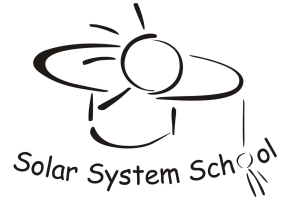




International Max Planck Research School  
on Physical Processes in the Solar System and Beyond  
at the Universities of Braunschweig and Göttingen



## FUNDAMENTALS OF MAGNETOHYDRODYNAMICS

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### 1. Introduction: the magnetohydrodynamic approximation (MHD).

- 1.1. Maxwell equations. Lorentz transformations.
- 1.2. Physical assumptions underlying the MHD approximation.
- 1.3. *Ohm's law* as a constitutive relation.

### 2. Kinematic preliminaries.

- 2.1. The continuity equation in the spatial and in the material representations. The Jacobian determinant; Euler's identity.
- 2.2. Material curves and material regions. Reynold's transport theorem for line and for volume integrals.
- 2.3. Magnetic flux tubes.

### 3. The magnetic induction equation.

- 3.1. Derivation of the induction equation of MHD.
- 3.2. Scale analysis: the magnetic Reynolds number.
- 3.3. The limit of infinite conductivity: Alfén's theorem on the conservation of magnetic flux. Walén's theorem on the 'freezing' of magnetic field lines.
- 3.4. Magnetic helicity and its topological significance. Gauss' linking number. Conservation of magnetic helicity.

### 4. The momentum equation.

- 4.1. Derivation of the equation of motion from the integral expression of momentum balance.
- 4.2. The Lorentz force. Magnetic pressure, magnetic tension and curvature force. Maxwell's stress tensor.
- 4.3. The theorem for the balance of mechanical energy. Mechanical power; the stress tensor and the deformation power.

### 5. The energy equation.

- 5.1. Poynting's theorem for the balance of electromagnetic energy. Poynting's vector and its meaning.
- 5.2. The principle of energy conservation. The heat-flux-density vector; heating power. Statement of the 'First principle of Thermodynamics' for a MHD plasma. Ohmic dissipation in resistive MHD.

- 5.3. The energy equation expressed in terms of entropy. Viscous dissipation, Ohmic dissipation and heat conduction as entropy sources. Isentropic motions.
- 5.4. Alternative forms of writing the energy equation in MHD in different thermodynamic representations.

## **6. Magnetostatics.**

- 6.1. Equation of magnetostatic equilibrium. Scale analysis of the different terms.
- 6.2. Force-free magnetic fields. Theorems.

## **7. Alfvén waves.**

- 7.1. Alfvén waves in an ideal plasma. Obtention of the wave equation. The Alfvén speed. Polarization properties. Equipartition of energy.
- 7.2. Alfvén waves in a plasma with electrical resistivity. Damped oscillations.

## **8. Jump relations across a MHD shock transition.**

- 8.1. The basic equations of MHD in conservation form.
- 8.2. The basic equations in a frame of reference comoving with the shock discontinuity. Continuity of the normal component for a solenoidal vector or tensor quantity.
- 8.3. Jump relations for mass conservation, momentum balance and energy conservation. Jump relations for the magnetic and the electric fields.
- 8.4. Jump condition for the entropy flux density across the shock discontinuity. Implications of this inequality on the direction of change of the other physical quantities across the shock.
- 8.5. Classification of MHD shocks.

## **9. Elementary introduction to stellar dynamo theory.**

- 9.1. Formal posing of the dynamo problem.
- 9.2. Averages and deviations. Derivation of the kinematic dynamo equations.
- 9.3. Differential rotation and  $\Omega$ -effect. Mean induced electric field and  $\alpha$ -effect. Enhanced magnetic diffusivity.
- 9.4. The concept of  $\alpha\Omega$ ,  $\alpha^2$  and  $\alpha^2\Omega$  dynamos.
- 9.5. A simple *constitutive relation* introducing the scalar functions  $\alpha$  ( $\alpha$ -effect) and  $\beta$  (*turbulent diffusivity*). Parker's original heuristic model for a simple  $\alpha\Omega$ -dynamo; dynamo waves.