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Exercise for Solar Physics (2008) - Part 4

Chapter 8: Magnetic fields and atmospheric dynamics

8.1 Magnetic pressure

Where does the expression for magnetic pressure come from? Can you see why it called magnetic pressure? Another term results from this as well, what is its function?

You will need here the MHD momentum equation ($\rho\frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j}\times\mathbf{B} + \rho\mathbf{g}$), Ampère's law ($\mathbf{j} = \frac{1}{\mu_0}\nabla\times\mathbf{B}$) and a certain triple vector identity ($\mathbf{A}\times(\nabla\times\mathbf{A}) = (\nabla\mathbf{A})\cdot\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{A} = \frac{1}{2}\nabla(\mathbf{A}\cdot\mathbf{A}) - (\mathbf{A}\cdot\nabla)\mathbf{A}$).

8.2 Flux tubes and the canopy

In an isothermal system the external pressure, p_e, and the pressure inside a small flux tube, p_i, vary vertically with the same scale height, H: $p_e = p_{e0}e^{-z/H}$ and ditto for p_i. If we assume a pressure balance exists between the tube and its surroundings ($p_{tot,e} = p_{tot,i}$), how does the magnetic field strength vary with height? How about the radius of the tube (note that the magnetic flux in the tube must be conserved)?

If we have a collection of these tubes with a density of n tubes per surface area, at what height will the tubes merge, i.e., above which height is the entire volume filled with magnetic field? How does it depend on B_0 and the spatial average magnetic field strength on the surface?

On the next page you can find a figure of a semi-empirical plane-parallel solar atmospheric model which mimics the quiet Sun (VALC, Vernezza et al., *Astrophys. J. Suppl.*, *45*, 635, 1981). From this you can get a rough estimate for what the scale height is. Now compute the merging height if B_0 is 1.5 KG assuming that the average magnetic field is 4 G (quiet areas) or 200 G (plage).

8.3 Plasma-β

Plasma-β measures the ratio of the gas and magnetic pressures. Where in the VALC model is the plasma-β equal unity if the magnetic field has a constant strength of a) 1000 G, b)100 G, c)10 G? How do you think the $\beta = 1$ layer looks like in the real Sun?

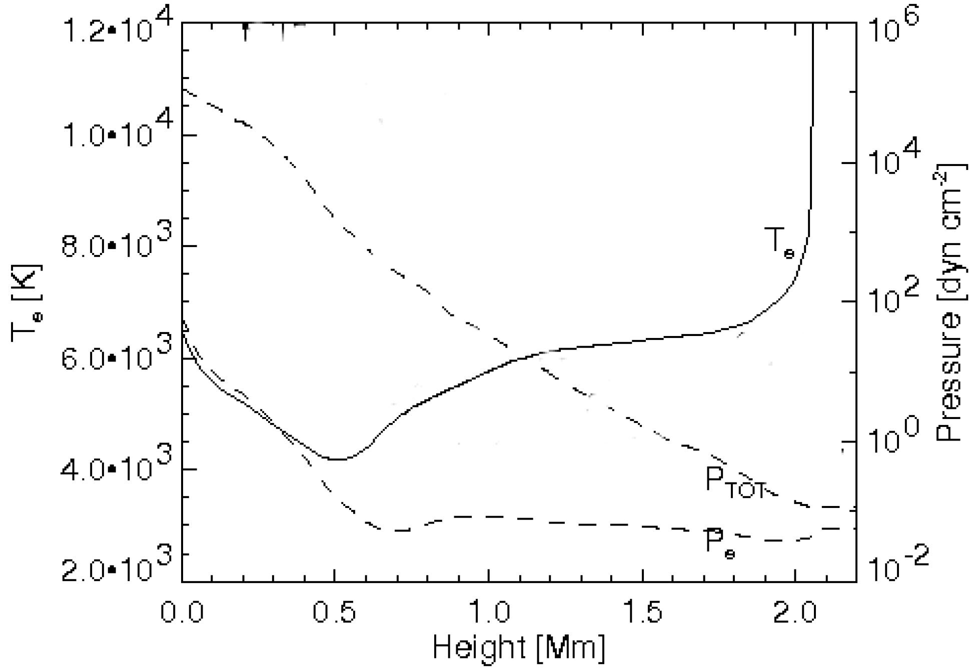

Figure 1: VALC atmospheric model (Vernazza et al., 1981).

8.4 Dynamical time scales

Making realistic simulations of the solar atmosphere is computationally very expensive, especially in the chromosphere, so simplifications and short cuts are needed. One such short cut is assuming statistical equilibrium for the level populations, i.e,. the populations adjust immediately to changing conditions. Is this a valid approximation? (Think in terms of dynamic timescales, e.g., timescale for hydrogen ionization/recombination increases from 1 s in the photosphere to 10^5 s in the mid-chromosphere where it starts to decrease again becoming 10^2 s at the base of the transition region.) Why is hydrogen sometimes referred to as a thermostat?