Max Planck Institute for Solar System Research
Prof. S. Solanki with assistants R. Cameron, J. Graham, Y. Narita, and A. Pietarila

Solutions to Exercise Solar Physics (2008)

Chap.8 Magnetic fields and atmospheric duynamics

Magnetic pressure

The Lorenz force $\vec{j} \times \vec{B}$ gives two terms when coupled to the Ampère's law (neglecting the displacement current)

$$\vec{j} \times \vec{B} = \frac{1}{\mu_0}(\nabla \times \vec{B}) \times \vec{B} \quad (1)$$

$$= \frac{1}{\mu_0}(\vec{B} \cdot \nabla)\vec{B} - \nabla \left(\frac{B^2}{2\mu_0} \right). \quad (2)$$

Here the first term is called the magnetic tension (the force acting at curved magnetic field, trying to straighten the field), and the second term is called the magnetic pressure as it resembles the form of the pressure gradient.

8.2 Flux tubes and the canopy

Total pressure is the gas and the magnetic pressure, $P_{\rm tot} = P_{\rm gas} + P_{\rm mag}$. We treat the total pressure constant at each height z. If we apply the constant total pressure inside and outside the flux tube at base ($z = 0$), we have

$$\frac{B_0^2}{2\mu_0} + P_{i0} = P_{e0}, \quad (3)$$

where the left hand side is the total pressure inside the tube and the right hand side outside the tube. The gas pressure itself, on the other hand, is in a hydrostatic equilibrium (to the gravity) and hence it decays exponentially to the scale height H. At height z the total pressure balance reads

$$\frac{B^2(z)}{2\mu_0} + P_{i0}e^{-z/H} = P_{e0}e^{-z/H}. \quad (4)$$

Using the above two equations, we obtain

$$\frac{B(z)}{B_0} = e^{-z/2H}. \quad (5)$$

The magnetic field decreases also with height (but more slowly because the effective scale height is doubled).

Conservation of the magnetic field flux is

$$\Phi = \pi r^2 B = \text{const}, \tag{6}$$

where r denotes the radius of the tube. We compare the flux at different heights,

$$\pi r_0^2 B_0 = \pi r^2 B, \tag{7}$$

where the lhs is at base ($z = 0$) and the rhs is at height z. This gives a scaling of the tube radius as a function of the height

$$\frac{r}{r_0} = e^{+z/4H}, \tag{8}$$

that is, the radius becomes larger with height.

The height of merging is estimated as follows. The average separation of the tubes (from center to center) is about $1/\sqrt{n}$, where n is the density of the tubes. They will merge (contact each other) when the doubled radius (or diameter) reaches the separation distance,

$$r = \frac{1}{2\sqrt{n}}. \tag{9}$$

If we use now the scaling of the tube radius, $r/r_0 = e^{+z/4H}$, we obtain

$$r_0 e^{z/4H} = \frac{1}{2\sqrt{n}}, \tag{10}$$

or when squared,

$$r_0^2 e^{z/2H} = \frac{1}{4n}. \tag{11}$$

Therefore the merging height is

$$z = -2H \log(4nr_0^2), \tag{12}$$

or in terms of B,

$$z = -2H \log\left(\frac{4\langle B\rangle}{\pi B_0^2}\right), \tag{13}$$

where we used the averaged magnetic field

$$\langle B\rangle = n\pi r_0^2 B_0. \tag{14}$$

If we read the scale height H from the plot of P_{tot}, $H \simeq 100$ [km]. Note that at $z = H$ the pressure decreases by $1/e \simeq 0.37$ from the base. For $B_0 = 1500$ [G] and $\langle B\rangle = 4$ [G] (quiet areas) we obtain $z \simeq 1100$ [km]. For $\langle B\rangle = 200$ [G] we obtain $z \simeq 350$ [km]. In both cases the merging height is larger than the scale height H.

8.3 Plasma Beta

The plasma parameter β measures the ratio of the gas to the magnetic pressure,

$$\beta = \frac{P_{\text{gas}}}{P_{\text{mag}}}. \tag{15}$$

At $\beta = 1$ the total pressure can be written only with the magnetic field strength,

$$
\begin{aligned}
P_{\rm tot} &= 2P_{\rm mag} && (16)\\
&= \frac{B^2}{\mu_0} && (17)\\
&= \frac{B^2}{4\pi \times 10^7}\ [\mathrm{Pa}] && (18)\\
&= \frac{B^2}{4\pi \times 10^7} \times 10\ [\mathrm{dyn/cm^2}], && (19)
\end{aligned}
$$

where B is in units of T (Tesla). From the plot of $P_{\rm tot}$ we obtain the heights of $\beta = 1$ about 100 [km], 500 [km], and 1200 [km] for 1000 [G], 100 [G], and 10 [G], respectively.

While the surface with the constant total pressure may be smooth, the surface of "(gas pressure) = (magnetic pressure)" is warped and distorted very much in the real Sun (e.g. flux tubes, sunspots). The surface changes not only spatially but also temporally.

8.4 Dynamical time scale

Dynamical time scale in the chromosphere is typically 200-400 [s] (3-minute oscillations in internetwork and 5-7 minutes in network), while the time scale of hydrogen ionization and recombination is of the order 10^5 [s]. The system is therefore constantly evolving, trying to catch up with the dynamics. This results in hydrogen fluctuations that are smaller than those derived from the statistical equilibrium, i.e., the dynamics vary more than the populations. For hydrogen the ionization potential is high and relaxation time to an equilibrium is long. The concept of thermostat applies to the temperature plateau, where energy is used up to ionize hydrogen, which in turn leads to a high specific heat and also releases electrons that can through collisions excite other elements and thus the energy is radiated away instead of a temperture rise. The plateau ends when hydrogen is fully ionized and the transition region begins. On the other hand, the effect during shocks has an opposite sense to the thermostat: because of the long ionization time scales, hydrogen does not have enough time to be ionized during the shock compression phase and energy is used to increase the temperature, i.e., the increase of temperature over shock fronts is sharper when statistical equilibrium is *not* assumed.