Introduction to Solar Physics

Sami K. Solanki

IMPRS lectures

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Structure of lectures I

- Introduction and overview
- Core and interior: energy generation and standard solar model
- Solar oscillations and helioseismology
- Solar rotation
- Solar radiation and spectrum
  - Solar spectrum
  - Radiative transfer
  - Formation of absorption and emission lines
- Convection: The convection zone and granulation etc.

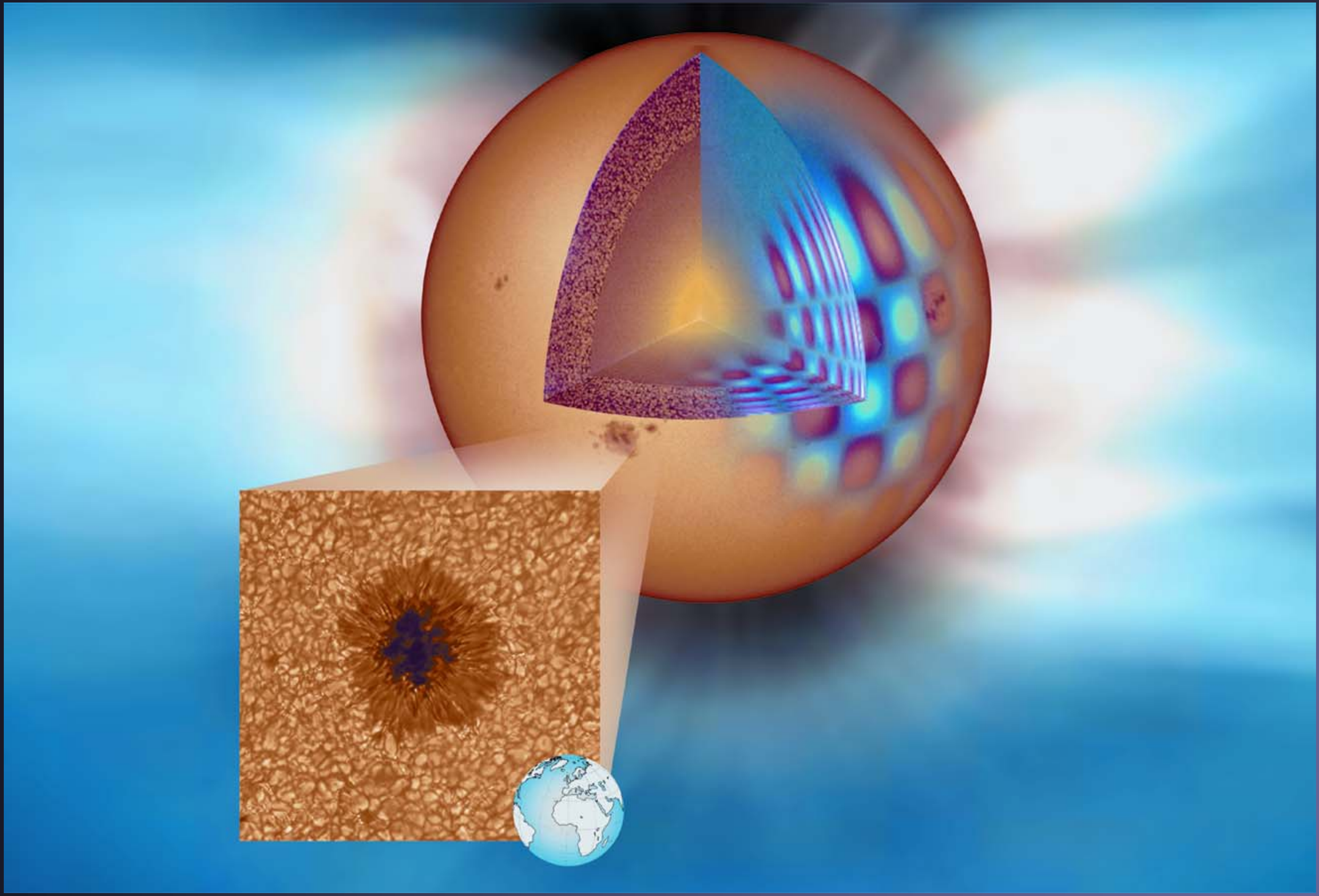
Structure of lectures II

- The solar atmosphere: structure
  - Photosphere
  - Chromosphere
  - Transition Region
  - Corona
  - Solar wind and heliosphere
- The magnetic field
  - Zeeman effect and Unno-Rachkovsky equations
  - Magnetic elements and sunspots
  - Chromospheric and coronal magnetic field
  - The solar cycle
  - Coronal heating

Structure of lectures III

- Solar cycle and solar dynamo
- Atmospheric heating
- Explosive and eruptive phenomena
  - Flares
  - CMEs
  - Explosive events
- Sun-Earth connection
  - CMEs and space weather
  - Longer term variability and climate
- Solar-stellar connection (not this time)
  - Activity-rotation relationship
  - Sunspots vs. starspots

The Sun: a brief overview



The Sun, our star

- The Sun is a normal star: middle aged (4.5 Gyr) main sequence star of spectral type G2
- The Sun is a special star: it is the only star on which we can resolve the spatial scales on which fundamental processes take place.
- The Sun is a special star: it provides almost all the energy to the Earth
- The Sun is a special star: it provides us with a unique laboratory in which to learn about various branches of physics.

The Sun: a few numbers

- Mass = 1.99 10^{30} kg (= 1 $M_{\odot}$)
- Average density = 1.4 g/cm^3
- Core density = 1.5×10^2 g/cm^3
- Luminosity = 3.84 10^{26} W (= 1 $L_{\odot}$)
- Effective temperature = 5777 K (G2 V)
- Core temperature = 15 10^6 K
- Surface gravitational acceleration g = 274 m/s^2
- Age = 4.55 10^9 years (from meteorite isotopes)
- Radius = 6.96 10^5 km
- Distance = 1 AU = 1.496 (+/-0.025) 10^8 km (≈8 light minutes)
- 1 arc sec = 722±12 km on solar surface (elliptical Earth orbit)
- Rotation period = 27 days at equator (synodic, i.e. as seen from Earth; Carrington rotation)

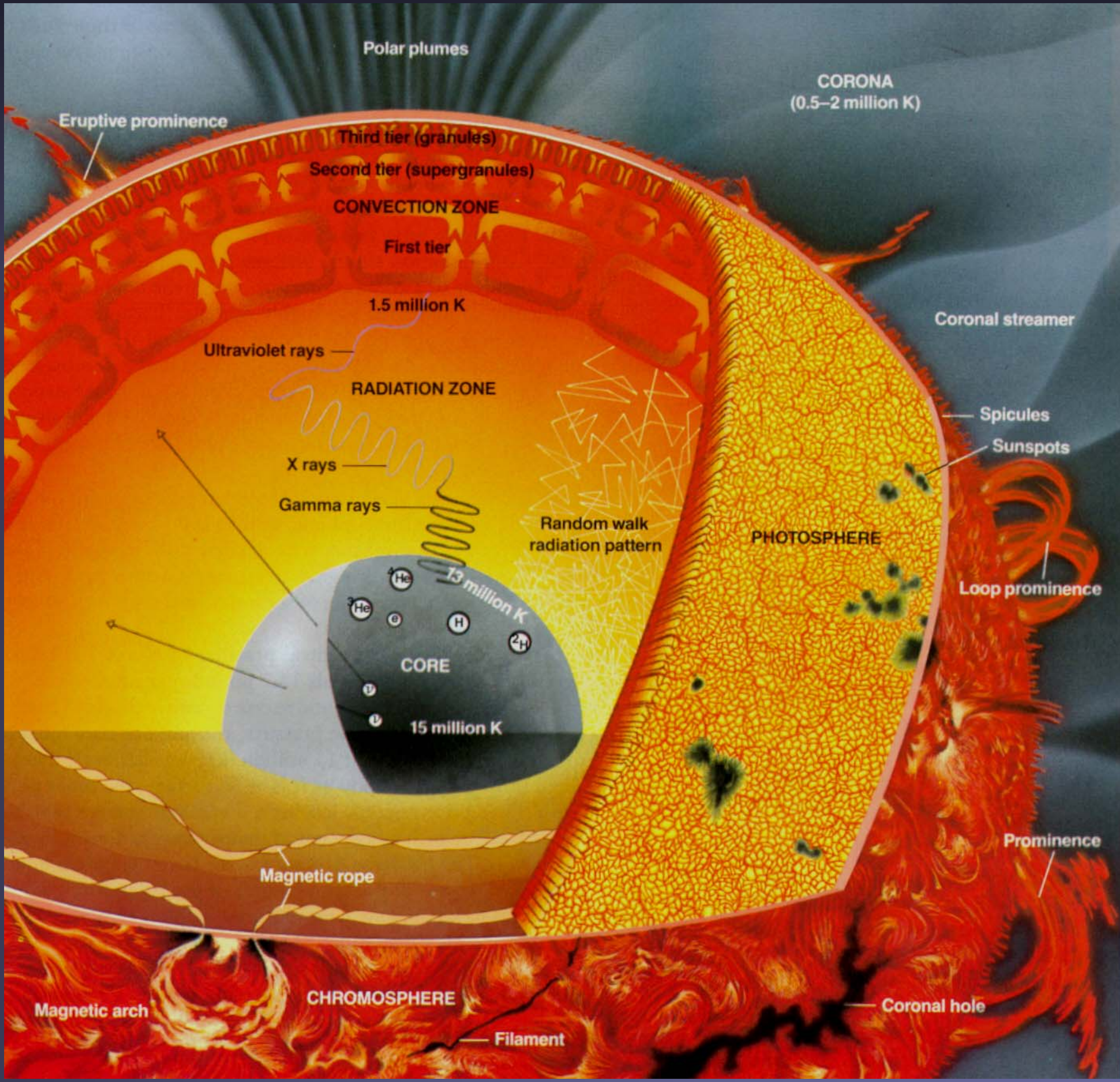
The Sun's Structure

Solar interior:

- Everything below the Sun's (optical) surface
- Divided into hydrogen-burning core, radiative and convective zones

Solar atmosphere:

- Directly observable part of the Sun.
- Divided into photosphere, chromosphere, corona, heliosphere

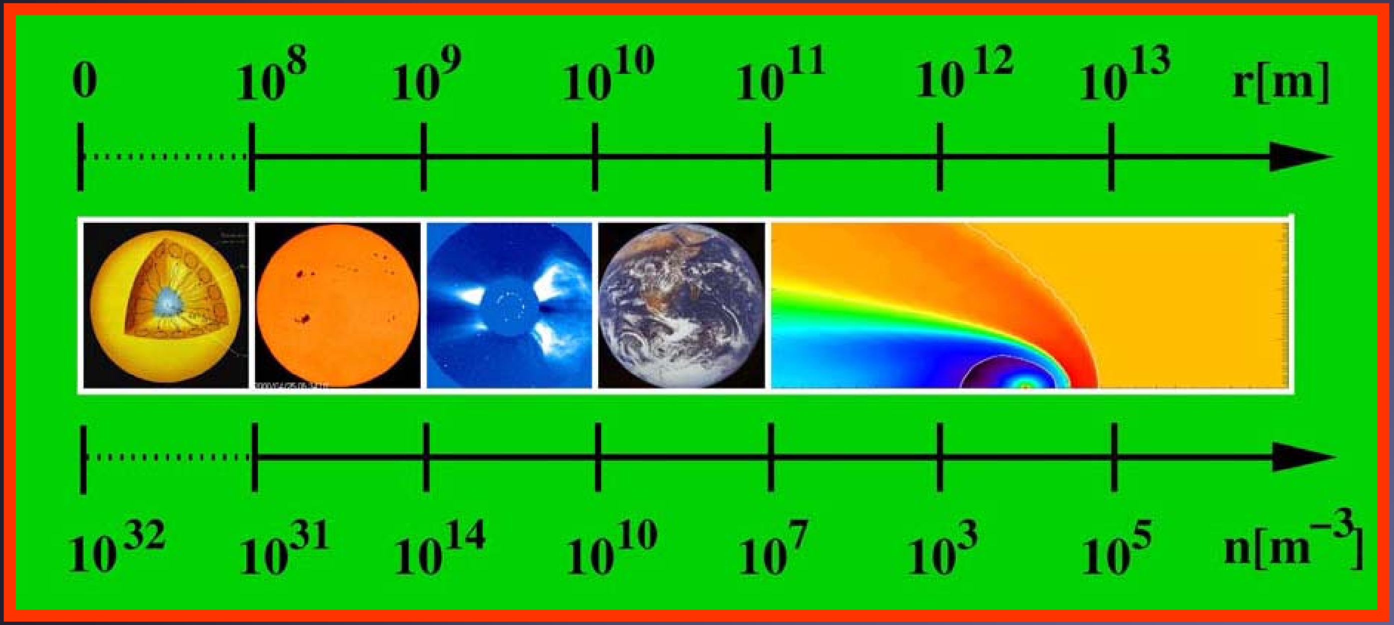


The solar surface

- Solar material exhibits no phase transition (e.g. from solid or liquid to gaseous as on Earth) ➔ define solar surface through its radiation.
- Photons in solar interior make a random walk, since they are repeatedly absorbed & reemitted. Mean free path increases rapidly with radial distance from the solar core (density and opacity decrease).
- Solar surface: where average vertical mean free path becomes so large that photons escape from Sun. Surface corresponds to optical depth $\tau = 1$. Its height depends on λ.
- Often $\tau = 1$ at $\lambda = 5000$ Å is used as standard for the solar surface.

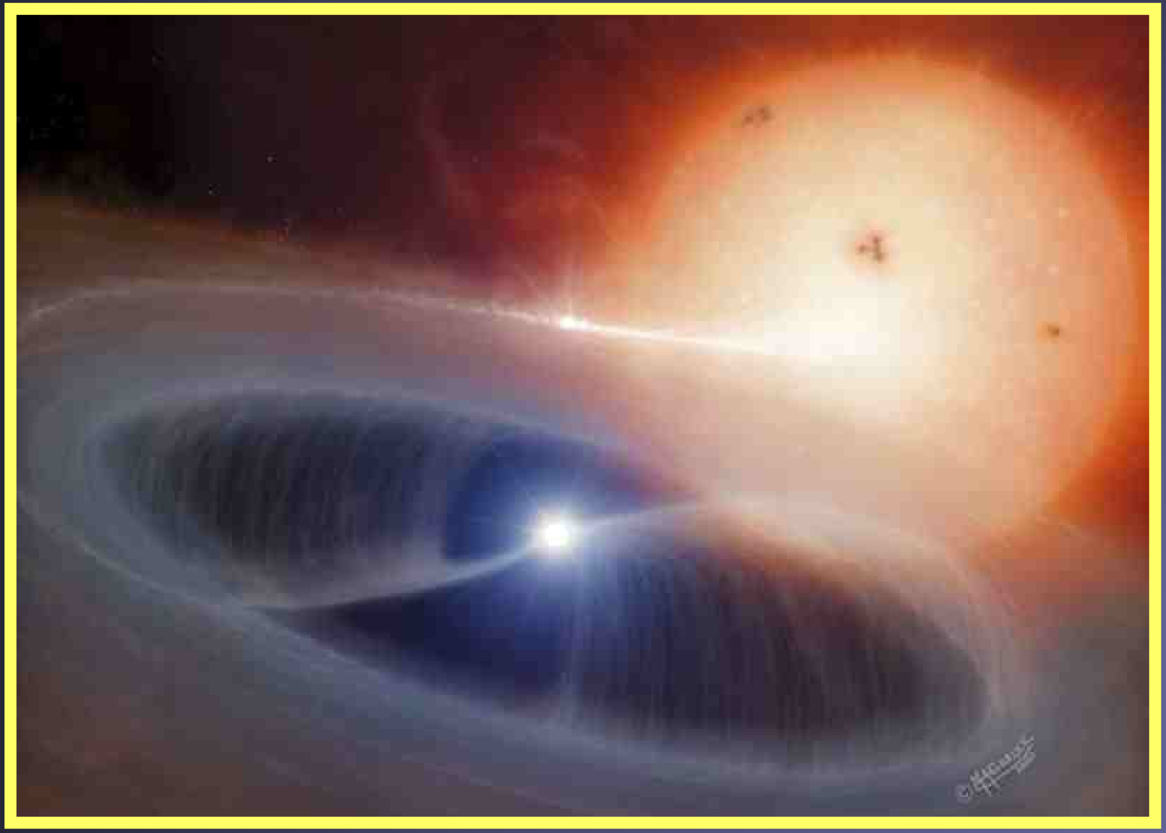
Wide range of physical parameters

- Between core and corona Sun presents a wide variety of physical phenomena and processes.
- E.g. Gas density varies by ≈ 30 orders of magnitude, temp. T by 4 orders, relevant time scales from 10^{-10} sec to 10 Gyr



➔ Different observational and theoretical techniques needed to study different parts of Sun, e.g. helioseismology & nuclear physics for interior, polarimetry & MHD for magnetism, etc.

Solar physics in relation to other branches of physics



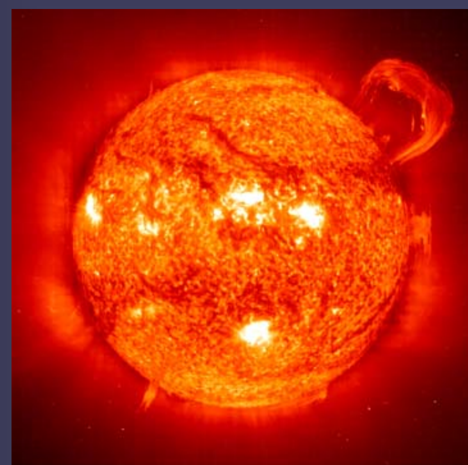
Solar Physics in Relation to Other Fields

Sun-Earth relations: climate, space weather

Turbulence, dynamos

Cosmic rays, local interstellar medium

Plasma physics



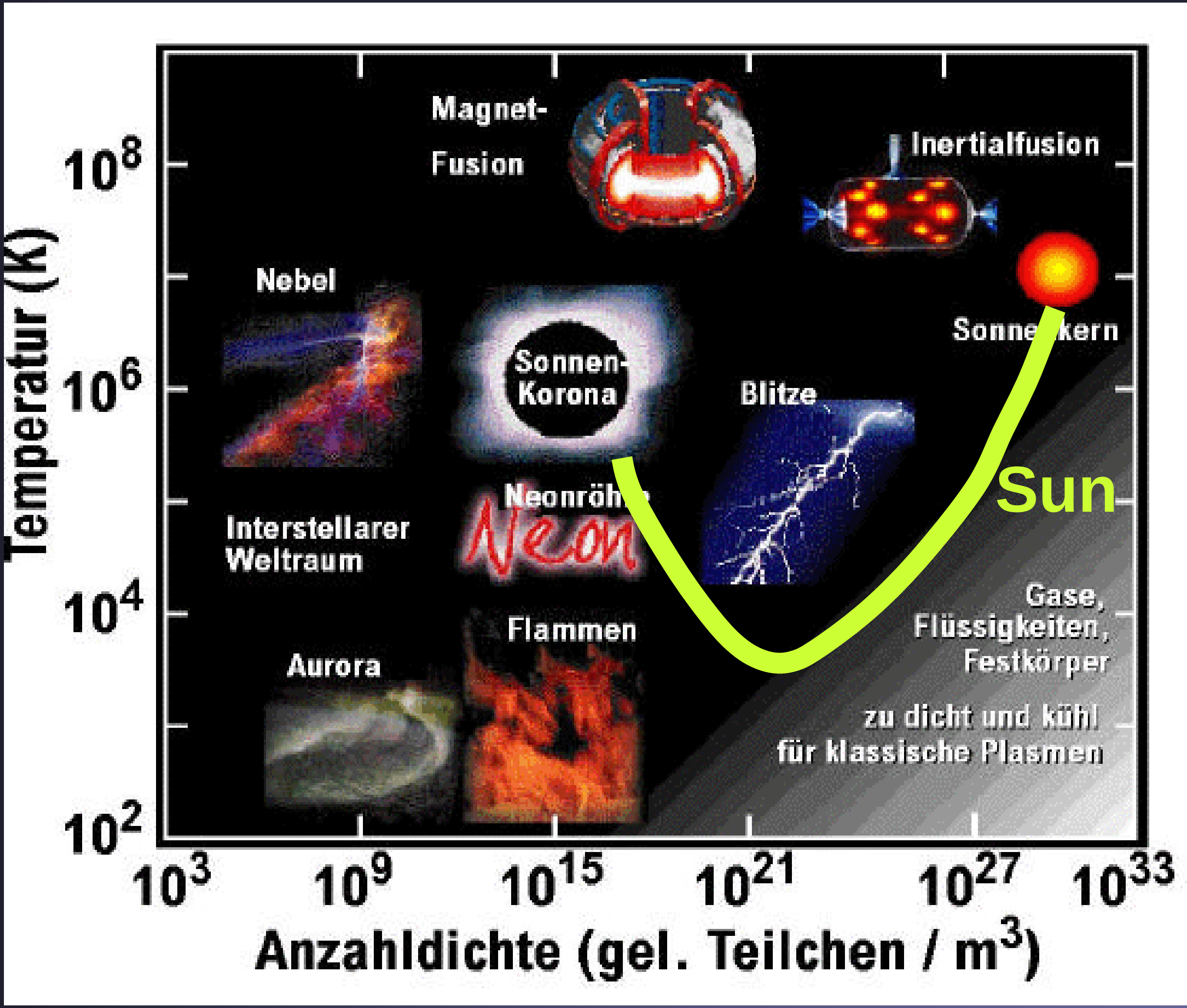
Solar planets, extrasolar planets

Atomic/molecular physics

Cool stars: activity, structure & evolution

Fundamental physics: Neutrinos, Gravitation

The Sun as a plasma physics lab.

