International Max Planck Research School
on Physical Processes in the Solar System and Beyond
at the Universities of Braunschweig and Göttingen

FUNDAMENTALS OF MAGNETOHYDRODYNAMICS

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1. **Introduction: the magnetohydrodynamic approximation (MHD).**
   - 1.1. Maxwell equations. Lorentz transformations.
   - 1.2. Physical assumptions underlying the MHD approximation.
   - 1.3. *Ohm's law* as a constitutive relation.
2. **Kinematic preliminaries.**
   - 2.1. The continuity equation in the spatial and in the material representations. The Jacobian determinant; Euler's identity.
   - 2.2. Material curves and material regions. Reynold's transport theorem for line and for volume integrals.
   - 2.3. Magnetic flux tubes.
3. **The magnetic induction equation.**
   - 3.1. Derivation of the induction equation of MHD.
   - 3.2. Scale analysis: the magnetic Reynolds number.
   - 3.3. The limit of infinite conductivity: Alfén's theorem on the conservation of magnetic flux. Waléns theorem on the 'freezing' of magnetic field lines.
   - 3.4. Magnetic helicity and its topological significance. Gauss' linking number. Conservation of magnetic helicity.
4. **The momentum equation.**
   - 4.1. Derivation of the equation of motion from the integral expression of momentum balance.
   - 4.2. The Lorentz force. Magnetic pressure, magnetic tension and curvature force. Maxwell's stress tensor.
   - 4.3. The theorem for the balance of mechanical energy. Mechanical power; the stress tensor and the deformation power.
5. **The energy equation.**
   - 5.1. Poynting's theorem for the balance of electromagnetic energy. Poynting's vector and its meaning.
   - 5.2. The principle of energy conservation. The heat-flux-density vector; heating power. Statement of the 'First principle of Thermodynamics' for a MHD plasma. Ohmic dissipation in resistive MHD.

5.3. The energy equation expressed in terms of entropy. Viscous dissipation, Ohmic dissipation and heat conduction as entropy sources. Isentropic motions.

5.4. Alternative forms of writing the energy equation in MHD in different thermodynamic representations.

6. Magnetostatics.

6.1. Equation of magnetostatic equilibrium. Scale analysis of the different terms.

6.2. Force-free magnetic fields. Theorems.

7. Alfvén waves.

7.1. Alfvén waves in an ideal plasma. Obtention of the wave equation. The Alfvén speed. Polarization properties. Equipartition of energy.

7.2. Alfvén waves in a plasma with electrical resistivity. Damped oscillations.

8. Jump relations across a MHD shock transition.

8.1. The basic equations of MHD in conservation form.

8.2. The basic equations in a frame of reference comoving with the shock discontinuity. Continuity of the normal component for a solenoidal vector or tensor quantity.

8.3. Jump relations for mass conservation, momentum balance and energy conservation. Jump relations for the magnetic and the electric fields.

8.4. Jump condition for the entropy flux density across the shock discontinuity. Implications of this inequality on the direction of change of the other physical quantities across the shock.

8.5. Classification of MHD shocks.

9. Elementary introduction to stellar dynamo theory.

9.1. Formal posing of the dynamo problem.

9.2. Averages and deviations. Derivation of the kinematic dynamo equations.

9.3. Differential rotation and Ω-effect. Mean induced electric field and α-effect. Enhanced magnetic diffusivity.

9.4. The concept of $\alpha\Omega$, α^2 and $\alpha^2\Omega$ dynamos.

9.5. A simple *constitutive relation* introducing the scalar functions α (α-effect) and β (*turbulent diffusivity)*. Parker's original heuristic model for a simple $\alpha\Omega$-dynamo; dynamo waves.