rotation rate of the magnetic field)
 - Doppler shifts of the gas
 - Coronal holes (not plotted) rotate rigidly



Surface differential rotation

Description:

 $\mathbf{\Omega} = A + B \sin^2 \psi + C \sin^4 \psi$

where ψ is the latitude, $A = \Omega$ at the equator and $A+B+C = \Omega$ at the poles.

Different tracers give different *A*, *B*, *C* values. E.g. spots rotate faster than the surface gas.

How come different rotation laws?

- Are different tracers anchored at different depths in the convection zone?
 - Evidence in support comes from sunspots: young spots rotate faster than older spots (→ older spots are slowed down by the surrounding gas)
- How come coronal holes rotate rigidly, while the underlying photospheric magnetic field rotates differentially?
 - Individual magnetic features must move in and out of coronal holes
 - Support: evidence for enhanced magnetic reconnection at the edges of coronal holes

Internal differential rotation

Method: Helioseismic inversions

- In a non-rotating star the individual modes of oscillation, described by "quantum numbers" n,l,m are degenerate in that their frequency depends only on n and l, but not on m.
- Similar to Zeeman effect. Note that *m* distinguishes between the surface distribution of oscillation nodes. For a spherically symmetric star (no rotation) all these modes must have same frequency.
- In a rotating Sun the degeneracy is removed and modes with different *m* have slightly different frequency.
- Since modes with different / sample the solar latitudes in different ways, it is possible to determine not just vertical, but also latitudinal differential rotation by helioseismology.

Internal differential rotation II

Structure of internal rotation deduced from MDI dataNote: differential rotation in CZ, solid rotation below



Internal differential rotation III : tachocline





What about rotation of solar core?

- Rotation rate of solar core is not easy to determine, since p-modes are rather insensitive to the innermost part of the Sun.
- Different values in the literature for the core's rotation rate: $\Omega(r=0) = \Omega(r=R_{\odot}) \dots 2 \Omega(r=R_{\odot})$
- One way to set limits on Ω(r=0): quadrupole moment of Sun.
- Solar rotation leads to oblateness, i.e. diameter is larger at equator than between the poles.
- If core rotates more rapidly than surface, then oblateness will be larger than expected due to surface rotation rate.

Solar oblateness

Oblateness = ΔR/R_☉

- Direct measurements: ΔR/R_☉ ≈ 10⁻⁵
 - Very tricky, since oblateness 10⁻⁵ corresponds to ΔR = 14 km (best spatial resolution achievable: 100 km).
 - Systematic errors due to concentration of magnetic activity to low latitudes → affects measurements of solar diameter, since shape of limb is distorted.
 - Initial measurements due to Dicke & Goldenberg (1967) gave $\Delta R/R_{\odot} \approx 5 \times 10^{-5} \rightarrow required change of general$ relativity to explain motion of Mercury's perihelion (butwas consistent with Brans-Dicke gravitation theory)
- Helioseismic measurements give for the acoustic radius of the Sun (which is not the same as the optical radius, but similar): $\Delta R/R_{\odot} \approx 10^{-5}$ (Redouane Mecheri)

Evolution of solar rotation

- Young stars are seen to rotate up to 100 times faster than the Sun.
- Did the Sun also rotate faster when it was young?
- Skumanich law: $\Omega \sim t^{-1/2}$, where t is the age of the star (deduced from observing stars in clusters of different ages).
- Sun also rotated faster as a young star.
- Question: where did all the angular momentum go?

Evolution of solar rotation II

- Question: where did all the angular momentum go?
- Answer part 1: Solar wind! The solar wind carries away angular momentum with it. Torque *j*, i.e. rate of change in angular momentum, exerted by solar wind (without magnetic field):

$j = \Omega R_{\odot} dm/dt$

- Here *dm/dt* is the solar mass-loss rate (mass carried away by solar wind)
- Problem: *j* is 2-3 orders of magnitude too small to cause a significant braking of solar rotation...

Evolution of solar rotation III

Answer part 2: Magnetic field!

Solar wind is channeled by magnetic field up to the Alfven radius R_A , i.e. point where wind speed > Alfven speed. Up to that radius, the wind rotates rigidly with the solar surface (forced to do so by rigid field lines), i.e. it only carries angular momentum away beyond R_A . Proper expression for Torque:

$j = \Omega R_A dm/dt$

R_A typically is 10-20 times larger than R_☉.

Evolution of rotation IV

$= j = \Omega R_A dm/dt \sim \Omega$

→ The faster the star rotates, the quicker it spins down.

- Additional corrections:
 - *dm/dt* depends on Ω (more rapidly rotating star, more magnetic field, hotter the corona, larger the *dm/dt*)
 - *R_A* depends on Ω (although not in a straightforward manner: more rapidly rotating star, more magnetic field, but also larger the density and velocity of wind).
- In general $j = k\Omega^{\alpha}$, where α typically > 1, although there are signs that for very large Ω , the α value becomes very small (saturation).