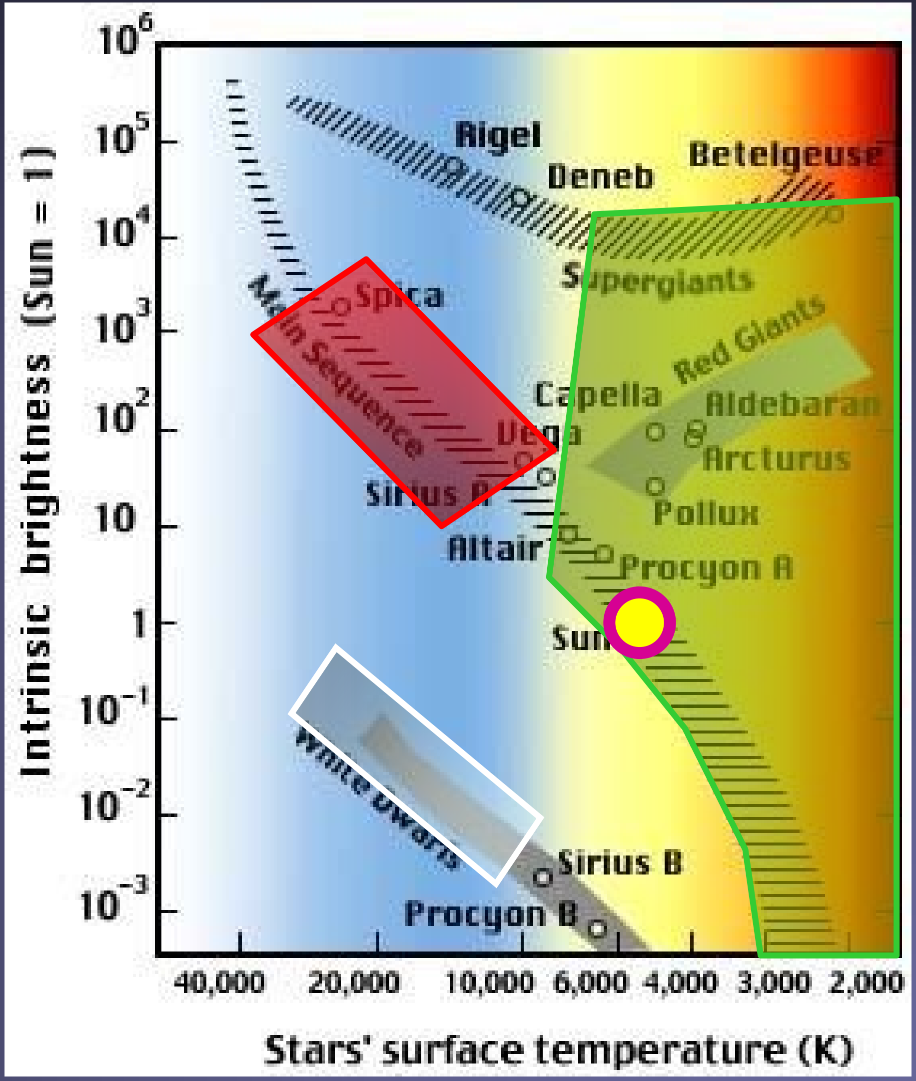


Solar Tests of Gravitation Physics

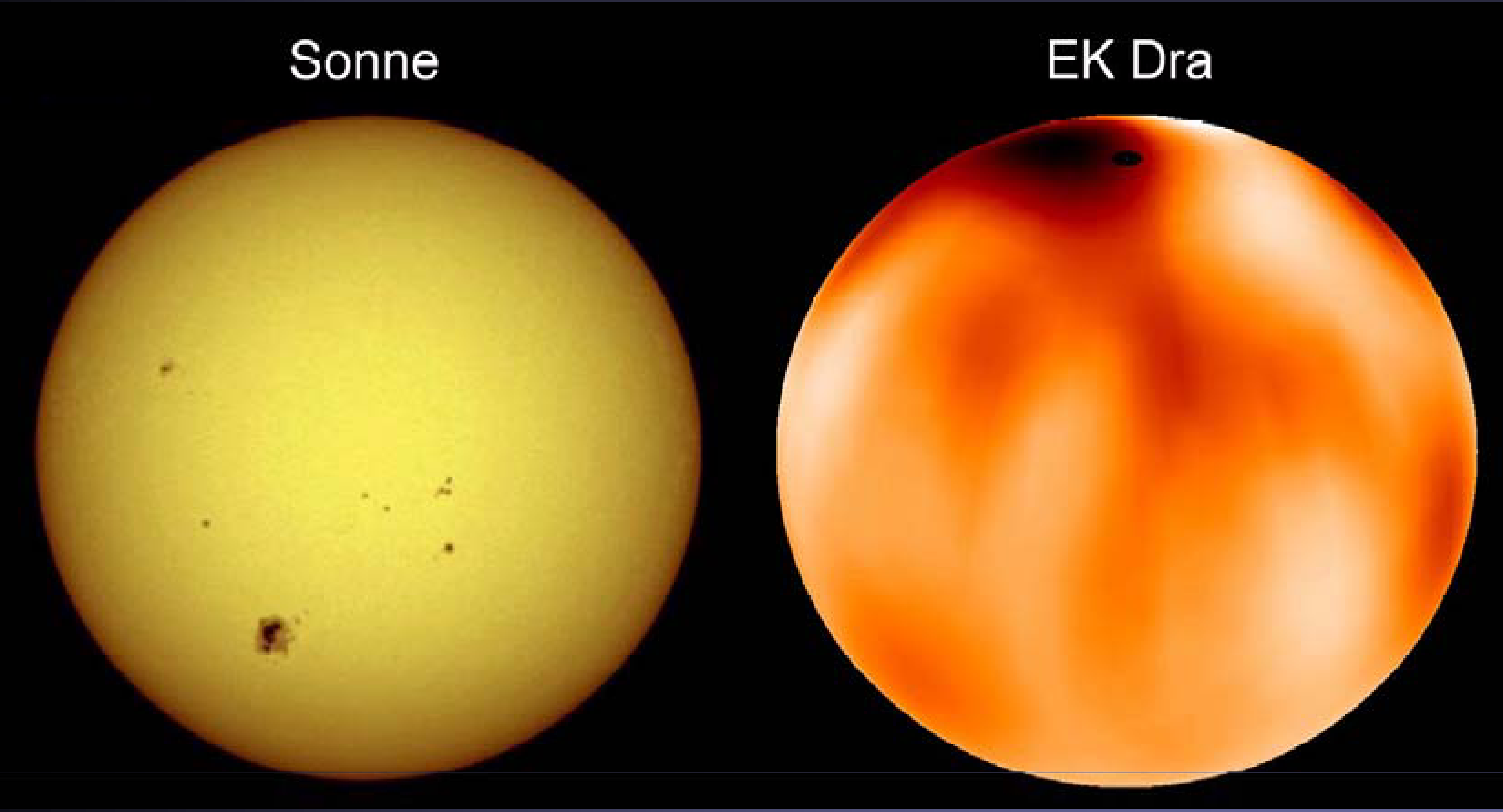
- Curved light path in solar gravitational field ➔ Test of General Relativity
- Red shift of solar spectral lines ➔ Test of EEP
- Oblate shape of Sun ➔ Quadrupol moment of solar gravitational field: Test of Brans-Dicke theory
- Comparison of solar evolution models with observations ➔ Limits on evolution of fundamental constants
- Polarization of solar spectral lines: Tests gravitational birefringence ➔ Tests of equivalence principle & alternative theories of gravity

Which stars have magnetic fields or show magnetic activity?

- Best studied star: Sun
- F, G, K & M stars (outer convection zones) show magnetic activity & have <B> fields of G-kG.
- Early type stars: Ap, Bp, (kG-100kG), Be (100G)
- White dwarfs have B ≈ kG-10^9 G, no activity
- Not on diagram: pulsars

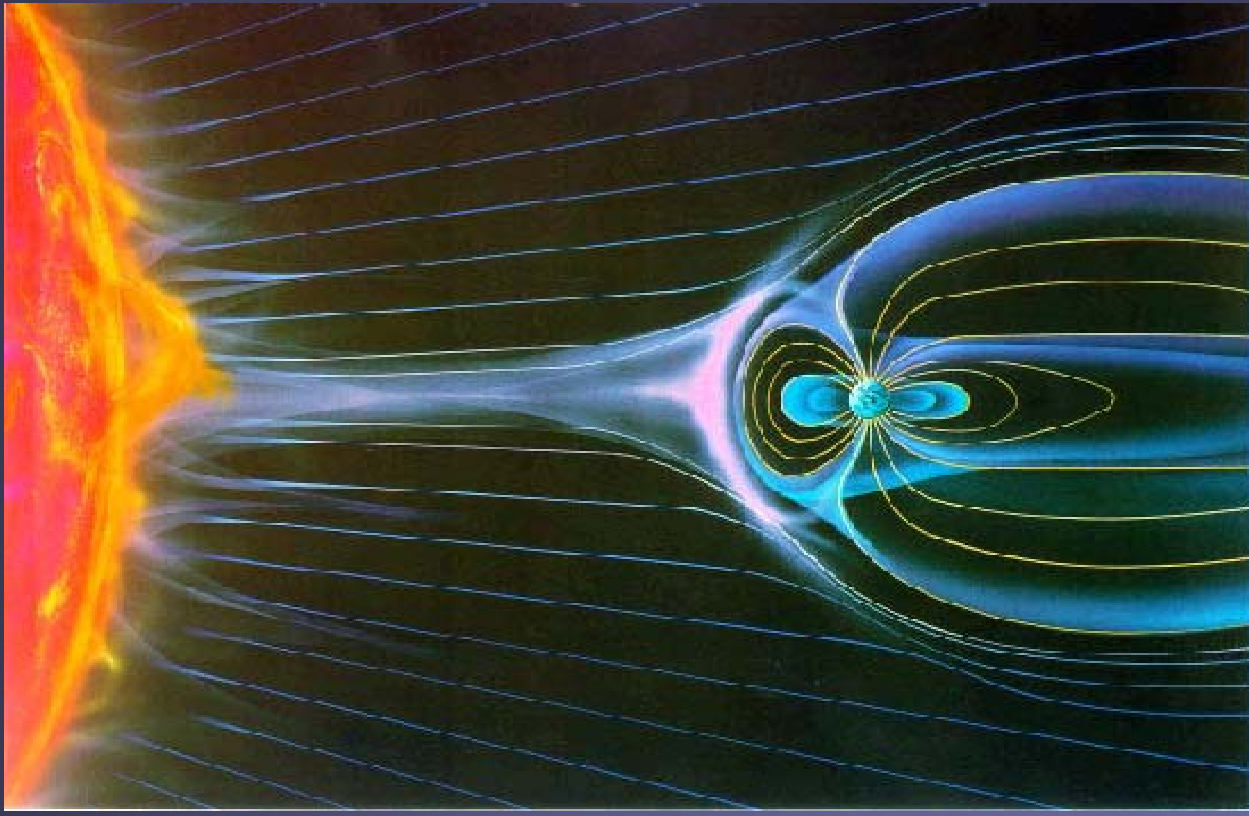


The Sun compared with an active star



Sun, Earth and planets

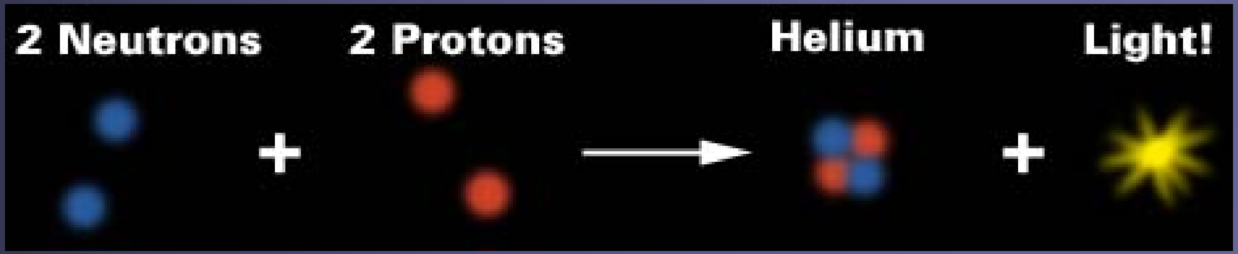
- Solar output affects the magnetospheres and atmospheres of planets
- Solar energy is responsible for providing a habitable environment on Earth
- Solar evolution and liquid water on Mars...



The solar interior

The Sun’s core

- In the Sun’s core, mass is turned into energy.
- Nuclear reactions burn 7×10^{11} kg/s of hydrogen into helium.
- In core, particle density and temperature are high ➔ individual protons ram into each other at sufficient speed to overcome the Coulomb barrier, forming He nuclii (α) and releasing energy



Nuclear reactions in cores of stars

- Sun gains practically all its energy from the reaction $4p \rightarrow \alpha + 2e^{+} + 2\nu = {}^{4}He + 2e^{+} + 2\nu$
- Two basic routes
  - p-p chain: yields about 99% of energy in Sun
  - CNO cycle : 1% of energy released in present day Sun (but dominant form of energy release in hotter stars)
- Both chains yield a total energy Q of 26.7 MeV per He nucleus, mainly in the form of γ-radiation Q_γ (which is absorbed and heats the gas) and neutrinos Q_ν (which escapes from the Sun).
- 0.7% of each protons mass is converted into energy

Nuclear reactions of pp-chain

- p=proton
- d=deuterium
- α=Helium
- γ=radiation
- ν=neutrino
- 2nd reaction replaces step 3 of 1st reaction

Table 2.1. Nuclear reactions of the pp chains. Energy values according to Bahcall and Ulrich (1988) and Caughlan and Fowler (1988)

	Reaction	Q'[MeV]	Q_ν[MeV]	Rate symbol
ppI	$p(p,e^+\nu)d$	1.177 (x2)	0.265	λ_{pp}
	$d(p,\gamma)^3He$	5.494 (x2)		λ_{pd}
	$^3He(^3He,2p)\alpha$	12.860		λ_{33}
ppII	$^3He(\alpha,\gamma)^7Be$	1.586		λ_{34}
	$^7Be(e^-,\nu\gamma)^7Li$	0.049	0.815	λ_{e7}
	$^7Li(p,\alpha)\alpha$	17.346		λ'_{17}
ppIII	$^7Be(p,\gamma)^8B$	0.137		λ_{17}
	$^8B(,e^+\nu)^8Be^*$	8.367	6.711	λ_8
	$^8Be^*(,\alpha)\alpha$	2.995		λ'_8

- 3rd reaction replaces steps 2+3 of 2nd reaction
- Branching ratios:
  - 1st vs. 2nd + 3rd 87 : 13
  - 2nd vs. 3rd → 13 : 0.015

Nuclear reactions of CNO-cycle

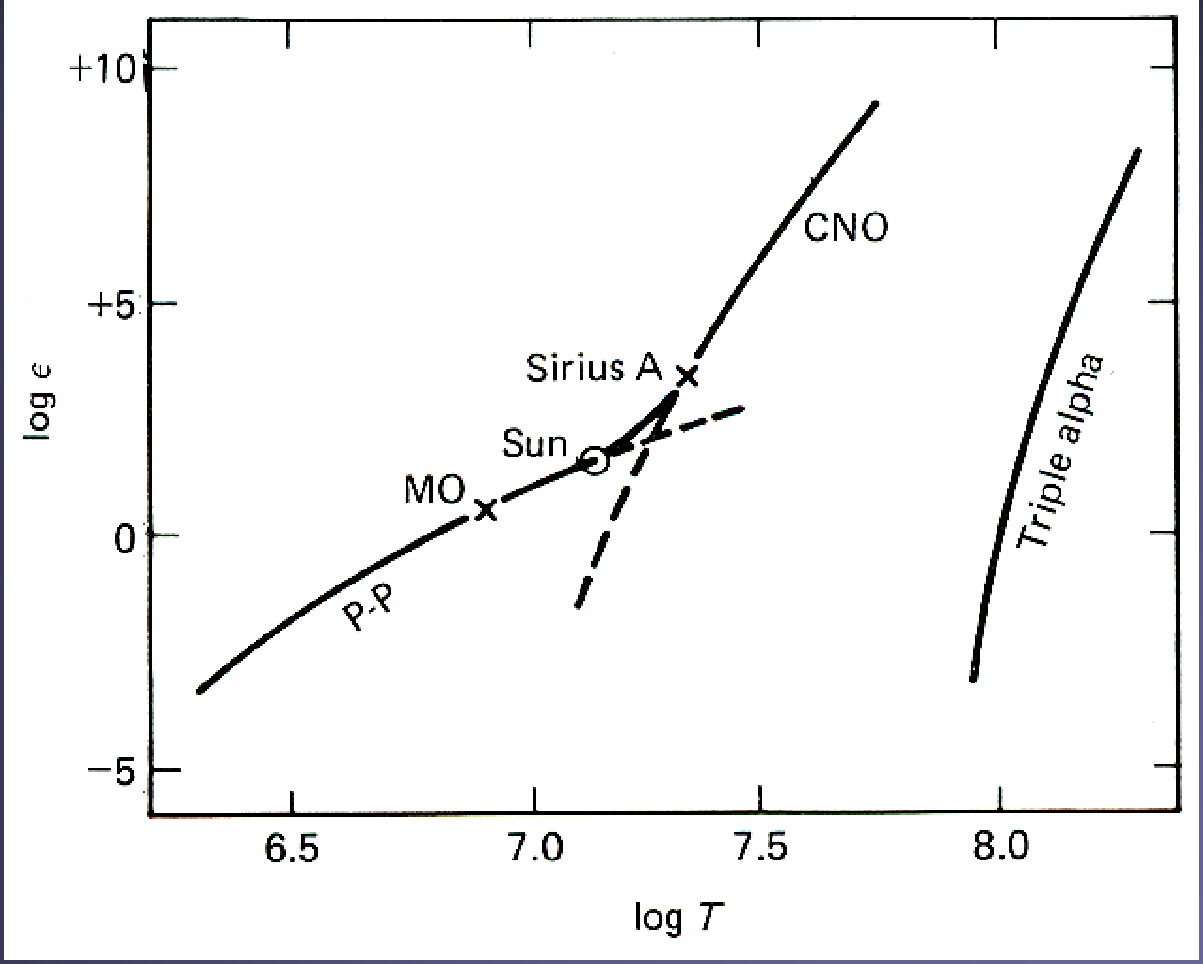
- C, N and O act only as catalysts: Basically the same things happens as with proton chain.

Table 2.2. Nuclear reactions of the CNO cycle. Energy values according to Bahcall and Ulrich (1988) and Caughlan and Fowler (1988)

Reaction	Q' [MeV]	Q_ν [MeV]	Rate symbol
$^{12}C(p,\gamma)^{13}N$	1.944		λ_{p12}
$^{13}N(,e^+\nu)^{13}C$	1.513	0.707	λ_{13}
$^{13}C(p,\gamma)^{14}N$	7.551		λ_{p13}
$^{14}N(p,\gamma)^{15}O$	7.297		λ_{p14}
$^{15}O(,e^+\nu)^{15}N$	1.757	0.997	λ_{15}
$^{15}N(p,\alpha)^{12}C$	4.966		λ_{p15}

Temperature dependence of pp-chain and CNO cycle

- p-p chain in cool main-sequence stars
- CNO cycle in hot main-sequence stars
- Triple alpha process in red giants: 3He → C
- T dependence of p-p chain:: $\eta(T/10^6)^4$, where η depends on many other parameters.



