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Exercise for Solar Physics (2008) - Part 3

Chapter 5: Radiation and spectrum

5.1 Optical depth

Give a physical interpretation of optical depth.

5.2 Limb darkening

(A) What is limb darkening?

(B) The limb darkening can be approximated by $I(\theta) \sim \cos\theta$ to the first order. Here $I(\theta)$ denotes the intensity at the angle to the local vertical direction θ (for the optical depth $\tau = 0$).

1. Derive the surface intensity at $\tau = 0$ in the form

$$I(0,\theta) = \int_0^\infty S(\tau) e^{-u} du$$

from the radiative transfer equation

$$\cos\theta \frac{dI(\tau,\theta)}{d\tau} = I(\tau,\theta) - S(\tau)$$

by multiplying by an integrating factor $\exp(-\tau/\cos\theta) = \exp(-u)$. $S(\tau)$ is called the source function.

2. Assume a linear dependence for the source function

$$S(\tau) = a + b\tau$$

and obtain the cosine dependence for the surface intensity

$$I(0,\theta) = a + b\cos\theta,$$

where a and b are constants.

Chapter 6: Convection

6.1 Onset of convection

The onset of the convection can be derived from the argument of entropy. A fluid element is convectively unstable if its entropy decreases in a certain direction (outward in the radial direction in the solar physics context), viz.,

$$ds < 0 \quad \text{(radially outward)},$$

where the entropy is defined as

$$s = \frac{P}{\rho^\gamma}.$$

Here P and ρ denote the pressure and the mass density of the fluid element, respectively, and γ the polytropic index (the ratio of specific heat). Note that we do not give the equation of state yet.

1. Derive the condition for convection in the form

$$\frac{d(\log P)}{dr} - \gamma \frac{d(\log \rho)}{dr} < 0 \quad \text{(radially outward)},$$

where dr denotes the line element in the radial direction.

2. Derive the condition for convection, on the assumption of the equation of state for ideal gas, in the form

$$\frac{d(\log T)}{d(\log P)} < \frac{\gamma - 1}{\gamma} \quad \text{(radially outward)},$$

where T denotes the temperature. In the solar physics the left hand side represents the temperature gradient given by radiative energy transport (∇_{rad}), and the right hand side the adiabatic gradient (∇_{ad}). In the radially *inward* direction (often used in the solar physics) the condition reads as $\nabla_{\text{rad}} > \nabla_{\text{ad}}$.

6.2 Granulation model

Consider a mass flow $\rho\vec{v}$ that horizontally varies like $\sin kx$, and has a vertical scale height H. Show that the ratio of horizontal to vertical velocity components scales as

$$\frac{v_h}{v_r} \simeq \frac{1}{kH}.$$

For given H, therefore, the ratio v_h/v_r increases with increasing cell size.

Chapter 7: Atmosphere

7.1 Corona temperature

In place of the strong Calcium (Ca^+) H and K lines (wavelengths $\lambda \simeq 400\,\mathrm{nm}$) rather shallow and broad dips (width $\Delta\lambda \simeq 20\,\mathrm{nm}$) can be noticed in the spectrum of K corona. Assume that the dips are in fact the Doppler broadening of the line spectrum and derive a temperature of particles. Note that Thomson scattering (photon-electron scattering) is one of the major processes in the K corona. In this way Grotrian (1931) first concluded that the corona might be hot.