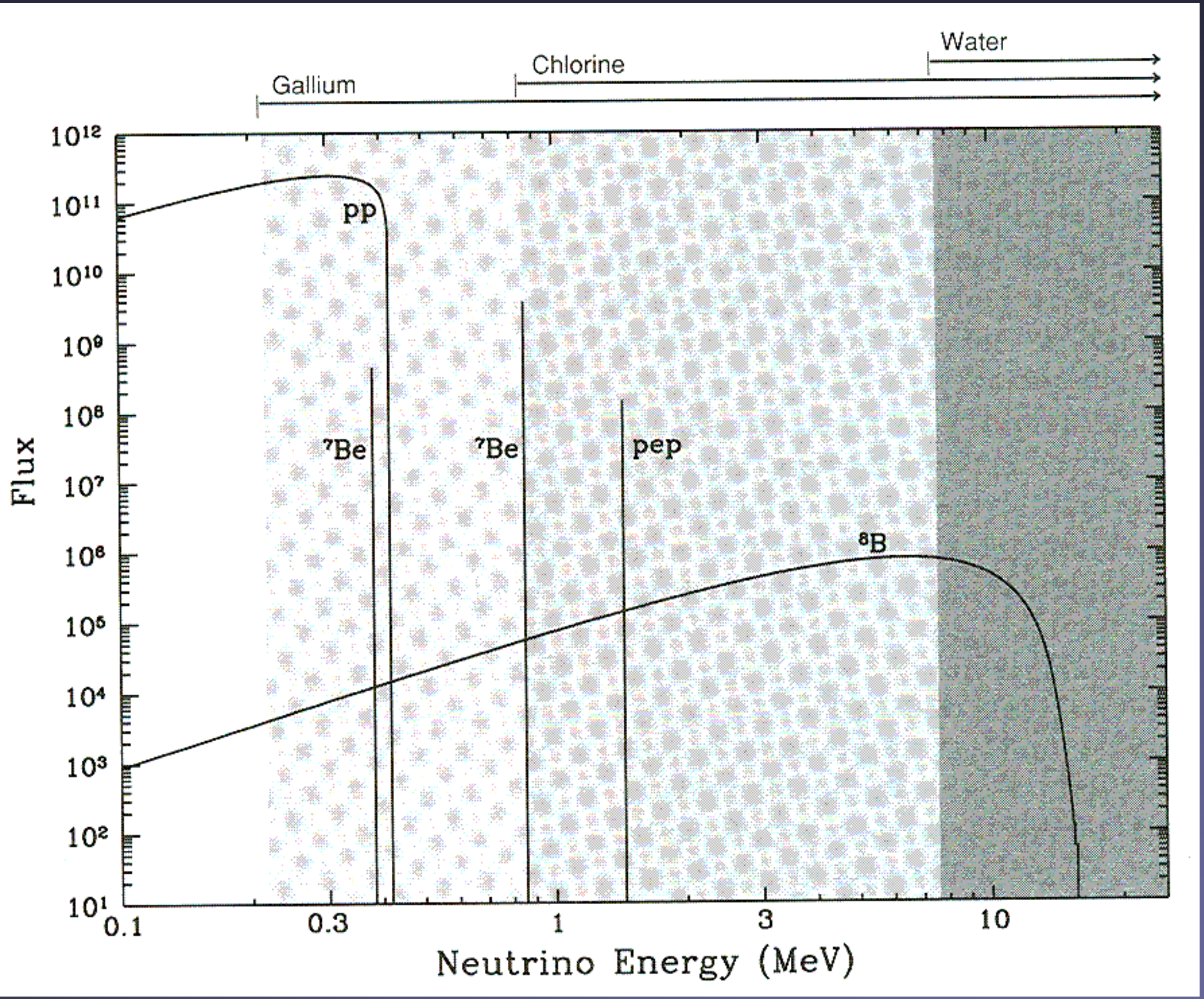
Solar neutrinos

- Neutrinos, ν, are produced at various stages of the pp-chain.
- Neutrinos are also produced by the reaction: p(p e$^-$, ν)d , so-called pep reaction. Being a 3-body reaction it is too rare to contribute to the energy, but does contribute to number of ν.

	Reaction	Q'[MeV]	Q_ν[MeV]
ppI	p(p,e$^+\nu$)d	1.177	0.265
	d(p,γ)^{3}He	5.494	
	^{3}He(^{3}He,2p)α	12.860	
ppII	^{3}He(α,γ)^{7}Be	1.586	
	^{7}Be(e$^-$, $\nu\gamma$)^{7}Li	0.049	0.815
	^{7}Li(p,α)α	17.346	
ppIII	^{7}Be(p,γ)^{8}B	0.137	
	^{8}B(,e$^+\nu$)^{8}Be*	8.367	6.711
	^{8}Be*(,α)α	2.995	

Solar neutrino spectrum

- Continua: number/(cm^2 s MeV)
- Lines: number/(cm^2 s)
- Bars at top & shading: sensitivity of different materials to ν



Solar neutrinos II

- Since 1968 the Homestake ^{37}Cl experiment has given a value of 2.1 ± 0.3 snu (1snu =1 ν / 10^{36} target atoms)
- Standard solar models predict: 7±2 snu

→ Solar Neutrino Problem!

- In 1980s & 90s water based Kamiokande and larger Superkamiokande detectors found that approximately half the rare, high energy ^{8}B ν were missing.
- ^{71}Ga experiments (GALLEX at Gran Sasso and SAGE in Russia) showed that the neutrino flux was too low, even including the $p(p,e^{+}\nu)d$ neutrinos.

Results of neutrino experiments

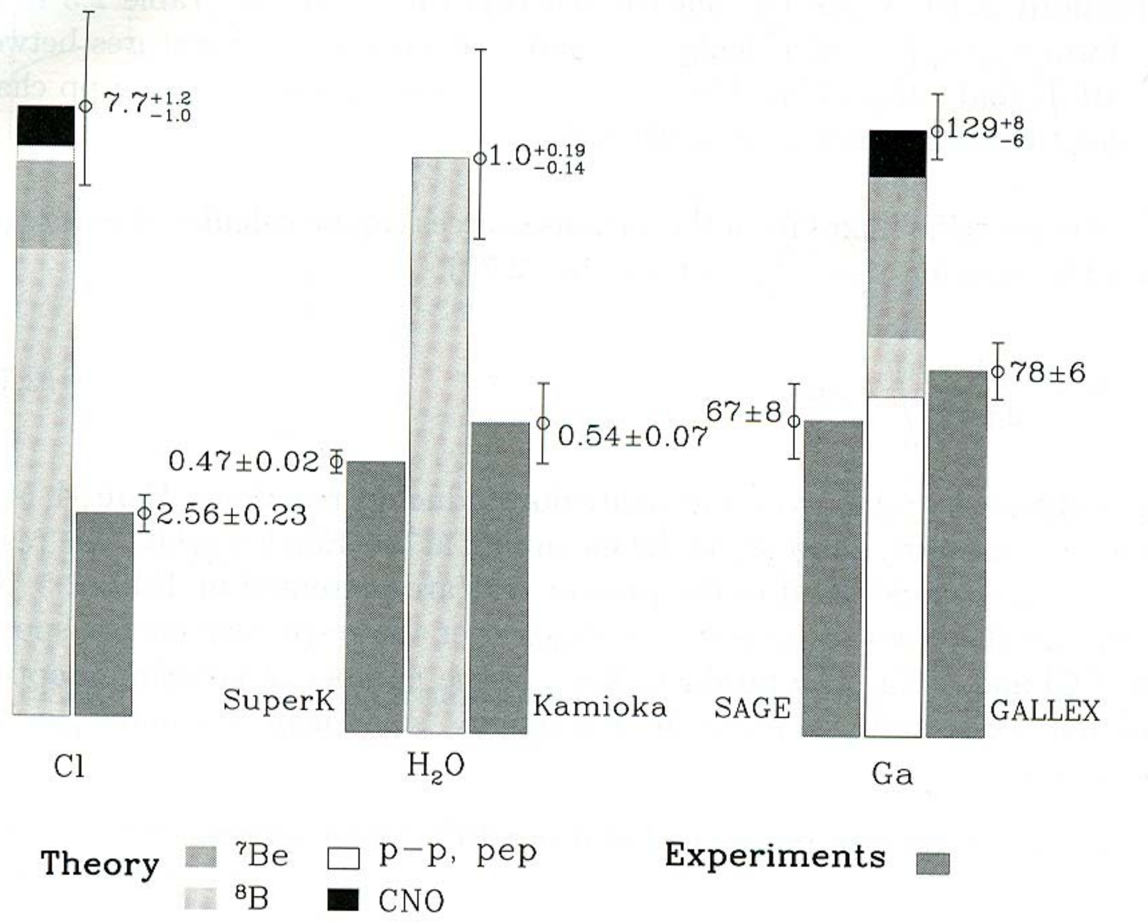
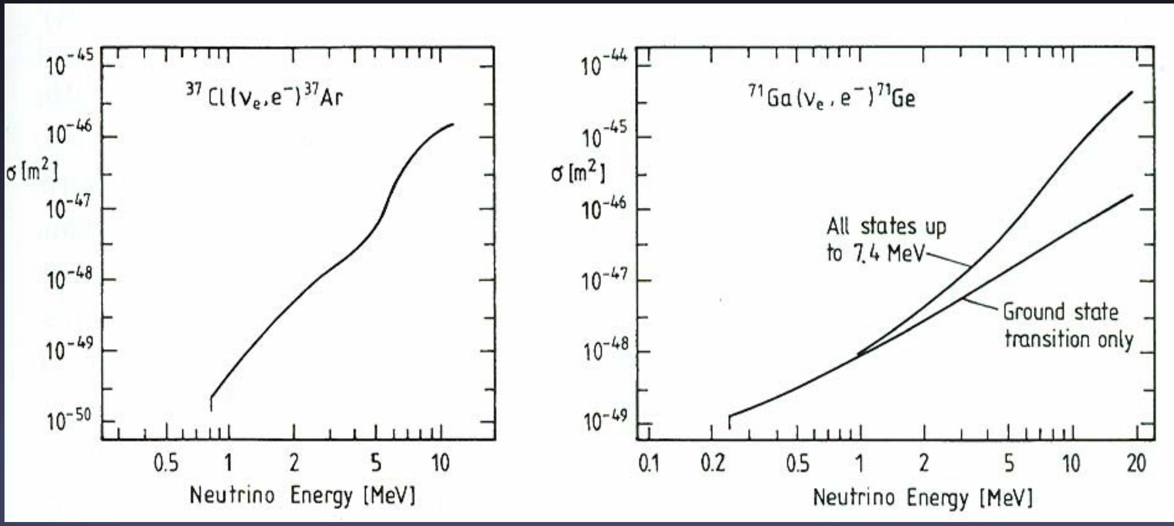


Fig. 2.13. Solar neutrinos: Prediction from the standard solar model BP98 of J. N. Bahcall and M. H. Pinsonneault (*high columns*) and experimental results (*lower columns*), for the ^{37}Cl (*left*), water (*middle*), and ^{71}Ga detectors (*right*). The *shading* indicates the contributions to the theoretical prediction from the diverse nuclear reactions in the Sun, as indicated at the bottom

Solar neutrinos III



- Sensitivity of H_2O, ^{71}Ga and ^{37}Cl to ν increases ~ exponentially with increasing ν energy.

➔ Homestake ^{37}Cl detector and (Super-) Kamiokande see mainly high-energy ν from rare β^+-decay of ^8B.

- Branching ratios between the various chains: central for predicting exact ν-flux detectable by ^{37}Cl & H_2O
- Branching ratios depend very sensitively on T(r=0), while total ν-flux depends only linearly on luminosity.
- Even ^{71}Ga experiments are sensitve largely to high energy ν.

Resolution of neutrino problem

- SNO (Sudbury Neutrino Observatory) in Sudbury, Canada uses D_2O and can detect not just the electron neutrino, but also μ and τ neutrinos

→ The neutrinos aren't missing, e^- neutrinos produced in the Sun just convert into μ and τ neutrinos

→ The problem lies with the neutrino physics.

- The neutrino has a small rest mass (10^{-8} m_e), which allows it to oscillate between the three flavours: e^- neutrino, μ neutrino and τ neutrino (proposed 1969 by russian theorists: Bruno Pontecorvo and Vladimir Gribov, … but nobody believed them)
- Confirmation by measuring anti-neutrinos from power plant (with Superkamiokande).

Resolution of neutrino problem II

- Lesson learnt: neutrinos have a multiple personality problem (J. Bahcall)
- Other lesson learnt: the “dirty” and difficult solar model turned out to be correct, the clean and beautiful standard theory of particle physics turned out to be wrong, or at least incomplete (J. Bahcall)
- 2002: Raymond Davis got Nobel prize for uncovering the neutrino problem

Hawkin’s theory of progress

Progress does not consist in replacing a theory that is wrong by one that is right. It consists in replacing a theory that is wrong by one that is more subtly wrong.

Standard solar model

- Ingredients: Conservation laws and material dependent equations
  - Mass conservation
  - Hydrostatic equilibrium (= momentum conservation in a steady state)
  - Energy conservation
  - Energy transport
  - Equation of state
  - Expression for entropy
  - Nuclear reaction networks and reaction rates → energy production
  - Opacity
- Assumptions: standard abundances, no mixing in core or in radiative zone, hydrostatic equilibrium, i.e. model passes through a stage of equilibria (the only time dependence is introduced by reduction of H and build up of He in core).

Equations describing solar interior

- Equations are given in spherical geometry
- They can be derived by considering a thin shell centred on the solar core

Mass m conservation:

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi\rho r^2}$$

$r =$ radial distance, $\rho =$ gas density

Hydrostatic equilibrium:

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$

$P =$ pressure

$G =$ Gravitational constant

Energy balance:

$$\frac{\partial L}{\partial m} = \varepsilon - T\frac{\partial S}{\partial T}$$

$\varepsilon =$ energy generation per unit mass

$T =$ temperature

$S =$ entropy

Equations describing solar interior II

- Also: equations describing
  - nuclear reaction networks
  - radiative transport of energy
  - convective energy transport
  - conductive energy transport
  - opacity
  - ionisation equilibria (for opacity)
  - diffusion “constants”
  - etc. etc.

Equation of state:

$\rho = \rho(P, T)$

or (for an ideal gas):

$$P_G = \frac{\rho \Re T}{\mu}$$

P_G = gas pressure

$\Re$ = gas constant

μ = mean molecular weight

Energy transport:

$$F = F_{\mathrm{R}} + F_{\mathrm{C}} + F_{\mathrm{cond}} = \frac{L}{4\pi r^2}$$

F = total energy flux

F_{R} = radiative flux

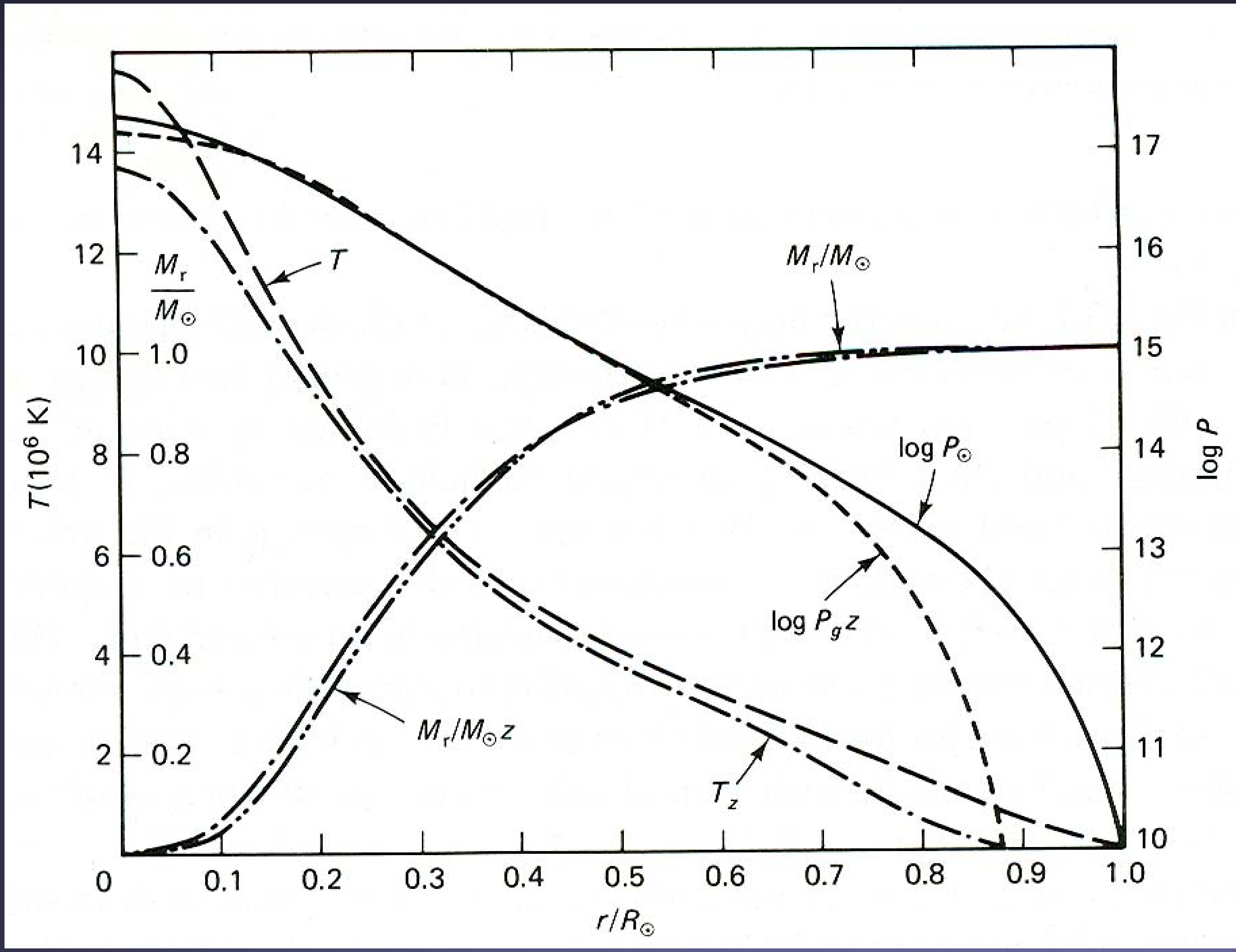
F_{C} = convective flux

F_{cond} = conductive flux

L = luminosity

Internal structure of the Sun

- Internal models shown for ZAMS Sun (subscript z) and for present day Sun (radius reaching out to 1.0, subscript ☉)



Elemental abundances

- **Photospheric values**
- Logarithmic (to base 10) abundances of the 32 lightest elements on a scale on which H has an abundance of 12
- Heavier elements all have low abundances
- In general solar photospheric abundances are very similar to those of meteorites. Exception: Li, which is depleted by a factor of 100.
- All elements except H, He and Li were produced within earlier generations of stars & released at their death

Element	Photosphere	Meteorites
1 H	12.00	–
2 He	10.93 ± 0.004	–
3 Li	1.10 ± 0.10	3.31 ± 0.04
4 Be	1.40 ± 0.09	1.42 ± 0.04
5 B	2.55 ± 0.30	2.79 ± 0.05
6 C	8.52 ± 0.06	–
7 N	7.92 ± 0.06	–
8 O	8.83 ± 0.06	–
9 F	4.56 ± 0.3	4.48 ± 0.06
10 Ne	8.08 ± 0.06	–
11 Na	6.33 ± 0.03	6.32 ± 0.02
12 Mg	7.58 ± 0.05	7.58 ± 0.01
13 Al	6.47 ± 0.07	6.49 ± 0.01
14 Si	7.55 ± 0.05	7.56 ± 0.01
15 P	5.45 ± 0.04	5.56 ± 0.06
16 S	7.33 ± 0.11	7.20 ± 0.06
17 Cl	5.5 ± 0.3	5.28 ± 0.06
18 Ar	6.40 ± 0.06	–
19 K	5.12 ± 0.13	5.13 ± 0.02
20 Ca	6.36 ± 0.02	6.35 ± 0.01
21 Sc	3.17 ± 0.10	3.10 ± 0.01
22 Ti	5.02 ± 0.06	4.94 ± 0.02
23 V	4.00 ± 0.02	4.02 ± 0.02
24 Cr	5.67 ± 0.03	5.69 ± 0.01
25 Mn	5.39 ± 0.03	5.53 ± 0.01
26 Fe	7.50 ± 0.05	7.50 ± 0.01
27 Co	4.92 ± 0.04	4.91 ± 0.01
28 Ni	6.25 ± 0.04	6.25 ± 0.01
29 Cu	4.21 ± 0.04	4.29 ± 0.04
30 Zn	4.60 ± 0.08	4.67 ± 0.04
31 Ga	2.88 ± 0.10	3.13 ± 0.02
32 Ge	3.41 ± 0.14	3.63 ± 0.04

Solar evolution

- Path of the Sun in the HR diagram, starting in PMS stage ‘P’ and ending at
  - Red giant stage
  - White dwarf stage
- Note the complex patch during the red giant phase due to various phases of Helium burning and core contraction and expansion, etc.
- η ~ mass loss rate

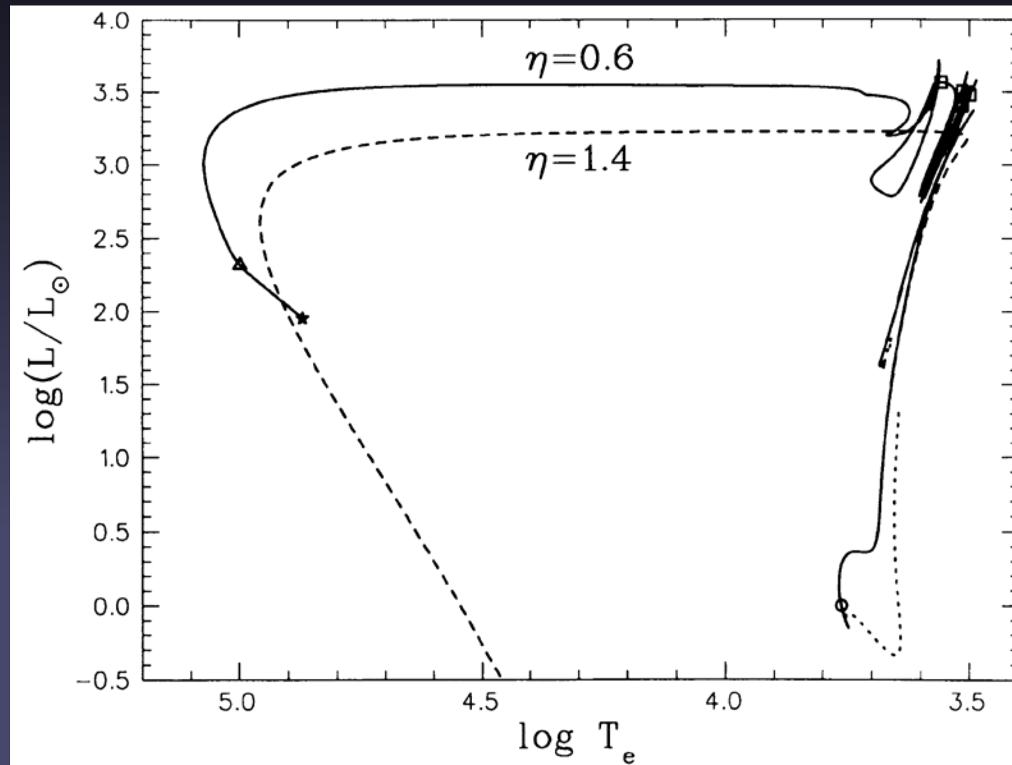
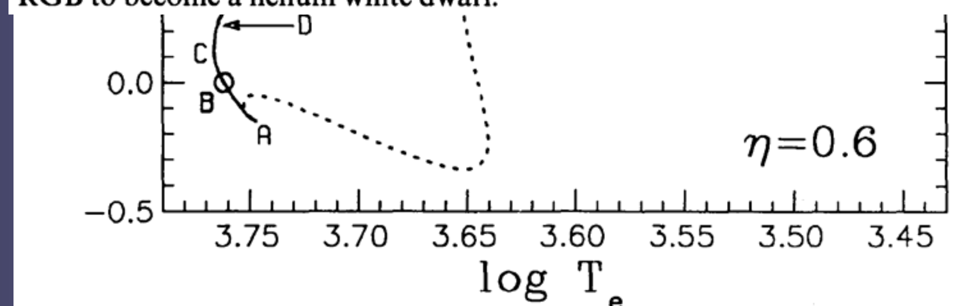
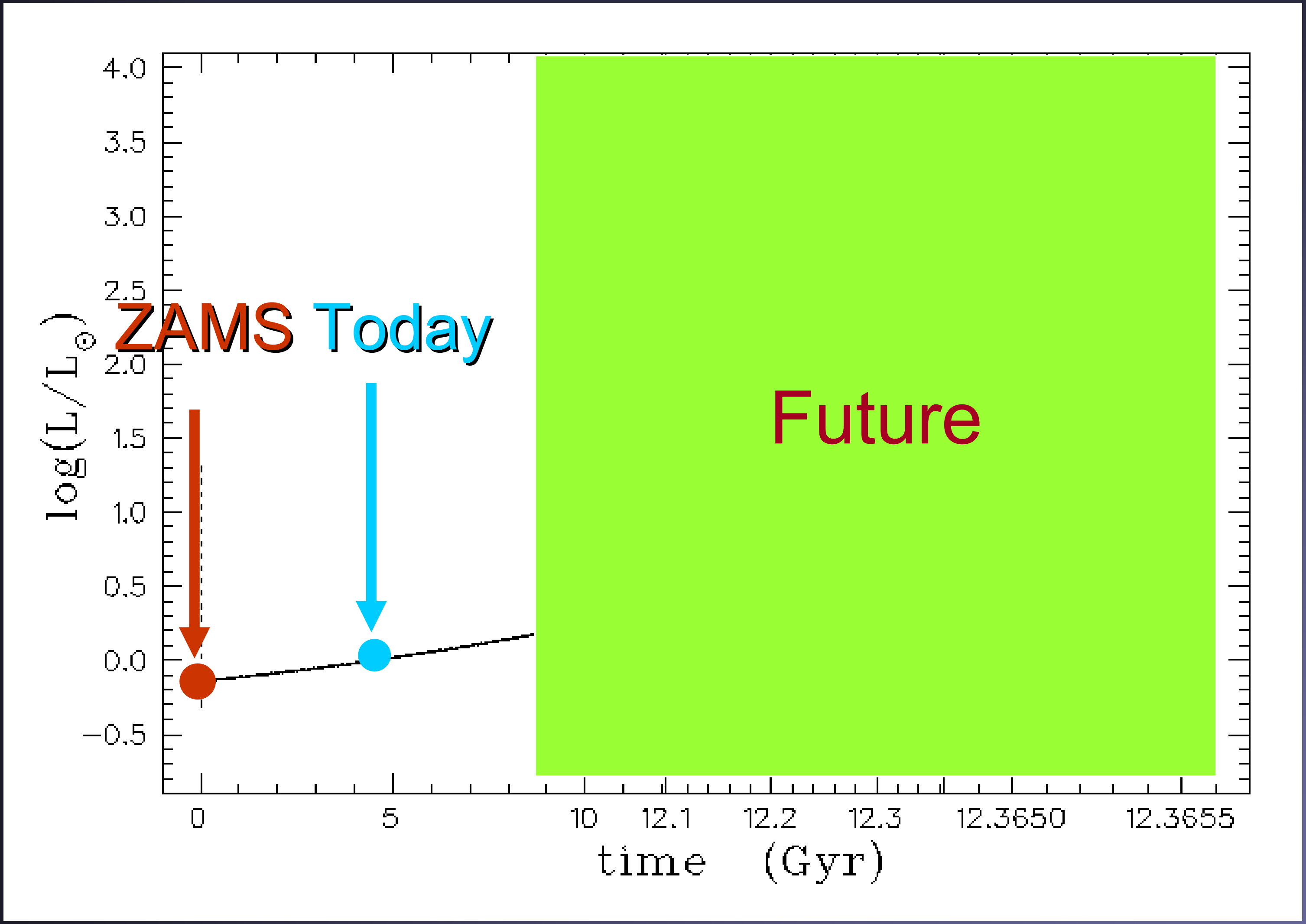


FIG. 5.—The Sun’s evolution in the HR diagram, from the pre–main-sequence state to the pre–white dwarf stage. For our preferred mass-loss case (*solid curve*: $\eta = 0.6$), the triangle indicates the beginning of the final helium shell flash, and the star its peak, where computations were terminated. The dashed curve shows our extreme mass-loss case ($\eta = 1.4$), which leaves the RGB to become a helium white dwarf.



Evolution of Sun's past luminosity

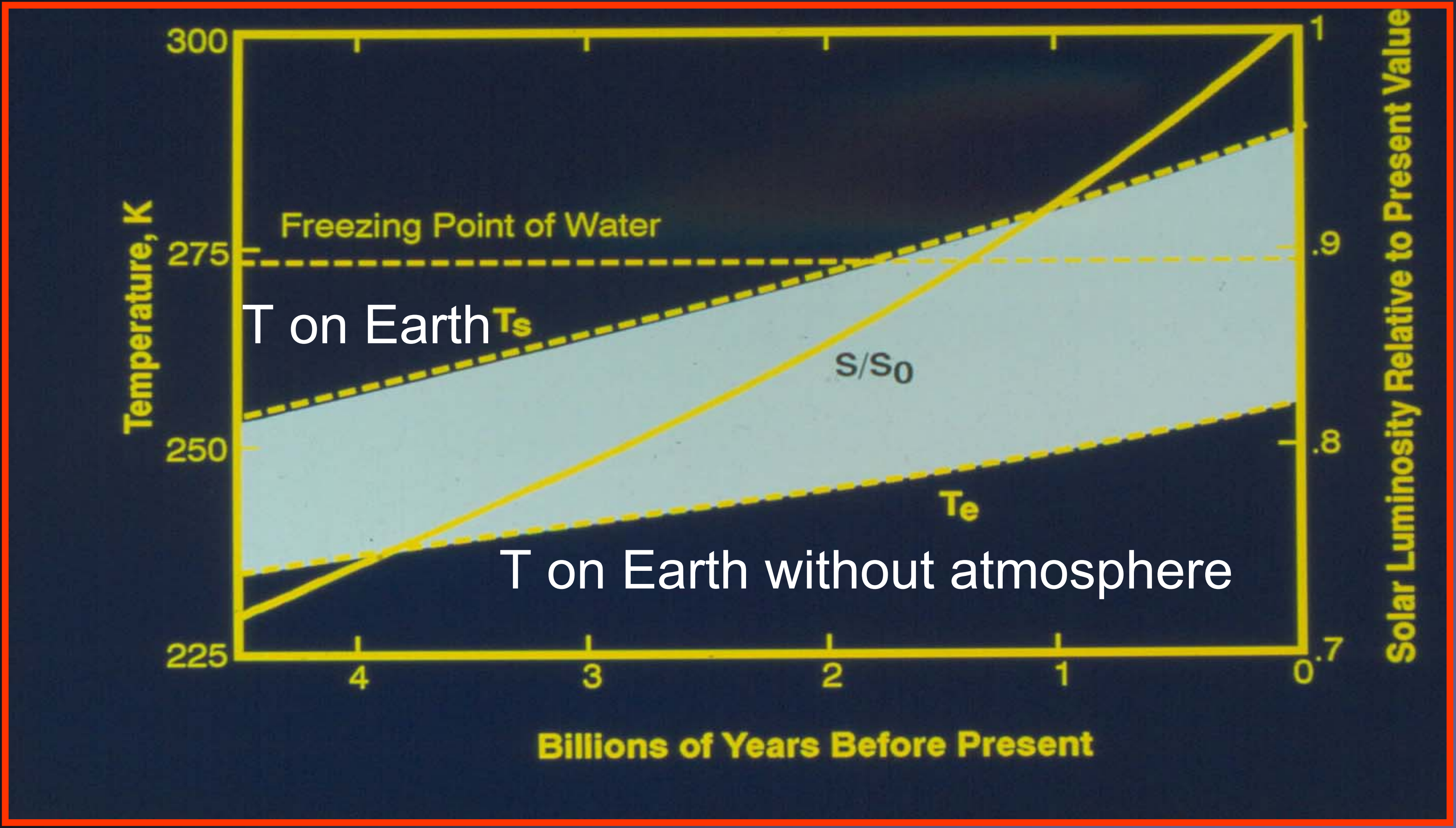


Faint young Sun paradox

- According to the standard solar model the Sun was approximately 30% less bright at birth than it is today
- Too faint to keep the Earth free of ice!
- Problem: Life started at the Earth's surface at least 3.5Million years ago
- Obviously the Earth was not covered with ice at that time
- So what is the solution?

- This and next 4 slides kindly provided by Piet Martens

A Faint Young Sun Leaves the Earth Frozen Solid



Kasting et al, Scientific American, 1988

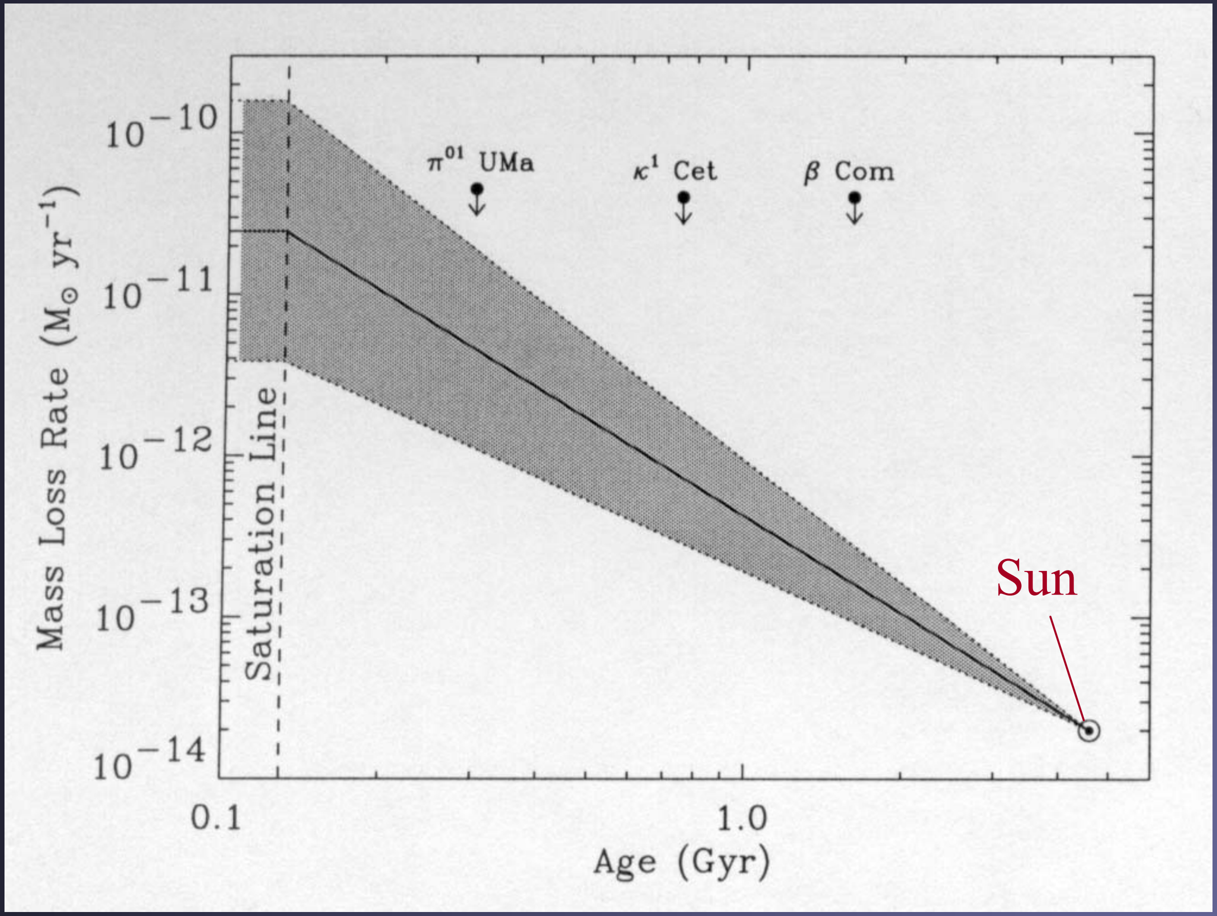
Where to look for a solution?

- Astrophysical Solutions: Young Sun was not faint
- Eartly Earth Atmosphere: Much more greenhouse gases
- Geology: Much more geothermal energy
- Biology: Life developed on a cold planet
- Fundamental Physics: e.g., gravitational constant has varied

Was the young Sun really faint?

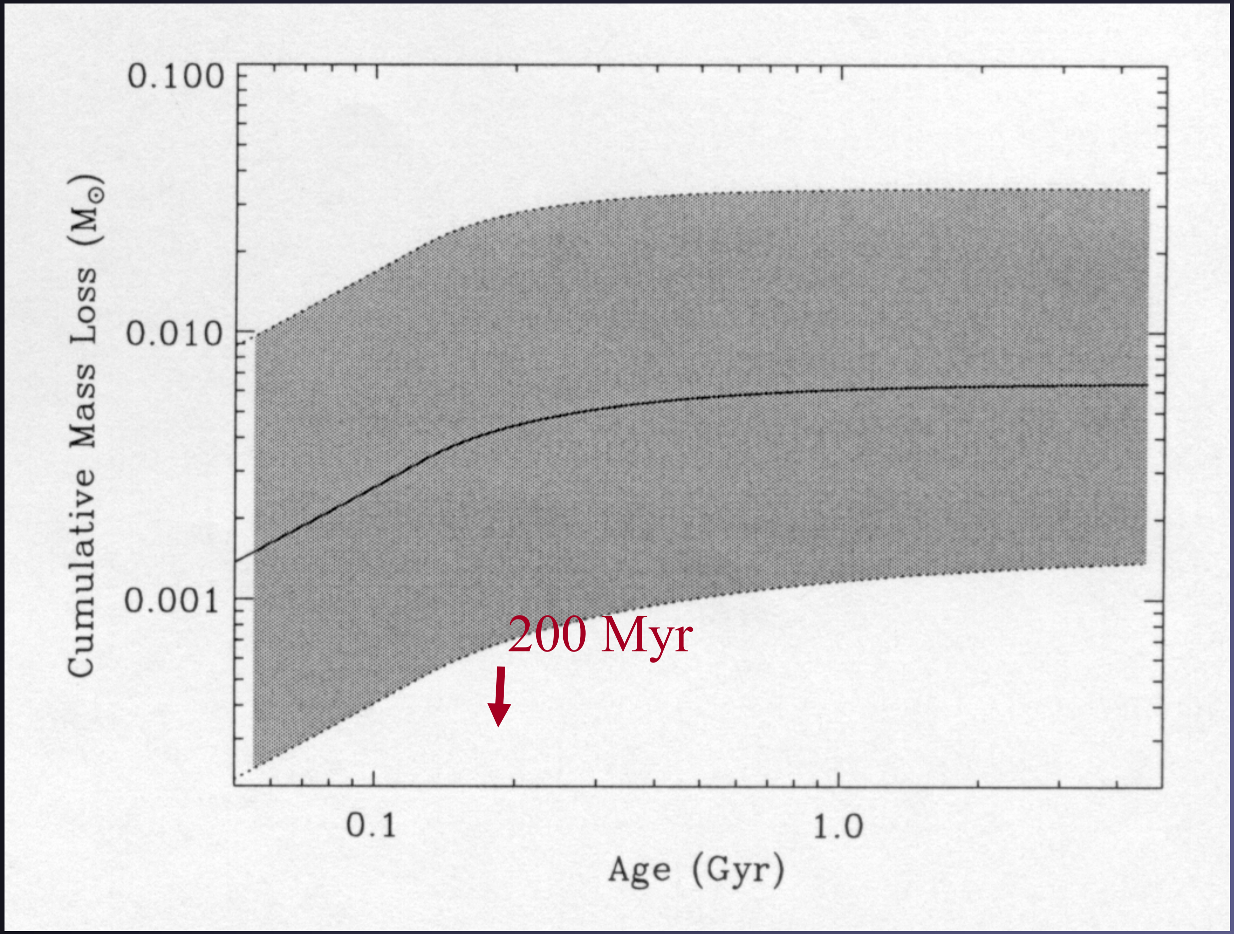
- Solar luminosity is a strong function of solar mass: $L_{\odot} \sim M_{\odot}^{4}$
- Planetary orbital distance varies inversely with solar mass: $a \sim M_{\odot}^{-1}$
- Solar flux varies inversely with orbital distance: $S \sim a^{-2}$
- Flux to the planets therefore goes as: $S \sim M_{\odot}^{6}$
- Even small increase in mass of young sun would have been enough to keep Earth (and Mars!) warm

Estimated mass loss rate vs. stellar age



Wood et al. (2002)

Integrated mass loss vs. time



% Changes

Mass loss	ΔS
0.6	3.6
1.0	6.2
2.0	13
3.0	19

Wood et al. (2002)

⇒ The Sun was probably back on the standard solar evolution curve by ~4.4 Ga (*i.e.*, 4.4 Gyr ago)

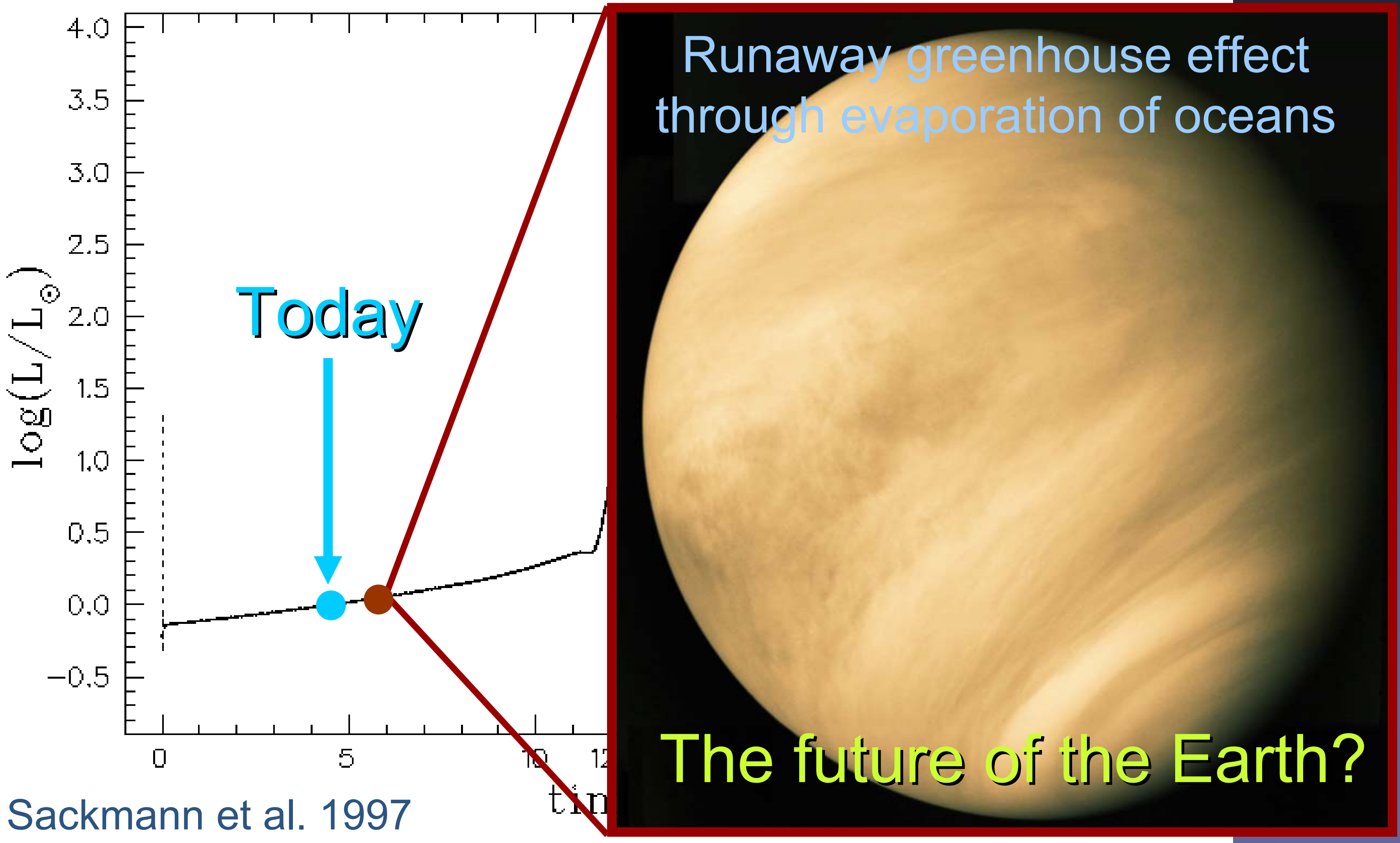
Where we stand regarding the faint young Sun paradox

- Young Sun was probably less luminous, yet its UV, EUV and X-ray emission was an order of magnitude larger (could that have changed the Earth's atmospheric chemistry & thus the climate?)
- Young Earth was probably warmer than today, and single-cell organisms were present from very early on
- No silver bullet has devised yet to reconcile these results

... and the future?

- Sun will continue to grow **brighter...** and **bigger**, first gradually, then rapidly.
- It will get 4000 times brighter than today!
- Will the Sun eventually cook the Earth, even evaporate it? When will it become too hot for life?

Evolution of solar luminosity

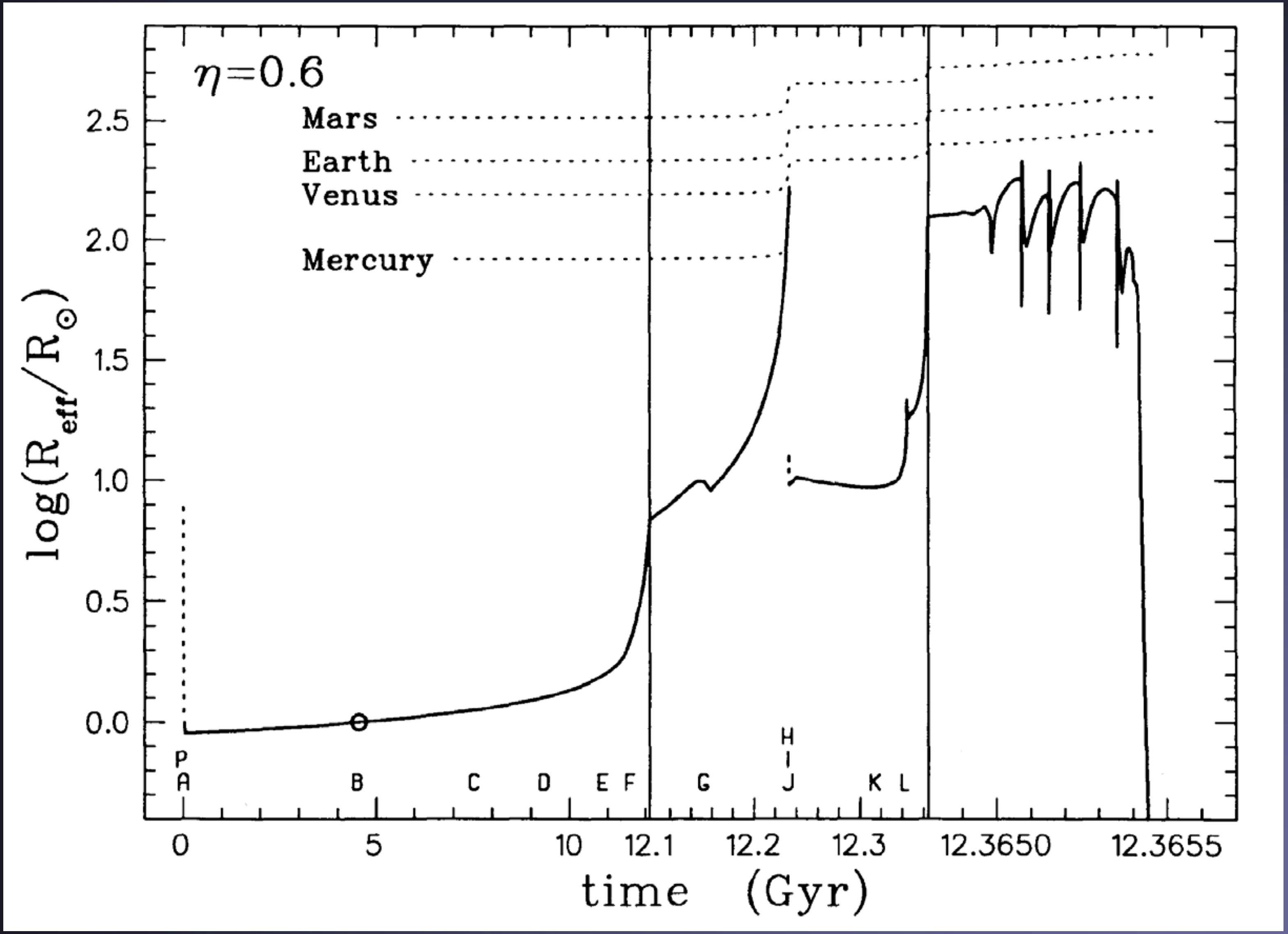


Sackmann et al. 1997

... and the future?

- Sun will continue to grow brighter... and bigger, first gradually, then rapidly.
- It will get 4000 times brighter than today!
- Will the Sun eventually cook the Earth, even evaporate it? When will it become too hot for life?
- It will eventually be so bloated that it will extend up to today's orbit of Earth!
- Will the Sun eventually swallow the Earth?

Will the Sun swallow the Earth?




Will the Earth really get away unharmed?

Oscillations and helioseismology

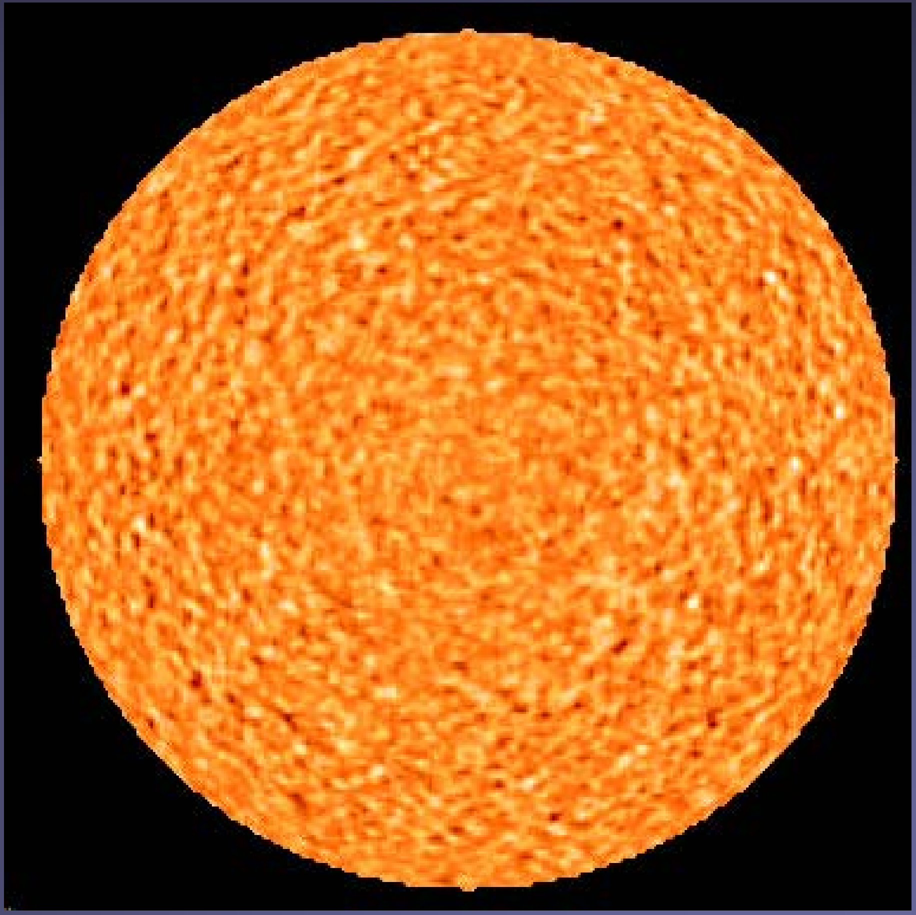
5-minute oscillations

- Entire Sun vibrates from a complex pattern of acoustic waves (called p-modes), with a period of around 5 minutes
- The oscillations are best seen as Doppler shifts of spectral lines, but are also visible as intensity variations.
- Spatio-temporal properties of oscillations best revealed by 3-D Fourier transforms (2-D space + 1-D time)

Hear the Sun sing!

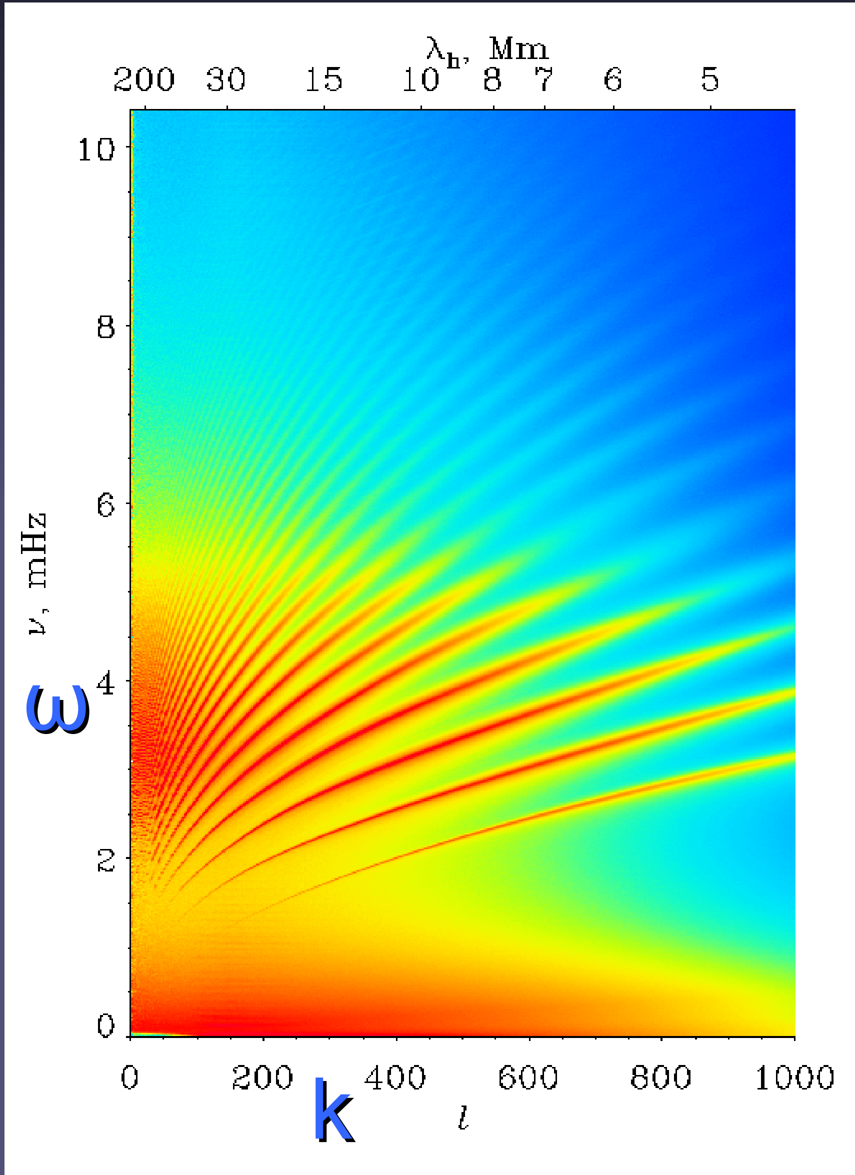
Sound waves speeded up 42,000 times

Doppler shift



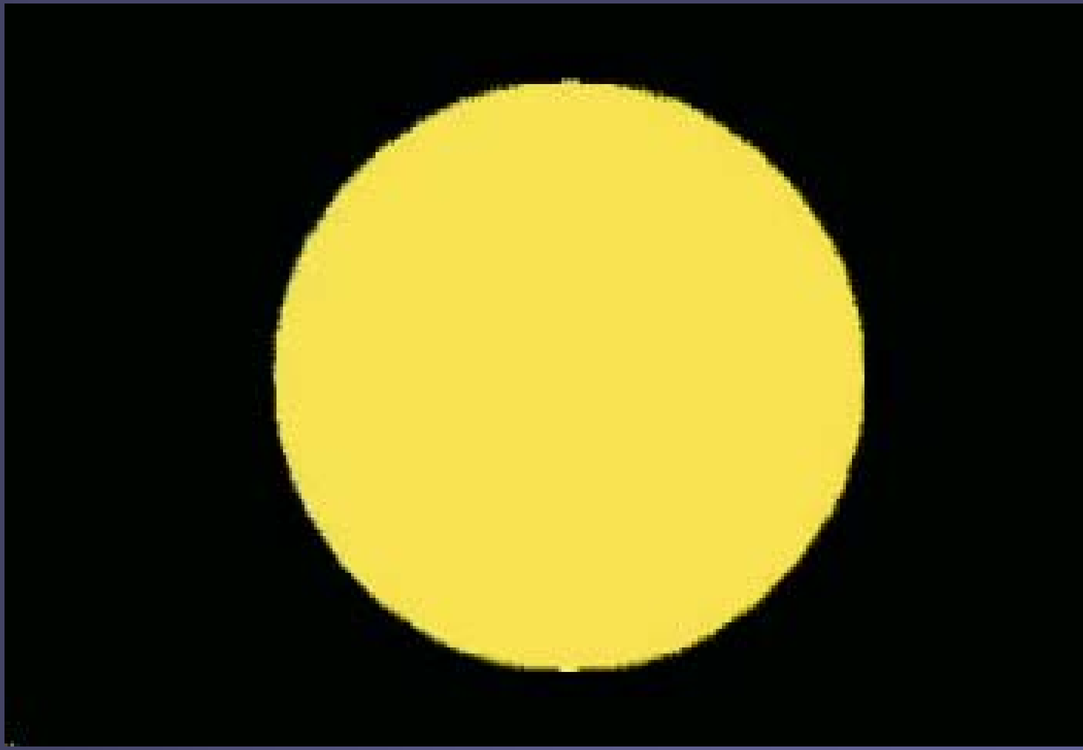
Solar Eigenmodes

- p-modes show a distinctive dispersion relation (k-ω diagram: k~ ω^2)
- Important: there is power only in certain ridges, i.e. for a given k^2 (= $k_x^2 + k_y^2$), only certain frequencies contain power.
- This discrete spectrum suggests the oscillations are trapped, i.e. eigenmodes of the Sun.



Global oscillations

- Sun's acoustic waves bounce (are reflected) at the solar surface, causing it to oscillate up and down.
- Modes differ in the depth to which they penetrate: they turn around due to refraction: because sound speed ($C_S \sim T^{1/2}$) increases with depth
- p-modes are influenced by conditions inside the Sun. E.g. they carry info on sound speed ~ $T^{1/2}$
- By observing these oscillations on the surface we can learn about the structure of the solar interior



Description of solar eigenmodes

- Eigen-oscillations of a sphere are described by spherical harmonics
- Each oscillation mode is identified by a set of three parameters:
  - n = number or radial nodes
  - l = number of nodes on the solar surface
  - m = number of nodes passing through the poles (next slide)

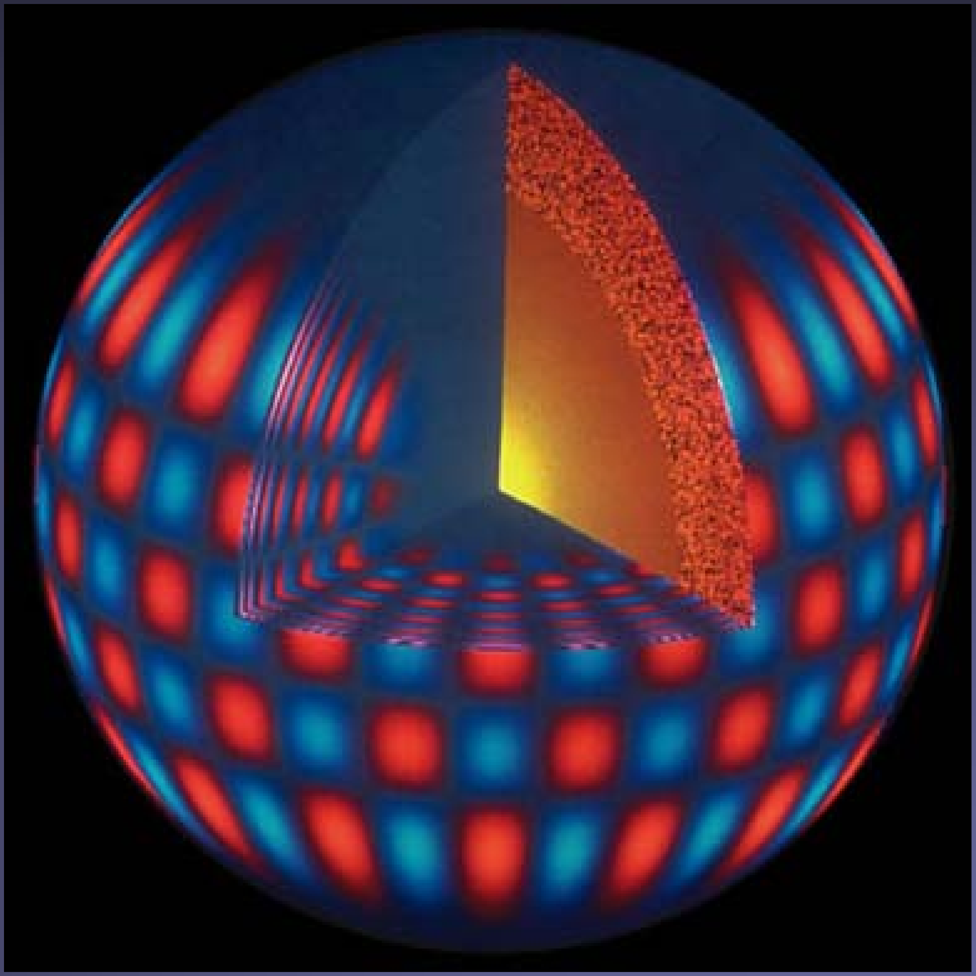
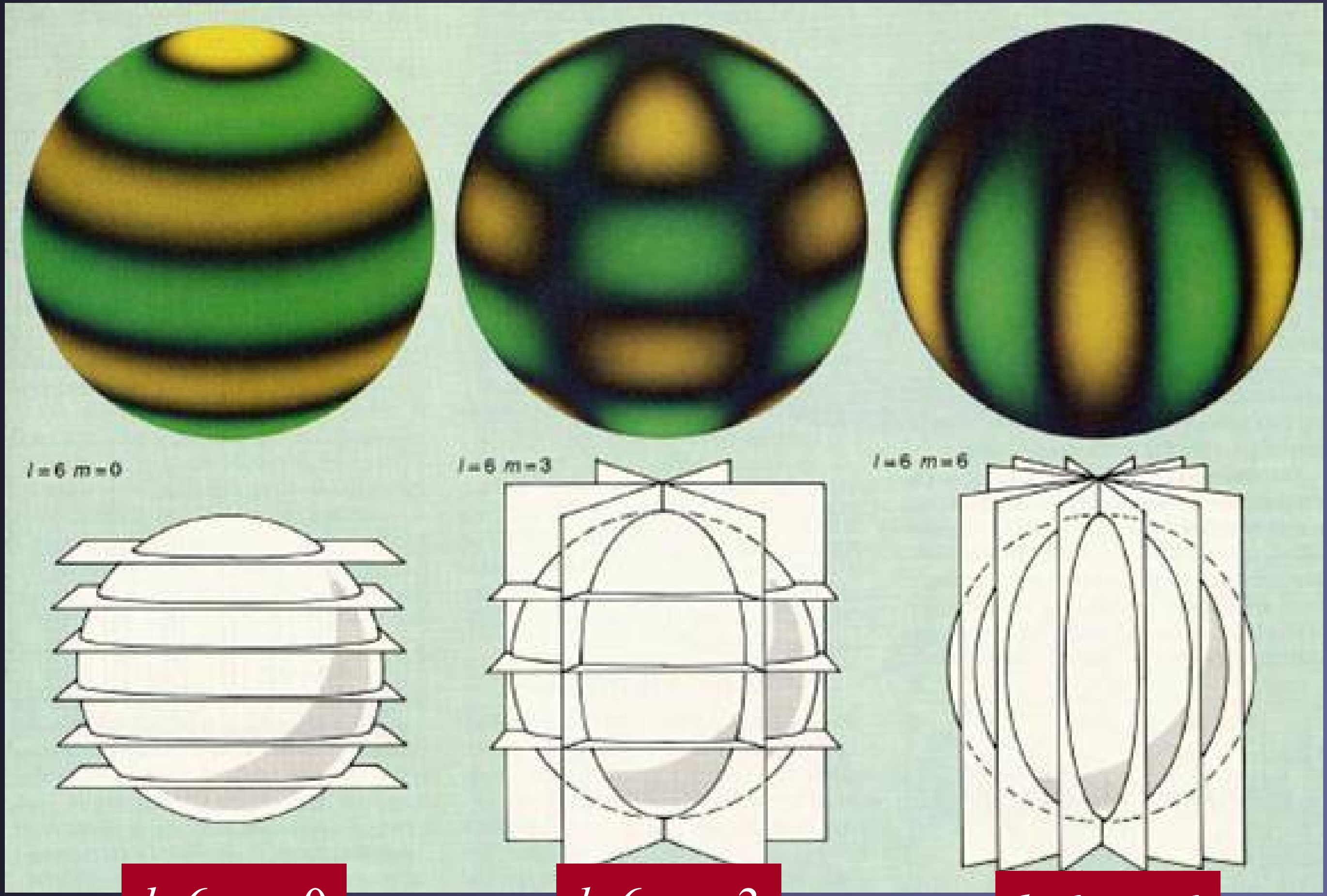


Illustration of spherical harmonics

- l = total number of nodes (in images: $l = 6$) = degree
- m = number of nodes connecting the “poles”



l=6, m=0

l=6, m=3

l=6, m=6

Spherical harmonics

- Let $v(\theta,\varphi,t)$ be the velocity, e.g. as measured at the solar surface over time t. Then:

$$v(\theta,\varphi,t)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}a_{lm}(t)\,Y_l^m(\theta,\varphi)$$

- The temporal dependence lies in a_{lm}, the spatial dependence in the spherical harmonic Y_l^m.

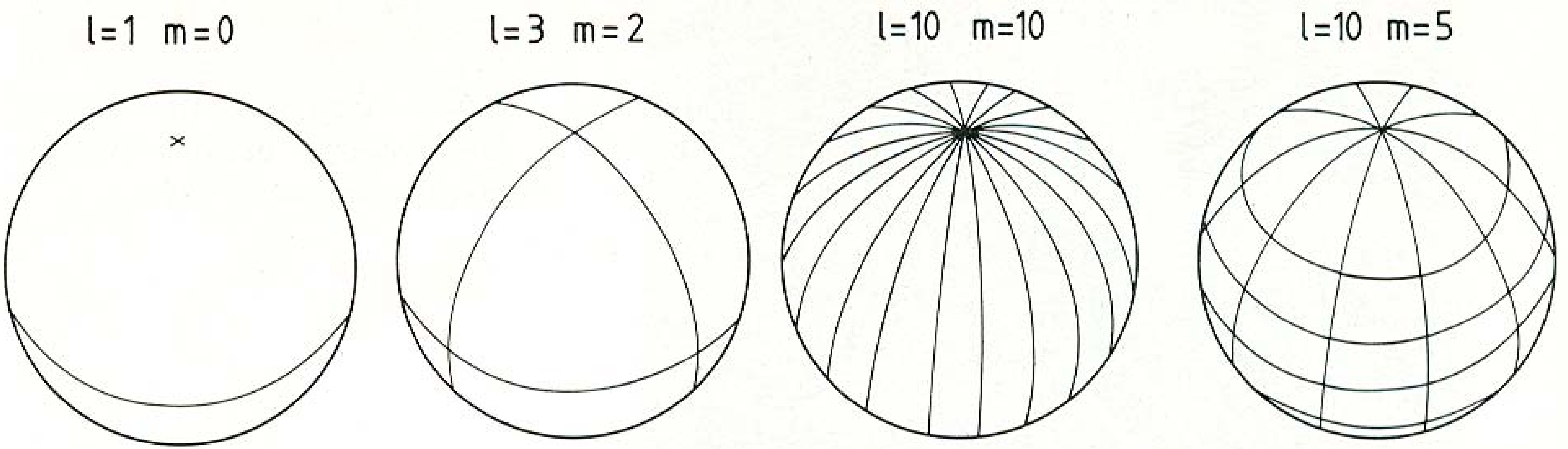
$$Y_l^m(\theta,\varphi)=P_l^{|m|}(\theta)\exp(im\varphi)$$

$$P_l^{|m|}(\theta)=\text{associated Legendre Polynomial}$$

- Due to the normalization of the spherical harmonic, the Fourier power is given by $F(a)F(a)^*$
- Here $F(a)$ is the Fourier transform of the amplitude a_{lm}, and $F(a)^*$ is its complex conjugate

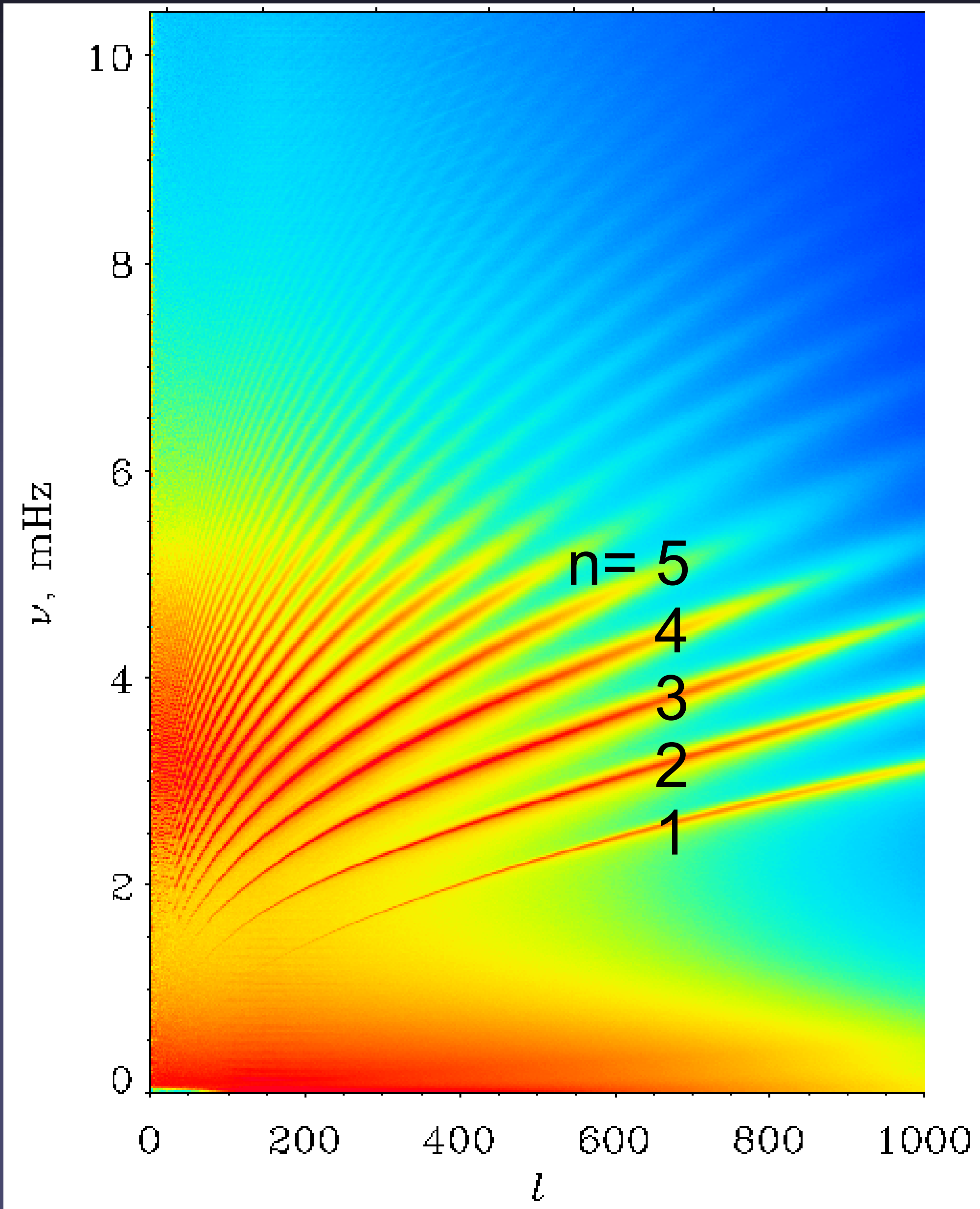
More examples and a problem with identifying spherical harmonics

- General problem: Since we see only half of the Sun, the decomposition of the sum of all 10^7 oscillations into spherical harmonics isn't unique.
- This results in an uncertainty in the deduced l and m



Interpretation of *k*-*ω* or *ν*-*l* diagram

- At a fixed *l*, different frequencies show significant power. Each of these power ridges belongs to a different order *n* (*n* = number of radial nodes), with *n* increasing from bottom to top.
- Typical are small values of *n*, but intermediate to large degree *l* (100-1000; with some spatial resolution dependence)



A few observational remarks

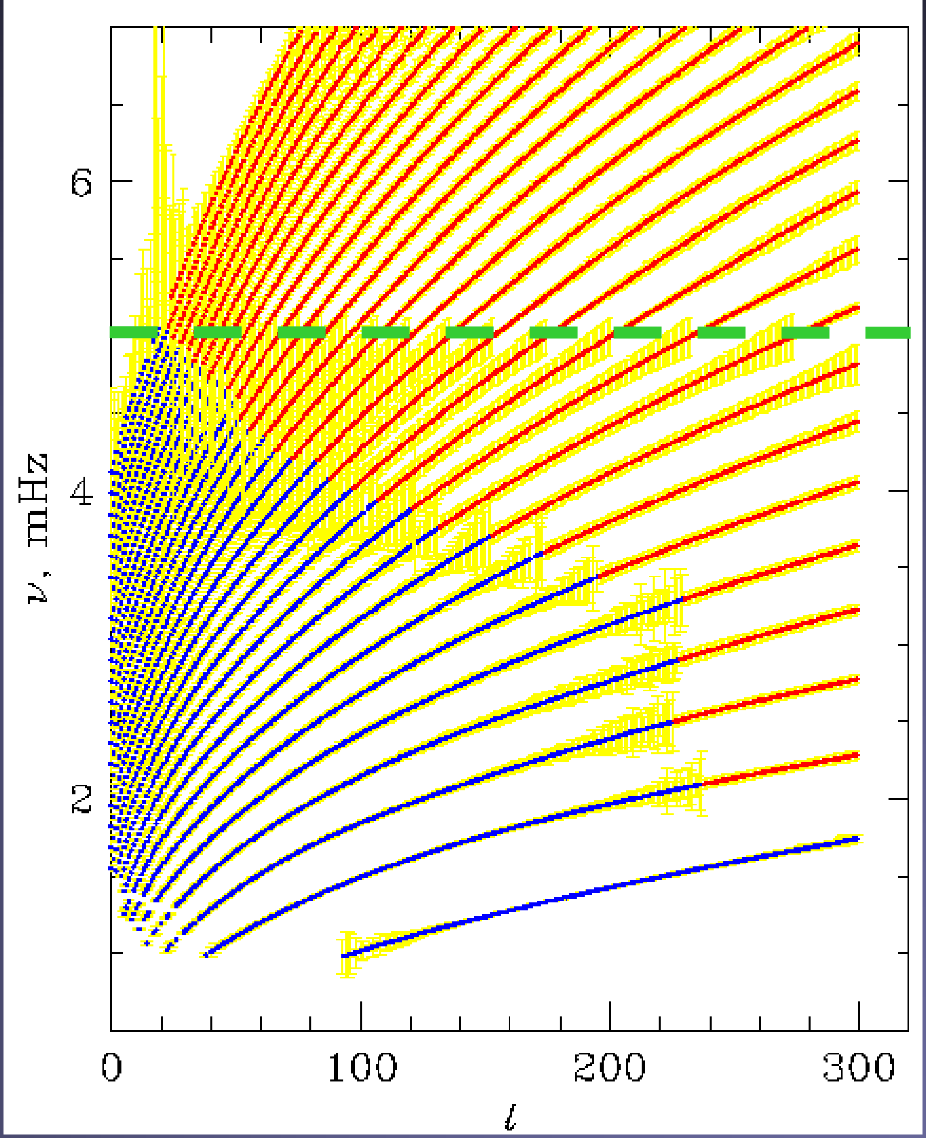
- 10^7 modes are present on the surface of the Sun at any given time (all interfering linearly with each other).
- Typical amplitude of a single mode: < 20 cm/s
- Total velocity of all 10^7 modes: a few 100 m/s
- Accuracy of current instruments: better than 1 cm/s
- Frequency resolution ~ length of time series (Heisenberg's uncertainty principle) ~ lowest detectable frequency

➔ Longer time series are better (even longer than maximum mode lifetime of a few weeks - months).

- Gaps in time series produce side lobes (i.e. spurious peaks in the power spectrum) ➔ best are uninterrupted obs.
- Highest detectable frequency ~ cadence of obs.

Accuracy of frequency measurements

- Plotted are identified frequencies and error bars (yellow). They correspond to 1000σ for blue freq., 100σ for red freq. below 5 mHz and 1σ for higher freq.)
- Best achievable freq. resolution: a few parts in 10^5; limit set by mode lifetime ~100 d



Frequency vs. amplitude

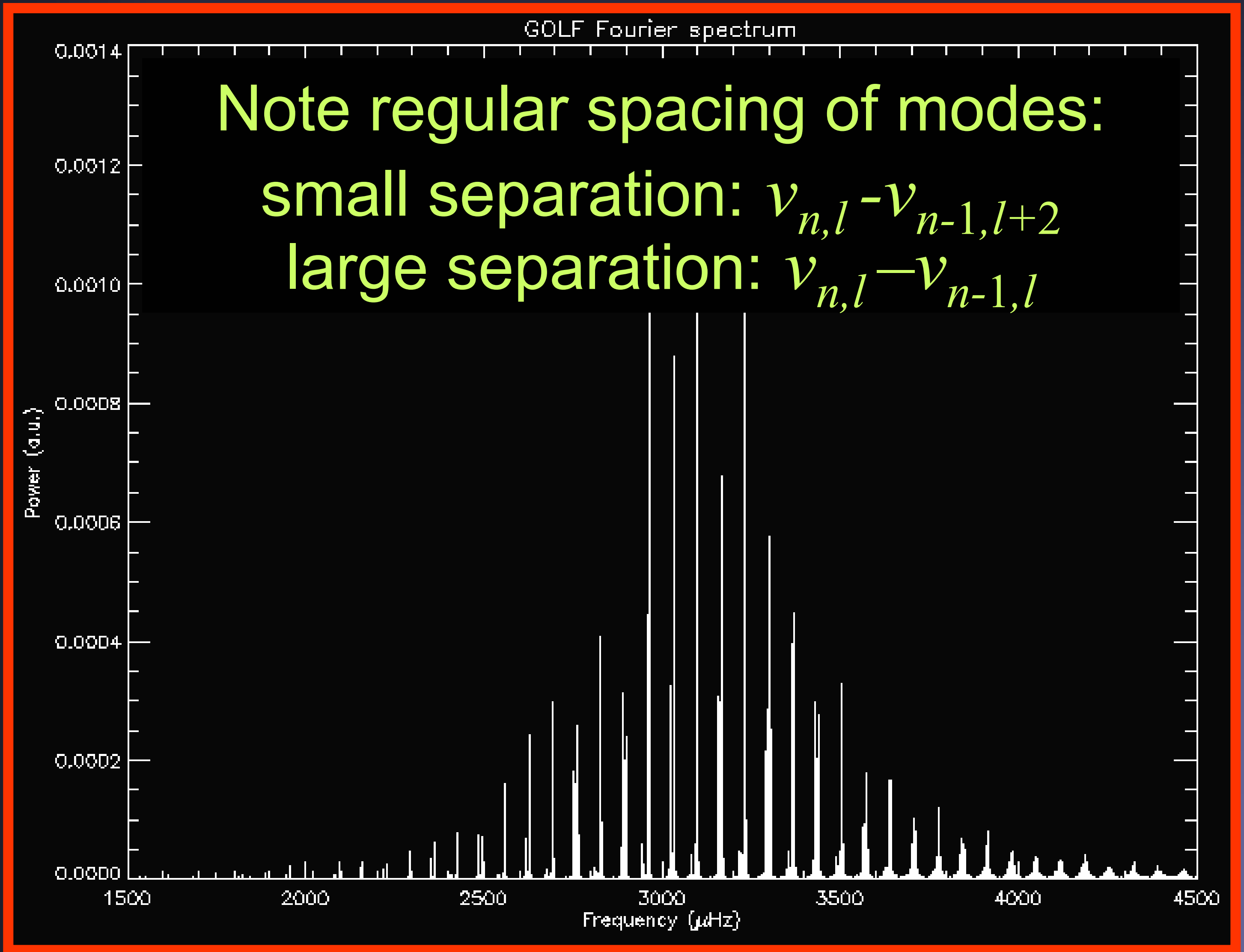
- Frequencies are the important parameter, more so than amplitudes of modes or of power peaks.
- Frequencies are more constant & are measured with greater accuracy.
- Amplitudes depend on the excitation, while the frequencies do not. They carry the main information on the structure of the solar interior.
- p-modes are excited by turbulence, which excites all frequencies. At eigenfrequencies of the Sun the modes reach large amplitudes (eigenmodes), at other frequencies they decay quickly.

The measured low-l eigenmode signal

- Sun seen as a star: Due to cancellation effects, only modes with l=0,1,2 are visible → simpler power spectrum.
- Low l modes are important for 2 reasons:
  - They reach particularly deep into the Sun (see cartoon on earlier slide).
  - These are the only modes measurable on other Sun-like stars.
- These modes are sometimes called “global” modes.

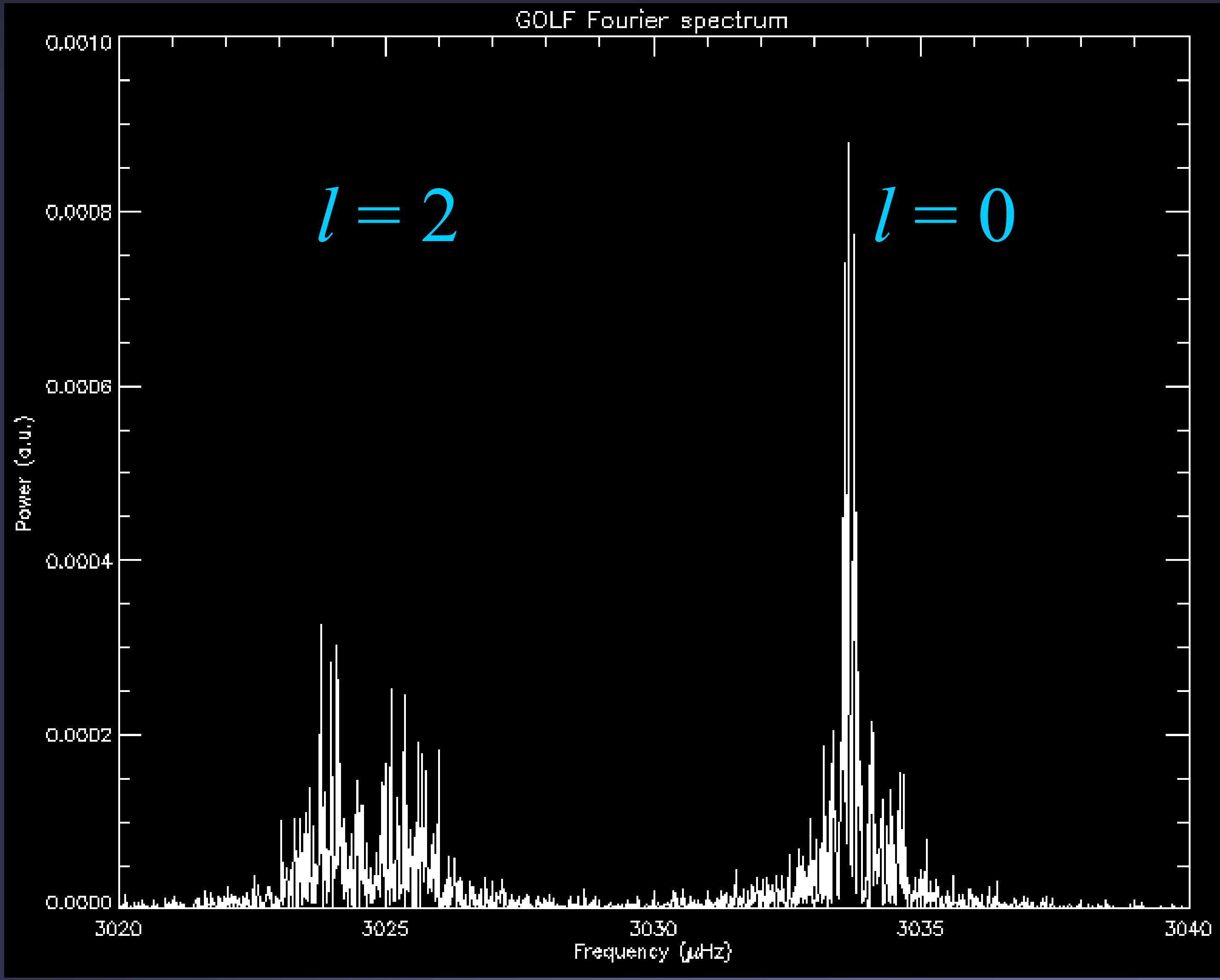
Best current low-*l* power spectrum

Given l has different peaks: different n values (n=15...25 typical)



Mode structure of low-l spectrum

- GOLF/SOHO observations showing a blowup of the power spectrum with an $l = 0$ and an $l = 2$ mode.
- The noise is due to random re-excitation of the oscillation mode by turbulence



Types of oscillations

- Solar eigenmodes can be of 2 types:
  - p-modes, where restoring force is pressure, i.e. normal sound waves
  - g-modes, where the restoring force is gravity (also called buoyancy modes)
- So far only p-modes have been detected on the Sun with certainty.
- They are excited by the turbulence associated with the convection, mainly the granulation near the solar surface (since there the convection is most vigorous).
- Being p-modes, they travel with the sound speed C_S. They dwell longest where C_S is lowest. Since $C_S \sim T^{1/2}$, this is at the solar surface.

p-modes vs. g-modes

- p-modes propagate throughout the solar interior, but are evanescent (later slide) in the solar atmosphere
- g-modes propagate in the radiative interior and in the atmosphere, but are evanescent in the convection zone (their amplitude drops exponentially there, so that very small amplitudes are expected at the surface). Convection means buoyancy instability (density blobs keep rising or falling); gravity-based oscillations require stability (density blobs oscillate)
- g-modes are expected to be most sensitive to the very core of the Sun, while p-modes are most sensitive to the surface
- Current upper limit on solar interior g-modes lies below 1 cm/s.

Solar oscillations: simple treatment

- Equations describing radial structure of adiabatic oscillations, neglecting any perturbations to the gravitational potential, are:

$$\frac{1}{r^2}\frac{d}{dr}(r^2\xi_r) - \frac{\xi_r}{c^2} + \frac{1}{\rho_0}\left(\frac{1}{c^2} - \frac{l(l+1)}{r^2\omega^2}\right)P_1 = 0$$

$$\frac{1}{\rho_0}\frac{dP_1}{dr} + \frac{g}{\rho_0 c^2}P_1 - \left(\omega^2 - N^2\right)\xi_r = 0$$

$$\text{where } N^2 = g\left(\frac{1}{\Gamma P_0}\frac{dP}{dr} - \frac{1}{\rho_0}\frac{d\rho_0}{dr}\right) = \text{Brunt - Vaisala frequency}$$

- Here ξ_r is radial displacement and P_1 is pressure perturbs. Quantities with subscript 0 refer to the unperturbed Sun. Γ is the adiabatic exponent: $\Gamma = \left(\frac{d\ln P}{d\ln\rho}\right)$

Solar oscillations: simplified treatment II

- Analytical solutions of these equations for an isothermal atmosphere are readily obtained:

$$\xi_r(r) \sim \rho_0^{-1/2} \exp(ik_r r)$$

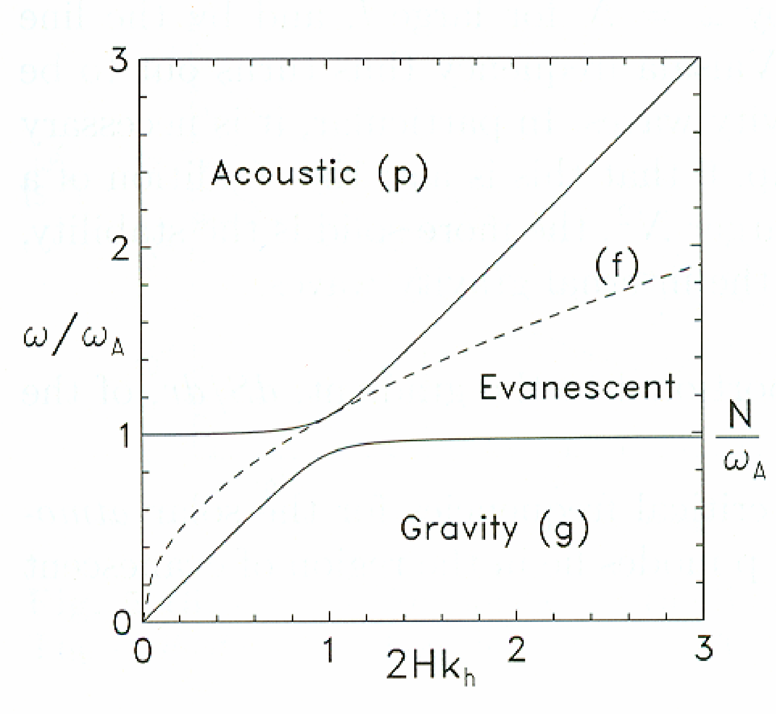
$$P_1(r) \sim \rho_0^{1/2} \exp(ik_r r)$$

- These oscillations are trapped in the body of the Sun. Since the time scale on which the atmosphere reacts to disturbances is low, waves which are travelling in the solar interior are evanescent in the atmosphere → They are present only for discrete frequencies (similar to the bound states in atomic physics).

Regimes of oscillation

- In regimes of acoustic and gravity waves $k_r^2 > 0$, while in regime of evanescent waves $k_r^2 < 0$ (exponential damping). The solid lines show $k_r^2 = 0$.

- Evanescent waves occur when the period is so long that the whole (exponentially stratified) medium has time to adapt to the perturbation, achieving a new equilibrium. Therefore the wave does not propagate, but rather the medium as a whole oscillates.



Cutoff frequency for acoustic waves in a stratified medium:

$$\omega_C = C_S/2H$$

Deducing internal structure from solar oscillations

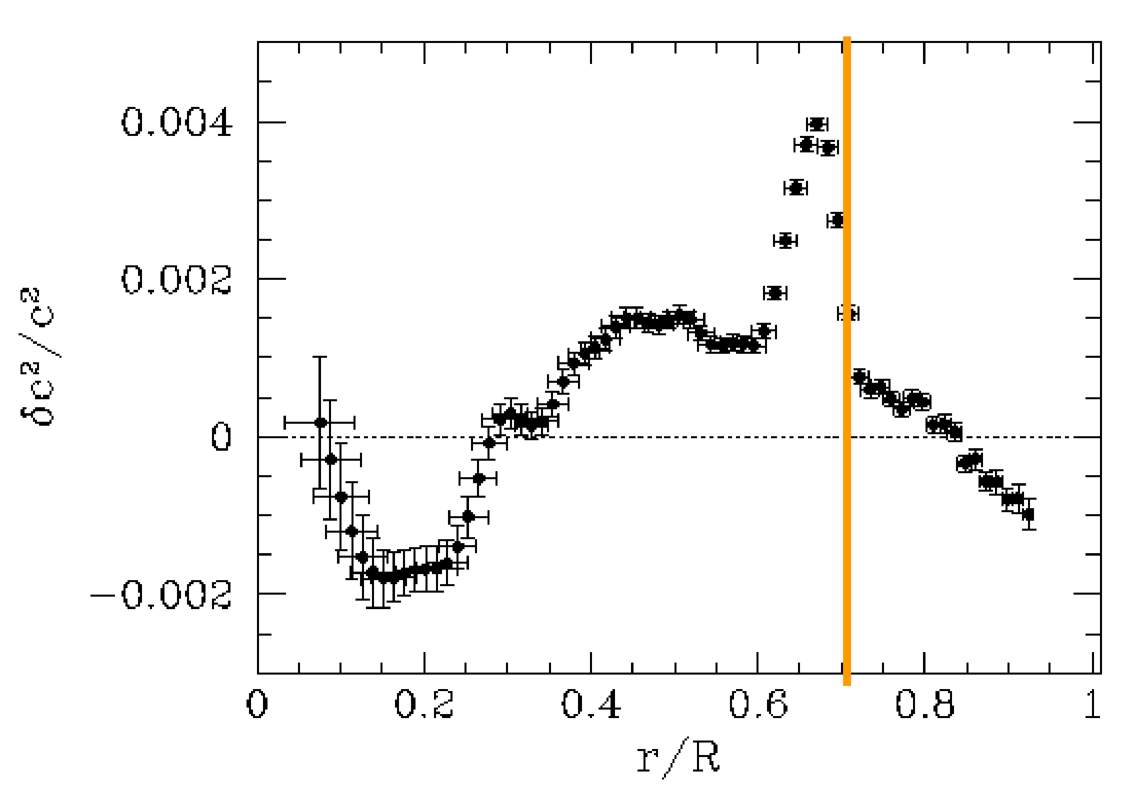
- Global helioseismology: Gives mainly the radial dependence of solar properties, although latitudinal dependence can also be deduced.
  - Radial structure of sound speed
  - Structure of differential rotation (torsional velocities)
- Local helioseismology: Allows in principle 3-D imaging of solar interior. E.g. time-distance helioseismology does not measure frequencies, but rather the time that a wave requires to travel a certain distance (relatively new)

Global helioseismology

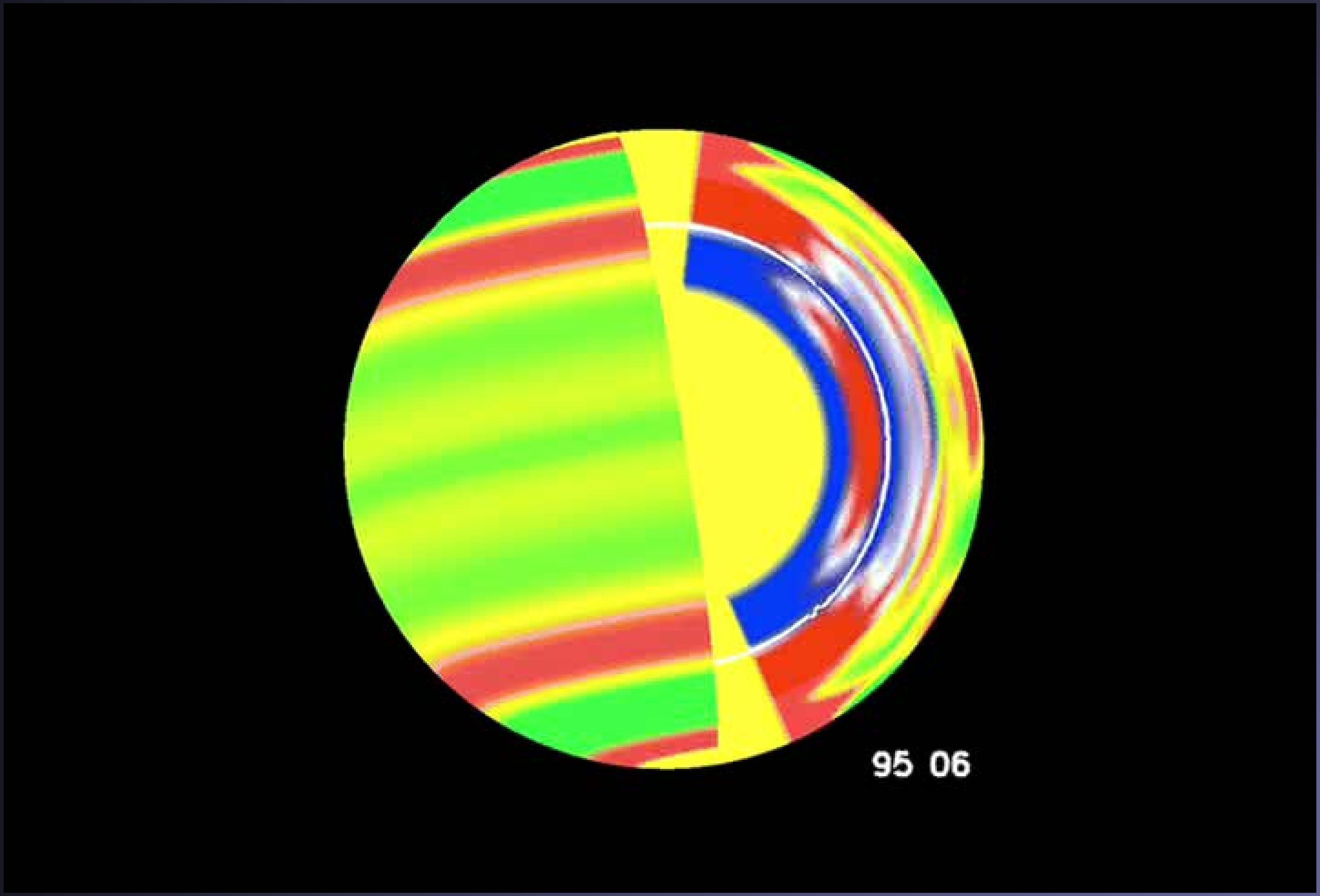
- Use frequencies of many modes.
- Basically two techniques for deducing information on the Sun's internal structure
  - Forward modelling: make a model of the Sun's internal structure (e.g. standard model discussed earlier), compute the frequencies of the eigenoscillations of the model and compare with observations
  - Inverse technique: Deduce the sound speed and rotation by inverting the oscillations (i.e. without any prior comparison with models)
- Note that forward modelling is required in order to first identify the modes. Only after that can inversions be carried out.

Testing the standard solar model: results of forward modelling

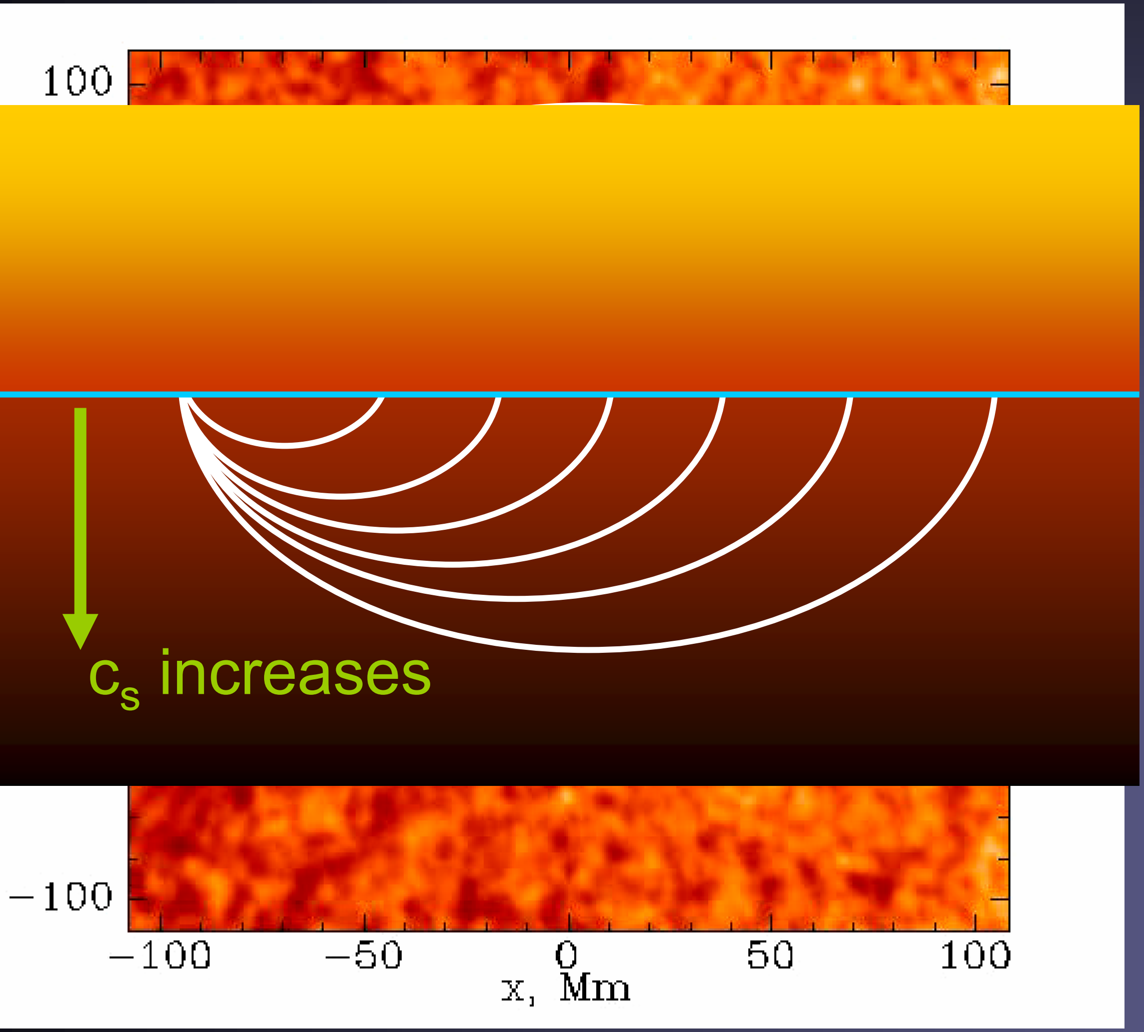
- Relative difference between C_S^2 obtained from inversions and from standard solar model plotted vs. radial distance from Sun centre.
- Typical difference: 0.002 ➔ good!
- Typical error bars from inversion: 0.0002 ➔ poor!
- Problem areas:
  - solar core
  - bottom of CZ
  - solar surface



Variation of the solar rotation velocity with period of 1.3 years



Local excitation of waves by a flare

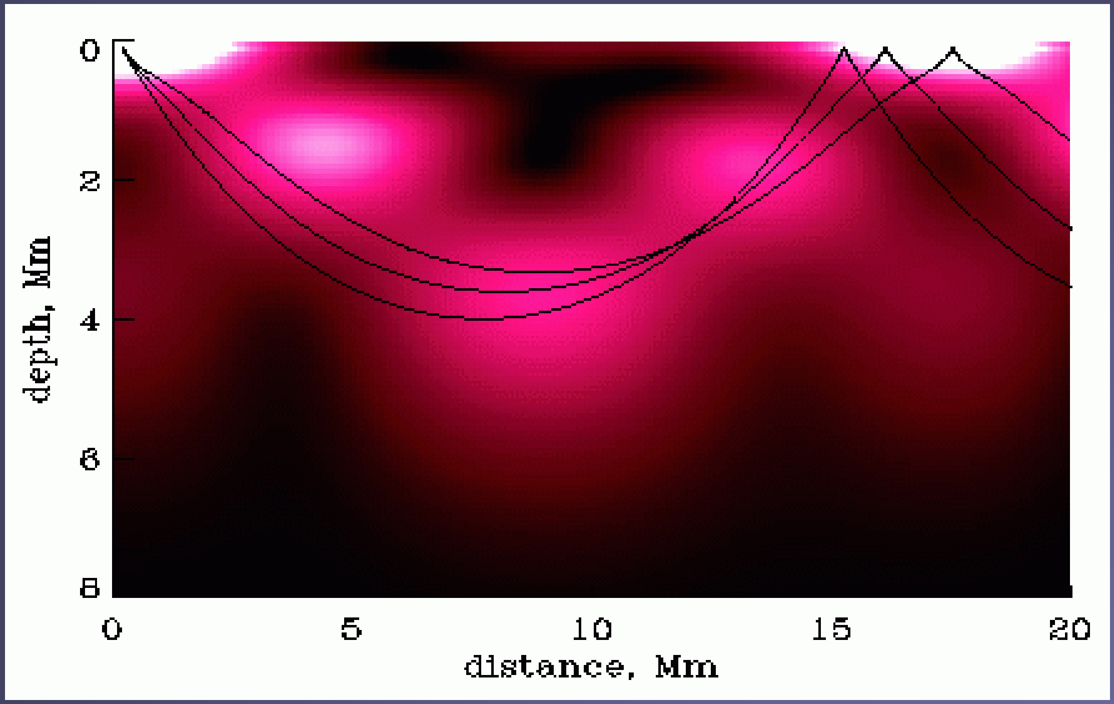


- Clear example of wave being triggered.
- The wave is not travelling at the surface, but rather reaching the surface further out at later times. Note how it travels ever faster. Why?

Local helioseismology

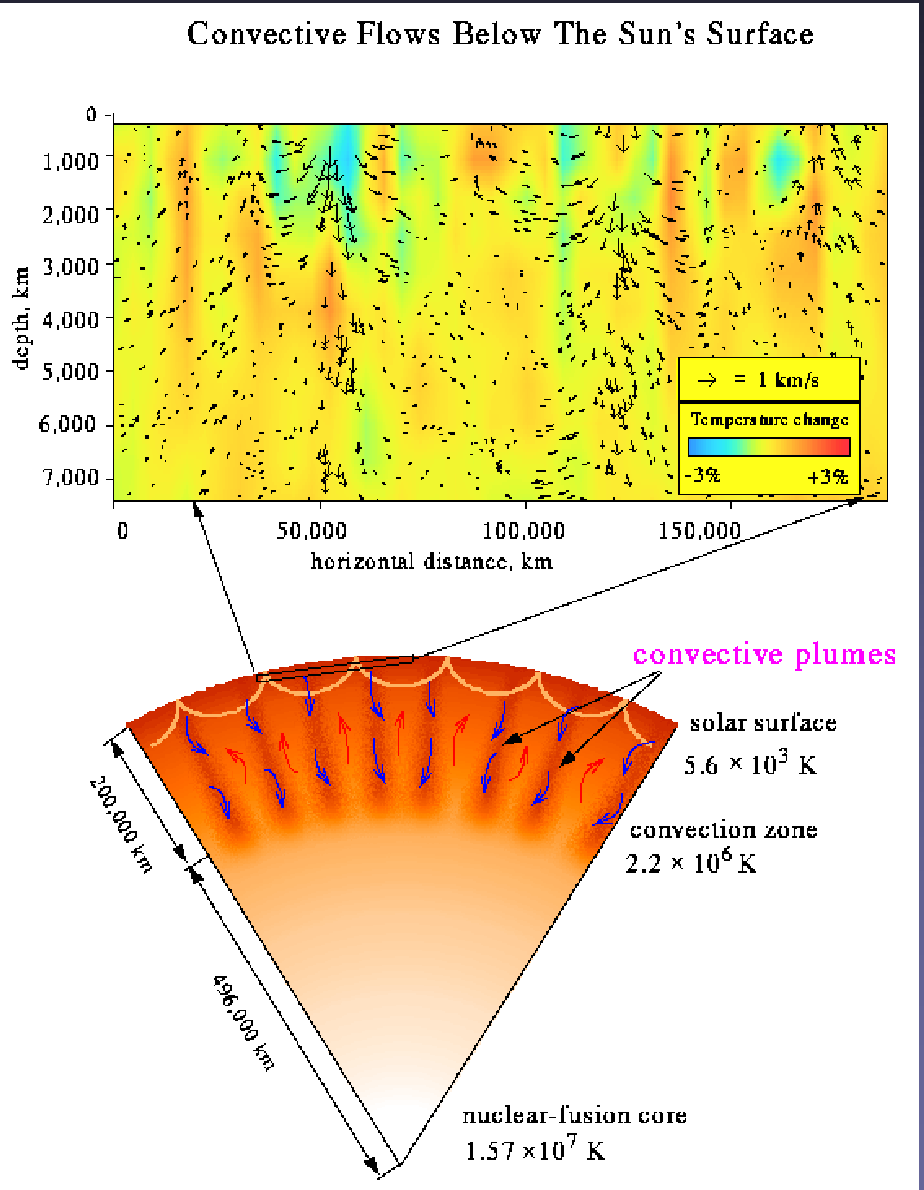
- Does not build upon measuring frequencies of eigenmodes, but rather measures travel times of waves through the solar interior, between two “bounces” at the solar surface (for particular technique of time-distance helioseismology).

- The travel time between source and first bounce depends on the structure of C_S below the surface. By considering waves following different paths inhomogeneous distributions of C_S can be determined.



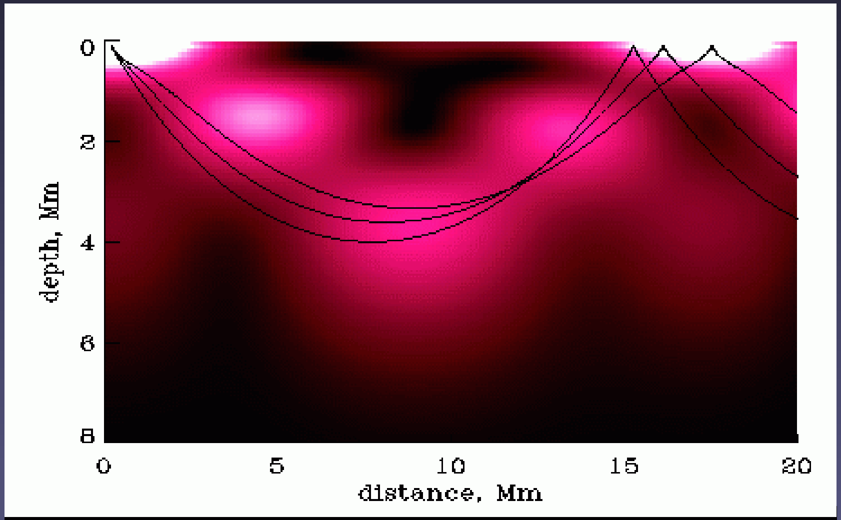
Local helioseismology II

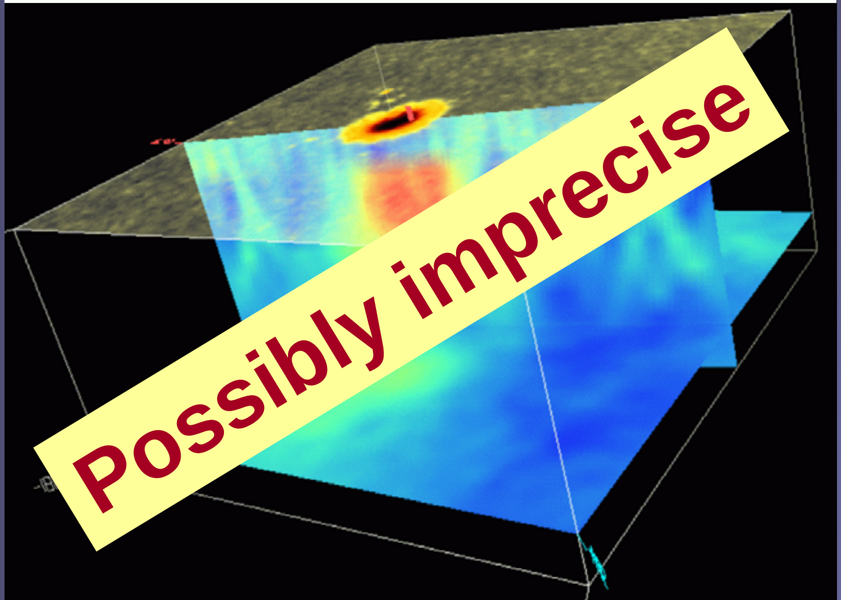
- Distinguish temperature and velocity structures
  - a wave will propagate faster along a flow than against it.
  - By considering waves passing in both directions it is possible to distinguish between T and velocity.
- At right: 1st images of convection zone of a star



Time-Distance Helioseismology of a sunspot

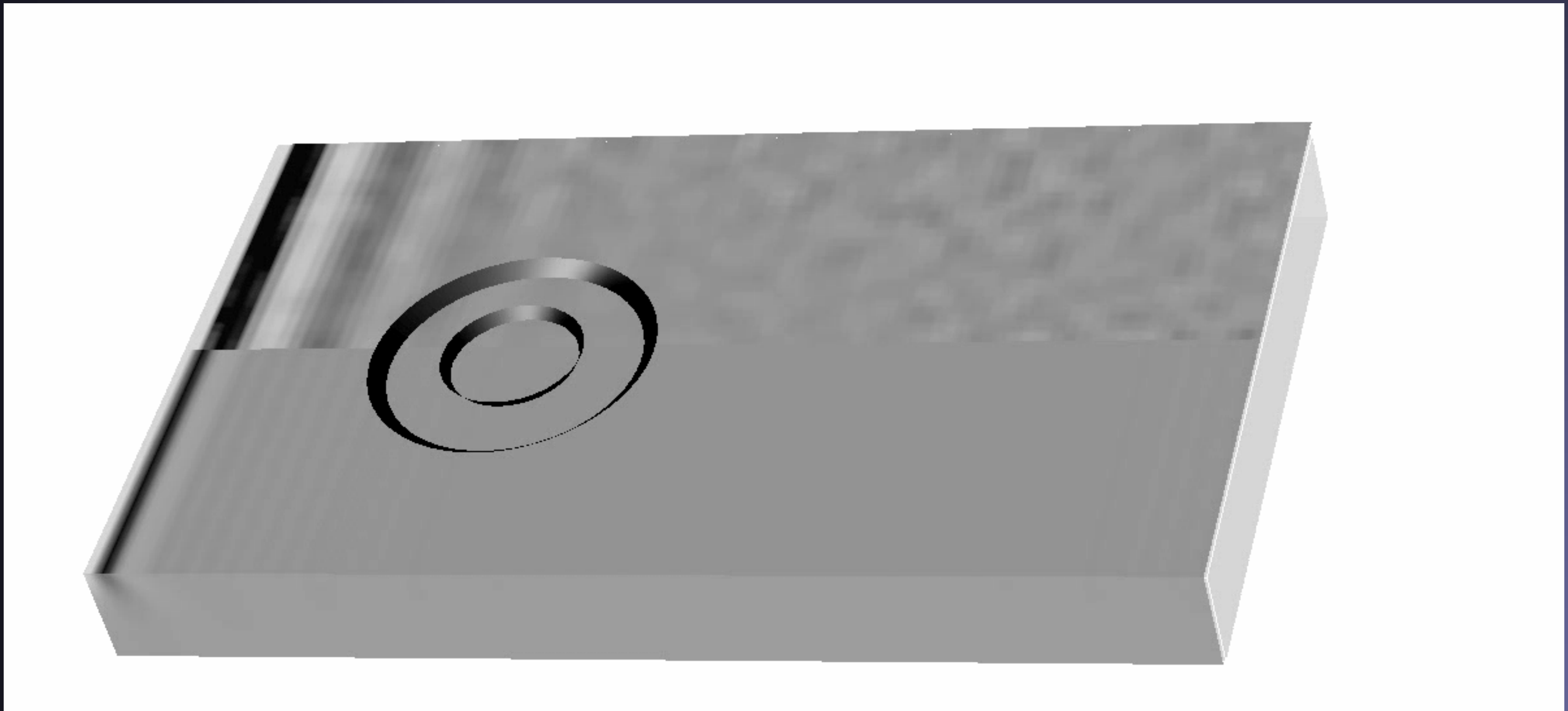
- Subsurface structure of sunspots
- Sunspots are good targets, due to the large temperature contrast AND large B
- Early work neglected influence of magnetic field on waves!





Wave-sunspot interaction: Comparison of simulations with observations

Top: observations (SOHO/MDI cross-covariance function)



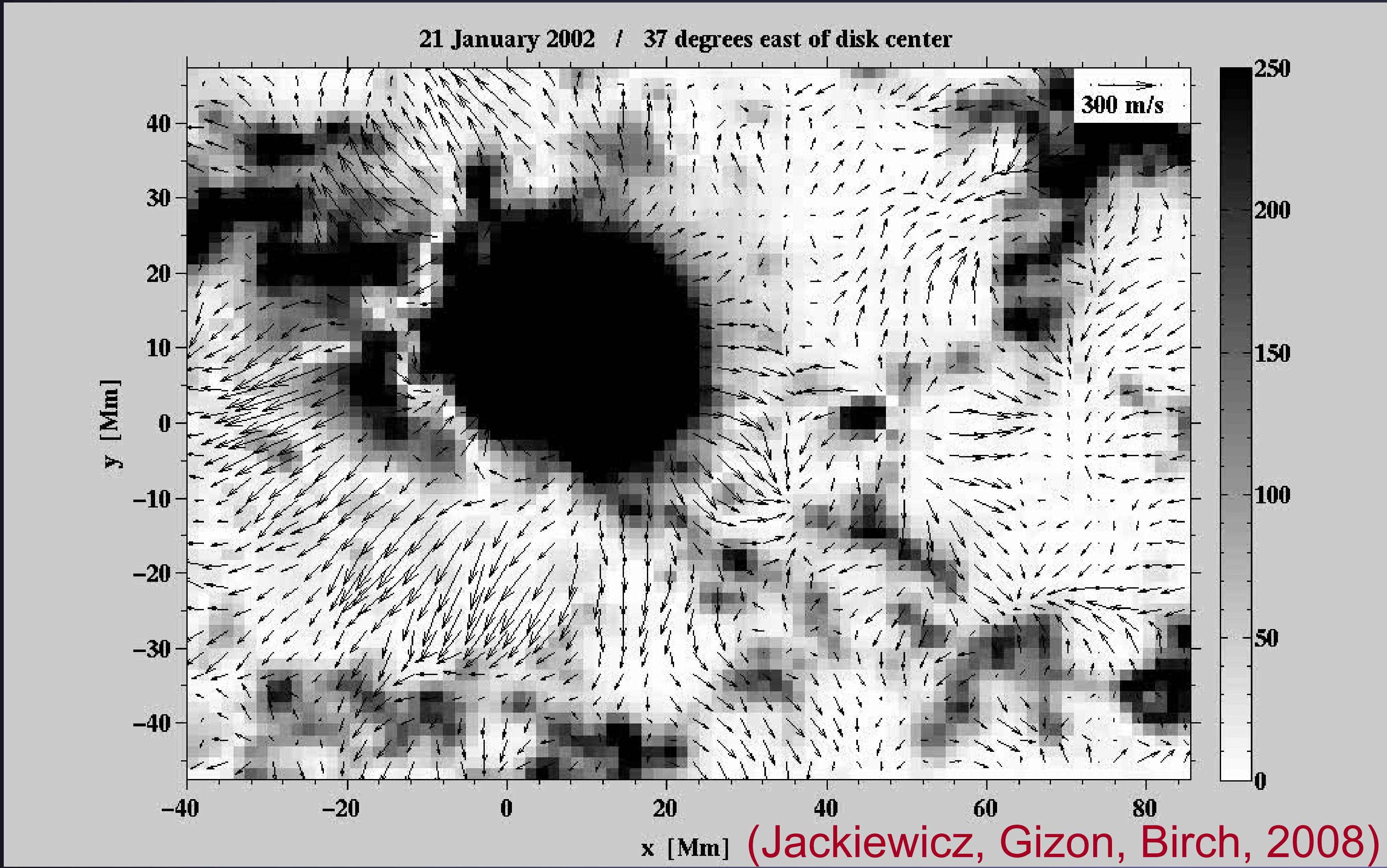
Bottom: SLiM numerical solution for Bz=3kG

Cameron, Gizon & Duvall (2007)

Subsurface Structure of Sunspots

1 Mm depth, longuest arrow corresponds to 400 m/s

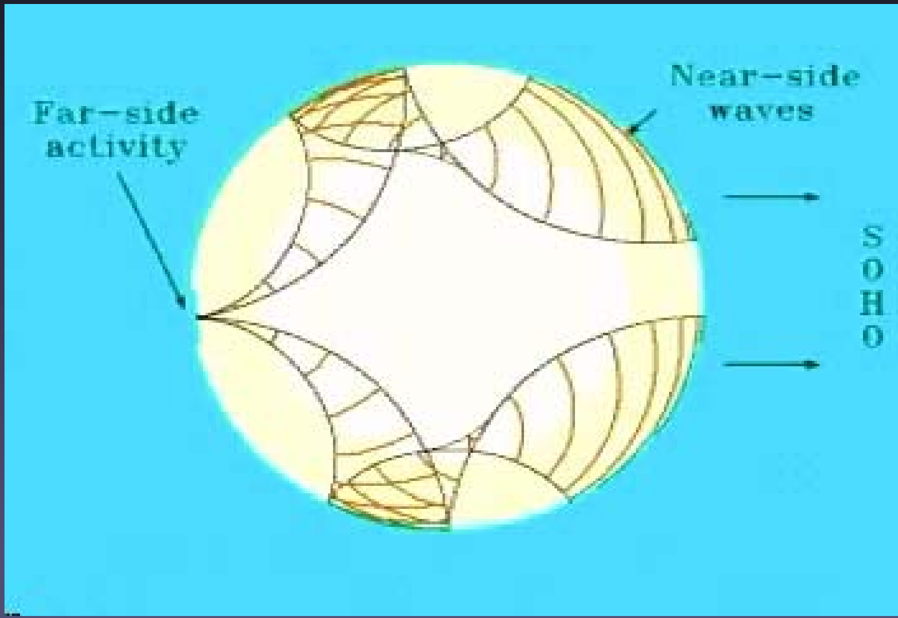
Dark shades: surface magnetic field

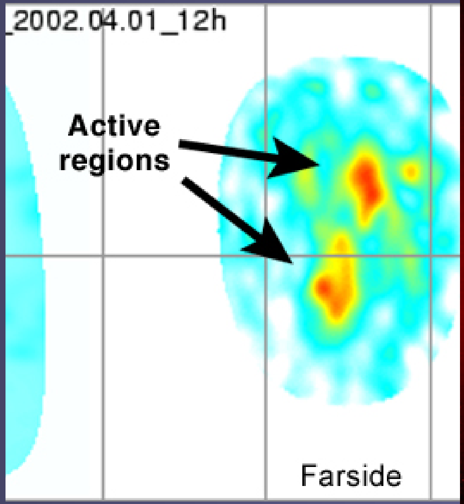


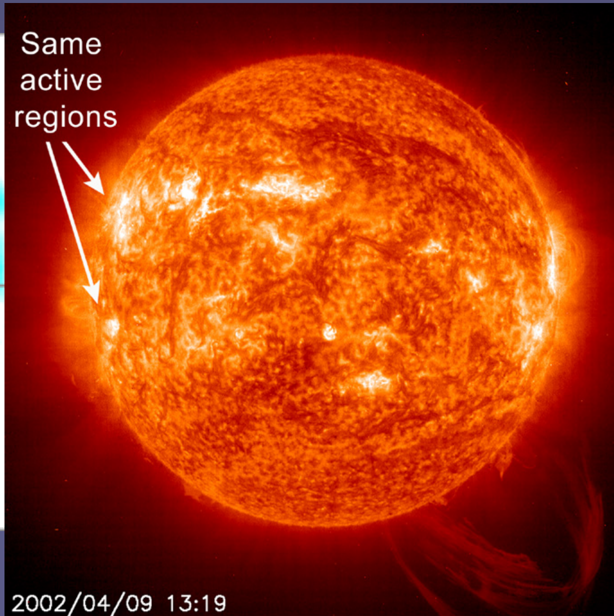
(Jackiewicz, Gizon, Birch, 2008)

Seeing right through the Sun

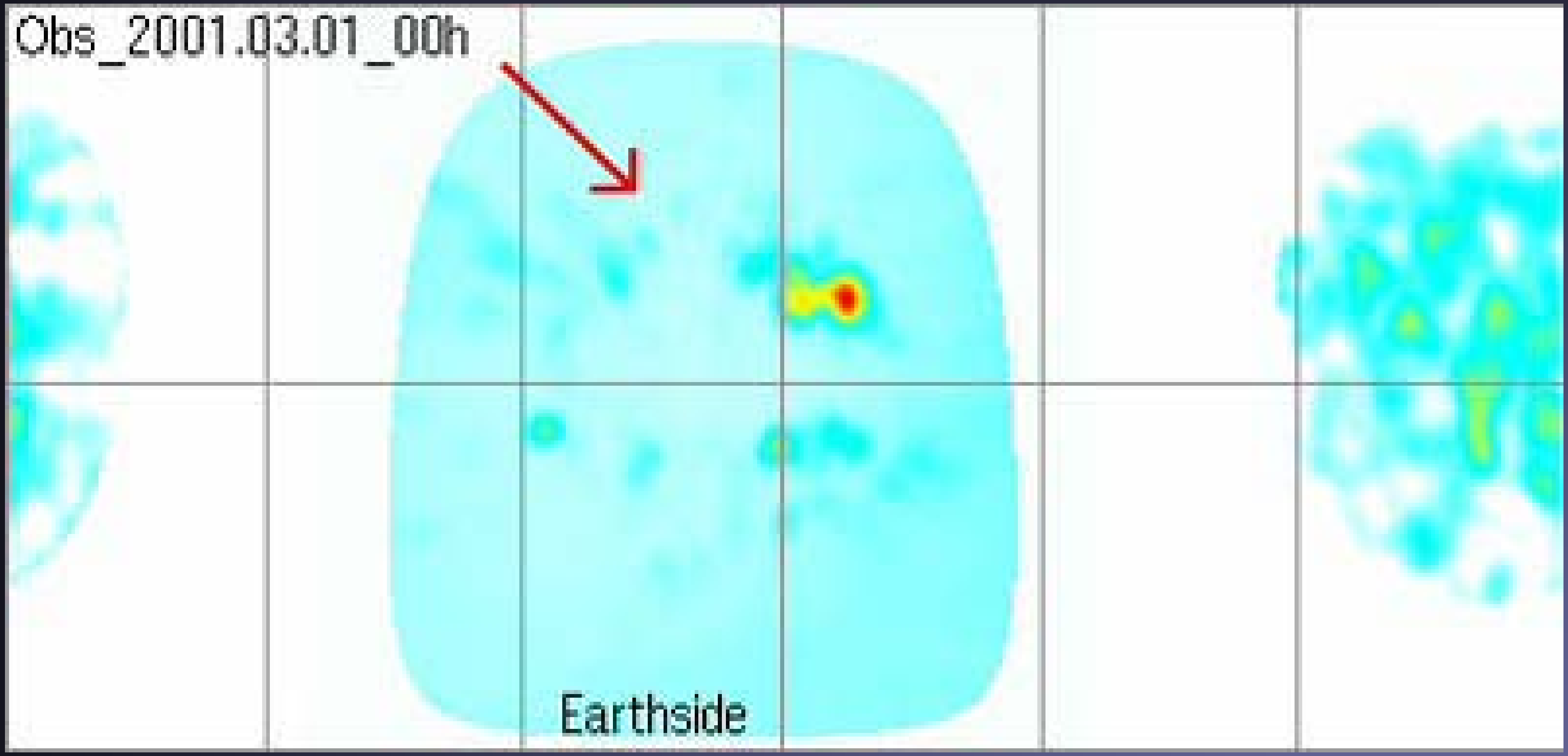
- A technique called two-skip far-side seismic holography gives images of the far side of the Sun
- Waves from back reach front, after skipping at the surface on the way
- Acoustic waves speed up in active regions (hotter subsurface layers)
- Sound waves delayed by ≈12 sec in a total travel time of 6 hours







Seeing right through the Sun II



Real-time far side images:
http://soi.stanford.edu/data/farside/index.html

Helioseismology instruments

- Needed:
  - uninterrupted, long time series of observations
  - Either high velocity sensitivity, or high intensity sensitivity (and extremely good stability)
  - Low noise
  - Spatial resolution better than 1” (for local helioseismology)
- Instruments are either:
  - Ground based global networks (GONG+, BiSON)
  - Space based instruments in special full-Sun orbits (advantage of lower noise relative to ground-based networks; MDI, GOLF, VIRGO on SOHO, HMI on SDO)
  - Usually filter instruments with high spectral or intensity fidelity

Asteroseismology

First reliable detection of oscillations on the near solar analogue, α Centauri, and other Sun-like stars. Note the shift in the p-mode frequency range to lower values for α Centauri, which is older than the Sun (note also factor 10^3 difference in ν scale)